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Boundary spectra in superspace σ-models

Thomas Quella[2,3], Volker Schomerus[1,3] and Thomas Creutzig[1]

[1] DESY Hamburg, Theory Group,
Notkestrasse 85, D–22607 Hamburg, Germany

[2] Institute for Theoretical Physics, University of Amsterdam,
Valckenierstraat 65, 1018 XE Amsterdam, The Netherlands

[3] Isaac Newton Institute for Mathematical Sciences,
20 Clarkson Road, Cambridge, CB3 0EH, United Kongdom

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Abstract

In this note we compute exact boundary spectra for D-instantons in σ-models on the supergroup PSL(2|2). Our results are obtained through an explicit summation of the perturbative expansion for conformal dimensions to all orders in the curvature radius. The analysis exploits several remarkable properties of the perturbation series that arises from rescalings of the metric on PSL(2|2) relative to a fixed Wess-Zumino term. According to Berkovits, Vafa and Witten, the models are relevant in the context of string theory on AdS_3 with non-vanishing RR-flux. The note concludes with a number of comments on various possible generalizations to other supergroups and higher dimensional supercoset theories.

e-mail: tquella@science.uva.nl, volker.schomerus@desy.de, thomas.creutzig@desy.de

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1 Introduction

The celebrated AdS/CFT correspondence [1, 2] has promoted the solution of string theory in Anti-de Sitter (AdS) spaces to one of the central problems of modern mathematical physics. Progress in this direction requires to construct new types of quantum field theories with internal Lie superalgebra symmetries. The precise model to be considered depends on the particular approach that is employed. Recent investigations have been based on certain gauge fixed versions of the Green-Schwarz superstring [3, 4, 5, 6, 7, 8], the pure spinor formalism [9, 10, 11, 12] and the hybrid formalism [13, 14].

Without much further comment on the precise relation with string theory (see some remarks below, however), we shall turn our attention to a particular class of quantum theories with internal supersymmetries, namely to non-linear sigma models on supergroups. They are characterized by the following simple action

$$\mathcal{S}_{f,k}[S] \;=\; -\frac{1}{2\pi f^2}\int_\Sigma d^2z\ \mathrm{str}\left(S^{-1}\partial S S^{-1}\bar\partial S\right) - \frac{k}{12\pi}\int_\Sigma d^{-1}\mathrm{str}\left(\left(S^{-1}dS\right)^3\right) \qquad (1.1)$$

with a suitably normalized supertrace str. Here, S is a map from the world-sheet Σ to some supergroup G. We have weighted the standard kinetic term with a coupling constant f^2 and also added a topological Wess-Zumino (WZ) term with coefficient k. For sigma models on bosonic groups, quantum conformal invariance requires $f^{-2} = k$. Once we have adjusted the coupling constants in this way, we are dealing with a Wess-Zumino-Novikov-Witten (WZNW) theory which can be solved using the algebraic techniques of 2-dimensional conformal field theory, exploiting the infinite dimensional current algebra symmetry of the WZNW model.

It is one of the intriguing features of certain supersymmetric target spaces that the requirement of quantum conformal invariance may not impose any restriction on f^{-2}, see e.g. [13, 15, 16, 17]. This happens whenever the supergroup G has vanishing dual Coxeter number. The latter condition is satisfied e.g. for the superconformal groups PSL(N|N) that appear in the AdS/CFT correspondence, but also for OSP(2N+2|N) and $D(2,1;\alpha)$. In these cases, the action (1.1) gives rise to a continuous family of conformal quantum field theories. All models share the same global target space symmetries. On the other hand, the WZ point with $f^{-2} = k$ is still distinguished by an enhancement of world-sheet symmetries. For generic values of f, one only expects to find a few chiral higher spin fields in addition to the Virasoro symmetry that comes with conformal invariance (see [15] for details). Whatever the precise chiral symmetry is, it will almost certainly not suffice for a full algebraic solution of generic supergroup sigma models. This insight has lead many scientists working in the field to discard conformal field theory techniques and to turn to other methods in integrable systems, such as the Bethe-Ansatz and generalizations thereof.

Though ultimately, computations in superspace sigma models may involve a variety of integrable techniques (see e.g. [18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29] for an incomplete collection of recent relevant ideas, a few results and many further references, in particular to the earlier literature), it seems to us that the real potential of conformal field theory methods has not been explored with sufficient care. In fact, we shall see below that a combination of algebraic techniques with conformal perturbation theory can provide powerful new results going far beyond the WZ point. To be more precise, we propose to consider the sigma models (1.1) as deformations of a WZNW model,

$$\mathcal{S}_{f,k}[S] \;=\; \mathcal{S}_k^{\rm WZNW}[S] \;-\; \frac{\lambda}{2\pi}\int_{\mathcal{H}} d^2z\ {\rm str}\left(S^{-1}\partial S S^{-1}\bar\partial S\right) \;=\; \mathcal{S}_k^{\rm WZNW}[S] + \mathcal{S}_\lambda[S] \quad . \qquad (1.2)$$

The deformation parameter λ is related to k and f through $\lambda = f^{-2} - k$. For reasons to be explained below, we shall often refer to this deformation of the WZNW model as a "RR-deformation". Note, however, that on the level of sigma models it simply changes the overall scale factor of the metric while leaving the magnetic background field invariant. Our approach is then to study the sigma model through conformal perturbation theory around the WZ point. In this note we restrict our attention to the simplest objects, namely to partition functions, leaving investigations of correlators etc. as an interesting problem for future research.

In order to explain our strategy, let us briefly look at simple torus compactifications. Suppose we are interested e.g. in the spectrum of strings on a 1-dimensional circle with arbitrary compactification radius r. At generic points in the 1-dimensional moduli space, the chiral symmetry of the model is generated by the U(1) current $i\partial X$ and its anti-holomorphic counterpart. With respect to these currents, the theory is not rational. But there exist some distinguished points in the moduli space at which the chiral symmetry is enhanced and the theory becomes rational once the additional chiral fields are taken into account. In particular, the moduli space contains one point, known as the self-dual radius $r_0 = r_{\rm SD}$, where the symmetry gets enhanced to an sl(2) current algebra at level $k = 1$. At this special radius, all spectra can be composed from a finite number of sectors. With later generalizations in mind, we consider the partition function on a strip or half-plane with Neumann boundary conditions which is simply given by the vacuum character of the sl(2) current algebra[1]

$$Z_N^{r_0}(q) \; = \chi_{0,k=1}^{su(2)} \; = \; \vartheta_3(q^2)/\eta(q) \; = \; \frac{1}{\eta(q)} \sum_{n\in\mathbb{Z}} q^{n^2} \quad . \tag{1.3}$$

Other points in the moduli space may be reached through a deformation with the perturbation $\mathcal{S}_\gamma = \frac{\gamma}{2\pi}\int d^2z \partial X \bar\partial X$. The perturbation series for the conformal dimensions of boundary fields can be summed up to all orders in perturbation theory. Our partition function (1.3) gets deformed to

$$Z_N^{r}(q) \; = \; \frac{1}{\eta(q)} \sum_{n\in\mathbb{Z}} q^{\frac{n^2}{1-\gamma}} \quad . \tag{1.4}$$

The result corresponds to the spectrum of a point-like brane on a circle with radius $r = r_0\sqrt{1-\gamma}$. In the perturbative treatment, the factor $1/(1-\gamma) = 1 + \gamma/(1-\gamma)$ arises

[1] At the self-dual radius there is no fundamental difference between a D-instanton and an extended brane since they can can be rotated continuously into the other, see e.g. [30].

from a geometric series as explained e.g. in [31]. Bulk spectra can also be computed, either directly or through modular transformation of the boundary partition function. Let us point out that the perturbative analysis is insensitive to the fact that the theory ceases to be rational once we move away from the self-dual radius. Of course, in this particular case the U(1) current algebra symmetry is sufficiently large for an algebraic construction of the theory at generic radii and such a construction is about as difficult as it is at the self-dual point. Hence, there is no good motivation to pass through a perturbative construction.

But there exists a better example to illustrate the enormous potential conformal perturbation theory may possess. It is provided by the 1-dimensional boundary sine-Gordon theory. In this model, a periodic potential is switched on along the boundary of a free field theory. As a consequence, the spectrum of boundary dimension develops gaps which can grow with the strength λ of the perturbation. Eventually, only a point-like spectrum remains. Given the complexity of the spectrum at intermediate values of λ, one might suspect that its precise form is very difficult to determine. Yet, the boundary partition function can be calculated rather easily in perturbation theory [32, 30, 33], for any value of the deformation parameter λ. In this example, the boundary potential reduces the chiral symmetry to the Virasoro algebra. In principle, the latter is still sufficiently large to allow for a standard CFT construction of the boundary sine-Gordon theory, but such an analysis is of the same level of difficulty as the solution of Liouville theory and it has never been carried out. Hence, the example of boundary sine-Gordon theory supports our claim that in some situations, conformal perturbation theory provides an easy route to complicated results that seem (almost) inaccessible through the usual algebraic methods. A similar picture will emerge from our study of boundary spectra on supergroup σ-models.

