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**On relative contributions of fusion and fragmentation mechanisms
in J/ψ photoproduction at high energy**

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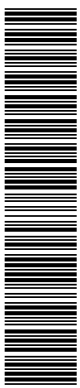
Abstract

We study J/ψ photoproduction via the fusion and fragmentation mechanisms at the HERA Collider within the frameworks of the collinear parton model and the quasi-multi-Regge kinematics approach using the factorization formalism of non-relativistic QCD at leading order in the strong-coupling constant α_s and the relative velocity v of the bound quarks. It is shown that the fusion production mechanism dominates over the fragmentation production mechanism at the all relevant J/ψ transverse momenta. The J/ψ meson p_T -spectra in the fragmentation and fusion production at the asymptotically large p_T have equal slopes in the quasi-multi-Regge kinematics approach, opposite the collinear parton model.

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I. INTRODUCTION

The phenomenology and the theory of processes involving the production of heavy quarkonia have been vigorously developed for last decade following the measurement of the transverse-momentum (p_T) spectra of prompt J/ψ mesons by the CDF Collaboration at the Tevatron Collider [1] and by the ZEUS and H1 Collaboration at the HERA Collider [2, 3]. The color-singlet model [4] previously proposed to describe a non-perturbative transition of a $Q\bar{Q}$ pair to final-state quarkonium was extended in a natural way within the formalism of nonrelativistic QCD (NRQCD) [5]. The extended model takes into account $Q\bar{Q}$ -pair production not only in the color-singlet state but also in the color-octet state.

The NRQCD formalism makes it possible to calculate consistently, by perturbation theory in two small parameters (strong coupling constant α_s at the scale of the heavy-quark mass and the relative velocity v of quarks in quarkonium), not only the parton cross sections for quarkonium production processes through the fusion of Q and \bar{Q} quarks but also the universal fragmentation functions for parton splitting into various quarkonium states [6].

Usually it is assumed that, in the region of large quarkonium transverse momenta ($p_T \gg M$, where M is the quarkonium mass), the fragmentation production mechanism is more adequate, because it takes into account effectively high order corrections in α_s , than the mechanism associated with the fusion of a heavy quark and a heavy antiquark produced in a hard subprocess, which is calculated in the lowest order (LO) in strong coupling constant α_s . However, the numerical value of the critical transverse momentum is unknown a priori.

In the energy region of the Tevatron and HERA Colliders the main contribution to the heavy quarkonium production cross sections comes from the gluon-gluon or the photon-gluon fusion at the small values of the argument x of the gluon distribution function. In the conventional collinear parton model [7], the initial-state gluon dynamics is controlled by the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation [8]. In this approach, it is assumed that $S > \mu^2 \gg \Lambda_{\text{QCD}}^2$, where \sqrt{S} is the invariant collision energy, μ is the typical energy scale of the hard interaction, and Λ_{QCD} is the asymptotic scale parameter of QCD. In this way, the DGLAP evolution equation takes into account only one big logarithm, namely $\ln(\mu/\Lambda_{\text{QCD}})$. In fact, the collinear-parton approximation is used, and the transverse momenta of the incoming gluons are neglected.

In the high-energy limit, the contribution from the partonic subprocesses involving t -

channel gluon exchanges to the total cross section can become dominant. The summation of the large logarithms $\ln(\sqrt{S}/\mu)$ in the evolution equation can then be more important than the one of the $\ln(\mu/\Lambda_{\text{QCD}})$ terms. In this case, the non-collinear gluon dynamics is described by the Balitsky-Fadin-Kuraev-Lipatov (BFKL) evolution equation [9]. In the region under consideration, the transverse momenta (k_T) of the incoming gluons and their off-shell properties can no longer be neglected, and we deal with reggeized t -channel gluons. The theoretical frameworks for this kind of high-energy phenomenology are the k_T -factorization approach [10, 11] and the (quasi)-multi-Regge kinematics (QMRK) approach [12, 13]. Last one is based on effective quantum field theory implemented with the non-abelian gauge-invariant action, as suggested a few years ago [14]. However, the k_T -factorization approach has well-known principal difficulties [15] at next-to-leading order (NLO). By contrast, the QMRK approach offers a conceptual solution of the NLO problems [16].

Our previous analysis of J/ψ meson production at the Fermilab Tevatron Colliders using the high-energy factorization scheme and the NRQCD formalism [17, 18, 19] has shown the dominant role of the color-octet intermediate state $^3S_1^{(8)}$ contribution at the large p_T region in the fusion production and in the fragmentation production with the gluon splitting. We found also the good agreement between the values of non-perturbative matrix element (NME) $\langle \mathcal{O}^{J/\psi} [^3S_1^{(8)}] \rangle$ obtained by the fit in collinear parton model and in the QMRK approach. The similar analysis for the J/ψ photoproduction at the high energy is the subject of study in this paper. The situation should be different because of the c -quark fragmentation via the color-singlet state dominates over the gluon fragmentation via the color-octet state in J/ψ photoproduction.

This paper is organized as follows. In Sec. II we discuss basic ideas and formulas of the fragmentation approach, including the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations for the c -quark and gluon fragmentation functions into J/ψ meson. In Sec. III we present relevant LO squared amplitude in the QMRK approach and master formulas for the differential cross sections. In Sec. IV we discuss our results for the J/ψ photoproduction at the HERA Collider. In Sec. V we summarize our conclusions.

