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Reinterpretation of the Starobinsky model

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Abstract

The Starobinsky model of inflation, consistent with Planck 2015, has a peculiar form of the action, which contains the leading Einstein term R , the R^2 term with a huge coefficient, and negligible higher order terms. We propose an explanation of this form based on compactification of extra dimensions. Once tuning of order 10^{-4} is accepted to suppress the linear term R , we no more have to suppress higher order terms, which give nontrivial corrections to the Starobinsky model. We show our predictions of the spectral index, its runnings, and the tensor-to-scalar ratio. Finally, we discuss quantum gravity may appear at the scale $\Lambda \gtrsim 5 \times 10^{15}$ GeV.

1 Introduction

The precise CMB observations favor the plateau-type inflaton potentials. Although the combined analysis of BICEP–Keck–Array–Planck resulted in a finite value of tensor-to-scalar ratio, $r = 0.048_{-0.032}^{+0.035}$ [1], Planck 2015 itself has not found any evidence of detecting it, $r < 0.103$ (Planck TT + lowP) [2]. In fact, combining these results in a stringent limit, $r < 0.08$ (Planck TT+lowP+BKP) [2]. There are many models, including Starobinsky model [3], Higgs inflation model [4], and cosmological attractors (see Ref. [5] and references therein), whose predictions are at the center of the Planck constraint. Among others, the Starobinsky model of inflation has specific features that it does not require introduction of an inflaton field by hand: the inflaton degree of freedom emerges from a higher order gravitation term.

The model in the well-known form¹ is [7]

$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{2} M_{\text{P}}^2 R + \frac{M_{\text{P}}^2}{12m^2} R^2 \right), \quad (1)$$

where $M_{\text{P}} \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass, R is the Ricci scalar, and m is a mass-dimensional parameter (actually the inflaton mass). In the IR limit, $R \ll m^2$, it reduces to the General Relativity (with the cosmological constant, which should be fine-tuned to be a small number and hence we ignore here), which is well established in wide scales. On the other hand, when R becomes comparable with m^2 , the second term becomes important. Introducing an auxiliary scalar field and applying Weyl transformation and scalar field redefinition, the model is recast in the form of Einstein gravity with a canonically normalized scalar field ϕ with the following scalar potential [8–10],

$$V_{\text{Starobinsky}} = \frac{3}{4} m^2 M_{\text{P}}^2 \left(1 - e^{-\sqrt{2/3}\phi/M_{\text{P}}} \right)^2, \quad (2)$$

where m is interpreted as the inflaton mass at the vacuum.

If one interprets m^2 in eq. (1) as the expansion parameter of the theory, there are no reasons to expect absence of even higher order terms like R^3 and R^4 (aside from terms involving Ricci and Riemann tensors and derivatives, which we neglect because they introduce negative norm states (ghosts) [12]) with negative powers of m^2 . That is, eq. (1) should be augmented by the higher order terms as follows,

$$S = M_{\text{P}}^2 \int d^4x \sqrt{-g} \left(-\frac{1}{2} R + \sum_{n=2}^{\infty} a_n m^2 \left(\frac{R}{m^2} \right)^n \right), \quad (3)$$

where $a_2 = 1/12$, and a_n 's ($n \geq 3$) are naively expected to be of order one. These terms, however, easily spoil the success of the inflationary model by substantially modifying the inflaton

¹ The original formulation involves all quadratic curvature invariants such as $R^{\mu\nu} R_{\mu\nu}$ and $R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}$. In conformally flat spacetime, effects of these terms are represented by the scalar curvature term as in eq. (1). Note also that the de Sitter expansion in $f(R)$ gravity was discussed in Ref. [6].

potential (2). So in any way, the higher order terms must be sufficiently suppressed to maintain the predictions of the model.

If the higher order terms involving negative powers of m^2 are suppressed by phenomenological reasons, what is the scale of the suppression? The “next-to-natural” expectation would be that it is the reduced Planck scale M_{P} , since there are no other scales in the theory. In this case, the action is expanded by the Planck scale M_{P} with order one coefficients, but then the coefficient of the second term R^2 must be somewhat very large ($a_2 \simeq 5 \times 10^8$). This is the well-known peculiarity of the Starobinsky model, which we try to partially explain here.

