

# $W$ Boson Production at Large Transverse Momentum<sup>1</sup>

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## Abstract

The production of  $W$  bosons at large transverse momentum in  $p\bar{p}$  collisions is studied. The next-to-leading order cross section in this region is dominated by threshold soft-gluon corrections. The transverse momentum distribution of the  $W$  at the Tevatron is modestly enhanced when next-to-next-to-leading-order soft-gluon corrections are added, and the dependence on the factorization and renormalization scales is significantly reduced.

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# 1 Introduction

$W$  boson hadroproduction is a process of importance in testing the Standard Model and in estimates of backgrounds to new physics, such as associated Higgs boson production at the Tevatron. In Refs. [1, 2] the complete next-to-leading-order (NLO) cross section for  $W$  hadroproduction at large transverse momentum was first calculated. The NLO corrections contribute an enhancement of the differential distributions in transverse momentum  $Q_T$  of the  $W$  boson and considerably stabilize the dependence of the cross section on the factorization and renormalization scales.

$W$  boson production at the Tevatron receives important contributions from the near-threshold kinematical region. The calculation of hard-scattering cross sections near partonic threshold involves corrections from the emission of soft gluons from the partons in the process. At each order in perturbation theory there are large logarithms arising from incomplete cancellations near partonic threshold between graphs with real emission and virtual graphs due to the limited phase space available for real gluon emission. These threshold corrections exponentiate as a result of the factorization properties [3, 4, 5, 6] of the cross section and have been successfully resummed for a large number of processes [7].

In Ref. [8] the resummation of threshold logarithms for  $W$  boson hadroproduction at large transverse momentum was first studied, and the expansion of the resummed cross section at next-to-next-to-leading order (NNLO) and next-to-next-to-leading logarithmic (NNLL) accuracy was presented. Following Ref. [9], the accuracy of the theoretical prediction was increased in Ref. [10] to include next-to-next-to-next-to-leading logarithms (NNNLL) and phenomenological studies for  $W$  production at the Tevatron were also presented. We note that the theoretical and numerical results are similar to those found for direct photon production [11].

## 2 Theoretical results

For the process  $h_A(P_A) + h_B(P_B) \longrightarrow W(Q) + X$ , we can write:

$$E_Q \frac{d\sigma_{h_A h_B \rightarrow W(Q)+X}}{d^3Q} = \sum_f \int dx_1 dx_2 \phi_{f_a/h_A}(x_1, \mu_F^2) \phi_{f_b/h_B}(x_2, \mu_F^2) \\ \times E_Q \frac{d\hat{\sigma}_{f_a f_b \rightarrow W(Q)+X}}{d^3Q}(s, t, u, Q, \mu_F, \alpha_s(\mu_R^2)),$$

where  $E_Q = Q^0$ ,  $\phi_{f/h}$  are the parton distributions,  $\hat{\sigma}$  is the parton-level cross section,  $\mu_F$  is the factorization scale,  $\mu_R$  is the renormalization scale, and  $s, t, u$  are parton-level kinematical invariants. The lowest-order parton level subprocesses are  $q(p_a) + g(p_b) \longrightarrow W(Q) + q(p_c)$  and  $q(p_a) + \bar{q}(p_b) \longrightarrow W(Q) + g(p_c)$ . The variable  $s_2 = (p_a + p_b - Q)^2$  is defined as the invariant mass of the system recoiling against the  $W$  at the parton level. It parametrizes the inelasticity of the parton scattering, being zero for one-parton production.

For the  $n$ -th order corrections in the strong coupling  $\alpha_s$ , the partonic cross section  $\hat{\sigma}$  includes distributions with respect to  $s_2$  of the type  $[\ln^m(s_2/Q_T^2)/s_2]_+$ , with  $m \leq 2n - 1$ . Leading logarithms (LL) are those with  $m = 2n - 1$ , next-to-leading logarithms (NLL) with  $m = 2n - 2$ ,

next-to-next-to-leading logarithms (NNLL) with  $m = 2n - 3$ , and next-to-next-to-next-to-leading logarithms (NNNLL) with  $m = 2n - 4$ .

The NLO soft and virtual, and the NNLO soft-gluon corrections are presented in the  $\overline{\text{MS}}$  scheme as given in Ref. [10]. Here we show the formulae for the  $qg \rightarrow Wq$  subprocess. The  $q\bar{q} \rightarrow Wg$  subprocess is calculated in a similar way [10].