II. FRAGMENTATION MODEL

Within the framework of the NRQCD, the fragmentation function for quarkonium H production can be expressed as a sum of terms, which are factorized into a short-distance coefficient and a long distant matrix element [5]:

$$D(a \rightarrow H) = \sum_n D(a \rightarrow Q\bar{Q}[n])\langle\mathcal{O}^H[n]\rangle. \quad (1)$$

Here the n denotes the set of color and angular momentum numbers of the $Q\bar{Q}$ pair, in which the fragmentation function is $D(a \rightarrow Q\bar{Q}[n])$. The last one can be calculated perturbatively in the strong coupling constant α_s . The non-perturbative transition of the $Q\bar{Q}$ pair into \mathcal{H} is described by the NMEs $\langle\mathcal{O}^H[n]\rangle$, which can be extracted from experimental data. To LO in v , we need to include the $c\bar{c}$ Fock states $n = {}^3S_1^{(1)}, {}^3S_1^{(8)}, {}^1S_0^{(8)}, {}^3P_J^{(8)}$ if $\mathcal{H} = J/\psi(\psi')$. The recipes of calculations for heavy quarkonium fragmentation functions at the initial scale $\mu = \mu_0 = 2m_c$ in LO NRQCD formalism are well known [6] and detailed formulas for the gluon and c -quark fragmentation via the different intermediate states are presented, for example, in Ref.[20].

To calculate the J/ψ production spectra, we need the fragmentation functions at the factorization scale $\mu \gg M$. Then, large logarithms in μ/M appear, which have to be resummed. It is achieved by using the DGLAP equations

$$\frac{\mu^2 d}{d\mu^2} D_{a \rightarrow H}(\xi, \mu) = \sum_b \int_\xi^1 \frac{dx}{x} P_{ba}\left(\frac{\xi}{x}, \alpha_s(\mu)\right) D_{b \rightarrow H}(x, \mu), \quad (2)$$

where $P_{ba}\left(\frac{\xi}{x}, \alpha_s(\mu)\right)$ are the timelike splitting functions of parton a into parton b . The initial conditions $D_{a \rightarrow H}(\xi, \mu_0)$ for $a = c, g$ and $H = J/\psi(\psi')$ are taken from Ref. [20]. The initial light-quark fragmentation functions are set equal to zero. They are generated at larger scale μ . However, in the case of J/ψ production at the HERA Collider their contribution is very small. Here, $a = g, c, \bar{c}$. To illustrate the ξ and μ dependence of the relevant fragmentation functions we show in Figs. 1 and 2 the c -quark and the gluon fragmentation functions at the different values of μ . We use LO in α_s approximation for the splitting functions. The dashed curves are obtained neglecting the $c - g$ coupling during the evolution ($a = c$ or $a = g$), the solid curves are obtained using the system (2), which takes into account cross transitions between gluons and c -quarks during the evolution.

III. BASIC FORMULAS

Because we take into account the gluon and c -quark fragmentation into the J/ψ mesons, we need to sum contributions from LO partonic subprocesses in which these ones are produced.

In the collinear parton model we take into consideration the following subprocesses:

$$\gamma + g \rightarrow c + \bar{c}, \quad (3)$$

$$\gamma + q(\bar{q}) \rightarrow g + q(\bar{q}), \quad (4)$$

where $q = u, d, s$. The squared amplitudes of the subprocesses (3) and (4) are well known.

In the case of QMRK approach we have only one dominant LO partonic subprocess:

$$\gamma + R \rightarrow c + \bar{c}, \quad (5)$$

which is described by the Feynman diagrams shown in Fig. 3. Here R is the reggeized gluon. Accordingly [13], amplitude of the subprocess (5) can be presented as follows:

$$\begin{aligned} \mathcal{A}(\gamma + R \rightarrow c + \bar{c}) = & -ig_s e_c e T^a \bar{U}(p_1, m_c) \left[\gamma^\alpha \frac{\hat{p}_1 - \hat{q}_1 + m_c}{(p_1 - q_1)^2 - m_c^2} \gamma^\beta + \right. \\ & \left. + \gamma^\beta \frac{\hat{p}_1 - \hat{q}_2 + m_c}{(p_1 - q_2)^2 - m_c^2} \gamma^\alpha \right] V(p_2, m_c) \varepsilon^\alpha(q_1) (n^-)^\beta, \end{aligned} \quad (6)$$

where $(n^-)^\mu = P_N^\mu / E_N$, $P_N^\mu = E_N(1, 0, 0, 1)$ is the four-momentum of the proton, E_N is the energy of the proton, $\varepsilon^\alpha(q_1)$ is the polarization four-vector of the photon. For any four-momentum k^μ , we define $k^- = (k \cdot n^-)$. Note, that reggeized gluon four-momentum can be written as

$$q_2^\mu = x_2 E_N (n^-)^\mu + q_{2T}^\mu, \quad (7)$$

where $q_{2T}^\mu = (0, \mathbf{q}_{2T}, 0)$, $q_2^2 = q_{2T}^2 = -\mathbf{q}_{2T}^2$ and x_2 is the fraction of the proton longitudinal momentum passed on to the reggeized gluons.

