

Two-loop virtual top-quark effect on Higgs-boson decay to bottom quarks

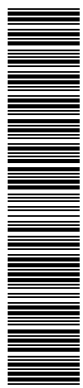
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Abstract

In most of the mass range encompassed by the limits from the direct search and the electroweak precision tests, the Higgs boson of the standard model preferably decays to bottom quarks. We present, in analytic form, the dominant two-loop electroweak correction, of $\mathcal{O}(G_F^2 m_t^4)$, to the partial width of this decay. It amplifies the familiar enhancement due to the $\mathcal{O}(G_F m_t^2)$ one-loop correction by about +16% and thus more than compensates the screening by about -8% through strong-interaction effects of order $\mathcal{O}(\alpha_s G_F m_t^2)$.

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The standard model (SM) of elementary-particle physics, whose fermion and gauge sectors have been impressively confirmed by an enormous wealth of experimental data, predicts the existence of a last undiscovered fundamental particle, the Higgs boson H , whose mass M_H is a free parameter of the theory. The direct search for the Higgs boson at the CERN Large Electron-Positron Collider LEP 2 led to a lower bound of $M_H > 114$ GeV at 95% confidence level (CL) [1]. On the other hand, high-precision measurements, especially at LEP and the SLAC Linear Collider SLC, were sensitive to the Higgs-boson mass via electroweak radiative corrections, yielding to the value $M_H = (85^{+39}_{-28})$ GeV together with an upper limit of $M_H < 166$ GeV at 95% CL [2]. The vacuum-stability and triviality bounds suggest that $130 \lesssim M_H \lesssim 180$ GeV if the SM is valid up to the grand-unification scale (for a review, see Ref. [3]). If the Higgs mechanism of spontaneous symmetry breaking, as implemented in the SM, is realized in nature, then we are now being on the eve of a groundbreaking discovery, to be made at the CERN Large Hadron Collider (LHC), which will go into operation in a just few months from now. After finding a new scalar particle, the burning question will be whether it is in fact the Higgs boson of the SM, or lives in some extended Higgs sector. Therefore, it is indispensable to know the SM predictions for the production and decay rates of the SM Higgs boson with high precision. Its decay to a bottom-quark pair, $H \rightarrow b\bar{b}$, is of paramount interest, as it is by far the dominant decay channel for $M_H \lesssim 140$ GeV (see, *e.g.*, Ref. [4]). On the other hand, the inverse process, $b\bar{b} \rightarrow H$, was identified to be a crucial hadroproduction mechanism, appreciably enhancing the yield due to gluon fusion [5]. Precise knowledge of the bottom Yukawa coupling is also requisite for reliable predictions of associated hadroproduction of Higgs bosons and bottom quarks [6].

The purpose of this Letter is to fill a long-standing gap in our knowledge of the quantum corrections to the partial width Γ_b of the $H \rightarrow b\bar{b}$ decay, by providing, in analytic form, the dominant two-loop electroweak correction, of $\mathcal{O}(G_F^2 m_t^4)$, where G_F is Fermi's constant and m_t is the top-quark mass. This correction also applies to the cross section of $b\bar{b} \rightarrow H$. Surprisingly, it turns out to be more than twice as large as the $\mathcal{O}(\alpha_s G_F m_t^2)$ one, which is formally enhanced by one power of the strong-coupling constant α_s . In the discussion of virtual top-quark effects, it is useful to distinguish between universal corrections, which are independent of the produced fermion flavor, and non-universal corrections, which are specific for the $H \rightarrow b\bar{b}$ decay because bottom is the weak-isospin partner of top. Here, we have to consider both types.

Prior to going into details with our calculation, we briefly review the current status of the radiative corrections to Γ_b in the intermediate mass range, defined by $M_W < M_H < 2M_W$. As for effects arising solely from quantum chromodynamics (QCD), the full m_b dependence is known in $\mathcal{O}(\alpha_s)$ [7]. In $\mathcal{O}(\alpha_s^2)$, the leading [8] and next-to-leading [9] terms of the expansion in m_b^2/M_H^2 of the Feynman diagrams without top quarks are available. Those involving top quarks either contain gluon self-energy insertions or represent cuts through three-loop double-triangle diagrams; the former contribution is exactly known [10], while the four leading terms of the expansion in M_H^2/m_t^2 are known in the latter case [11]. In $\mathcal{O}(\alpha_s^3)$, the diagrams containing only light degrees of freedom were evaluated directly [12], while those involving the top quark were treated in the framework of an

appropriate effective field theory [13]. As for purely electroweak corrections, the one-loop result is completely known [14]. At two loops, the dominant universal correction, of $\mathcal{O}(G_F^2 m_t^4)$, was already studied in Ref. [15], while the non-universal one is considered here for the first time. As for mixed corrections, the universal [16] and non-universal [17] $\mathcal{O}(\alpha_s G_F m_t^2)$ terms at two loops and the universal [18] and non-universal [19] $\mathcal{O}(\alpha_s^2 G_F m_t^2)$ terms at three loops are available.