Even though most of the ideas and technical steps we are about to explain hold quite generally, we shall carry them out in a particular example, namely for the supergroup PSL(2|2). This allows our presentation to be very concrete. Furthermore, our results apply to string theory in $AdS_3 \times S^3$ whose solution has been reduced to the construction of sigma models on the supergroup PSL(2|2) through the hybrid approach developed by Berkovits, Vafa and Witten [13]. In this context, the WZNW model corresponds to a background with pure NSNS 3-form flux. Switching on an additional RR field is modelled by the marginal perturbation with $\mathcal{S}_\lambda$ which is why we often refer to this term

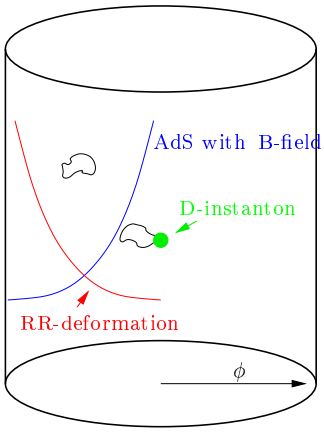

Figure 1: The influence of NSNS and RR potentials on strings in the bulk and on an instantonic brane.

as RR-deformation. Sigma models on PSL(2|2) and closely related target superspaces have been investigated by several groups [13, 15, 34, 35, 36]. For our analysis, the studies by Bershadsky et al. have been particularly useful.

With the example of strings in AdS_3 in mind, we may re-evaluate our optimistic hopes to compute exact spectra through perturbation theory. Let us think of the target space as a 3-dimensional solid cylinder. Since AdS_3 is curved, the corresponding sigma model is interacting. At the WZ point, the interaction falls off exponentially towards the boundary of the cylinder. This has several effects on the bulk spectrum. In particular, the spectrum is continuous and there exist so-called long string states that can stretch along the boundary [37]. The RR-deformation now adds another term to the interaction which increases exponentially near the boundary. Obviously, such a new term must have drastic effects on the spectrum. Certainly, long string states disappear. In addition, the spectrum is expected to become discrete since closed strings are now moving in a box between the two exponential walls. The dramatic effects of the RR-deformation may raise doubts that perturbative computations could be successfully performed. And indeed, it is most likely true that the bulk spectrum of the theory is not amenable to a perturbative expansion in λ. But the situation changes if we consider the boundary spectrum [38, 39] on a D-instanton instead. Suppose, the instanton has been placed at the center of the solid cylinder. Open strings that end on such a D-instanton must be very highly excited in order to penetrate into the region close to the boundary where the RR-background flux can be felt. Therefore a D-instanton spectrum might be accessible through a perturbative computation. Below we shall see that this intuition is indeed correct. In fact, we are able to determine the exact spectrum of a D-instanton for any value of λ. The same calculation fails at one step when we try to apply it to the bulk or to spectra on non-compact branes.

Our main new result is a complete computation of the boundary spectrum for maximally symmetric, point-like branes in sigma models on the supergroup PSL(2|2). The partition function of such a system was argued in [40] to be of the general form

$$\begin{aligned} Z^{\mathrm{PSL}(2|2)}_{DI;\lambda}(z_1,z_2;q) &= \mathrm{str}_{\mathcal{H}}\left(q^{L_0-\frac{c}{24}}\, z_1^{K_1^0}\, z_2^{K_2^0}\right) \\ &= \sum_{j_1\neq j_2} a^{\lambda}_{j_1 j_2}(q)\,\chi_{[j_1,j_2]}(z_1,z_2) + \sum_j b^{\lambda}_j(q)\,\chi_{\mathcal{P}[j]}(z_1,z_2) \quad . \end{aligned} \tag{1.5}$$

Here, K_1^0 and K_2^0 are two Cartan elements in the bosonic subalgebra $\mathrm{sl}(2)\oplus\mathrm{sl}(2)$ of $\mathrm{psl}(2|2)$ and we have denoted the characters of the contributing finite dimensional $\mathrm{psl}(2|2)$ representations by χ (see Appendix A for explicit formulas). The branching functions b_j and a_{j_1,j_2} at the WZ point $\lambda = 0$ were also determined in [40]. Our aim in this work is to show that the branching functions b_j are independent of the deformation parameter λ while

$$a^{\lambda}_{j_1,j_2}(q) \;=\; q^{-\,C_2^{j_1,j_2}\,\frac{\lambda}{k(k+\lambda)}}\; a^{\lambda=0}_{j_1,j_2}(q) \qquad \text{with} \qquad C_2^{j_1,j_2} \;=\; j_2(j_2+1) - j_1(j_1+1)\;. \tag{1.6}$$

Let us already point out that the dependence of the conformal weights on the deformation parameter λ is very similar to the one found in free field theory (see eq. (1.4)). We shall see that this is due to some peculiar features of the Lie superalgebra $\mathrm{psl}(2|2)$.

Our formulas (1.5) and (1.6) contain a surprising wealth of information. Let us unravel some of that through a few selected cases. Consider, for example, the boundary current $J^\mu(x)$ where μ runs through some 14-dimensional basis of $\mathrm{psl}(2|2)$. Under the action of the global $\mathrm{psl}(2|2)$, the currents transform in the adjoint representation which is part of the atypical module $\mathcal{P}[0]$ (see Appendix A). Since the branching functions b_j are independent of λ, states transforming in any of the $\mathcal{P}[j]$ do not receive corrections. Hence, the currents J^μ continue to possess dimension $h = 1$, as expected. Things become more interesting once we proceed to products $J^\mu J^\nu$ of currents. These form a 196-dimensional subspace of fields transforming in the 48-dimensional representations $[0,1]$, $[1,0]$ and various subspaces of $\mathcal{P}[j]$. Hence, under the deformation, the weight of 96 fields gets lifted while 100 fields remain at conformal weight $h = 2$.

Formula (1.6) passes a few interesting test. To begin with, we observe that the energy shift is positive for states with sufficiently large momentum j_1 in the radial direction of AdS_3. This is in line with our geometric intuition: Only states that are highly excited in the radial direction can penetrate to the region near the boundary of AdS_3 where

their energy gets lifted due to the RR perturbation. It is also interesting to evaluate our formula in the semi-classical regime, i.e. for large values of the level k. Inserting the relation $\lambda = f^{-2} - k$ in (1.6) and sending k to infinity, the spectrum of boundary conformal weights is seen to coincide with the spectrum of $f^2 C_2$ up to the usual integer shifts. The eigenvalues of $f^2 C_2$ may be interpreted as energies for a particle moving on PSL(2|2). Hence, at large level k and modulo integers, the spectrum of the sigma model on PSL(2|2) agrees with the minisuperspace approximation, as it is supposed to. A much more detailed investigation of the approach to minisuperspace spectra for supersphere models is included in a very interesting upcoming paper by Candu and Saleur [41].

The plan of this work is as follows. In the next section we collect some background material, partly from our earlier paper [40]. This includes a careful discussion of maximally symmetric, point-like branes in the WZNW model on PSL(2|2). The ones that are relevant for our analysis are located at the group unit e of the bosonic base and they extend in all eight fermionic directions. The associated boundary partition function is discussed in section 2.2 along with more details on the Casimir decomposition (1.5) at the WZ point. Section 2.3 contains a construction of the perturbing field in terms of currents and a new proof of its exact marginality. Most of our novel results are obtained in section 3 which begins with a few comments on 2-point functions. Section 3.2 lists several observations concerning the perturbative series generated by $\mathcal{S}_\lambda$. We shall show that the RR-deformation, while being non-abelian and non-constant on PSL(2|2) in general, simplifies drastically in the evaluation of psl(2|2) invariant quantities, such as conformal weights. In fact, the RR-deformation turns out to be quasi-abelian, i.e. its combinatorics is no more complex than it is for constant shifts of the closed string background fields in a flat target space. There remains a mixing problem, however, that we can only overcome when the general results are applied to boundary conformal weights of a point-like D-instanton. This is explained in section 3.3 before we combine all our results into an exact computation of boundary weights, following closely the steps of a similar computation in [31]. Our concluding section includes extensive comments on possible generalizations, applications and consequences.