The squared amplitude obtained from (6) reads:

$$\overline{|\mathcal{A}(\gamma + R \rightarrow c + \bar{c})|^2} = \frac{64\pi^2 e_c^2 \alpha_s \alpha}{\tilde{t}^2 \tilde{u}^2} [2m_c^2 (\tilde{t} p_1^- - \tilde{u} p_2^-)^2 + \mathbf{q}_{2T}^2 \tilde{u} \tilde{t} ((p_1^-)^2 + (p_2^-)^2)], \quad (8)$$

where the bar indicates average (summation) over initial-state (final-state) spins and colors, $\tilde{u} = \hat{u} - m_c^2$, $\tilde{t} = \hat{t} - m_c^2$, $\hat{u} = (q_1 - p_2)^2$, $\hat{t} = (q_1 - p_1)^2$, $\alpha = e^2/4\pi$, $\alpha_s = g_s^2/4\pi$, $e_c = +2/3$. The obtained result (8) satisfies gauge invariant conditions,

$$\lim_{\mathbf{q}_{2T}^2 \rightarrow 0} |\mathcal{A}(\gamma + R \rightarrow c + \bar{c})|^2 = 0,$$

and agrees with the k_T -factorization formula from Ref. [25] if we take into account relation between amplitudes in the QMRK and the k_T -factorization approach [24]:

$$|\mathcal{A}_{KT}(\gamma + R \rightarrow c + \bar{c})|^2 = \frac{(x_2 E_N)^2}{\mathbf{q}_{2T}^2} |\mathcal{A}(\gamma + R \rightarrow c + \bar{c})|^2. \quad (9)$$

At HERA, the cross section of prompt J/ψ production was measured in a wide range of the kinematic variables $W^2 = (P_N + q_1)^2$, $Q^2 = -q_1^2$, $x_1 = (P_N \cdot q_1)/(P_N \cdot k)$, $z = (P_N \cdot p)/(P_N \cdot q_1)$, and p_T , where P_N^μ , k^μ , k'^μ , $q_1^\mu = k^\mu - k'^\mu$, and p^μ are the four-momenta of the incoming proton, incoming lepton, scattered lepton, virtual photon, and produced J/ψ meson, respectively, both in photoproduction [2], at small values of Q^2 , and deep-inelastic scattering (DIS) [3], at large values of Q^2 . At sufficiently large values of Q^2 , the virtual photon behaves like a point-like object, while, at low values of Q^2 , it can either act as a point-like object (direct photoproduction) or interact via its quark and gluon content (resolved photoproduction). Resolved photoproduction is only important at low values of z and p_T . The subject of our study is large p_T photoproduction processes at the $Q^2 \approx 0$ only.

In the region of $p_T \gg m_c$ we consider c -quark and J/ψ -meson as massless particles, so that the fragmentation parameter ξ is related to the c -quark and J/ψ -meson four-momenta as follows:

$$p^\mu = \xi p_1^\mu. \quad (10)$$

We only used non-zero c -quark mass in the initial factorization scale $\mu_0 = M = 2m_c$ and, correspondingly, in the definition of the $\mu = \sqrt{p_T^2 + 4m_c^2}$.

Let us first present the relevant formula for the double differential cross sections of J/ψ direct photoproduction in the collinear parton model via the partonic subprocess (3):

$$\frac{d\sigma^{ep}}{dz dp_T^2} = \int_{x_{1,min}}^1 dx_1 f_{\gamma/e}(x_1) \int_{\xi_{min}}^1 \frac{d\xi}{\xi} D_{c \rightarrow J/\psi}(\xi, \mu) \frac{|\mathcal{M}(\gamma + g \rightarrow c + \bar{c})|^2 x_2 G(x_2, \mu)}{16\pi z(\xi - z)(x_1 x_2 S)^2}, \quad (11)$$

where

$$x_2 = \frac{p_T^2}{z x_1 S(\xi - z)}, \quad x_{1,min} = \frac{p_T^2}{z S(1 - z)}, \quad \xi_{min} = z + \frac{p_T^2}{x_1 z S},$$

$G(x_2, \mu)$ is the gluon collinear distribution function in the proton, for which we use numerical CTEQ parameterization [21] and $f_{\gamma/e}(x_1)$ is the quasi-real photon flux. In the Weizäcker-Williams approximation, the latter takes the form

$$f_{\gamma/e}(x_1) = \frac{\alpha}{2\pi} \left[\frac{1 + (1 - x_1)^2}{x_1} \ln \frac{Q_{\max}^2}{Q_{\min}^2} + 2m_e^2 x_1 \left(\frac{1}{Q_{\min}^2} - \frac{1}{Q_{\max}^2} \right) \right], \quad (12)$$

where $Q_{\min}^2 = m_e^2 x_1^2 / (1 - x_1)$ and Q_{\max}^2 is determined by the experimental set-up, *e.g.* $Q_{\max}^2 = 1 \text{ GeV}^2$ [2].