In this letter, we take a view that the large coefficient of the R^2 term is actually an overall coefficient of the action. As we will see, such a large overall factor naturally emerges in theories with extra spacetime dimensions. Although we have to suppress the coefficient of the linear term R by tuning of order 10^{-4} , we do not have to additionally suppress the higher order terms. Moreover, it leads to a Starobinsky-like model with interesting observational consequences. At the end, we predict inflationary observables, and obtain the lower bound on the fundamental scale of the underlying higher dimensional theory.

2 Starobinsky-like model from extra dimensions

Suppose that the underlying gravitational theory lives in D spacetime dimensions with a characteristic energy scale Λ . Its effective action is described by

$$S = \Lambda^D \int d^D x \sqrt{-g} \sum_{n=0} b_n \left(\frac{R_D}{\Lambda^2} \right)^n, \quad (4)$$

where b_n 's are dimensionless coefficients and R_D is the D -dimensional Ricci scalar. Here we require the absence of ghosts, and hence assume the $f(R)$ -type theory (see *e.g.* Ref. [13] for a review of $f(R)$ gravity). Also, we neglect possible non-minimal couplings with matter fields for simplicity. Assuming $b_2 > 0$, we may set $b_2 = 1$ by redefinition of the scale Λ . Upon the compactification to four dimensions, the action becomes

$$S = c \int d^4 x \sqrt{-g} \sum_{n=0} b_n \Lambda^4 \left(\frac{R}{\Lambda^2} \right)^n, \quad (5)$$

where $c \equiv V_{D-4} \Lambda^{D-4}$ is the overall dimensionless factor, V_{D-4} is the volume of the compactified extra dimensions, and R is the 4-dimensional Ricci scalar². For example, if we take $D = 10$ (*c.f.* superstring theory) and the compactification radius $L \equiv V_6^{1/6}$ which satisfies $L \simeq 30/\Lambda$, we can naturally obtain the large overall factor $c \simeq 5 \times 10^8$.

² The above compactification assumes flat extra dimensions so that $R_D = R$. For generic extra dimensions, we have $R_D = R + \mathcal{O}(1/L^2)$ where L is the typical size of extra dimensions (compactification radius). Thus, coefficients b_n receive only corrections like $b_n \rightarrow b_n + \mathcal{O}((1/L^2 \Lambda^2))$. Since we take $L\Lambda$ large, these corrections are neglected.

Basically, all the coefficients b_n are expected to be order one, but b_0 should be fine-tuned to suppress the cosmological constant. Furthermore, we require that b_1 also happens to be very small. Otherwise, the situation is similar to eq. (3) with m replaced by Λ . Since the limit $b_1 \rightarrow 0$ does not enhance symmetries of the theory, it is regarded as tuning. To reproduce the Starobinsky model, we set

$$cb_1\Lambda^2 = -\frac{M_{\text{P}}^2}{2}, \quad c = \frac{M_{\text{P}}^2}{12m^2} \simeq 5 \times 10^8, \quad (6)$$

with $|b_1| \ll 1$. The eq. (5) becomes

$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{2}M_{\text{P}}^2 R + \frac{M_{\text{P}}^2}{12m^2} \left(R^2 + \sum_{n=3}^{\infty} b_n \left(-\frac{6m^2}{b_1} \right)^{2-n} R^n \right) \right), \quad (7)$$

so the suppression scales for $n \geq 3$ become larger than the inflation scale m as we tune b_1 to be small. This is the form of the action we advocate.

The extension of the Starobinsky model with an R^n term was studied in Ref. [14], which gives us a constraint on each coefficient,

$$\left| nb_n \left(\frac{b_1}{2} \right)^{n-2} \right| \lesssim 10^{-2n+2.6} \quad (n \geq 3). \quad (8)$$

The constraint for $n = 3$ is $|b_1| \lesssim 10^{-3.6}$. With $|b_1|$ satisfying this bound, the constraint (8) is satisfied also for $n \geq 4$. Using the results of Planck 2015, $n_s = 0.9655 \pm 0.0062$ (Planck TT + low P) [2], we obtain more stringent 95% CL bounds on b_1 :

$$-2.5 \times 10^{-4} \lesssim b \lesssim 1.3 \times 10^{-4} \quad (N_e = 50), \quad (9)$$

$$-1.4 \times 10^{-4} \lesssim b \lesssim 2.2 \times 10^{-4} \quad (N_e = 60), \quad (10)$$

where $b = b_3 b_1$. Note that $b_1 < 0$, and b_3 is expected to be of order one, so the bound on b_1 is roughly $|b_1| \lesssim 2 \times 10^{-4}$.

