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Determining Heavy Mass Parameters in Supersymmetric SO(10) Models

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Abstract

Extrapolations of soft scalar mass parameters in supersymmetric theories can be used to explore elements of the physics scenario near the grand unification scale. We investigate the potential of this method in the lepton sector of SO(10) which incorporates right-handed neutrino superfields. The method is exemplified in two models by exploring limits on the precision that can be expected from coherent LHC and e^+e^- collider analyses in the reconstruction of the fundamental scalar mass parameters at the unification scale and of the D-terms related to the breaking of grand unification symmetries. In addition, the mass of the third-generation right-handed neutrino can be estimated in seesaw scenarios. Even though the models are simplified and not intended to account for all aspects of a final comprehensive SO(10) theory, they provide nevertheless a valid base for identifying essential elements that can be inferred on the fundamental high-scale theory from high-energy experiments.

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1 Introduction

The observation of neutrino oscillations has provided experimental proof for non-zero neutrino masses [1]. When right-handed neutrinos, not carrying any Standard Model gauge charges, are included in the set of leptons and quarks, the symmetry group SO(10) is naturally suggested as the grand unification group [2]. For theories formulated in a supersymmetric framework to build a stable bridge between the electroweak scale and the Planck scale, a scalar R-neutrino superfield is added to the spectrum of the minimal supersymmetric standard model.

A natural explanation of the very light neutrino masses in relation to the electroweak scale is offered by the seesaw mechanism [3]. For right-handed Majorana neutrino masses $M_{\nu_{Ri}}$ in a range close to the grand unification (GUT) scale, small neutrino masses can be generated quite naturally by this mechanism: $m_{\nu_i} \sim m_{q_i}^2/M_{\nu_{Ri}}$, with m_{q_i} denoting up-type quark masses. In parallel, the R-sneutrino masses are very heavy too.

In the present analysis we will first focus on a simple model incorporating one-step symmetry breaking from SO(10) down to the Standard Model SM,

$$\text{(I)}\quad \mathrm{SO}(10) \underset{\Lambda_{\mathcal{U}}}{\rightarrow} \mathrm{SM}$$

with $\Lambda_{\mathcal{U}} \approx 2 \cdot 10^{16}$ GeV denoting the usual GUT scale with apparent unification of the SM gauge couplings in supersymmetric theories. While it is not intended to account for all aspects such a model must finally cover, we will concentrate on the analysis of a few key points expected to be characteristic for results on the comprehensive SO(10) theory from precision analyses at high-energy colliders. The report expands on earlier work in Ref. [4] by including a systematic analysis of neutrino mixing.

In a subsequent section we will add the analysis of a specific two-step breaking chain, *cf.* Ref. [5],

$$\text{(II)}\quad \mathrm{SO}(10) \underset{\Lambda_{\mathcal{O}}}{\rightarrow} \mathrm{SU}(5) \underset{\Lambda_{\mathcal{U}}}{\rightarrow} \mathrm{SM}\,,$$

with $\Lambda_{\mathcal{O}} > \Lambda_{\mathcal{U}}$ denoting the SO(10) breaking scale and $\Lambda_{\mathcal{U}}$ fixed again by the unification scale of the gauge couplings. Though this extension will be formulated in a simple scheme, it confronts us with problems in phenomenological analyses of SO(10) scenarios more strongly than the one-step scheme. Nevertheless, this hypothetical chain may serve as an interesting

