

# FERMIONS AND SUPERSYMMETRY BREAKING IN THE INTERVAL

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## Abstract

We study fermions, such as gravitinos and gauginos in supersymmetric theories, propagating in a five-dimensional bulk where the fifth dimensional component is assumed to be an interval. We show that the most general boundary condition at each endpoint of the interval is encoded in a single complex parameter representing a point in the Riemann sphere. Upon introducing a boundary mass term, the variational principle uniquely determines the boundary conditions and the bulk equations of motion. We show the mass spectrum becomes independent from the Scherk-Schwarz parameter for a suitable choice of one of the two boundary conditions. Furthermore, for any value of the Scherk-Schwarz parameter, a zero-mode is present in the mass spectrum and supersymmetry is recovered if the two complex parameters are tuned.

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A common feature of five-dimensional supersymmetric models are fermions propagating in the bulk of the extra dimension. In order to extract physical predictions at low energies, the four dimensional mass spectrum of those fermions has to be known. For instance, supersymmetry breaking is determined by the mass spectrum of the gravitino, the existence of a zero mode signalling unbroken supersymmetry. Similarly, when gauge multiplets propagate in the bulk supersymmetry breaking is intimately linked to the existence of gaugino zero modes. In particular if supersymmetry breaking is implemented by non-trivial twist conditions, or Scherk-Schwarz mechanism [1], it acts in the same way both in the gravitino and the gaugino sectors.

The aim of this letter is to study fermions propagating in a flat five-dimensional space-time, with coordinates  $(x^\mu, y)$ , where the compact fifth dimension (with radius  $R$ ) has two four-dimensional boundaries located at  $y = 0$  and  $y = \pi R$ . Often this space is constructed as the orbifold  $S^1/\mathbb{Z}_2$ , identifying points on the circle related by the reflection of the fifth coordinate  $y \rightarrow -y$ . Fields with odd parity with respect to the  $\mathbb{Z}_2$  reflections are zero at the fixed points, while the normal derivative of even fields is forced to vanish. The treatment of fermions is complicated in the presence of brane actions localized at the boundaries. In the orbifold approach, these brane actions are introduced with a delta-function distribution, peaked at the location of the orbifold fixed point. The latter induces discontinuities in the wave functions of the fermions which take different values at the fixed point and infinitesimally close to it [2, 3]. A possible way to avoid these jumps is to give up the rigid orbifold boundary conditions and instead enforce the fields to be continuous, while the boundary conditions are determined by the boundary action itself. This is called the *interval* approach and leads to physically equivalent spectra as those of the orbifold approach without any need of using, as the latter, singular functions <sup>6</sup>. To summarize, in the orbifold approach one imposes fixed (orbifold) boundary conditions while the brane action induces jumps, whereas in the interval approach one imposes continuity and the brane action induces the boundary conditions.

In this letter we will follow the interval approach and show how the boundary action can give rise to consistent boundary conditions for the fermions. In a forthcoming publication [4] we will give a detailed treatment of how to translate the two pictures into each other. In a manifold  $\mathcal{M}$  with a boundary the dynamics is determined by two equally important ingredients: the bulk equations of motion and the boundary conditions (BC's). An economical way to determine a set of consistent BC's together with the bulk equations of motion is the action principle <sup>7</sup>: under a variation of the dynamical fields the action must be stationary. This in general translates into two separate conditions: the vanishing of the variation of the action in the bulk and the vanishing of the variation at the boundary  $\partial\mathcal{M}$ . Contributions to the action variation at the boundary come from integration by parts of bulk variation and, if present, from varying the boundary part

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<sup>6</sup>The interval approach is sometimes called “downstairs” approach while the orbifold approach is called “upstairs” approach.

<sup>7</sup>For an alternative approach see [5].

of the action (see [6] for a recent application to symmetry breaking). In the following we will consider the five-dimensional (5D) manifold  $\mathcal{M}$  as the direct product of the four dimensional Minkowski space  $\mathcal{M}_4$  and the interval  $[0, \pi R]$ .