3 Implications for inflationary observables

For self-completeness and with the latest Planck data, let us discuss effects of the extra terms in the action to the inflationary observables, namely the scalar spectral index n_s , its running α_s , and the tensor-to-scalar ratio r . As we saw above, effects of higher order ($n \geq 4$) terms are more suppressed than the $n = 3$ term, so we neglect the higher order terms in the following analyses. The addition of R^3 term has been studied since early times [11, 15], and recent discussions include Refs. [14, 18]. Here, we summarize the properties of the scalar potential in the Einstein frame and inflationary observables, and compare with the latest observational data. Considering

observation of the 21 cm line from hydrogen atoms, we obtain the opposite conclusion to that in the literature [14].

Under the standard procedure, the Jordan frame action (7) up to the third term is transformed into the Einstein frame action with the following potential for a canonical scalar field ϕ ,

$$V = \frac{m^2}{9b^2} e^{-2\sqrt{2/3}\phi} \left(\sqrt{1 + 3b \left(e^{\sqrt{2/3}\phi} - 1 \right)} - 1 \right) \left(1 + 6b \left(e^{\sqrt{2/3}\phi} - 1 \right) - \sqrt{1 + 3b \left(e^{\sqrt{2/3}\phi} - 1 \right)} \right). \quad (11)$$

Here and hereafter, we take the reduced Planck unit $M_{\text{P}} = 2.4 \times 10^{18} \text{GeV} = 1$. When we set $b = 0$, it reduces to the Starobinsky potential (2). If b is negative, the potential blows up in the large field region. If b is positive, the potential has a run-away behavior, and there is a possibility of topological inflation as discussed in Ref. [16]. If we retain b up to the leading nontrivial order, the potential is

$$V = V_{\text{Starobinsky}} \times \left(1 - \frac{b}{2} e^{\sqrt{2/3}\phi} \left(1 - e^{-\sqrt{2/3}\phi} \right) \right) + \mathcal{O}(b^2), \quad (12)$$

where $V_{\text{Starobinsky}}$ is given in eq. (2). Of course, the large field behavior ($\phi \rightarrow \infty$) depends also on the higher order terms [17, 18], but the leading order is enough for our purpose.

In the leading order of the deformation parameter b , the spectral index n_s , the tensor-to-scalar ratio r , the running of the spectral index α_s , and its running β_s are obtained as

$$1 - n_s \simeq \frac{2}{N} \left(1 + \frac{16}{27} b N^2 \right) = \frac{2}{N} + \frac{32}{27} b N, \quad (13)$$

$$r \simeq \frac{12}{N^2} \left(1 - \frac{16}{27} b N^2 \right) = \frac{12}{N^2} - \frac{64}{9} b, \quad (14)$$

$$\alpha_s \simeq -\frac{2}{N^2} \left(1 - \frac{16}{27} b N^2 \right) = -\frac{2}{N^2} + \frac{32}{27} b, \quad (15)$$

$$\beta_s \simeq -\frac{4}{N^3} \left(1 + \frac{4}{9} b N \right) = -\frac{4}{N^3} - \frac{16}{9N^2} b, \quad (16)$$

where higher order terms in $1/N$ are also neglected. The results for n_s , r , and α_s are consistent with the $n = 3$ case in Ref. [14], and we additionally obtain the expression of β_s .