We now outline the course of our calculation and exhibit the structure of our results. Full details will be presented in a forthcoming communication [20]. For convenience, we work in 't Hooft-Feynman gauge. As usual, we extract the ultraviolet divergences by means of dimensional regularization, with $D = 4 - 2\epsilon$ space-time dimensions and 't Hooft mass scale μ . We do not encounter ambiguities related to the treatment of γ_5 in D dimensions and are thus entitled to use the anti-commuting definition. We adopt Sirlin's formulation of the electroweak on-shell renormalization scheme [21], which uses G_F and the physical particle masses as basic parameters. We take the Cabibbo-Kobayashi-Maskawa quark mixing matrix to be unity, which is well justified because the third quark generation is, to good approximation, decoupled from the first two [22]. For convenience, we renormalize the Higgs sector by introducing counterterm vertices involving tadpole and Higgs-boson mass counterterms, δt and δM_H , respectively [23]. Specifically, δt is adjusted so that it exactly cancels the sum of all one-particle-irreducible tadpole diagrams.

Detailed inspection reveals that, to the orders considered here, the amputated matrix element of $H \rightarrow b\bar{b}$ exhibits the simple structure

$$\mathcal{A} = A + B (\not{p} - \not{\bar{p}}) \omega_-, \quad (1)$$

where $\omega_{\pm} = (1 \pm \gamma_5)/2$ are the helicity projection operators, p and \bar{p} are the four-momenta of b and \bar{b} , respectively, and A and B are Lorentz scalars. Including the wave-function renormalizations of the external particles and employing the Dirac equation, we find the transition matrix element to be

$$\mathcal{T} = \sqrt{Z_H} \left(\sqrt{Z_{b,L} Z_{b,R}} A + m_b Z_{b,L} B \right) s, \quad (2)$$

where $s = \bar{u}(p, r) v(\bar{p}, \bar{r})$, with r and \bar{r} being spin labels. Owing to parity violation, the left- and right-handed components of the bottom-quark field, $b_{L,R} = \omega_{\mp} b$, participate differently in the electroweak interactions and thus receive different wave-function renormalizations, $Z_{b,L/R}$. At tree-level, we have $A^{(0)} = -m_b/v$ and $B^{(0)} = 0$, where $v = 2^{-1/4} G_F^{-1/2}$ is the Higgs vacuum expectation value. Here and in the following, superscripts enclosed in parentheses denote the loop order. In Sirlin's formulation of the electroweak on-shell scheme, where Fermi's constant is introduced to the SM through a charged-current process, namely muon decay, the SU(2) gauge coupling $g = 2M_W/v$ does not receive power corrections in m_t , so that [24]

$$\frac{M_{W,0}}{v_0} = \frac{M_W}{v} \quad (3)$$

to the orders considered here, which implies that the renormalization of v is reduced to the one of M_W . Here and in the following, bare quantities carry the subscript 0. It hence

follows that we need to perform a genuine two-loop renormalization of Z_H , m_b , $Z_{b,L/R}$, and M_W , while a one-loop renormalization of M_H and m_t is sufficient. As usual, we denote the sums of all one-particle-irreducible H , f ($f = b, t$), and W self-energy diagrams at four-momentum transfer q as $i\Sigma_H(q^2)$, $i[q(\omega_{-\Sigma_{f,L}}(q^2) + \omega_{+\Sigma_{f,R}}(q^2)) + m_{f,0}\Sigma_{f,S}(q^2)]$, and $-i[(g^{\mu\nu} - q^\mu q^\nu/q^2)\Sigma_{W,T}(q^2) + (q^\mu q^\nu/q^2)\Sigma_{W,L}(q^2)]$, and split the bare masses as $M_{H/W,0}^2 = M_{H/W}^2 + \delta M_{H/W}^2$ and $m_{f,0} = m_f + \delta m_f$. Imposing the on-shell renormalization conditions on the dressed propagators then yields

$$\begin{aligned}
\delta M_H^2 &= \Sigma_H(M_H^2), \\
Z_H &= \frac{1}{1 + \Sigma'_H(M_H^2)}, \\
\frac{\delta m_f}{m_f} &= \frac{1}{\sqrt{f(m_f^2)}} - 1, \\
Z_{f,L/R} &= \frac{1}{\left(1 + \Sigma_{f,L/R}(m_f^2)\right) \left(1 - m_f^2 \frac{f'(m_f^2)}{f(m_f^2)}\right)}, \\
\delta M_W^2 &= \Sigma_{W,T}(M_W^2),
\end{aligned} \tag{4}$$

where

$$f(q^2) = \frac{(1 - \Sigma_{f,S}(q^2))^2}{(1 + \Sigma_{f,L}(q^2))(1 + \Sigma_{f,R}(q^2))}. \tag{5}$$