2 Collection of background material

The purpose of the following section is mainly to provide the background material that our subsequent perturbative evaluation of boundary partition functions is based upon. In the first part we gear up to explain the structure of the boundary partition function we are about to deform. We start with a few comments on brane geometries in WZNW models on PSL(2|2), extending our previous analysis of branes in the gl(1|1) WZNW model [42].[2] One of the instantonic D-branes we find, possesses exactly the spectrum that was anticipated in [40]. The full field theory partition function and its so-called Casimir decomposition is reviewed in the second subsection. We then turn to a more detailed analysis of the perturbing field, mostly following our previous discussion in [40]. On this occasion, we propose a new argument for the exact marginality of Φ which differs a bit in character from the original derivation [15, 13]. Rather than using the vanishing of the dual Coxeter number of psl(2|2), our reasoning exploits the position of Φ at the bottom of a logarithmic multiplet (see also [43]). Most of the results we describe below are not new and the impatient or experienced reader may skip forward to section 3, at least on first reading.

2.1 Branes at the WZ point: gluing conditions and geometry

As we shall explain in great detail below, the success of our subsequent exact computation of a boundary partition function for the sigma model on PSL(2|2) hinges on three key properties of the imposed boundary condition. To begin with, it (i) must preserve some combination of left and right regular psl(2|2) transformations. At the WZ point, maximally symmetric boundary conditions are associated with so-called twisted conjugacy classes (see [44] and [42] for the supersymmetric case). Explicit formulas for the deformation of the partition function can only be found if (ii) the corresponding twisted conjugacy class is point-like localized on the bosonic base and (iii) it is delocalized in all the fermionic directions. Later we shall rephrase these conditions as inherent features of the boundary conformal field theory. Our aim here is to describe a boundary condition which meets all these requirements and to determine the relevant boundary partition function at the WZ point.

In the WZNW model, the global symmetries of the PSL(2|2) sigma model are gener-

[2] This first subsection is based on unpublished notes of TC on branes in supergroup WZNW models.

ated by the zero modes of chiral currents

$$J(z) \;:=\; -k\partial S S^{-1} \quad , \quad \bar{J}(\bar{z}) \;:=\; kS^{-1}\bar{\partial}S \;\; . \tag{2.1}$$

A boundary WZNW model is scale invariant if the Sugawara stress tensor obeys $T(z) = \bar{T}(\bar{z})$ all along the boundary $z = \bar{z}$. Such a conformal boundary theory preserves a global psl(2|2) symmetry provided that the currents satisfy the following gluing condition

$$J^\mu(z) \;=\; \Omega\bar{J}^\mu(\bar{z}) \quad \text{for} \;\; z = \bar{z} \;\; . \tag{2.2}$$

Here, Ω is a metric preserving automorphism of the Lie superalgebra. It determines the precise combination $J_0 + \Omega\bar{J}_0$ of global psl(2|2) charges that remains unbroken by the boundary condition. In the case of bosonic groups, the geometry underlying maximally symmetric boundary conditions in WZNW models was unravelled in [44] (see also [45, 46] for various generalizations and [47] for a review). There it was shown that a boundary condition in which left and right moving currents are identified with a trivial gluing automorphism $\Omega = \mathrm{id}$ correspond to branes whose world-volume is localized along conjugacy classes. When Ω is nontrivial, the relevant geometric objects are twisted conjugacy classes

$$\mathcal{C}_u^\Omega \;=\; \{\, h \;\in\; G \,|\, h \;=\; gu\Omega(g^{-1}) \,\}$$

where u is an element in G and we have lifted the automorphism Ω from the Lie algebra to the group. As explained in [42], the derivation of [44] carries over to WZNW models on supergroups (see also [48] for a general analysis).

Having outlined the link between boundary conditions and conjugacy classes we are now searching for a pair (u, Ω) such that $\mathcal{C}_u^\Omega$ meets the requirements (ii) and (iii) we have listed in the introductory paragraph to this subsection. We shall not conduct our search systematically. Instead, let us simply argue that the choice $u = e$ and $\Omega(X) = (-1)^{|X|}X$ does the job. The corresponding twisted conjugacy class $\mathcal{C}_u^\Omega$ is localized at the unit element e of the bosonic group and it extends in all fermionic directions, i.e. along those tangent vectors $X \in \mathrm{psl}(2|2)$ which have degree $|X| = 1$. It is easy to see that $\Omega(X) = (-1)^{|X|}X$ is consistent with the Lie superalgebra structure and the metric. Hence, it extends to a gluing automorphism on the entire current algebra. Moreover, parametrizing elements g of the supergroup in the form $g = \exp(F)\exp(B)$ where $F(B)$ is any linear combination of odd (even) elements, we find

$$\mathcal{C}_u^\Omega \;=\; \{\, h \;\in\; G \,|\, h \;=\; e^F e^B \Omega\left(e^{-B}e^{-F}\right) \;=\; e^{2F} \,\} \;\; .$$

Indeed, the bosonic coordinates have dropped out and we remain with a superconjugacy class of superdimension $0|8$ which extends merely along the 8 fermionic directions. We conclude that the space of functions on the corresponding brane is given by

$$f \;=\; f(\eta_a, \bar{\eta}_b) \tag{2.3}$$

where $\eta_1, \ldots, \eta_4$ and their bared counterparts are four fermionic coordinates that parametrize the odd generators F. The relevant action of $\mathrm{psl}(2|2)$ on this 2^8-dimensional space is obtained by restricting the Ω-twisted adjoint action of $\mathrm{psl}(2|2)$ on the supergroup $\mathrm{PSL}(2|2)$ to a point in the bosonic submanifold, or more explicitly,

$$\begin{aligned} A_1^a \;&=\; \bar{\partial}^a - \frac{1}{2}\epsilon^{abcd}\eta_b(\eta_c\partial_d - \eta_d\partial_c) - \epsilon^{abcd}\eta_b(\bar{\eta}_c\bar{\partial}_d - \bar{\eta}_d\bar{\partial}_c)\ , \\ A^{ab} \;&=\; -i\eta^a\partial^b + i\eta^b\partial^a - i\bar{\eta}^a\bar{\partial}^b + i\bar{\eta}^b\bar{\partial}^a \qquad , \qquad A_2^a \;=\; \partial^a\ . \end{aligned} \tag{2.4}$$

The curious reader can find explicit formulas for the generators of the left and right regular action L_X and R_X in [40]. When combined as $A_X = L_X + (-1)^{|X|}R_X$, they result in the twisted adjoint action that is relevant for the minisuperspace description of the symmetry of our brane.

Under the twisted adjoint action A_X, the 2^8-dimensional space of ground states (2.3) may be seen to transform according to the representation

$$\mathcal{B}(0,0) \;:=\; \mathrm{Ind}_{\mathfrak{g}^{(0)}}^{\mathfrak{g}} V_{(0,0)} \;=\; \mathcal{U}(\mathfrak{g}) \otimes_{\mathfrak{g}^{(0)}} V_{(0,0)} \;\cong\; \mathcal{P}_0 \;\oplus\; [1,0] \;\oplus\; [0,1]\ .$$

Here, $\mathfrak{g}^{(0)}$ denotes the bosonic subalgebra of the Lie superalgebra $\mathfrak{g} = \mathrm{psl}(2|2)$ and we introduced $V_{(0,0)}$ for the trivial 1-dimensional representation of $\mathfrak{g}^{(0)}$. According to general mathematical results, the module $\mathcal{B}(0,0)$ is projective. Hence, it is guaranteed to decompose into a direct sum of projective modules. The corresponding decomposition is spelled out on the right hand side. Here, the symbols $[0,1]$ and $[1,0]$ denote 48-dimensional irreducible typical representations (long multiplets) of $\mathrm{psl}(2|2)$. These are generated from the two 3-dimensional representations of $\mathrm{sl}(2) \oplus \mathrm{sl}(2)$ by the application of four fermionic generators. In addition, there appears the 160-dimensional projective cover $\mathcal{P}[0]$ of the trivial representation $[0]$. It is an indecomposable representation that is built up from irreducible atypicals (short multiplets) of $\mathrm{psl}(2|2)$ according to the following diagram

$$\mathcal{P}[0]:\ [0] \;\longrightarrow\; 3[1/2] \;\longrightarrow\; 2[1] \oplus 6[0] \;\longrightarrow\; 3[1/2] \;\longrightarrow\; [0]\ .$$

This so-called composition series tells us that $\mathcal{P}[0]$ contains the trivial representation $[0]$ as a true subrepresentation. Its representation space is spanned by the unique invariant element in $\mathcal{P}[0]$. We call this subrepresentation $[0]$ the socle of $\mathcal{P}[0]$. At the other end of the diagram, i.e. in the so-called head of $\mathcal{P}[0]$, we find another copy of $[0]$. It is associated with the factor space of $\mathcal{P}[0]$ which is obtained if we divide the projective cover by its maximal non-trivial subrepresentation. A brief summary of the representation theory of psl(2|2) is provided in appendix A. Many more details can be found in [49, 40]. We advise readers who are unfamiliar with indecomposable representations of Lie superalgebras to consult those references or other mathematical literature.