Since we are mainly interested in supersymmetric theories, we will take the fermions to be symplectic-Majorana spinors, although a very similar treatment holds for the case of fermionic matter field associated to Dirac fermions. In particular we will consider the gaugino case, the treatment of gravitinos being completely analogous. The 5D spinors  $\Psi^i$  satisfy the symplectic-Majorana reality condition and we can represent them in terms of two chiral 4D spinors according to <sup>8</sup>

$$\Psi^i = \begin{pmatrix} \eta_\alpha^i \\ \bar{\chi}^{i\dot{\alpha}} \end{pmatrix}, \quad \bar{\chi}^{i\dot{\alpha}} \equiv \epsilon^{ij} (\eta_\beta^j)^* \epsilon^{\dot{\alpha}\dot{\beta}}. \quad (1)$$

where  $\epsilon_{ij} = i(\sigma_2)_{ij}$  and  $\epsilon^{im}\epsilon_{jm} = \delta_j^i$ . Consider thus the bulk Lagrangian

$$\mathcal{L}_{\text{bulk}} = i \bar{\Psi} \gamma^M D_M \Psi = \frac{i}{2} \bar{\Psi} \gamma^M D_M \Psi - \frac{i}{2} D_M \bar{\Psi} \gamma^M \Psi. \quad (2)$$

where the last equation is not due to partial integration but holds because of the symplectic-Majorana property, Eq. (1). The derivative is covariant with respect to the  $SU(2)_R$  automorphism symmetry and thus contains the auxiliary gauge connection  $V_M$ . The field  $V_M$  is non propagating and appears in the off-shell formulation of 5D supergravity [7]. A vacuum expectation value (VEV) <sup>9</sup>

$$V_M = \delta_M^5 \frac{\omega}{R} \vec{q} \cdot \vec{\sigma}, \quad \vec{q}^2 = 1 \quad (3)$$

implements a Scherk-Schwarz (SS) supersymmetry breaking mechanism [1] in the Hosotani basis [8,9]. The standard form of the SS mechanism, originally introduced for circle compactification, can be recovered by a gauge transformation  $U$  that transforms away  $V_M$  but twists the periodicity condition for fields charged under  $SU(2)_R$  on the circle. As we will see later in the interval a SS breaking term is equivalent to a suitable modification of the BC's at one of the endpoints. The unitary vector  $\vec{q}$  points toward the direction of SS breaking. We supplement the bulk action by the following boundary terms at  $y = y_f$  ( $f = 0, \pi$ ) with  $y_0 = 0$  and  $y_\pi = \pi R$

$$\mathcal{L}_f = \frac{1}{2} \bar{\Psi} (T^{(f)} + \gamma^5 V^{(f)}) \Psi = \frac{1}{2} \eta^i M_{ij}^{(f)} \eta^j + \text{h.c.}, \quad (4)$$

where  $T^{(f)}$  and  $V^{(f)}$  are matrices acting on  $SU(2)$  indices,

$$M^{(f)} = i\sigma_2 (T^{(f)} - iV^{(f)}) \quad (5)$$

and we have made use of the decomposition (1). Notice that the mass matrix is allowed to have complex entries. Without loss of generality we take it to be symmetric, which enforces  $T^f$  and  $V^f$  to be spanned by Pauli matrices.

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<sup>8</sup>We use the Wess-Bagger convention [10] for the contraction of spinor indices.

<sup>9</sup>Consistent with the bulk equation of motion  $d(\vec{q} \cdot \vec{V}) = 0$  [7].

The total bulk + boundary action is then given by

$$S = S_{\text{bulk}} + S_{\text{boundary}} = \int d^5x \mathcal{L}_{\text{bulk}} + \int_{y=0} d^4x \mathcal{L}_0 - \int_{y=\pi R} d^4x \mathcal{L}_\pi \quad . \quad (6)$$

The variation of the bulk action gives

$$\delta S_{\text{bulk}} = \int d^5x i (\delta \bar{\Psi} \gamma^M D_M \Psi - D_M \bar{\Psi} \gamma^M \delta \Psi) - \int d^4x [\delta \eta^i \epsilon_{ij} \eta^j + h.c.]_0^{\pi R} \quad , \quad (7)$$

where the boundary piece comes from partial integration. One now has to add the variation of the boundary action. Enforcing that the total action  $S = S_{\text{bulk}} + S_{\text{boundary}}$  has zero variation we get the standard Dirac equation in the bulk provided that all the boundary pieces vanish. The latter are given by

$$[\delta \eta^i (\epsilon_{ij} + M_{ij}^{(f)}) \eta^j + h.c.]|_{y=y_f} = 0 \quad . \quad (8)$$