The formulae in the case of the partonic subprocess (4) looks like (11). The J/ψ mesons are produced also via decays of excited charmonium states and B -mesons. In fact, only the contribution of the decays $\psi' \rightarrow J/\psi X$ is large at the relevant kinematic condition. In the approximation used here we can estimate this one simply:

$$d\sigma(ep \rightarrow J/\psi X) = d\sigma^{\text{direct}}(ep \rightarrow J/\psi X) + d\sigma^{\text{direct}}(ep \rightarrow \psi' X) \text{Br}(\psi' \rightarrow J/\psi X), \quad (13)$$

where $\text{Br}(\psi' \rightarrow J/\psi X)$ is the inclusive decay branching fractions, $d\sigma^{\text{direct}}(ep \rightarrow \psi' X)$ can be obtained formally from $d\sigma^{\text{direct}}(ep \rightarrow J/\psi X)$ after replacement $\langle \mathcal{O}^{J/\psi}[n] \rangle \rightarrow \langle \mathcal{O}^{\psi'}[n] \rangle$.

In the QMRK approach the double differential cross sections of J/ψ direct photoproduction reads:

$$\begin{aligned} \frac{d\sigma^{ep}}{dz dp_T^2} &= \frac{1}{8(2\pi)^2} \int d\phi_2 \int \frac{d\mathbf{q}_{2T}^2}{\mathbf{q}_{2T}^2} \int dx_1 f_{\gamma/e}(x_1) \int \frac{d\xi}{\xi} D_{c \rightarrow J/\psi}(\xi, \mu) \times \\ &\times \frac{(x_2 E_N)^2 \Phi(x_2, \mathbf{q}_{2T}^2, \mu)}{z(\xi - z)(x_1 x_2 S)^2} |\mathcal{A}(\gamma + R \rightarrow c + \bar{c})|^2, \end{aligned} \quad (14)$$

where ϕ_2 is the angle between \mathbf{q}_{2T} and the transverse momentum \mathbf{p}_T of the J/ψ meson, $\Phi(x_2, \mathbf{q}_{2T}^2, \mu)$ is the unintegrated reggeized gluon distribution function. The collinear and the unintegrated gluon distribution functions are formally related as:

$$x_2 G(x_2, \mu) = \int_0^{\mu^2} d\mathbf{q}_{2T}^2 \Phi(x_2, \mathbf{q}_{2T}^2, \mu), \quad (15)$$

so that the normalizations of Eqs. (14) and (11) agree. In our numerical calculations, we use the unintegrated gluon distribution function by Kimber, Martin, and Ryskin (KMR) [22], which gives the best description in the framework of QMRK approach [17, 24] of the heavy quarkonium production spectra measured at the Tevatron [1, 23].

To compare predictions of the fragmentation and the fusion mechanisms we need to calculate J/ψ photoproduction cross section using the fusion model too. In the case of the LO collinear parton model calculation we use squared amplitudes for the color-singlet contribution from Ref. [4] and for the color-octet contribution from Ref. [26]. Working in the QMRK approach with the fusion model we use analytical results obtained in Refs. [17, 28]. All numerical values of parameters are the same as in the Ref. [17].

IV. RESULTS

We now present and discuss our numerical results.

Let us compare J/ψ meson p_T spectra obtained using the fragmentation and fusion models within the QMRK approach. As it was shown recently [17, 28], we can describe the experimental data from HERA Collider [2, 3] in the fusion model without the color-octet contribution, g.e. in the color-singlet model [4] (see the curve 1 in the Fig. 4). The agreement is good for the all measured J/ψ transverse momenta up to $p_T^2 = 50 \text{ GeV}^2$. The prediction of the fragmentation model is shown in Fig. 4 for $p_T^2 > 10 \text{ GeV}^2$ only (the curve 2). We see that the curve 2 tends to the curve 1 as p_T^2 increases but it is being lower at all relevant p_T^2 till $p_T^2 = 10^3 \text{ GeV}^2$. The result is the same as in the hadronic J/ψ production [17, 18] although in the photoproduction the c -quark fragmentation via the color-singlet state $^3S_1^{(1)}$ is dominant, oppocite to the hadroproduction where the gluon fragmentation via the color-octet state $^3S_1^{(8)}$ is dominant. So, we see that in the QMRK approach the slope of the p_T spectra sufficiently depends on the initial reggeized gluon transverce momentum, which is ordered by the unintegrated distribution function, and both production mechanisemes, the fusion and the fragmentation, have similar p_T -slopes at the large transverse momentum of the J/ψ meson.

The situation in the relevant LO calculations within the collinear parton model was studied earlier in Refs. [20, 27]. It was shown that at approximately $p_T^2 \geq 100 \text{ GeV}^2$ the contribution of the c -quark fragmentation production exceeds the contribution of the fusion mechanism with the color-singlet LO amplitudes. Note, that in both of the discussed approaches, the fragmentation and the fusion within the CSM in LO, do not describe data. However, if we take into account the color-octet contribution, with the values of the color-octet NMEs fixed at the Tevatron, we see that the fusion model describes data well, as it is shown in Fig. 5 (the curve 1). The contribution of the fragmentation model increases too, because of the additional piece coming from the gluon fragmentation via $^3S_1^{(8)}$ state in the partonic subprocesses $\gamma + q \rightarrow g + q$ (the curve 6 in Fig. 5). Moreover, at $p_T^2 > 500 \text{ GeV}^2$ the gluon fragmentation dominates over the c -quark fragmentation (the curve 5), but their sum (the curve 4) is still less than the fusion model prediction in the NRQCD approach up to $p_T^2 = 2 \cdot 10^3 \text{ GeV}^2$.