2.2 Boundary partition function and its Casimir decomposition

After this brief discussion of brane geometry and the space of ground states, let us analyze the excited states which arise through application of current algebra modes. By construction, these states transform in representations that emerge from a product of a projective module with some power of the adjoint and which, by abstract mathematical results, can be decomposed into projectives. Explicit formulas for the involved characters were provided in [40]. Since we do not need the details below, we refrain from reproducing these formulas here. In [40] we also explained how sectors erected over projective modules can be decomposed into representations of the Lie superalgebra psl(2|2). The result can be expressed in the form

$$\begin{aligned} Z_{D0}^{\mathrm{PSL}(2|2)}(z_1,z_2;q) &= \chi_{\mathcal{P}[0]}(z_1,z_2;q)+\chi_{[1,0]}(z_1,z_2;q)+\chi_{[0,1]}(z_1,z_2;q) \\ &= \sum_{j_1\neq j_2}\Big(a_{j_1j_2}^{\mathcal{P}[0]}(q)+a_{j_1j_2}^{[1,0]}(q)+a_{j_1j_2}^{[0,1]}(q)\Big)\ \chi_{[j_1,j_2]}(z_1,z_2) \\ &\quad+\sum_{j}\Big(b_{j}^{\mathcal{P}[0]}(q)+b_{j}^{[1,0]}(q)+b_{j}^{[0,1]}(q)\Big)\ \chi_{\mathcal{P}[j]}(z_1,z_2) \end{aligned} \tag{2.5}$$

where $\chi_{[j_1,j_2]}$ and $\chi_{\mathcal{P}[j]}$ are supercharacters of the Lie superalgebra psl(2|2) (see Appendix A for explicit formulas). Formula (2.5) is known as the Casimir decomposition of the partition function. The various branching coefficients a_{ij} and b_j count how many times a projective psl(2|2) multiplet appears on a given energy level. These numbers may be determined with the help of the Racah-Speiser algorithm. A detailed explanation can be found in [40] along with a few explicit expressions for the branching of the representation $\hat{\mathcal{P}}[0]$. Here it suffices to recall that the lowest conformal weight h_{j_1,j_2}^{ϖ} among

all the multiplets $[j_1, j_2]$ that are generated out of ground states in the representations $\varpi \cong \mathcal{P}[0], [0,1], [1,0]$ satisfies

$$h^{\varpi}_{j_1,j_2} \;=\; C_2(\varpi)/k + n(j_1,j_2) \quad \text{with} \;\; n(j_1,j_2) \;\in\; \mathbb{N}$$

where we denoted the eigenvalue of the quadratic Casimir element in the representation ϖ by $C_2(\varpi)$.[3] The same formula with $j_1 = j_2$ applies to the projective covers $\mathcal{P}[j]$. Note that at the WZ point the spectrum has huge degeneracies because many different representations of psl(2|2) can appear on the same level of the state space. We shall see how the RR-deformation partially removes this degeneracy.

2.3 The RR-perturbation and its exact marginalilty

The most important actress of this work certainly is the perturbing field Φ that generates the deformation away from the WZ point. So, it is important to fully appreciate its structure and properties. The following discussion is mostly borrowed from our paper [40] which in turn was based upon [15, 13]. The deformation we are interested in is generated by the field

$$\Phi(z,\bar{z}) \;=: \mathrm{str}\left(S^{-1}\partial S S^{-1}\bar{\partial} S\right) :=\; -\frac{1}{k^2}\; : J^{\mu}(z)\,\phi_{\mu\nu}(z,\bar{z})\,\bar{J}^{\nu}(\bar{z}) : \tag{2.6}$$

The second formulation involves the left and right invariant (anti-)holomorphic currents $J^{\mu}(z)$ and $\bar{J}^{\nu}(\bar{z})$ along with some degenerate primary fields $\Phi_{\mu\nu}(z,\bar{z})$ that transform in the (atypical) adjoint representation $[1/2]$ of psl(2|2), i.e.

$$J^{\mu}(z)\,\phi_{\nu\rho}(w,\bar{w}) \;=\; \frac{i f^{\mu}{}_{\nu}{}^{\sigma}}{z-w}\;\phi_{\sigma\rho}(w,\bar{w}) + \ldots \quad , \tag{2.7}$$

$$\bar{J}^{\mu}(\bar{z})\,\phi_{\nu\rho}(w,\bar{w}) \;=\; \frac{i f^{\mu\sigma}{}_{\rho}}{\bar{z}-\bar{w}}\;\phi_{\nu\sigma}(w,\bar{w}) + \ldots \quad . \tag{2.8}$$

Let us stress that the vertex operators $\phi_{\mu\nu}$ possess zero conformal weight, as all vertex operators that are associated with the atypical sector of the theory. According to [15], the field Φ generates a truly marginal perturbation S^{Φ}_{λ} of the WZNW model. By construction, the field Φ has conformal weights $h = \bar{h} = 1$ but in principle its dimension could change when we perturb the theory, i.e. Φ could be marginally relevant. This is not the case. We

[3] The Casimir element is non-diagonalizable in $\mathcal{P}[0]$. Its generalized eigenfunctions possess vanishing eigenvalue.

shall establish true marginality of Φ through a new argument, simpler and of a somewhat different character than the one used in [15].

Our key observation is that all N-point functions of Φ vanish identically. Recall from [40] that the entire bulk spectrum of the PSL(2|2) WZNW model is organized in projective modules with respect to global PSL(2|2) (left or right) transformations. Hence, our perturbing field Φ is part of an indecomposable PSL(2|2) multiplet $\mathcal{P}[0]$. Since Φ is invariant, i.e.

$$X\Phi(z,\bar{z}) \;:=\; [J_0^X, \Phi(z,\bar{z})] \;=\; 0 \qquad \text{for all} \quad X \;\in\; \text{psl}(2|2)\;,$$

it is associated with the bottom (socle) of the projective cover $\mathcal{P}[0]$. Consequently, there must exist another bulk field $\Psi = \Psi(z,\bar{z})$ along with a (fermionic) symmetry generator Q such that

$$\Phi(z,\bar{z}) \;=\; Q\Psi(z,\bar{z}) \;=\; [J_0^Q, \Psi(z,\bar{z})] \;\;.$$

The rest of the argument is now rather standard. Let us consider the N-point function of Φ. By the previous comment we can represent one of the N fields as $\Phi = Q\Psi$ and obtain

$$\Big\langle \Phi(z_1,\bar{z}_1) \prod_{\nu=2}^{N} \Phi(z_\nu,\bar{z}_\nu) \Big\rangle \;=\; \Big\langle Q\Psi(z_1,\bar{z}_1) \prod_{\nu=2}^{N} \Phi(z_\nu,\bar{z}_\nu) \Big\rangle \tag{2.9}$$

$$=\; \sum_{\nu=2}^{N} \Big\langle \Psi(z_1,\bar{z}_1)\Phi(z_2,\bar{z}_2)\ldots Q\Phi(z_\nu,\bar{z}_\nu)\ldots\Phi(z_n,\bar{z}_N) \Big\rangle \;=0 \;\;. \tag{2.10}$$

We have used the psl(2|2) invariance of the expectation value to re-shuffle Q from Ψ to the other fields. The resulting $N-1$ terms in the second row all vanish because Q now acts on one of the invariant field insertions Φ. Since all N-point functions of Φ vanish, there is no need to ever renormalize the perturbing field. Hence, its scaling dimension remains unaltered.

3 Deformation of the boundary partition function

With the proper preparation from the previous section we now come to the central aim of this work: To compute the conformal weights of boundary fields on our point-like brane as we go beyond the WZ point. After a few remarks on the general structure of 2-point functions we shall discuss several remarkable features of the RR-deformation for conformal weights. These lead to drastic simplifications of the relevant perturbative

expansions. In fact, their combinatorics is no more complex than the combinatorics of radius deformations in torus compactifications! There remains a mixing problem, however, that we can only overcome for the boundary spectra of point-like localized branes. The relevant argument is presented in the third subsection. Finally, all the pieces are collected and the conformal weights of boundary fields are computed explicitly, following a closely related computation in [31].

3.1 The boundary 2-point function

A boundary partition function stores all information about the conformal weights of boundary fields. The latter are also encoded in the boundary 2-point functions which is the place from which we are going to read them off. In logarithmic conformal field theories, such as the WZNW model on PSL(2|2), the 2-point functions contains additional data that we are not interested in and, in fact, cannot compute perturbatively. Since the reader may not be so familiar with these issues, we shall briefly discuss the general structure of 2-point functions in RR-deformations of the WZNW model on PSL(2|2).