Varying the value of the parameter b , we obtain the prediction of the model as curves on the (n_s, r) -plane in Fig. 1. The region of positive b corresponds to the left to the large point (the Starobinsky model, $b = 0$), and negative to the right. This is because the positive b makes the potential flatter (ϵ smaller) and more curved ($|\eta|$ larger), so both of n_s and r smaller. The constraint on n_s gives constraint on b as the inequalities (9) and (10). The correction from the extra term does not drastically change the value of r to improve the detection prospect of r . The running in the model (11) with the rising correction ($b < 0$) was discussed in Ref. [18] in the context of power suppression in low multipoles.³

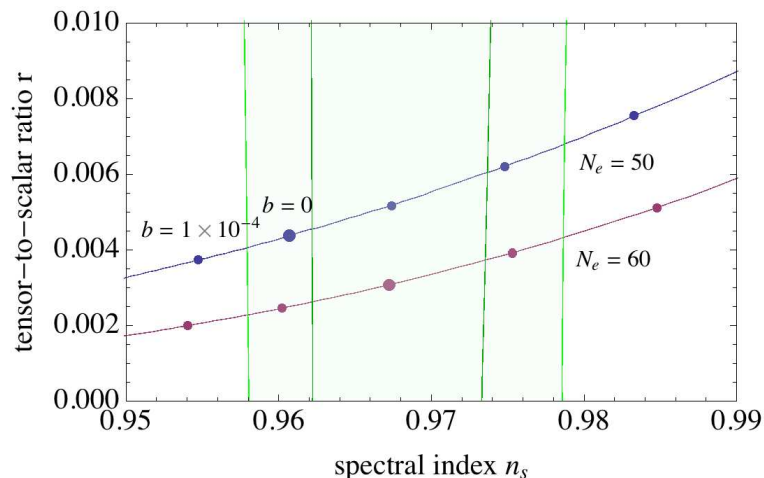


Figure 1: Prediction of the model in the (n_s, r) -plane as we vary the parameter $b = b_3 b_1$. The blue (top) line and red (bottom) lines correspond to $N_e = 50$ and $N_e = 60$, respectively. The large dots show the prediction of the Starobinsky model ($b = 0$), and the small dots correspond to the points of $b = n \times 10^{-4}$ with an integer n . The green contours are the Planck TT+lowP+BKP+lensing+BAO+JLA+ H_0 constraints (traced from Fig. 21 in Ref. [19]).

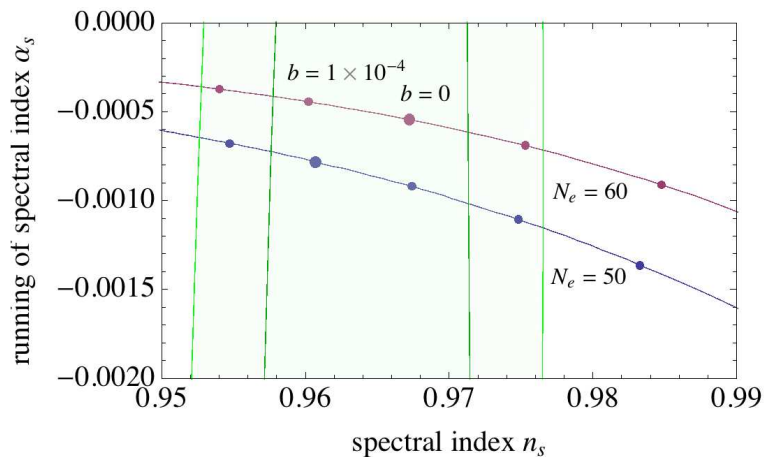


Figure 2: Prediction of the model in the (n_s, α_s) -plane as we vary the parameter $b = b_3 b_1$. The blue (bottom) line and red (top) lines correspond to $N_e = 50$ and $N_e = 60$, respectively. The large dots show the prediction of the Starobinsky model ($b = 0$), and the small dots correspond to the points of $b = n \times 10^{-4}$ with an integer n . The green contours are the Planck TT, TE, EE + low P constraints (traced from Fig. 4 in Ref. [2]).

The prediction of the model in the (n_s, α_s) -plane is shown in Fig. 2. As can be seen from the Figure and eq. (15), the positive b makes the absolute value of the running smaller. The future prospect of precision of the running α_s by 21 cm line and CMB observation will be 3×10^{-4} [20], so it will become possible to distinguish the Starobinsky model ($b = 0$) and our extension if b is of order 10^{-4} .

On the other hand, the running of running β_s is of order 10^{-5} in our case, and it cannot be measured by the near future observations [20]. In other words, our predictions can be falsified by detection of the running of running β_s .