For completeness, we present here the z -spectra of J/ψ calculated in the QMRK ap-

proach (Fig. 6) and in the collinear parton model (Fig. 7) with the experimental data from HERA Collider at the $p_T > 3$ GeV [3]. We see that in both approaches the fusion mechanism is dominant for all z , excepting the region of very small $z < 0.1$. It is well known that at small z the resolved J/ψ meson photoproduction becomes important. However, it is evident that in the resolved photoproduction the relation between the fusion and fragmentation mechanisms will be the same as in the direct photoproduction.

V. CONCLUSIONS

It was shown that working within NRQCD both in the collinear parton model and the QMRK approach, we have predicted the dominant role of the fusion mechanism as compared to the fragmentation one in the J/ψ meson photoproduction at all relevant values of charmonium transverse momenta. Therefore, it is impossible to extract the c -quark or gluon fragmentation function into J/ψ meson from the photoproduction data similar as in the hadroproduction processes, as it was shown previously [18]. Opposite to the assumption in the collinear factorization, that the fragmentation contribution is enhanced by powers of (p_T^2/m_c^2) relative to the contribution of the fusion model, we have demonstrated that such an enhancement is absent in the non-collinear factorization approach, QMRK. We have obtained equal slopes of the p_T -spectra for both production mechanisms at the asymptotically large p_T in the QMRK approach.

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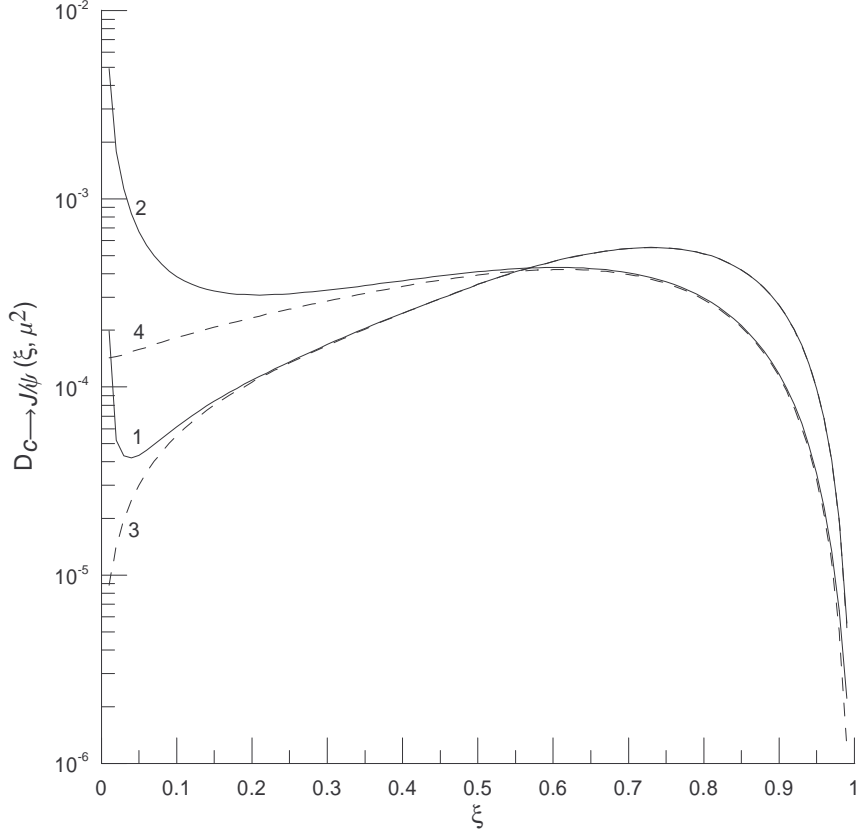


FIG. 1: The fragmentation function $D_{c \rightarrow J/\psi}(z, \mu^2)$ at the $\mu^2 = 10 \text{ GeV}^2$ (the curves 1 and 3) and $\mu^2 = 300 \text{ GeV}^2$ (the curves 2 and 4). The solid lines are results of Eqs. (2), the dashed are obtained neglecting the $c - g$ coupling.