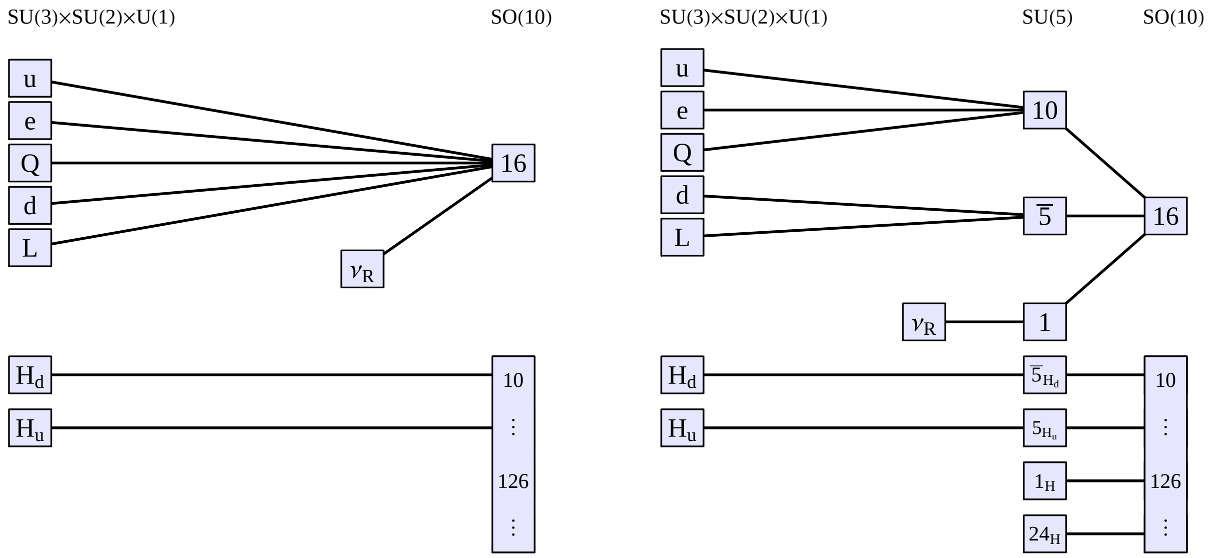

Figure 1: *Potential decompositions of SO(10) representations,* Left: *one-step breaking* $SO(10) \rightarrow SU(3) \times SU(2) \times U(1)$*;* Right: *two-step breaking* $SO(10) \rightarrow SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$*. The dots denote Higgs fields for generating masses of matter fields and breaking the SO(10) gauge symmetry in addition to those which are indicated explicitly in standard notation.*

example for elucidating to what extent the high complexity encountered in SO(10) scenarios can be controlled eventually.

(I) In the first, one-step scenario the scalar soft SUSY breaking sector, *cf.* Fig. 1(left), is parametrized by the gravity induced mass parameters for the matter superfields and for the Higgs superfields at the unification scale $\Lambda_{\mathcal{U}}$. Two Higgs fields generate masses separately for up- and down-type fields at the electroweak scale. In general they evolve into a superposition of iso-doublet components in an ensemble of Higgs fields at the grand unification scale [6,7]. An extended ensemble of Higgs fields in large representations is needed in renormalizable theories to account for the mass matrices of all three generations, *cf.* Refs. [6–9] [Generally, though, this does not solve the fine-tuning problem of the light Higgs fields]. This expansion however is not necessarily required for the approximate description of the third generation which dominates our analysis as a result of the large Yukawa couplings, and we will assume the standard Higgs-10 field to be dominant. Alternatively the Higgs sector may be supplemented by Planck-scale suppressed operators to account for mass spectra and mixing phenomena [10, 11]. In the present context the mechanism for generating the masses of the right-handed Majorana neutrinos needs not be specified in detail as long as the scale is close to the grand unification scale. A popular choice is a Higgs-126 field with small couplings to the ν_R fields [12].

The scalar mass parameters are assumed universal:

$$m_{16} = m_{10} = ... = M_0 \,. \tag{1}$$

The GUT scale $\Lambda_{\mathcal{U}}$ is defined technically by the minimum distance of the gauge couplings within the triangle built by the running couplings near the unification point. At $\Lambda_{\mathcal{U}}$ the soft scalar mass parameters are shifted, non-universally, by D-terms associated with the SO(10) breaking to the lower-rank SM group [13,14]. Starting at $\Lambda_{\mathcal{U}}$, the mass parameters evolve, following the renormalization group (RG) [15], down to the electroweak scale. They define the Lagrangian parameters at the supersymmetry scale $\tilde{M}$, chosen at 1 TeV according to the SPA convention [16]. The observed masses of sleptons and squarks, charginos/neutralinos and Higgs bosons can be expressed in terms of these Lagrangian parameters [17,18]. Once the masses are measured, the RG evolution from the Terascale upwards will allow us to reconstruct the physics scenario at the GUT scale [4,19,20].