Relations that, to the order of our analysis, are equivalent to Eq. (4) were found in Ref. [25] using an alternative approach.

Performing a loop expansion and eliminating all bare masses, we thus obtain

$$\begin{aligned}
\frac{\mathcal{T}^{(0)}}{s} &= A^{(0)}, \\
\frac{\mathcal{T}^{(1)}}{s} &= A^{(1)} + m_b B^{(1)} + A^{(0)} \left(\delta_u^{(1)} + X^{(1)} \right), \\
\frac{\mathcal{T}^{(2)}}{s} &= A^{(2)} + m_b B^{(2)} + A^{(1)} X^{(1)} + m_b B^{(1)} \delta Z_{b,L}^{(1)} \\
&\quad + \left(A^{(1)} + m_b B^{(1)} + A^{(0)} X^{(1)} \right) \left[\delta_u^{(1)} + 2(1 - \epsilon) \frac{\delta m_t^{(1)}}{m_t} \right. \\
&\quad \left. - \frac{\delta M_W^{2(1)}}{M_W^2} \right] + A^{(0)} \left[\delta_u^{(2)} + X^{(2)} + \frac{1}{2} \frac{\delta m_b^{(1)}}{m_b} \left(\delta Z_{b,L}^{(1)} \right. \right. \\
&\quad \left. \left. + \delta Z_{b,R}^{(1)} \right) - \frac{1}{8} \left(\delta Z_{b,L}^{(1)} - \delta Z_{b,R}^{(1)} \right)^2 \right],
\end{aligned} \tag{6}$$

where

$$\delta_u^{(1)} = \frac{1}{2} \delta Z_H^{(1)} - \frac{1}{2} \frac{\delta M_W^{2(1)}}{M_W^2},$$

$$\begin{aligned} \delta_u^{(2)} = & \frac{1}{2}\delta Z_H^{(2)} - \frac{1}{2}\frac{\delta M_W^{2(2)}}{M_W^2} + \delta_u^{(1)} \left[-\frac{1}{2}\delta_u^{(1)} + 2(1-\epsilon) \right. \\ & \left. \times \frac{\delta m_t^{(1)}}{m_t} - 2\frac{\delta M_W^{2(1)}}{M_W^2} \right] - \frac{1}{2} \left(\frac{\delta M_W^{2(1)}}{M_W^2} \right)^2 \end{aligned} \quad (7)$$

are the universal corrections and

$$X^{(i)} = \frac{\delta m_b^{(i)}}{m_b} + \frac{1}{2} (\delta Z_{b,L}^{(i)} + \delta Z_{b,R}^{(i)}). \quad (8)$$

The Feynman diagrams contributing to $A_0^{(2)}$ and $B_0^{(2)}$ are depicted in Fig. 1. They are generated and drawn using the program `FeynArts` [26] and evaluated using the program `MATAD` [27], which is written in the programming language `FORM` [28], by applying the asymptotic-expansion technique (for a careful introduction, see Ref. [29]). Here, χ and ϕ denote the neutral and charged Higgs-Kibble ghosts with masses M_Z and M_W , respectively. The crosses in Figs. 1(s) and (t) indicate the insertions of the Higgs-boson mass and tadpole counterterms $i\delta t/v_0$ and $-i(\delta t/v_0 + \delta M_H^2)/v_0$ in a ϕ -boson line and a $H\phi\phi$ vertex, respectively. In the soft-Higgs limit, $M_H \ll m_t$, which is underlying our analysis, the diagrams in Figs. 1(a)–(s) can also be evaluated by applying a low-energy theorem (see Ref. [30] and references cited therein) to the corresponding b -quark self-energy diagrams that emerge by removing the external Higgs-boson line. This provides a powerful check for our calculation. Apart from the diagrams in Fig. 1, we also need to calculate the relevant one-particle-irreducible H , b , and W self-energy diagrams at two loops. Furthermore, we need to expand all the relevant one-loop diagrams through $\mathcal{O}(\epsilon)$.