The NLO soft and virtual corrections for  $qg \rightarrow Wq$  are

$$E_Q \frac{d\hat{\sigma}_{qg \rightarrow Wq}^{(1)}}{d^3Q} = F_{qg \rightarrow Wq}^B \frac{\alpha_s(\mu_R^2)}{\pi} \left\{ c_3^{qg} \left[ \frac{\ln(s_2/Q_T^2)}{s_2} \right]_+ + c_2^{qg} \left[ \frac{1}{s_2} \right]_+ + c_1^{qg} \delta(s_2) \right\},$$

where  $F_{qg \rightarrow Wq}^B$  is the Born term. The LL  $[\ln(s_2/Q_T^2)/s_2]_+$  term and the NLL  $[1/s_2]_+$  term are the soft gluon corrections. The  $\delta(s_2)$  term gives the virtual corrections. In the discussion below “NLO-NLL” denotes when, at NLO, all the LL and NLL soft-gluon terms, and also the scale-dependent terms in  $\delta(s_2)$ , are included. The NLO coefficients,  $c_1^{qg}$ ,  $c_2^{qg}$  and  $c_3^{qg}$ , can be found in Ref. [10].

Using the conventions in Ref. [10], the NNLO soft and virtual corrections are

$$E_Q \frac{d\hat{\sigma}_{qg \rightarrow Wq}^{(2)}}{d^3Q} = F_{qg \rightarrow Wq}^B \frac{\alpha_s^2(\mu_R^2)}{\pi^2} \hat{\sigma}_{qg \rightarrow Wq}^{(2)}$$

with

$$\begin{aligned} \hat{\sigma}_{qg \rightarrow Wq}^{(2)} &= \frac{1}{2} (c_3^{qg})^2 \left[ \frac{\ln^3(s_2/Q_T^2)}{s_2} \right]_+ + \left[ \frac{3}{2} c_3^{qg} c_2^{qg} - \frac{\beta_0}{4} c_3^{qg} + C_F \frac{\beta_0}{8} \right] \left[ \frac{\ln^2(s_2/Q_T^2)}{s_2} \right]_+ \\ &+ \left\{ c_3^{qg} c_1^{qg} + (c_2^{qg})^2 - \zeta_2 (c_3^{qg})^2 - \frac{\beta_0}{2} T_2^{qg} + \frac{\beta_0}{4} c_3^{qg} \ln \left( \frac{\mu_R^2}{s} \right) + (C_F + C_A) K \right. \\ &\quad \left. + C_F \left[ -\frac{K}{2} + \frac{\beta_0}{4} \ln \left( \frac{Q_T^2}{s} \right) \right] - \frac{3}{16} \beta_0 C_F \right\} \left[ \frac{\ln(s_2/Q_T^2)}{s_2} \right]_+ \\ &+ \left\{ c_2^{qg} c_1^{qg} - \zeta_2 c_2^{qg} c_3^{qg} + \zeta_3 (c_3^{qg})^2 - \frac{\beta_0}{2} T_1^{qg} + \frac{\beta_0}{4} c_2^{qg} \ln \left( \frac{\mu_R^2}{s} \right) + \mathcal{G}_{qg}^{(2)} \right. \\ &+ (C_F + C_A) \left[ \frac{\beta_0}{8} \ln^2 \left( \frac{\mu_F^2}{s} \right) - \frac{K}{2} \ln \left( \frac{\mu_F^2}{s} \right) \right] - C_F K \ln \left( \frac{-u}{Q_T^2} \right) - C_A K \ln \left( \frac{-t}{Q_T^2} \right) \\ &\quad \left. + C_F \left[ \frac{\beta_0}{8} \ln^2 \left( \frac{Q_T^2}{s} \right) - \frac{K}{2} \ln \left( \frac{Q_T^2}{s} \right) \right] - \frac{3}{16} \beta_0 C_F \ln \left( \frac{Q_T^2}{s} \right) \right\} \left[ \frac{1}{s_2} \right]_+ \\ &+ \left\{ R_{qg}^{\mu, (2)} + R_{qg}^{(2)} \right\} \delta(s_2), \end{aligned}$$