4 Discussion

In this letter, we proposed a new interpretation of the Starobinsky model as a low-energy effective theory of a higher dimensional theory whose characteristic energy scale is denoted by Λ . Compactification of extra dimensions naturally introduces the large overall factor. With the tuning of $|b_1| \lesssim 2 \times 10^{-4}$ (in addition to the one for the cosmological constant), we obtain Starobinsky model augmented with higher order terms suppressed enough to be consistent with the Planck 2015 results. Compared to taking a large parameter only in front of the R^2 term, taking a small parameter is regarded less unnatural in the sense that it may happen by accidental cancellation of several contributions. If the deformation is indeed of order 10^{-4} , the model can be distinguished from the original Starobinsky model ($b = 0$) by future observations of CMB and 21 cm line.

Predictions on inflationary observables studied in the previous section are consequences of the action (5), but does not crucially depend on the underlying assumption (4). Here, we briefly discuss another possibility to obtain the advocated action (5). One of the reasons of the unnatural expansion of the Starobinsky model action may reside in the fact that we regard the Einstein term as the fundamental term, and the other terms are “secondary” in the sense that they are originated by quantum corrections. In contrast, we can take a view that the fundamental or main term is the second term R^2 rather than R (see *e.g.* Refs. [24–29] in this kind of direction). The pure R^2 theory, $S = c \int d^4x \sqrt{-g} R^2$, does not have a dimensionful constant, and it is scale invariant.⁴ Inflation in this theory is in the pure de Sitter universe, and it eternally inflates. Note that the coefficient c of the action cannot be absorbed into R by scale transformation simply because the action is scale invariant, and it is legitimate to take the coefficient as a huge or minuscule number. The former eventually corresponds to the Starobinsky model. If the scale symmetry is spontaneously broken, perhaps after coupling to matter sector, then a scale Λ is generated. This will lead to the form of the action (5). The fact that b_1 should be suppressed is

³ It is interesting to note that such steepening arises also in the (old-minimal) supergravity embedding of Starobinsky model for some initial conditions [21–23].

⁴ Aspects of quadratic gravity was recently revisited in Ref. [30], and supergravity embedding of the pure R^2 theory was studied in Ref. [31]. See also the inflation scenario based on broken scale invariance in Ref. [32].

unchanged, and we have the same predictions (13), (14), (15), and (16).

It would also be useful to discuss a possible explanation for the tuning of b_1 (the cosmological constant also). From the effective theory point of view, the first few terms in the low energy expansion have very small values in our model. Indeed, such a situation sometimes occurs in the effective action of the order parameter near the phase transition point (e.g. the Lifshitz point of the Nambu-Jona-Lasinio model [33]) [34]. Based on this analogy, let us think of the metric as some “order parameter” and explore the “phase structure” of $f(R)$ gravity, using the action $S = \int d^4x \sqrt{-g} c (-d_0 - d_1 R + R^2)$ with $c > 0$. Under the standard procedure, we rewrite this as a theory of canonical scalar field in Einstein frame. The potential is bounded below as long as $d_1^2 - 4d_0 > 0$. The shape of the scalar potential dramatically changes depending on the sign of d_1 . (i) For $d_1 = 0$, the potential is constant, and its sign (dS, AdS, or Minkowski) is determined by the value of d_0 . (ii) For $d_1 > 0$, just as in the Starobinsky model, the potential has a minimum as well as the flat region. Inflation is therefore realized. (iii) For $d_1 < 0$, the minimum of the potential is at $\phi = \infty$ (runaway potential). The inflaton slow-rolls toward $\phi = \infty$ and the eternal inflation in the approximate dS is realized. It would be interesting if we could interpret inflation as a consequence of the phase transition from the phase (ii) through (i) to (iii).

Finally, let us estimate the characteristic scale Λ in the higher dimensional theory. Using the upper bound on $|b_1|$, its lower bound is given by

$$\Lambda = m \sqrt{\frac{6}{|b_1|}} \gtrsim 5 \times 10^{15} \text{ GeV}. \quad (17)$$

This scale is close to the Grand Unified Theory scale, so it is tempting to relate the higher dimensional theory with Grand Unification. Moreover, the Planck scale M_P is not the fundamental scale in our viewpoint, and quantum gravity effects may appear at this scale. In the context of superstring theory, we can identify the scale Λ as the string scale m_s (see the discussion after eq. (5)). This implies that we may see the footprints of quantum gravity or string theory in the sky.

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