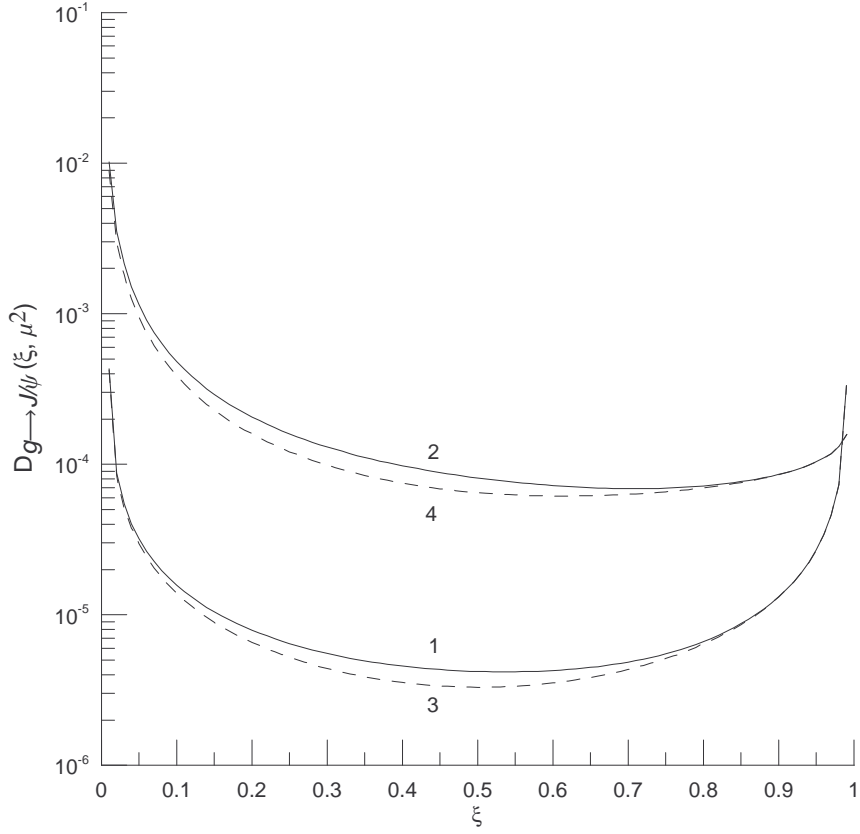


FIG. 2: The fragmentation function $D_{g \rightarrow J/\psi}(z, \mu^2)$ at the $\mu^2 = 10 \text{ GeV}^2$ (the curves 1 and 3) and $\mu^2 = 300 \text{ GeV}^2$ (the curves 2 and 4). The solid lines are results of Eqs. (2), the dashed are obtained neglecting the $c - g$ coupling.

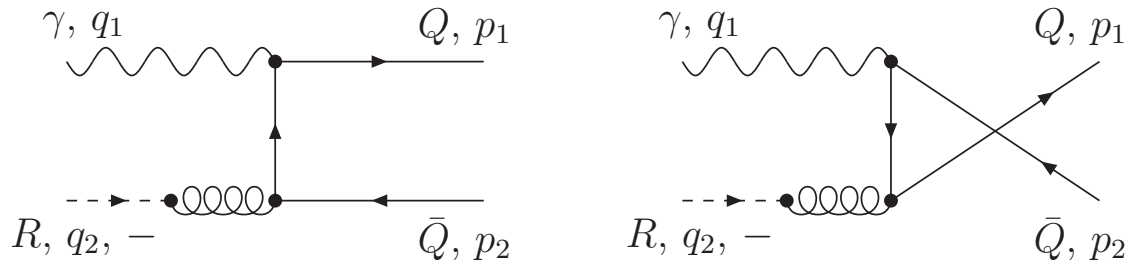


FIG. 3: The Feynman diagrams of the process $\gamma + R \rightarrow c + \bar{c}$

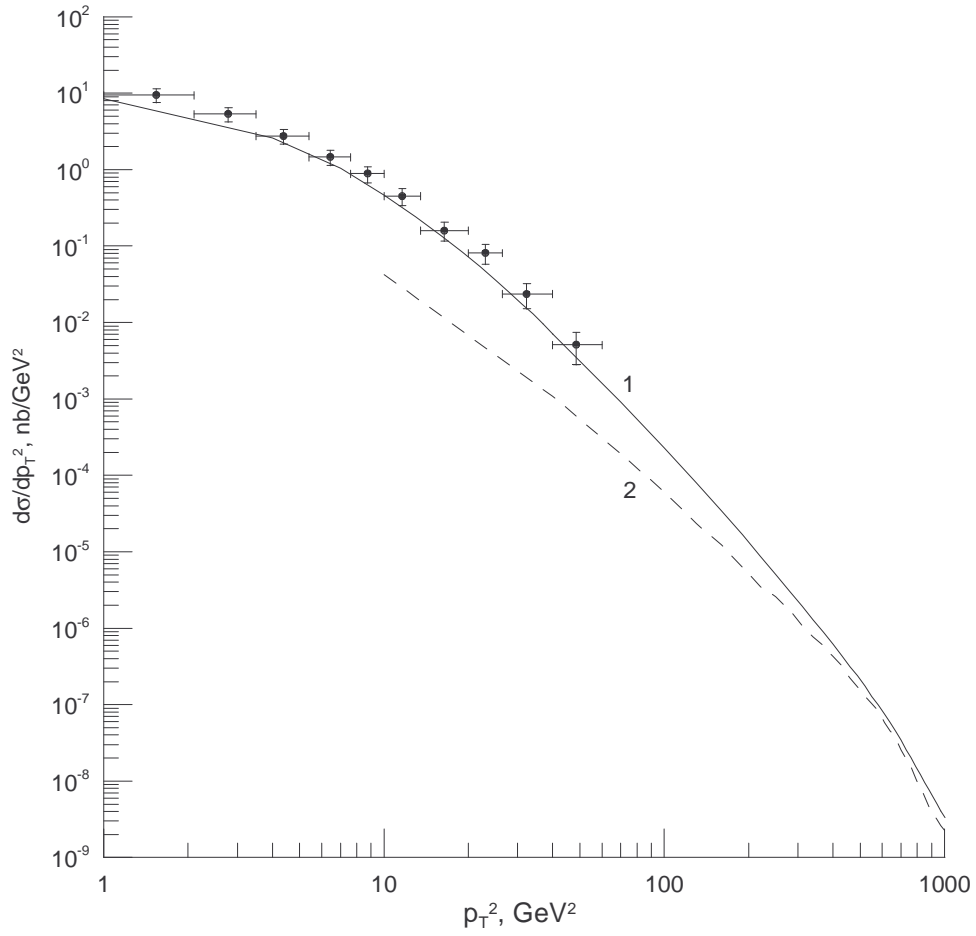


FIG. 4: p_T spectrum of prompt J/ψ in ep scattering with $E_p = 820$ GeV, $E_e = 27.5$ GeV, $60 \text{ GeV} < W < 240$ GeV, $Q^2 < 1 \text{ GeV}^2$, and $0.3 < z < 0.9$. The fusion production mechanism (curve 1), and the fragmentation production mechanism (curve 2) in the QMRK approach.