Let us recall that our boundary conditions were chosen such that they preserve a global psl(2|2) symmetry. This remains unbroken by the RR-deformation and hence all quantities in the deformed theory are organized in psl(2|2) multiplets. We shall label the boundary fields by $\Psi^\pi(x)$ with a superscript π that refers to the psl(2|2) representation the field transforms in. As we have reviewed above, boundary fields on our instantonic brane can only transform in projective modules π of psl(2|2). These can be either typical long multiplets or the projective covers of atypical short multiplets. In the following discussion we do not have to distinguish between these two possibilities. The form of the 2-point functions is strongly constrained by the usual Ward identities expressing conformal invariance and global psl(2|2) symmetry,

$$\langle\, \Psi^{\pi_1}(x_1)\, \Psi^{\pi_2}(x_2)\, \rangle_\lambda \;=\; \frac{1}{(x_1-x_2)^{\Delta_1(\lambda)+\Delta_2(\lambda)}}\; C^{12}(\lambda) \quad . \tag{3.1}$$

Here, the symbol $C^{12}(\lambda)$ denotes an intertwiner from the carrier space of the tensor product $\pi_1 \otimes \pi_2$ to the trivial representation. Let us note in passing that the space of such intertwiners may be multi-dimensional. The objects $\Delta = \Delta(\Psi^\pi)$ act on the carrier space of the representation π. They describe the action of L_0 on the field multiplets Ψ^π. Therefore, they clearly commute with the action of psl(2|2). We may split Δ into a term

that is proportional to the identity and a nil-potent contribution,

$$\Delta(\lambda) \;=\; h(\lambda)\cdot \mathbf{1}_\pi \,+\, \delta(\lambda)$$

where some finite power of δ vanishes. If the nilpotent part δ is non-zero for one of the fields Ψ^{π_1} or Ψ^{π_2} then the 2-point function contains logarithmic singularities. It is important to stress that all the quantities we have introduced, namely the constants h and the maps δ, C^{12} depend on the deformation parameter λ. For reasons that will soon become clear, we are not able to say anything useful about the λ-dependence of δ and C^{12}. On the other hand, we shall compute $h(\lambda)$ exactly, to all orders in perturbation theory. For a field $\Psi = \Psi^\pi$, the results is

$$h_\Psi(\lambda) \;=\; h_\Psi(0) - \frac{C_2^\pi}{k}\frac{\lambda}{k+\lambda} \;=\; h_\Psi(0) - C_2^\pi/k + C_2^\pi\, f^2 \quad . \tag{3.2}$$

Here, C_2^π is the (generalized) eigenvalue of the quadratic Casimir in the representation π, i.e. $C_2^\pi = j_2(j_2+1) - j_1(j_1+1)$ for $\pi = [j_1, j_2]$ and $C_2^\pi = 0$ whenever π is one of the projective covers $\mathcal{P}[j]$. Note that the shift of the conformal weight only depends on the transformation behaviour of $\Psi = \Psi^\pi$ under the action of psl(2|2). The simple result (3.2) is rather remarkable. Let us stress again that the numbers $h(\lambda)$ provide exactly the information that is encoded in the boundary partition function. In particular, the trace over state space is blind to any nilpotent terms $\delta(\lambda)$ so that our ignorance concerning their λ dependence does not really matter as long as we don't attempt to go beyond computing partition functions.

There is one more comment that might be worth adding. As we have seen in section 2.3 already, logarithmic conformal field theories contain many vanishing correlators. In particular, suppose that Ψ_1 and Ψ_2 are two fields that are associated with states in the socle of a projective cover. Then their 2-point function is bound to vanish by the same arguments we explained in section 2.3. A related observation was made by Bershadsky et al. in [15]. The authors of that work then went on to conclude that the conformal weights of fields in atypical representations could not be read off from their 2-point functions. We see now that this conclusion is incorrect. For each field in an atypical multiplet there exists some field such that the associated 2-point function is non-zero. If we pick Ψ_1 from the socle of a projective cover, for instance, then we can find an appropriate field Ψ_2 in the head of the dual projective cover.

3.2 Perturbative expansion for conformal weights

The perturbative computation of $h_i(\lambda)$ may seem like a daunting task at first, yet alone because of the very involved combinatorics of perturbation theory in curved backgrounds. In this subsection we shall list three observations that will allow us to drop most of the terms in the expansion for conformal weights. In fact, the terms that can safely be ignored are precisely the ones that arise from the curvature of PSL(2|2). Such simplifications, however, only apply to computations of psl(2|2) invariant quantities such as conformal weights etc. The reader is warned never to use the rules we are about to derive for computations of other structure constants.

All observations made in this subsection are based on a simple mathematical result that was first formulated and exploited in the work of Bershadsky et. al. [15]. Consider some psl(2|2) invariant A and suppose that A may be written as $A = C_{abc} f^{abc}$ where f^{abc} are the structure constants of psl(2|2) and C_{abc} are some numbers. Then A can be shown to vanish, i.e. $A = 0$. Since the supporting argument provided in [15] lacks a bit of mathematical precision, we have included a full proof and further discussion in appendix B of this paper. Bershadsky and collaborators applied the vanishing of A to a perturbative construction of the psl(2|2) invariant β-function. We shall exploit the same result in our computation of the numbers h_Ψ which are psl(2|2) invariants as well. A similar vanishing criterion is not satisfied for intertwiners Δ between two indecomposables or for maps C from the tensor product of indecomposables to the trivial representation (see also further comments in Appendix B). Therefore, we are not able to compute the full 2-point function of boundary fields, as mentioned before.

Let us now apply this mathematical lemma to our computation of conformal weights. The perturbative treatment we have in mind requires to evaluate correlators with insertions of the perturbing field Φ. Recall that Φ was composed from the vertex operators $\phi_{\mu\nu}$ and currents $J^\mu, \bar{J}^\nu$. An initial step is to remove all the current insertions through current algebra Ward identities. In the process, pairs of currents get contracted using

$$J^\mu(z)\, J^\nu(w) \;=\; \frac{i f^{\mu\nu}{}_\sigma}{z-w}\, J^\sigma(w) + \frac{k\kappa^{\mu\nu}}{(z-w)^2} + \ldots \;\sim\; \frac{k\kappa^{\mu\nu}}{(z-w)^2} \tag{3.3}$$

The first equality is the usual operator product for psl(2|2) currents. Since we are only interested in computing the invariants h_Ψ, we can drop all terms that involve the structure constants f of the Lie superalgebra psl(2|2). This applies to the first term in the above

operator product which distinguishes the non-abelian currents from the abelian algebra of flat target spaces. Here and in the following we shall use the symbol $\sim$ to mark equalities that are true up to terms involving structure constants. In conclusion, we have seen that, as far as the computation of conformal dimensions is concerned, we may neglect the non-abelian nature of the currents J^μ. Obviously, this leads to first drastic simplifications of the perturbative expansion.

Currents are not only contracted with other currents. They can also act on the vertex operators $\phi_{\mu\nu}$. The relevant operator product expansions have already been displayed in eq. (2.7) when we first introduced $\phi_{\mu\nu}$. With our new sensitivity for the appearance of structure constants we notice immediately that these operator products are proportional to f. Hence, we conclude

$$J^\mu(z)\ \phi_{\nu\rho}(w,\bar{w}) \ = \ \frac{if^\mu{}_\nu{}^\sigma}{z-w}\,\phi_{\sigma\rho}(w,\bar{w}) + \ldots \ \sim \ 0 \quad . \tag{3.4}$$

Consequently, we can simply ignore all terms in which a current acts on one of the vertex operators $\phi_{\mu\nu}$. In this respect, $\phi_{\mu\nu}$ does no longer behave like a vertex operator, but rather mimics the behavior of a constant background field.

Of course, $\phi_{\mu\nu}$ still is a non-trivial field and it therefore has possibly singular operator products with other fields in the theory. Such non-trivial operator products of the fields $\phi_{\mu\nu}$ could threaten a successful computation of conformal dimension. Here is where a third observation comes to our rescue. Note that shifts of the insertion point of the field $\phi_{\mu\nu}$ are controlled by the following operator version of the Knizhnik-Zamolodchikov equation

$$\partial_z\phi_{\mu\nu}(z,\bar{z}) \ = \ \frac{i}{k}\,f_{\sigma\mu}{}^\rho \ :J^\sigma(z)\phi_{\rho\nu}(z,\bar{z}): \sim \ 0 \ . \tag{3.5}$$

This means that in computations of invariants we can treat $\phi_{\mu\nu}$ as a *function* of conformal weight zero. Let us stress again that the operator products of $\phi_{\mu\nu}$ can certainly contain singularities. Relation (3.5) only asserts that all singular terms may be dropped in computations of conformal dimensions.

The rules (3.3) to (3.5) are the main results of this subsection. They will be employed at the end of this section when we compute boundary conformal weights. Related observations for the background field expansion of sigma models on PSL(N|N) were formulated in [15]. A successful computation of conformal weights requires one more important ingredient, though, that is novel to our analysis. This is what we are going to address next.