Since we are considering unconstrained variations of the fields, the BC's we obtain from Eqs. (8) are given by

$$(\epsilon_{ij} + M_{ij}^{(f)}) \eta^j|_{y=y_f} = 0 \quad . \quad (9)$$

These equations only have trivial solutions (are overconstrained) unless

$$\det(\epsilon_{ij} + M_{ij}^{(0)}) = \det(\epsilon_{ij} + M_{ij}^{(\pi)}) = 0 \quad . \quad (10)$$

Imposing these conditions, we get the two complex BC's which are needed for a system of two first order equations. Note that this means that an arbitrary brane mass matrix does not yield viable BC's; in particular a vanishing brane action is inconsistent<sup>10</sup> since  $\det(\epsilon_{ij}) \neq 0$ <sup>11</sup>. However this does not imply that the familiar orbifold BC  $\eta_1 = 0$  ( $\eta_2 = 0$ ) can not be achieved; in the interval approach they correspond to  $M = \sigma^1$  ( $M = -\sigma^1$ ).

The BC's resulting from Eqs. (9) are of the form

$$(c_f^1 \eta^1 + c_f^2 \eta^2)|_{y=y_f} = 0 \quad , \quad (11)$$

where  $c_f^{1,2}$  are complex parameters or, setting  $z_f = -(c_f^1/c_f^2)$

$$(\eta^2 - z_f \eta^1)|_{y=y_f} = 0, \quad z_f \in \mathbb{C} \quad . \quad (12)$$

Physically inequivalent BC's span a complex projective space  $\mathbb{C}P^1$  homeomorphic to the Riemann sphere. In particular,  $z_f = 0$  leads to a Dirichlet BC for  $\eta_2$ , and the point at infinity  $z_f = \infty$  leads to a Dirichlet BC for  $\eta_1$ . Notice that these BC's come from  $SU(2)_R$  breaking mass terms. Special values of  $z_f$  correspond to cases when these terms preserve part of the symmetry of the original bulk Lagrangian. In particular when both the SS and

<sup>10</sup>In the sense that the action principle does not provide a consistent set of BC's as boundary equations of motion.

<sup>11</sup>Notice that this agrees with the methods recently used in Ref. [11].

the preserved symmetry are aligned those cases can lead to a *persistent* supersymmetry as we will see. Once (10) is satisfied, the values of  $z_f$  in terms of the brane mass terms are given by

$$z_f = -\frac{M_{11}^{(f)}}{1 + M_{12}^{(f)}} = \frac{1 - M_{12}^{(f)}}{M_{22}^{(f)}} \quad (13)$$

where the second equality holds due to the condition (10).

The mass spectrum is found by solving the EOM with the boundary conditions (12). To simplify the bulk equations of motion it is convenient to go from the Hosotani basis  $\Psi^i$  to the SS one  $\Phi^i$ , related by the transformation

$$\Psi = U \Phi, \quad U = \exp\left(-i \vec{q} \cdot \vec{\sigma} \omega \frac{y}{R}\right). \quad (14)$$

In the SS gauge the bulk equations read

$$i \gamma^M \partial_M \Phi = 0 \quad . \quad (15)$$

We now decompose the chiral spinor  $\eta^i(x, y)$  in the Hosotani basis as  $\eta^i(x, y) = \varphi^i(y)\psi(x)$ , with  $\psi(x)$  a 4D chiral spinor. Setting  $\varphi = U\phi$  we get the following equations of motion in the SS basis

$$m \phi^i - \epsilon^{ij} \frac{d\bar{\phi}_j}{dy} = 0, \quad m \bar{\phi}_j \epsilon^{ij} + \frac{d\phi^i}{dy} = 0. \quad (16)$$

The parameter  $m$  in Eq. (16) is the Majorana mass eigenvalue of the 4D chiral spinor <sup>12</sup>

$$i\sigma^\mu \partial_\mu \bar{\psi} = m \psi, \quad i\bar{\sigma}^\mu \partial_\mu \psi = m \bar{\psi} \quad . \quad (17)$$

As a consequence of the transformation (14) the SS parameter  $\omega$  manifests itself only in the BC at  $y = \pi R$  <sup>13</sup>:

$$\zeta_0 \equiv \frac{\phi^2}{\phi^1} \Big|_{y=0} = z_0, \quad \zeta_\pi \equiv \frac{\phi^2}{\phi^1} \Big|_{y=\pi R} = \frac{\tan(\pi\omega)(iq_1 - q_2 - iq_3 z_\pi) + z_\pi}{\tan(\pi\omega)(iq_1 z_\pi + q_2 z_\pi + iq_3) + 1}, \quad (18)$$

where  $\zeta_f$  are the BC's in the SS basis. In particular the boundary condition  $\zeta_\pi$  is a function of  $\omega$ ,  $\vec{q}$  and  $z_\pi$ . From this it follows that we can always gauge away the SS parameter  $\omega$  in the bulk Lagrangian going into the SS basis through (14). However now in the new basis  $\omega$  reappears in one of the BC's.