We are now in a position to present our final results for the universal correction parameter δ_u and the relative correction to Γ_b . They read

$$\begin{aligned} \delta_u = & x_t N_c \frac{7}{6} + x_t^2 N_c \left(\frac{29}{2} - 6\zeta(2) + N_c \frac{49}{24} \right) \\ & + x_t \frac{\alpha_s}{\pi} C_F N_c \left(\frac{19}{12} - \frac{\zeta(2)}{2} \right), \end{aligned} \quad (9)$$

$$\begin{aligned} \frac{\Gamma_b}{\Gamma_b^{(0)}} = & x_t \left(-6 + N_c \frac{7}{3} \right) + x_t^2 \left[-20 + N_c (29 - 12\zeta(2)) \right. \\ & \left. + N_c^2 \frac{49}{9} \right] + x_t \frac{\alpha_s}{\pi} C_F \left[-36 + N_c \left(\frac{157}{12} - \zeta(2) \right) \right], \end{aligned} \quad (10)$$

where $N_c = 3$ and $C_F = (N_c^2 - 1)/(2N_c) = 4/3$ are color factors, $x_t = (G_F m_t^2)/(8\pi^2 \sqrt{2})$, $\zeta(2) = \pi^2/6$, and

$$\Gamma_b^{(0)} = \frac{\sqrt{2} N_c G_F M_H m_b^2}{8\pi} \left(1 - \frac{4m_b^2}{M_H^2} \right)^{3/2}. \quad (11)$$

If we convert Eq. (9) to a mixed renormalization scheme which uses the on-shell definitions for the particle masses and the definitions of the modified minimal-subtraction ($\overline{\text{MS}}$)

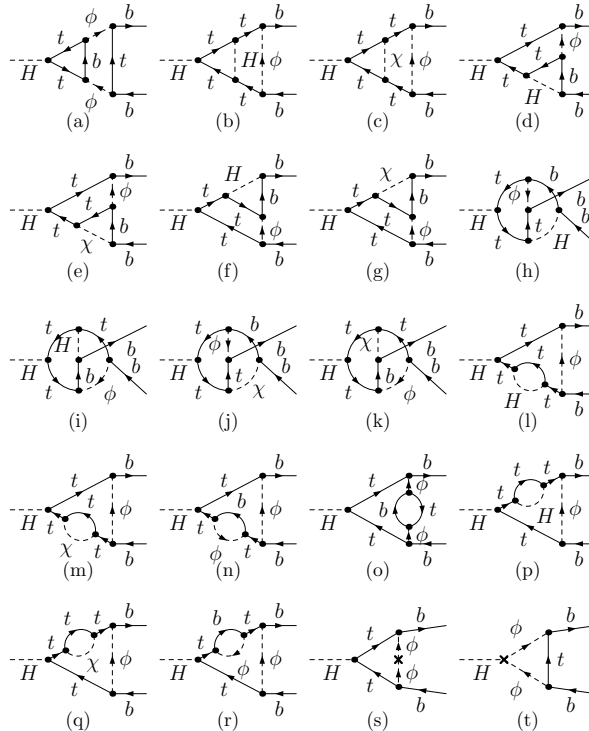


Figure 1: Diagrams contributing to $H \rightarrow b\bar{b}$ at $\mathcal{O}(G_F^2 m_t^4)$.

scheme for all other basic parameters, then we find agreement with Eq. (15) for $x = 0$ in the paper by Djouadi et al. [15]. However, the corresponding result for the electroweak on-shell scheme presented in their Eq. (27) for $x = 0$ disagrees with our Eq. (9). We can trace this discrepancy to the absence in their Eq. (25) of the additional finite term $\hat{\delta}_u^{(1)} \Delta\rho^{(1)}$ which arises from the renormalization of the one-loop result in their Eq. (7) according to the prescription in their Eq. (18). The $\mathcal{O}(G_F^2 m_t^4)$ term in Eq. (10) represents a new result.

In Eqs. (9) and (10), we have also included the two-loop $\mathcal{O}(\alpha_s G_F m_t^2)$ corrections [16,17], which we reproduced using our calculational techniques. As for the QCD renormalization, it is understood that m_b appearing in Eq. (11) is defined in the $\overline{\text{MS}}$ scheme as $m_b = \overline{m}_b(M_H)$, while the electroweak part of the renormalization remains in the on-shell scheme. This modification ensures that large logarithms of the type $\ln(M_H^2/m_b^2)$ that would otherwise appear already at $\mathcal{O}(\alpha_s)$ and spoil the convergence behavior of the perturbation expansion are properly resummed according to the renormalization group (RG) [7]. Since we wish to treat m_t on the same footing as m_b , we adopt this mixed scheme for m_t as well. The analysis at $\mathcal{O}(\alpha_s^2 G_F m_t^2)$ [18,19] reveals that Eqs. (9) and (10) may be further RG-improved by taking m_t and α_s to be $m_t = \overline{m}_t(m_t)$ and $\alpha_s = \alpha_s^{(6)}(m_t)$, respectively.