3.3 Perturbation of boundary conformal weights

Our arguments up to this point have made no use of the fact that we were setting off to compute conformal dimensions of boundary fields for a very particular boundary condition. In fact, everything we have stated applies to whatever conformal dimension we would like to compute, bulk or boundary. But there remains an issue that we cannot overcome in such a general context. According to the results of the previous subsection our vertex operators $\phi_{\mu\nu}$ behave like a matrix of functions rather than fields. This simplifies things immensely. But even multiplication with a set of functions can be a rather involved operation which we would have to diagonalize explicitly on field space before we could spell out conformal dimensions. In other words, there still exists a potentially complicated mixing problem to be solved. Here is where our special choice of boundary conditions comes in. As we shall see, it is chosen such that we can effectively replace $\phi_{\mu\nu}$ by a constant. Thereby, the mixing problem disappears.

While the reasoning to be detailed below is somewhat technical, the basic idea is rather simple: Before the bulk field $\phi_{\mu\nu}$ can act on boundary fields, it must be restricted to the world-volume of the brane. Since our brane is point-like localized at the group unit of the bosonic base, the restriction of $\phi_{\mu\nu}$ contains no further dependence on the bosonic coordinates and hence should have a rather simple action on boundary fields.

In order to make this geometric intuition more precise, let us look at the bulk-boundary operator product expansion of the vertex operator $\phi_{\mu\nu}(z,\bar{z})$. As the world-sheet coordinate approaches the point x on the boundary of the upper half-plane, we can re-expand the bulk field in terms of operators $\Psi(x)$ on the boundary. The leading terms of this expansion read

$$\phi_{\mu\nu}(z,\bar{z}) \;=\; \frac{1}{|z-\bar{z}|^{2/k}}\, C^{[1,0]}\, \Psi^{[1,0]}(x) + C^{\mathcal{P}[0]}\, \Psi^{\mathcal{P}[0]}(x) \;+\ldots\;. \tag{3.6}$$

On the boundary, the field with smallest conformal weight is the multiplet $\Psi^{[1,0]}$ that is associated with the ground states in the 48-dimensional typical representation $[1,0]$. In addition, there is one multiplet $\Psi^{\mathcal{P}[0]}$ of fields with vanishing conformal weight. All other fields possess positive scaling dimension and we have not displayed them in the expansion. The structure constants $C^{[1,0]}$ and $C^{\mathcal{P}[0]}$ are largely determined by psl(2|2) symmetry. Under the action of the unbroken global psl(2|2), the bulk multiplet $\phi_{\mu\nu}$ transforms in the 2-fold twisted[4] tensor product $[1/2]\otimes_\Omega[1/2]$ of the adjoint representation. Consequently,

[4]All tensor products in this subsection are constructed with the action $X \to X\otimes 1 + (-1)^{|X|}1\otimes X$

$C^{\mathcal{P}[0]}$ intertwines between $[1/2] \otimes_\Omega [1/2]$ and the projective cover $\mathcal{P}[0]$ etc.

Let us recall from the previous subsection that, in all computations of conformal dimensions, the bulk field $\phi_{\mu\nu}$ behaves like a set of functions on target space. Thereby, we are allowed to drop all terms from the bulk boundary operator product (3.6) which contain a non-trivial dependence on world-sheet coordinates, i.e.

$$\phi_{\mu\nu}(z,\bar{z}) \;\sim\; C^{\mathcal{P}[0]}\, \Psi^{\mathcal{P}[0]}(x) \quad . \tag{3.7}$$

Here, $\sim$ has the same meaning as before, warning us that the relation (3.7) should only be used in computations of conformal weights.

Further progress now requires to turn attention to the intertwiner $C^{\mathcal{P}[0]}$ from the twisted tensor product $[1/2] \otimes_\Omega [1/2]$ to the projective cover $\mathcal{P}[0]$. The precise structure of $[1/2] \otimes [1/2] \cong [1/2] \otimes_\Omega [1/2]$ has been determined in [49]. There, the tensor product was shown to decompose into four indecomposable representations. These include the typical multiplets $[1,0]$ and $[0,1]$ along with the trivial representations $[0]$ and one atypical indecomposable whose socle consists of a single adjoint $[1/2]$. The result implies that the space of intertwiners from $[1/2] \otimes [1/2]$ to the projective cover $\mathcal{P}[0]$ is 1-dimensional. In fact, the only non-trivial intertwiner $C^{\mathcal{P}[0]}$ maps the invariant $[0]$ in $[1/2] \otimes_\Omega [1/2]$ to the socle of $\mathcal{P}[0]$. Transferred back to our bulk boundary operator product (3.7) we conclude that only the socle of the boundary multiplet $\Psi^{[0]}$ can arise. Since the corresponding boundary operator is the identity field, we conclude

$$\phi_{\mu\nu} \;\sim\; c_0\, (-1)^{|\mu|} \kappa_{\mu\nu}\, \mathbf{1} \quad .$$

Here, we have used that every intertwiner from $[1/2] \otimes_\Omega [1/2]$ to the trivial representation $[0]$ is related to the metric by $(-1)^{|\mu|}\kappa_{\mu\nu}$ with $|\mu| = |X_\mu|$ as before. Since the field $\phi^\mu{}_\nu$ is a quantum analogue of the representation matrix $R_{\mathrm{ad}}(g)^\mu{}_\nu$ and since we are evaluating the latter at the unit element, $g = e$, we obviously have $c_0 = 1$. Consequently, in all computations of boundary conformal weights we are allowed to set $\phi \sim (-1)^{|\mu|}\kappa_{\mu\nu}$. Let us stress that our arguments rely heavily on the fact that we analyze the boundary fields on point-like branes. In particular, we used that there was no boundary field that transforms in the atypical $[1/2]$ representation.

where the second term is twisted by the gluing automorphism Ω

3.4 Computation of boundary conformal weights

Let us now finally harvest the results of our careful analysis in the previous two subsections. As we have shown in the second subsection, the perturbation series for conformal dimensions is identical to the one that appears in an abelian theory with constant background fields. Put differently, the currents J^μ and $\bar{J}^\nu$ behave like $J^\mu \approx -i\sqrt{k}\partial X^\mu$ and $\bar{J}^\nu \approx i\sqrt{k}\bar{\partial} X^\nu$ in a theory of 14 free fields X^μ. Moreover, the matrix $\phi_{\mu\nu}$ can be treated as if it was a constant, similar to the parameter γ we introduced in our brief discussion of circle compactifications around eq. (1.3). Including our choices of normalization, the precise relation is read off from

$$-\frac{\lambda}{2\pi}\Phi(z,\bar{z}) \;=\; \frac{\lambda}{2\pi k^2}\,:J^\mu(z)\,\phi_{\mu\nu}(z,\bar{z})\,\bar{J}^\nu(\bar{z}):\sim\; \frac{\lambda}{2\pi k}\,\phi_{\mu\nu}(x)\partial X^\mu\bar{\partial}X^\nu \quad .$$

Here, we have used a lower case x in the argument of ϕ in order to stress that it behaves like a function on target space. On the other hand, there is no dependence on the fields X^μ. For our special choice of Ω, the gluing condition (2.2) mimics Dirichlet boundary conditions for the bosons and Neumann boundary conditions for the fermions in free field theory,

$$\partial X^\mu(z,\bar{z}) \;=\; -(-1)^{|\mu|}\bar{\partial}X^\mu(z,\bar{z}) \quad \text{for} \quad z \;=\; \bar{z} \quad .$$

Putting things together, our setup is essentially identical to the starting point of the perturbative analysis in [31], Hence, we can carry over all results from that paper and conclude that the change of boundary conformal dimensions can be determined from an effective perturbing bulk field of the form

$$S_\lambda \;\longrightarrow \frac{\lambda}{2\pi k}\int_{\mathcal{H}} dzd\bar{z}\;\left(\frac{1}{k+(-1)^F\lambda\phi}\right)_\mu^{\ \rho}\phi_{\rho\nu}J^\mu(z)\bar{J}^\nu(\bar{z}) \tag{3.8}$$

where $\mathcal{H}$ is the upper half-plane and we are no longer allowed to contract currents among each other or with the matrix valued fields $\phi = (\phi_{\mu\nu})$. The matrix $(-1)^F$ is defined by $(-1)^F_{\mu\nu} = (-1)^{|\mu|}\kappa_{\mu\nu}$. To leading order, the effective perturbation (3.8) agrees with the original perturbing term. Higher order contributions are encoded in a factor $k/(k+\lambda\phi(-1)^F)$ that resembles the familiar $1/(1-\gamma)$ in the circle compactification (see discussion after eq. 1.3). The signs in the denominator take care of the gluing condition we imposed. There are a few remarks we would like to add. To begin with, note that there is no need for any normal ordering in the previous formula, just as in free field theory with constant background fields. Our effective perturbation (3.8) has rather limit validity,

though. While in [31] the effective perturbation was used to compute both the change of conformal weights and of 3-point couplings, our entire derivation here was restricted to conformal weights! So, the formula (3.8) for the effective interaction should never be used in computations of structure constants. Let us finally point out that for the time being we only assumed that the left and right moving currents satisfy the gluing condition (2.2). Therefore, our result holds for all branes of this gluing type, including those cases in which the brane extends along some of the bosonic directions.