where the definitions of the functions entering this expression can be found in Ref. [10]. The soft corrections are the LL  $[\ln^3(s_2/Q_T^2)/s_2]_+$  term, the NLL  $[\ln^2(s_2/Q_T^2)/s_2]_+$  term, the NNLL  $[\ln(s_2/Q_T^2)/s_2]_+$  term, and the NNNLL  $[1/s_2]_+$  term. In  $\mathcal{G}_{qg}^{(2)}$ , in the NNNLL term, two-loop process-dependent contributions are not included, but, from related studies (direct photon production [11], top hadroproduction [12]) they are expected to give a small contribution. “NNLO-NLL” will indicate inclusion of the LL, NLL, and NNLL terms at NNLO. “NNLO-NNNLL” includes the NNNLL terms as well. The term  $R_{qg}^{\mu, (2)} \delta(s_2)$  denotes the scale-dependent virtual corrections which are given explicitly in Ref. [10].  $R_{qg}^{(2)} \delta(s_2)$  are the scale-independent virtual corrections which are currently unknown.

### 3 Numerical results

The MRST2002 approximate NNLO parton densities [13] are used in the evaluation of the numerical results. At the left hand side of Fig. 1 we plot the transverse momentum distribution,  $d\sigma/dQ_T^2$ , for  $W$  hadroproduction at the Tevatron Run I with  $\sqrt{S} = 1.8$  TeV in the high- $Q_T$  region setting  $\mu = Q_T$ , where  $\mu \equiv \mu_F = \mu_R$ . We plot Born, exact NLO [1, 2], NNLO–NNLL, and NNLO–NNLL results. The NLO corrections provide a significant enhancement of the Born cross section. The NNLO–NNLL corrections provide a further modest enhancement of the  $Q_T$  distribution. The more accurate NNLL contributions are negative forcing the NNLO–NNLL cross section to lie between the NLO and NNLO–NNLL results.

At the right hand side of Fig. 1 the scale dependence of the differential cross section for  $Q_T = 80$  GeV is shown. We plot the differential cross section versus  $\mu/Q_T$  over two orders of magnitude:  $0.1 < \mu/Q_T < 10$ . We note the good stabilization of the cross section when the NLO corrections are included, and the further improvement when the NNLO–NNLL corrections (which include all the soft and virtual NNLO scale terms) are added. The NNLO–NNLL result approaches the scale independence expected of a truly physical cross section.

In Fig. 2 we show results at the Tevatron Run II with  $\sqrt{S} = 1.96$  TeV. On the left we plot the Born, NLO, and NNLO–NNLL cross sections for two choices of scale,  $\mu = Q_T/2$  and  $2Q_T$ . We see that while the Born result varies a lot between the two scales, the NLO result is more stable, and the NNLO–NNLL is so stable that the two curves are indistinguishable. Note that all  $\mu = Q_T/2$  curves lie on top of each other. This is all as expected from the right plot in Fig. 1. On the right-hand side of Fig. 2 are shown the  $K$ -factors, i.e. the ratios of cross sections at various orders and accuracies to the Born cross section, all with  $\mu = Q_T$ . The NLO/NLO–NLL curve also shows that the NLO–NLL result is a very good approximation to the full NLO result, i.e. the soft–gluon corrections dominate the NLO cross section. The difference between NLO and NLO–NLL is only 2% for  $Q_T > 90$  GeV and less than 10% for lower  $Q_T$  down to 30 GeV. The fact that the soft–gluon corrections dominate the NLO cross section is a major justification for studying the NNLO soft gluon corrections to this process.

### 4 Conclusions

The NNLO soft–gluon corrections for  $W$  hadroproduction at large transverse momentum in  $p\bar{p}$  collisions at the Tevatron Run I and II have been presented. The NLO soft–gluon corrections dominate the NLO differential cross section in this region, while the NNLO soft–gluon corrections provide modest enhancements and further decrease the factorization and renormalization scale dependence of the transverse momentum distributions.