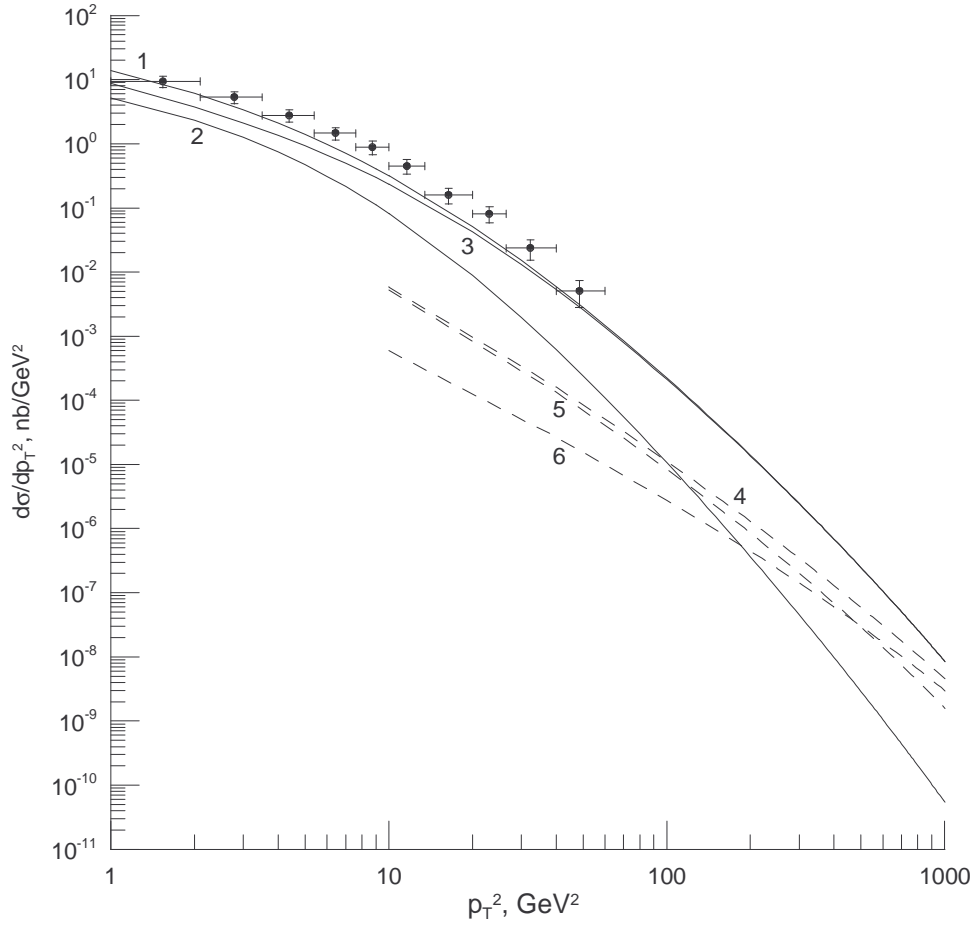


FIG. 5: p_T spectrum of prompt J/ψ in ep scattering with $E_p = 820$ GeV, $E_e = 27.5$ GeV, 60 GeV $< W < 240$ GeV, $Q^2 < 1$ GeV 2 , and $0.3 < z < 0.9$. The total contribution of the fusion mechanism (curve 1), the color-singlet part of the fusion mechanism (curve 2), the color-octet part of the fusion mechanism (curve 3), the total contribution of the fragmentation mechanism (curve 4), the color-singlet part of the fragmentation mechanism (curve 5), the color-octet part of the fragmentation mechanism (curve 6).