In the final step we specialize now to the instantonic brane that is located at the unit element e of the bosonic base. Using our results from the previous subsection we may then replace the functions $\phi_{\mu\nu}$ by constants, i.e. we insert $\phi = (-1)^F \mathbf{1}$ into the formula (3.8),

$$S_\lambda \;\longrightarrow\; \frac{\lambda}{2\pi k}\int_{\mathcal{H}} dz d\bar{z}\;\left(\frac{1}{k+\lambda}\right) J^\mu(z)(-1)^{|\mu|}\bar{J}^\mu(\bar{z}) \quad . \tag{3.9}$$

The change of the boundary conformal weights is determined by the logarithmic divergence in the regulated 2-point function which in turn arises from the simple poles of the operator products between the effective perturbing field and the boundary fields Ψ^π. With the usual normalizations, the resulting shift $\delta_\lambda h$ of conformal weights becomes

$$\delta_\lambda h(\Psi^\pi) \;=\; -2\pi\left(\frac{\lambda}{2\pi k}\frac{1}{k+\lambda}\;\pi(J^\mu J^\mu)\right) \;=\; -\frac{\lambda}{k(k+\lambda)}\;C_2^\pi \quad .$$

Note that the factor $(-1)^{|\mu|}$ in the effective perturbation is absorbed when we relate the anti-holomorphic current $\bar{J}^\mu$ with the boundary value of the holomorphic current J^μ. As a result, we have established the anticipated formula (3.2).

4 Conclusions and outlook

In this note we computed the full spectrum on a point-like brane in sigma models with target space PSL(2|2). The result was obtained by summing explicitly the perturbation series that is generated by the RR-deformation $\mathcal{S}_\lambda$. A non-vanishing topological WZ term was required in our analysis to guarantee that we could construct the spectrum directly at one point of the moduli space. We believe that this is merely a technical condition that can be overcome, at least in many examples (see next paragraph). A very decisive element was to focus on invariants of a Lie superalgebra to which the vanishing lemma (see appendix B) applies. This leaves ample room for generalization to other supergroup and coset spaces

with psl(N|N) or osp(2N+2|2N) symmetry. As explained in section 3.2, the vanishing lemma renders the perturbation series for conformal dimensions quasi-abelian. On the other hand, the effective perturbing operator (3.8) requires additional diagonalization whenever $\phi_{\mu\nu}$ is non-trivial. Here, we circumvented the issue with our special choice of instantonic boundary conditions which allowed us to replace $\phi_{\mu\nu}$ by a constant. Finally, to have sufficient control over the boundary partition function, a Casimir decomposition of the spectrum had to be performed. Such a decomposition is not always possible - it needs the brane to stretch out in all fermionic directions. Since branes in generic positions are fully delocalized along fermionic coordinates, no serious limitations should arise for generalizations to other backgrounds. In the following few paragraphs we shall go through all our assumptions in more detail, with an emphasis on general structures rather than the specific model we dealt with above.

To get our perturbative expansion started, we need the exact form of the boundary partition function at one point of the moduli space. In many cases, such an *initial condition* may come from a WZNW model. The solution of WZNW models on type I supergroups has been addressed in [50], based on similar studies of several concrete examples [51, 40, 52]. It may be interesting to stress that a point with non-abelian current algebra symmetry may exist in the moduli space even if no topological term appears in the action of the model under consideration. The simplest example is once more provided by circle compactification whose world-sheet symmetry gets enhanced to an su(2) current algebra at the self-dual radius. Similar phenomena are very likely to occur for many other principal chiral models on supergroups or cosets. For example, according to an intriguing conjecture of Candu and Saleur [41], there exists a particular choice of the coupling at which the principal chiral model on the supersphere $S^{3|2}$ coincides with a OSP(4|2) WZNW model at level $k = -1/2$. In general, such special points in moduli space and their exact properties are difficult to detect. But even if no points with current algebra symmetries are known to exist, exact spectra may still be accessible with different techniques, such as the use of lattice constructions etc. (see e.g. [18, 29, 41]).

Once the WZ point (or any other explicitly solvable point) is under control, we would like to deform the model. In most cases, summing up an entire perturbation series is a hopeless enterprise. Still, we have seen that explicit summation is possible for the RR-deformation of the PSL(2|2) sigma model, at least once we focus on appropriate quantities such as conformal weights of boundary fields. Drastic simplifications in the combinatorics

of the perturbative expansion resulted from three observations, (3.3) to (3.5), in section 3.2. None of them is specific to a target space with psl(2|2) symmetry. In fact, the underlying technical lemma (reviewed in appendix B) is closely related to the vanishing dual Coxeter number of psl(2|2), a property psl(2|2) shares with three families of Lie superalgebras, namely psl(N|N), osp(2N+2|N) and $D(2,1;\alpha)$. These describe the global symmetries of many interesting superspaces, ranging from odd dimensional superspheres $S^{2N+1|2N}$ to the coset spaces that are involved in the AdS/CFT correspondence. We wish to stress that a vanishing β function of the deformation and the quasi-abelianness of the perturbative expansion for conformal dimensions appear as two sides of the same coin. Indeed, they can both be traced back to the vanishing lemma.

Let us also point out once more that, even though the perturbation series simplifies for all spectra, we were only able to exploit this fact in the case of point-like branes. It seems to us that the absence of bosonic zero modes might be an important feature for the success of the computation, but whether it is decisive remains an interesting open problem. In particular, our brief discussion of bulk spectra in AdS_3 (see introduction) suggests that the remaining diagonalization for closed string modes could be more than a mere technical issue. In case the direct perturbative computation of bulk spectra turns out to be impossible, one might still be able to find bulk conformal dimensions indirectly through modular transformation of boundary partition functions. Approaching the bulk spectrum through open closed string duality would certainly require explicit formulas for the branching functions $a(q), b(q)$, going somewhat beyond their mere algorithmic construction [40]. Another potential hurdle to overcome are the modular properties of the branching functions $a(q), b(q)$ which might be difficult to control. Even if this is not possible in general, the branching functions might well combine into simpler objects for specific values of the deformation parameter λ. At points with an enhanced world-sheet symmetry one would expect an infinite number of branching functions to align such that they build the characters of a larger chiral algebra. The latter could well possess simpler modular properties. A systematic detection of points with enhanced symmetry along the line of deformations and the reconstruction of the bulk spectrum is a promising path for future research.

Two further comments concern the degeneracies we found in our D-instanton spectra. According to the results in [15], the chiral symmetry of sigma models on PSL(2|2) is generated by the psl(2|2) Casimir fields, and hence is much smaller than the full Casimir

algebra, see [40] for more explanation. Here, we found that the degeneracies of the boundary spectrum are determined by the Casimir decomposition. Hence, they are larger than one would have expected based on the chiral symmetry alone. This is a remarkable result which points towards the existence of some enhanced (possibly non-local) symmetry, at least for the boundary spectra we were concerned with in our work. It would certainly be very rewarding to uncover this symmetry. A second enhancement of degeneracies is found in the atypical sector of the model. In fact, the conformal weight of fields transforming in an atypical representation of psl(2|2) do not receive any corrections. Therefore such fields are guaranteed to possess an integer conformal weight. Similar phenomena have been encountered in recent work of Read and Saleur [53]. Following their analysis we believe that the large degeneracy in the atypical sector may be explained by the combined action of two commuting symmetries. One of them is the Lie superalgebra psl(2|2) of global transformations. The second should be closely related to the algebra of Casimir fields or some extension thereof.

Results on non-linear sigma models with super target spaces are currently not directly applicable to strings in AdS geometries other than via the hybrid approach for AdS_3. Nevertheless we believe that two rather general lessons can be inferred from our studies. First of all, conformal field theory techniques, and in particular conformal perturbation theory, can be rather powerful even in cases when the chiral symmetry is not sufficient to carry out a full-fledged algebraic construction of the model. Furthermore, models with a psl(2|2) symmetry can be much better behaved than one would expect after looking at any of their subsectors. In fact, supposedly simpler subsectors, such as e.g. those based on the bosonic sl(2), can lead to technical problems that are much more difficult and never encountered in the full psl(2|2) model. In this sense, subsector theories may turn out to be inappropriate as toy models for the kind of theories we are ultimately interested in.