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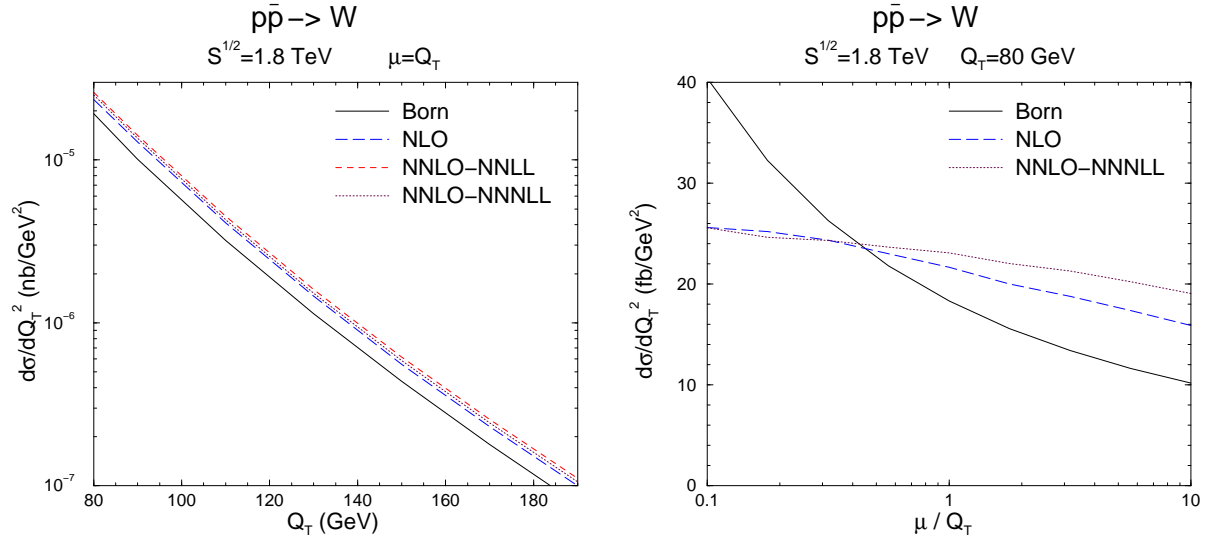


Figure 1:  $d\sigma/dQ_T^2$  for  $W$  production at the Tevatron with  $\sqrt{S} = 1.8$  TeV as a function of (left)  $Q_T$  with  $\mu = Q_T$ , and (right)  $\mu/Q_T$  with  $Q_T = 80$  GeV.

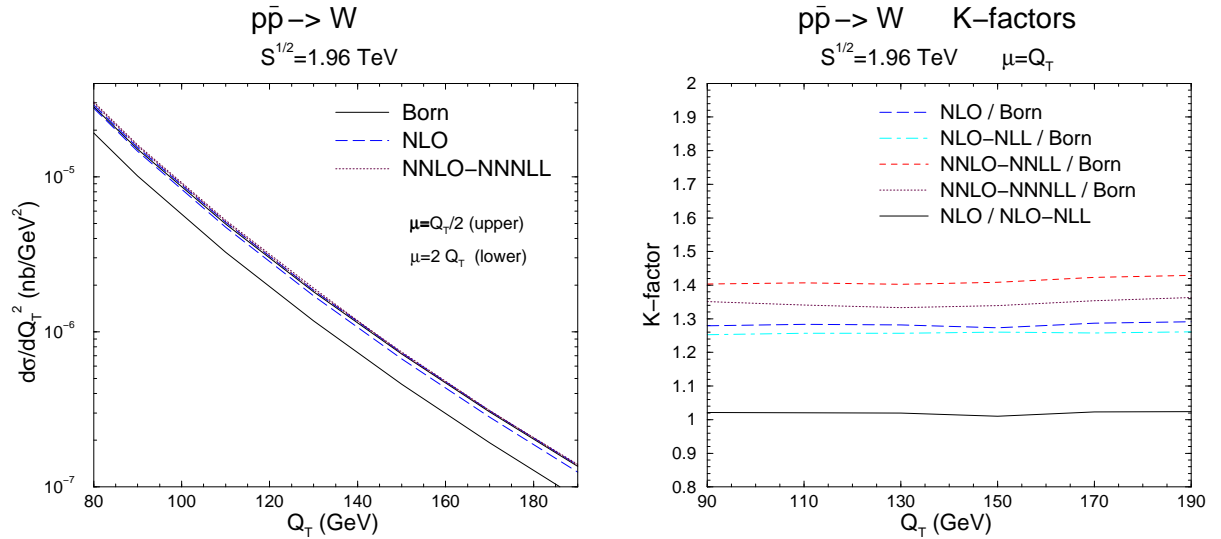


Figure 2: Left:  $d\sigma/dQ_T^2$  for  $W$  production at the Tevatron with  $\sqrt{S} = 1.96$  TeV as a function of  $Q_T$  with  $\mu = Q_T/2, 2Q_T$ . Right: the K-factors versus  $Q_T$  with  $\mu = Q_T$ .

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