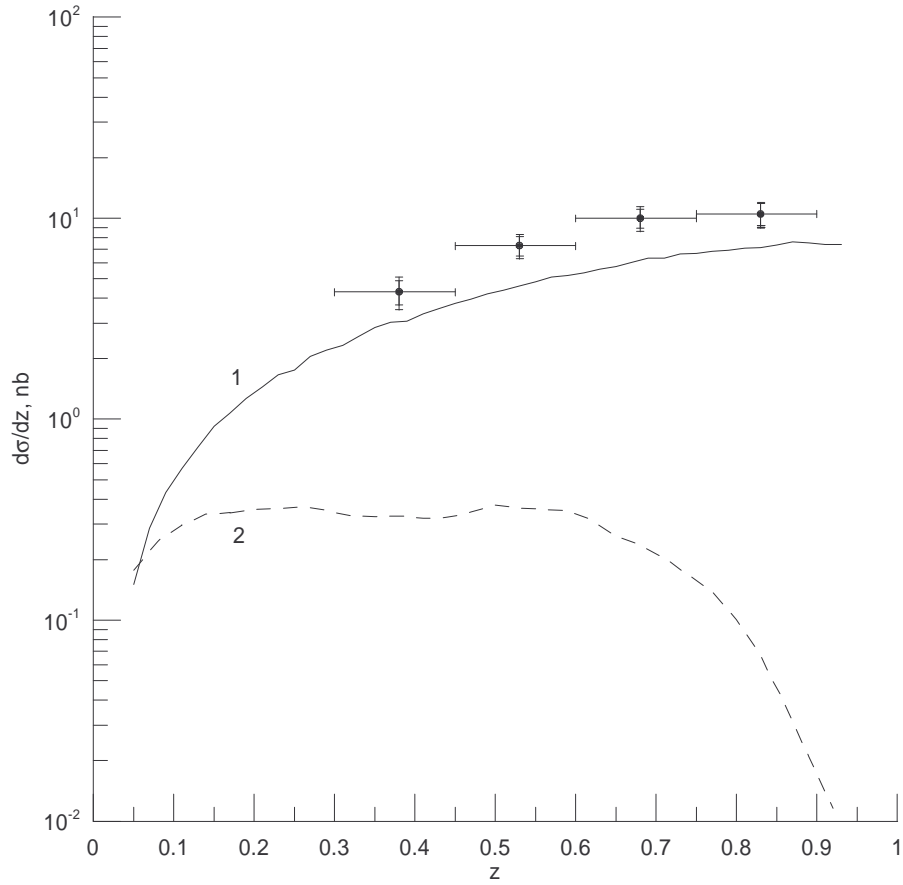


FIG. 6: z spectrum of prompt J/ψ in ep scattering with $E_p = 820$ GeV, $E_e = 27.5$ GeV, $60 \text{ GeV} < W < 240$ GeV, $Q^2 < 1 \text{ GeV}^2$, and $p_T > 3$ GeV. The fusion production mechanism (curve 1), and the fragmentation production mechanism (curve 2) in the QMRK approach

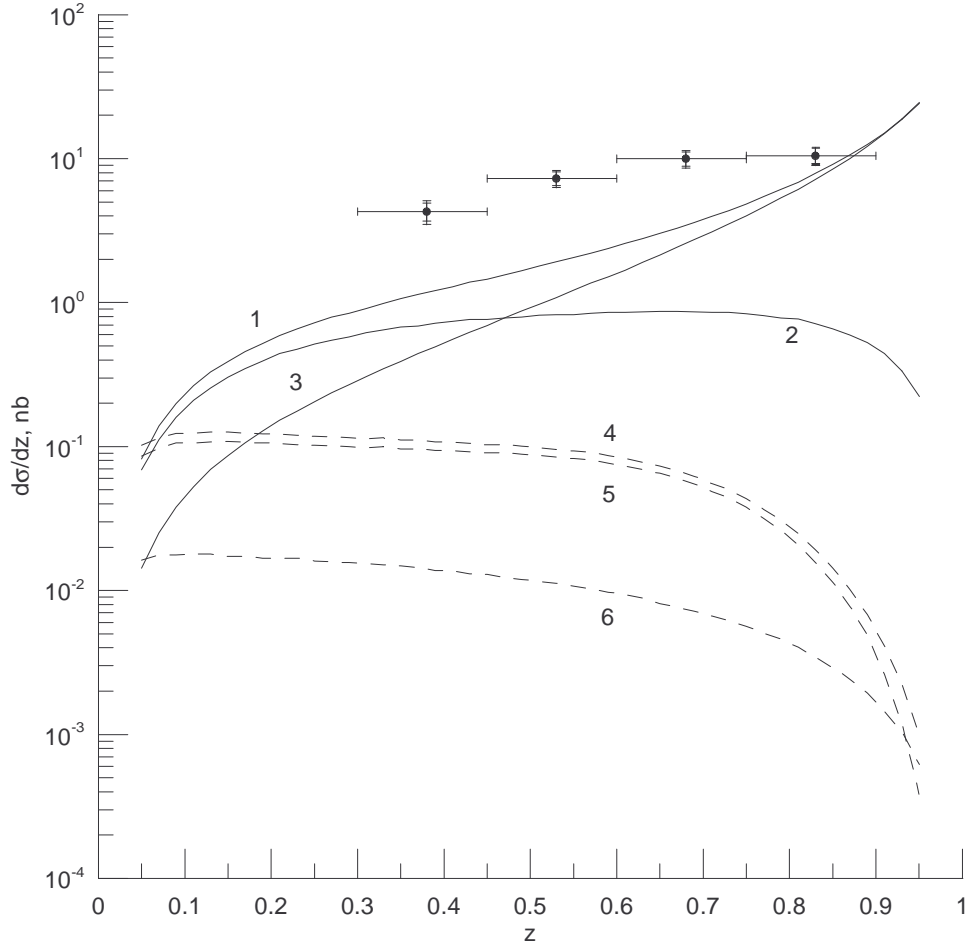


FIG. 7: z spectrum of prompt J/ψ in ep scattering with $E_p = 820$ GeV, $E_e = 27.5$ GeV, 60 GeV $< W < 240$ GeV, $Q^2 < 1$ GeV 2 , and $p_T > 3$ GeV. The total contribution of the fusion mechanism (curve 1), the color-singlet part of the fusion mechanism (curve 2), the color-octet part of the fusion mechanism (curve 3), the total contribution of the fragmentation mechanism (curve 4), the color-singlet part of the fragmentation mechanism (curve 5), the color-octet part of the fragmentation mechanism (curve 6).