Finally, we explore the phenomenological implications of our results. Adopting from Ref. [22] the values $G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2}$, $\alpha_s^{(5)}(M_Z) = 0.1176$, $M_Z = 91.1876 \text{ GeV}$,

Table 1: Relative corrections to Γ_τ , Γ_c , and Γ_b at $\mathcal{O}(G_F m_t^2)$, $\mathcal{O}(G_F^2 m_t^4)$, and $\mathcal{O}(\alpha_s G_F m_t^2)$.

Order	$\Gamma_\tau/\Gamma_\tau^{(0)}$	$\Gamma_c/\Gamma_c^{(0)}$	$\Gamma_b/\Gamma_b^{(0)}$
$\mathcal{O}(G_F m_t^2)$	+2.021%	+2.021%	+0.289%
$\mathcal{O}(G_F^2 m_t^4)$	+0.064%	+0.064%	+0.047%
$\mathcal{O}(\alpha_s G_F m_t^2)$	+0.060%	+0.452%	-0.022%

and $m_t^{\text{pole}} = 174.2$ GeV for our input parameters, so that $\alpha_s^{(6)}(m_t) = 0.1076$ and $m_t = 166.2$ GeV, we evaluate Eqs. (9) and (10) to $\mathcal{O}(G_F m_t^2)$, $\mathcal{O}(G_F^2 m_t^4)$, and $\mathcal{O}(\alpha_s G_F m_t^2)$. For comparison, we also evaluate the relative corrections to Γ_l and Γ_q , where $l = e, \mu, \tau$ and $q = u, d, s, c$, which, to the orders considered here, are given by

$$\frac{\Gamma_l}{\Gamma_l^{(0)}} = (1 + \delta_u)^2 - 1, \quad (12)$$

$$\frac{\Gamma_q}{\Gamma_q^{(0)}} = (1 + \Delta_{\text{QCD}})(1 + \delta_u)^2 - 1, \quad (13)$$

where [7]

$$\Delta_{\text{QCD}} = \frac{\alpha_s}{\pi} C_F \frac{17}{4} \quad (14)$$

is the $\mathcal{O}(\alpha_s)$ correction in the limit $m_q \ll M_H$.

The results are listed in Table 1. We observe that the $\mathcal{O}(G_F^2 m_t^4)$ correction to Γ_b increases the enhancement due to the $\mathcal{O}(G_F m_t^2)$ one by about 16% and has more than twice the magnitude of the negative $\mathcal{O}(\alpha_s G_F m_t^2)$ one. Also in the case of Γ_l , the $\mathcal{O}(G_F^2 m_t^4)$ correction exceeds the $\mathcal{O}(\alpha_s G_F m_t^2)$ one. The situation is quite different for the case of Γ_q , which is due to the additional appearance of the sizeable product term $2\Delta_{\text{QCD}}\delta_u^{(1)}$ in Eq. (13).

In conclusion, we analytically calculated the dominant electroweak two-loop correction, of order $\mathcal{O}(G_F^2 m_t^4)$, to the $H \rightarrow b\bar{b}$ decay width Γ_b of an intermediate-mass Higgs boson, with $M_H \ll m_t$. We performed various checks for our analysis. The ultraviolet divergences cancelled through genuine two-loop renormalization. Our final result is devoid of infrared divergences related to infinitesimal scalar-boson masses. We reproduced those $Hb\bar{b}$ vertex diagrams where the external Higgs boson is coupled to an internal top-quark line, which we had computed directly, through application of a low-energy theorem. After switching to a hybrid renormalization scheme, our $\mathcal{O}(G_F^2 m_t^4)$ result for the universal correction δ_u agrees with Ref. [15]. Using our computational techniques, we also recovered the $\mathcal{O}(\alpha_s G_F m_t^2)$ corrections to δ_u and Γ_b . The $\mathcal{O}(G_F^2 m_t^4)$ correction to Γ_b amplifies the familiar enhancement due to the $\mathcal{O}(G_F m_t^2)$ correction by about +16% and thus more than compensates the screening by about -8% through QCD effects of $\mathcal{O}(\alpha_s G_F m_t^2)$.

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