The bulk equations have the following generic solution

$$\phi(y) = \begin{pmatrix} \bar{a} \cos(my) + \bar{z}_0 a \sin(my) \\ -a \sin(my) + z_0 \bar{a} \cos(my) \end{pmatrix}, \quad (19)$$

where  $a$  is a complex number given in terms of  $z_0$  and  $\zeta_\pi$ :

$$a = \frac{z_0 - \zeta_\pi}{|z_0 - \zeta_\pi|} + \frac{1 + z_0 \bar{\zeta}_\pi}{|1 + z_0 \bar{\zeta}_\pi|}. \quad (20)$$

<sup>12</sup>The bar acting on a scalar quantity, as e.g.  $\bar{\phi}_i$ , and a chiral spinor, as e.g.  $\bar{\psi}$ , denotes complex conjugation.

<sup>13</sup>Notice that  $U(y=0) = 1$ . The roles of the branes and hence of  $z_\pi$  and  $z_0$  can be interchanged by considering the SS transformation  $U'(y) \equiv U(y - \pi R)$ .

The solution (19) satisfies the BC's Eq. (18) for the following mass eigenvalues

$$m_n = \frac{n}{R} + \frac{1}{\pi R} \arctan \left| \frac{z_0 - \zeta_\pi}{1 + z_0 \bar{\zeta}_\pi} \right|, \quad (21)$$

where  $n \in \mathbb{Z}$ . When  $z_0 = \zeta_\pi$  there is a zero mode and supersymmetry remains unbroken. When the only sources of supersymmetry breaking reside on the branes, setting them to cancel each other,  $z_0 = z_\pi$ , preserves supersymmetry [12]. Once supersymmetry is further broken in the bulk, an obvious way to restore it is by determining  $z_\pi$  as a function of  $z_0$  and  $\omega$  using the relation (18) with  $\zeta_\pi = z_0$ . This will lead to an  $\omega$ -dependent brane-Lagrangian at  $y = \pi R$ . In this case we could say that supersymmetry, that was broken by BC's (SS twist) is *restored* by the given SS twist (BC's) [13].

There is however a more interesting case: suppose the brane Lagrangian determines  $z_\pi$  to be

$$z_\pi = z(\vec{q}) \equiv \frac{\lambda - q_3}{q_1 - iq_2}. \quad (22)$$

with  $\lambda = \pm 1$ . This special value of  $z_\pi$  is a fixed point of the SS transformation, i.e.  $\zeta_f = z_f$ . For  $z_\pi = z(\vec{q})$  the spectrum becomes independent on  $\omega$ . In other words, for this special subset of boundary Lagrangians, the VEV for the field  $\vec{q} \cdot \vec{V}_5$  does not influence the spectrum. The reason for this can be understood by going back to the Lagrangian which we used to derive the BC's. From the relation (13) one can see that condition (22) is satisfied by the mass matrix

$$\begin{aligned} M_{12}^{(\pi)} &= \lambda q_3 \\ M_{11}^{(\pi)} &= -\lambda(q_1 + iq_2) \\ M_{22}^{(\pi)} &= \lambda(q_1 - iq_2) \end{aligned} \quad (23)$$