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A The superalgebra psl(2|2) and its representations

The Lie superalgebra psl(2|2) possesses six bosonic generators $K^{ab} = -K^{ba}$ with $a, b = 1, \ldots, 4$. They form the Lie algebra $so(4)$ which is isomorphic to $\mathrm{sl}(2) \oplus \mathrm{sl}(2)$. In addition, there are eight fermionic generators that we denote by S^a_α. They split into two sets ($\alpha = 1, 2$) each of which transform in the vector representation of $so(4)$ ($a = 1, \ldots, 4$) which is the $(1/2, 1/2)$ of $\mathrm{sl}(2) \oplus \mathrm{sl}(2)$. The relations of psl(2|2) are then given by

$$\begin{aligned}
[K^{ab}, K^{cd}] &= i\left[\delta^{ac}K^{bd} - \delta^{bc}K^{ad} - \delta^{ad}K^{bc} + \delta^{bd}K^{ac}\right] \\
[K^{ab}, S^c_\gamma] &= i\left[\delta^{ac}S^b_\gamma - \delta^{bc}S^a_\gamma\right] \\
[S^a_\alpha, S^b_\beta] &= i\,\epsilon_{\alpha\beta}\,\epsilon^{abcd}K^{cd} \quad .
\end{aligned} \tag{A.1}$$

Here, $\epsilon_{\alpha\beta}$ and ϵ^{abcd} denote the usual completely antisymmetric ϵ-symbols with $\epsilon_{12} = 1$ and $\epsilon^{1234} = 1$, respectively. An invariant metric is given by

$$\langle K^{ab}, K^{cd}\rangle = -\epsilon^{abcd} \qquad \langle S^a_\alpha, S^b_\beta\rangle = -2\epsilon_{\alpha\beta}\,\delta^{ab} \quad . \tag{A.2}$$

It is unique up to a scalar factor. The signs have been chosen in view of the real form psu(1, 1|2) which is considered in the main text. In order to define a root space decomposition of psl(2|2) we split the fermions into two sets of four generators

$$\mathfrak{g}^{(1)}_+ = \mathrm{span}\{S^a_1\} \quad , \quad \mathfrak{g}^{(1)}_- = \mathrm{span}\{S^a_2\} \quad .$$

As indicated by the subscripts $\pm$, we shall think of the fermionic generators S^a_1 as annihilation operators and of S^a_2 as creation operators.

Finite dimensional projective representations of psl(2|2) fall into two classes. The first one consists of all the long multiplets. These are labelled by two spins j_1, j_2 with $j_1 \neq j_2$ and their supercharacters read

$$\chi_{[j_1,j_2]}(z_1, z_2) = \mathrm{tr}\Big[(-1)^F z_1^{K^0_1}\, z_2^{K^0_2}\Big] = \chi_{j_1}(z_1)\,\chi_{j_2}(z_2)\,\chi_F(z_1, z_2) \quad . \tag{A.3}$$

where $\chi_j(z) = \sum_{l=-j}^{j} z^l$ are the standard characters for finite dimensional representations of the Lie algebra sl(2) and the fermionic factor χ_F is given by

$$\chi_F(z_1, z_2) = 4 + z_1^1 + z_1^{-1} + z_2 + z_2^{-1} - 2(z_1^{\frac{1}{2}} + z_1^{-\frac{1}{2}})(z_2^{\frac{1}{2}} + z_2^{-\frac{1}{2}}) \quad . \tag{A.4}$$

Let us also note in passing that the value $C_2([j_1, j_2])$ of the quadratic Casimir in typical representations may be expressed as

$$C_2\big([j_1, j_2]\big) = j_2(j_2 + 1) - j_1(j_1 + 1) \quad .$$

There exists a second class of projective representations $\mathcal{P}[j]$ whose members are labelled by a single spin j. They are built up from short multiplets such that their supercharacter becomes

$$\chi_{\mathcal{P}[j]} \;=\; \Big[2\chi_j(z_1)\chi_j(z_2) - \chi_{j+\frac{1}{2}}(z_1)\chi_{j+\frac{1}{2}}(z_2) - \chi_{|j-\frac{1}{2}|}(z_1)\chi_{|j-\frac{1}{2}|}(z_2)\Big]\,\chi_F(z_1,z_2)\;. \qquad \text{(A.5)}$$

The quadratic Casimir is non-diagonalizable in the projective covers, with Jordan cells up to rank five. Generalized eigenvalues of C_2 in $\mathcal{P}[j]$ are well known to vanish for all spins j. In this sense we shall write $C_2\big(\mathcal{P}[j]\big) = 0$.

The characters (A.3) and (A.5) are important ingredients in the Racah-Speiser algorithm that furnishes the Casimir decomposition for the partition function of a point-like brane, see [40] for details.

B Derivation of the main vanishing lemma

Our evaluation of the perturbative expansion for conformal weights is based on the fact that a psl(2|2)-invariant A vanishes whenever it is of the form $A = C_{abc}f^{abc}$. In order to make our presentation self-contained the vanishing lemma is derived below. We use this opportunity to clarify a few unsatisfactory issues in the original argument [15].

For the following discussion it is useful to consider A, C and f as intertwiners rather than a bunch of numbers. By definition, an invariant A is an intertwiner from the trivial representation to itself. Similarly, the structure constants f^{abc} may be considered as an intertwiner from the 3-fold tensor product of the adjoint $[1/2]$ to the trivial representation. The possible form of $[1/2]^{\otimes 3}$ can be severely constrained using results from [49]. The 2-fold tensor product $[1/2]\otimes[1/2]$ contains three irreducible representations $\mathcal{I} = [0]\oplus[1,0]\oplus[0,1]$ as well as a more complicated indecomposable $\pi^{\text{indec}}_{1/2,1/2}$. The tensor product of $\mathcal{I}$ with $[1/2]$ can easily be evaluated. Furthermore, the typical contributions to $\pi^{\text{indec}}_{1/2,1/2}\otimes[1/2]$ do not present any obstacle. This results in the decomposition

$$\begin{aligned}[1/2]^{\otimes 3} \;=\; & [1/2]\oplus 2\mathcal{P}_{1/2}\oplus 3\big([1,0]\oplus[0,1]\big)\oplus 4\big([3/2,1/2]\oplus[1/2,3/2]\big)\\ & \oplus\big([2,0]\oplus[0,2]\big)\oplus\cdots\end{aligned} \qquad \text{(B.1)}$$

The remaining terms "$\cdots$" are the atypical parts in the tensor product $\pi^{\text{indec}}_{1/2,1/2}\otimes[1/2]$. They are built by combining the following constituents

$$\big\{2[0]_1, 2[0]_3, 5[1/2]_1, 2[1/2]_3, 4[1]_2, [3/2]_1, [3/2]_3\big\} \qquad \text{(B.2)}$$

into a bunch of indecomposable representations.[5] The precise form of these indecomposables is currently not known to us. Nevertheless one can derive analytically that their socles can only contain the representations $[0]_1$ and $2[1/2]_1$. Due to the self-duality of $[1/2]^{\otimes 3}$, the same statement holds for the heads. One can also check that there is no true invariant in $[1/2]^{\otimes 3}$, i.e. that the head and the socle are formed by two different $[0]$'s. The argument rests on an explicit construction of the unique invariant state and the subsequent proof that it, in fact, lies in the image of the quadratic Casimir operator. Hence the unique invariant state has to be the socle of a larger indecomposable multiplet.

Given any representation of psl(2|2), the number of independent interwiners to the trivial representation may be obtained by counting the number of times $[0]$ appears as the head of an indecomposable sub-representation. In the case of $[1/2]^{\otimes 3}$, there is only one such occurrence of $[0]$, as we have just argued. Hence, the intertwiner to the trivial representation is unique up to normalization. This map is what we denote by f. Bershadsky et al. now continued to argue that the constants C_{abc} that are contracted with f^{abc} to form the invariant A must be proportional to f_{abc} (indices lowered with the metric) because of the uniqueness of f. A then vanishes because of the numerical identity $f_{abc}f^{abc} = 0$. We arrive at the same conclusion if we employ that f and C combine into an invariant A provided that C is a co-invariant, i.e. an intertwiner from the trivial representation to the 3-fold tensor product of the adjoint. Such co-invariants are in one to one correspondence with representations $[0]$ in the socle of $[1/2]^{\otimes 3}$. A glance back onto our argument above shows that there is a single such representation and hence C is unique. The reason for the vanishing of any invariant $A = C \circ f$ is that the image $\mathrm{Im}\, C$ of C, given by the socle of $[1/2]^{\otimes 3}$, is in the kernel of f, i.e. $\mathrm{Im}\, C$ has no component in the head of $[1/2]^{\otimes 3}$. The outcome of this analysis, namely the vanishing of an invariant $A = C \circ f$, is the crucial ingredient in our observations (3.3) to (3.5).

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