which can be translated into a mass term at the boundary  $y = y_\pi$  along the direction of the SS term, i.e.  $V^{(\pi)} = 0$  and  $T^{(\pi)} = -\lambda \vec{q} \cdot \vec{\sigma}$  in the notation of Eq. (4). In particular this brane mass term preserves a residual  $U(1)_R$  aligned along the SS direction  $\vec{q}$ . In other words, the SS-transformation  $U$  leaves both brane Lagrangians invariant and  $\omega$  can be gauged away. When we further impose  $z_0 = z(\pm \vec{q})$ , i.e.  $V^{(0)} = 0$  and  $T^{(0)} = \pm T^{(\pi)}$  the  $U(1)_R$  symmetry is preserved by the bulk. In particular if  $z_0 = z(\vec{q})$  supersymmetry remains unbroken, although the VEV of  $\vec{q} \cdot \vec{V}_5$  is nonzero. One could say that in this case the theory is *persistently* supersymmetric even in the presence of the SS twist, with mass spectrum  $m_n = n/R$ . On the other hand if  $z_0 = z(-\vec{q})$  the theory is (*persistently*) non-supersymmetric and independent on the SS twist: the mass spectrum is given by  $m_n = (n + 1/2)/R$ . In this case supersymmetry breaking amounts to an extra  $\mathbb{Z}'_2$  orbifolding [14].

Notice that we have not chosen the most general solution to Eq. (22) but one where  $V^{(f)} = 0$ . In the most general case the condition (10) leads to  $(\vec{T}^{(f)})^2 - (\vec{V}^{(f)})^2 = 1$  and  $\vec{T}^{(f)} \cdot \vec{V}^{(f)} = 0$ , and for  $\vec{V}^{(f)} \neq 0$  Eq. (22) has in general a two-parameter family of solutions. All of them should comply with the existence of *persistent* zero modes

(irrespective of the SS twist). However the condition for an (off-shell) supersymmetric action is only consistent with the solution with  $V^{(f)} = 0$ , as we will see below.

Something similar happens in the warped case [4]: when bulk cosmological constant and brane tensions are turned on, invariance of the action under local supersymmetry requires gravitino mass terms on the brane. In the tuned case, – i.e. in the Randall-Sundrum (RS) model – those brane mass terms precisely give rise to the BC  $z_0 = z_\pi = z(\vec{q})$  [15]. Note that there  $\vec{q} \cdot \vec{V}_5$  is replaced by  $A_5$ , the fifth component of the graviphoton. In fact, it has been shown that in this case there always exists a Killing spinor and supersymmetry remains unbroken [16, 17], consistent with the result that in RS supersymmetry can not be spontaneously broken<sup>14</sup> by the SS mechanism [15, 19]. This and other issues, such as the comparison between the interval and the orbifold approaches and how to relate them, will be presented elsewhere [4].

Up to now, we have focused on the fermion sector spectrum. Adding the complete vector multiplet does not invalidate our conditions for supersymmetry restoration as long as the supersymmetry breaking brane terms are of the form given by Eq. (4). We would like to show the invariance of our gaugino Lagrangian, Eq. (6), under (global) supersymmetry. To this end, let us focus on a simple abelian gauge multiplet. Clearly, since we are not imposing any a priori boundary condition on the fields in the action, we have to worry about the total derivatives which arise in the variation of the bulk action. The latter is given by<sup>15</sup>

$$S_{\text{bulk}}^{U(1)} = \int_{\mathcal{M}} \left( 2\vec{X} \cdot \vec{X} - \Sigma \partial^2 \Sigma - \frac{1}{2} \partial_M \Sigma \partial^M \Sigma + i\bar{\Psi} \not{\partial} \Psi - \frac{1}{4} G_{MN} G^{MN} \right). \quad (24)$$

Under a global supersymmetric transformation the Lagrangian transforms into a total derivative giving rise to the supersymmetry boundary-variation:

$$\delta_\epsilon S_{\text{bulk}}^{U(1)} = \int_{\partial\mathcal{M}} \bar{\epsilon} i \gamma^5 \rho, \quad \rho = \left( i\vec{X} \cdot \vec{\sigma} - \Sigma \not{\partial} - \frac{1}{4} \gamma^{MN} G_{MN} - \frac{1}{2} \not{\partial} \Sigma \right) \Psi. \quad (25)$$

To compensate for this, we add to it the brane action

$$S_{\text{brane}}^{U(1)} = \int_{\partial\mathcal{M}} \left( 2\vec{T}^{(f)} \cdot \Sigma \vec{X} + \frac{1}{2} \bar{\Psi} T^{(f)} \Psi \right) \quad (26)$$

which transforms into

$$\delta_\epsilon S_{\text{brane}}^{U(1)} = \int_{\partial\mathcal{M}} \bar{\epsilon} T^{(f)} \rho. \quad (27)$$

Now the supersymmetry variation at each boundary is proportional to  $(1 + i\gamma^5 T^{(f)})\epsilon(y_f)$ . Denoting with  $\xi$  [see Eq. (1)] the upper part of  $\epsilon$ , whenever  $(\vec{T}^{(f)})^2 = 1$  these variations can cancel provided the transformation parameter satisfies the BC's  $\xi^2 = z(\vec{T}^{(f)}) \xi^1$ . The only possibility is that  $T^{(0)} = T^{(\pi)}$ , since  $\epsilon$  is constant for global supersymmetry. Notice

<sup>14</sup>A discrete supersymmetry breaking by BC's,  $z_0 = z(-\vec{q})$ ,  $z_\pi = z(\vec{q})$ , was performed in Ref. [18].

<sup>15</sup>Besides the gauge field  $B_M$  with field strength  $G_{MN}$  and the gaugino  $\Psi$  the 5D vector multiplet contains the real scalar  $\Sigma$  and the auxiliary  $SU(2)_R$  triplet  $\vec{X}$ .

that according to Eqs. (10) and (5), this gives rise to the same BC's for the gaugino,  $\eta^2 = z(\vec{T}^{(f)})\eta^1$ . The remaining EOM then fix the boundary conditions  $G_{\mu 5} = \vec{X} = \Sigma = 0$ . The bottom line of the off-shell approach is that, in the presence of a boundary, at most one supersymmetry can be preserved. Global SUSY invariance for the action of a vector multiplet singles out a special boundary mass term for gauginos such that  $z_0 = z_\pi$  which is at origin of the zero mode in the spectrum [see Eq. (21) for  $\omega = 0$  <sup>16</sup>.] We expect there to be a locally supersymmetric extension of the action (24)+(26) for  $T^{(0)} \neq T^{(\pi)}$ . In this case the  $SU(2)_R$  auxiliary gauge connection  $\vec{V}_M$  from the supergravity multiplet gives an additional source of supersymmetry breaking. Notice that for a globally supersymmetric vacuum there must then be a solution to the Killing spinor equation

$$\gamma^5 D_5 \epsilon(y) = 0, \quad \xi^2(y_f) = z(\vec{T}^{(f)}) \xi^1(y_f), \quad (28)$$

where  $D_5$  is covariant with respect to  $SU(2)_R$ . These equations coincide with the zero mode condition for the gaugino considered above.

In conclusion we have studied in this letter the issues of fermion mass spectrum, and supersymmetry breaking in the presence of Scherk-Schwarz twists <sup>17</sup>, in the interval approach with arbitrary BC's fixed by boundary mass terms. If *alignment* occurs, i.e. BC's are invariant under the SS twist, the mass spectrum (supersymmetric or not) becomes independent on the SS parameter. If the BC's are identical for the different boundaries there appears a zero mode in the spectrum: supersymmetry is *restored* by a cancellation between BC's and the SS twist. When the two previous conditions are fulfilled, i.e. the BC's are equal at different boundaries and SS twist invariant, the mass spectrum is supersymmetric and independent on the SS parameter: supersymmetry is *persistent* in the presence of the SS twist. In this case the bulk + brane Lagrangian is invariant under a remaining  $U(1)_R$  symmetry. The conditions imposed on the brane Lagrangians in the *persistent* supersymmetry case can be regarded as technically natural, since once they are satisfied at tree level, they will not be upset by corrections coming from the bulk + brane Lagrangian to any order. Only after the addition of extra breaking terms, for example brane kinetic terms, supersymmetry would be broken in a controllable way. Those two conditions could have their origin on a higher dimensional completion of the theory, as it takes place at Horava's gaugino condensation model [12], and they would lead to *persistent* supersymmetry after compactification down to five dimensions. In our scenario, *alignment* would give rise to a model where supersymmetry could be broken, but the breaking scale would be completely fixed by the compactification scale  $1/R$  and the relative size of brane breaking terms  $z_f$ , irrespective of the SS-breaking scale  $\omega$ . This phenomenon opens new possibilities for model building whenever one needs to control the

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<sup>16</sup>In the global theory on the interval, all supersymmetry breaking is encoded in the  $T^{(f)}$ : there is no auxiliary field  $V_M$  whose VEV could contribute to the breaking .

<sup>17</sup>We have studied SS or Hosotani breaking in the bulk, but one could similarly consider radion  $F$ -term breaking [20].



effect of supersymmetry breaking in the bulk.

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