For the mass parameters of matter fields in the first two generations and the Higgs field H_d the RG flow from the Terascale to the GUT scale is, effectively, not interrupted by any intermediate thresholds. The first two generations and the Higgs H_d can therefore be exploited to extract the scalar GUT mass parameters m_{16} and m_{10}, and the D-term. By contrast, the running of the mass parameters in the third generation and of the second H_u Higgs mass parameter are affected by Yukawa interactions at the intermediate seesaw scale [4,20,21]. The resolution of ν_R-higgsino and $\tilde{\nu}_R$-Higgs loops gives rise to kinks in the evolution of the iso-doublet scalar L-mass parameter and the H_u Higgs mass parameter. Linking the measured $\tilde{\tau}$ and $\tilde{\nu}_\tau$ masses to the GUT parameter m_{16}, assumed universal, determines the position of the kink and thus allows us to measure the R-neutrino mass, i.e. the seesaw scale of the third generation.

(II) In the second scenario the GUT symmetry is broken down to the Standard Model in two consecutive steps, *cf.* Fig. 1(right). At the scale $\Lambda_{\mathcal{O}} > \Lambda_{\mathcal{U}}$ the SO(10) symmetry breaks to SU(5). Apart from the D-terms, the scalar mass parameter m_{16} splits at $\Lambda_{\mathcal{O}}$ to three separate parameters m_{10}, $m_{\bar{5}}$ and m_1 according to the decomposition of the matter multiplet $16 = 10 + \bar{5} + 1$. At the scale $\Lambda_{\mathcal{U}}$ the SU(5) gauge symmetry is broken to the SM symmetry by a $\{24\}$ Higgs field, for example. At $\Lambda_{\mathcal{U}}$ the scalar SU(5) mass parameters finally split to the MSSM parameters evolving down to the Terascale. To simplify the analysis, we assume all the Higgs mass parameters to be degenerate with m_{16} at $\Lambda_{\mathcal{O}}$. Such an assumption can be motivated by string theories, in which the scales Λ_{str} and $\Lambda_{\mathcal{O}}$ are identified [22]. The evolution between $\Lambda_{\mathcal{U}}$ and the Terascale is driven by *a priori* experimental information, modulo the effect of the heavy R-neutrino mass parameter of the third generation. By contrast, the evolution from $\Lambda_{\mathcal{O}} \to \Lambda_{\mathcal{U}}$ depends on the field content of the high-scale SU(5) theory. To determine, *a posteriori*, such elements of the physics scenario at the high scales by means of renormalization group techniques is the prime target of extrapolations from the experimentally accessible Terascale.

The analyses assume, implicitly, that a number of fundamental problems [23] are solved without interfering strongly with the present parametric analysis, *i.e.* mechanisms leading to doublet-triplet splitting and suppressing proton decay. Solutions, partial in general, are approached by means of extended Higgs sectors. Extended Higgs systems have also been shown to accommodate the fermion mass patterns in renormalizable theories, *cf.* Refs. [6–9]. Alternatively the Higgs system is supplemented by higher-dimensional, Planck-scale suppressed operators, *cf.* Refs. [10, 11]. In addition, if the survival principle is violated [24], potential contributions of intermediate states have to be taken into account. When these mechanisms are specified, also parametrically, it is straightforward to transcribe the results we will derive to any such system provided the additional interactions, giving rise to threshold effects on the observables, can be treated *ad hoc* as a perturbation. If not, the evolution of the scalar masses must be reformulated by including the new degrees of freedom from the beginning. Once such a system is defined properly, the analysis can be performed strictly in parallel to the procedure elaborated in the present paper.

Though only a limited set of elements in the high-scale scenario can be focused on, the potential of such analyses in exploring the high-scale scenario can nevertheless be elucidated. Refinements and modifications can be incorporated if the theoretical frame for solving the problems mentioned above, is specified *in toto*. In this context, the results presented can serve as a crucial intermediate step for the analysis of a comprehensive SO(10) theory.

Given the multitude of potential physics scenarios near the Planck scale, these experimental projections – supplemented by other observations in the neutrino sector, proton decay, lepton flavor violating processes, and cosmological observations – will be a valuable ingredient for reconstructing the fundamental high-scale theory.

2 One-Step SO(10) $\to$ SM Breaking

In the SO(10) model which we will analyze, the matter superfields of the three generations belong to 16-dimensional representations of SO(10) and the standard Higgs superfield coupling to the up-type matter fields of the third generation is embedded in a Higgs-10 field at the unification scale. No reference is needed to the Yukawa couplings of the first two generations. The mass of the heavy R-neutrino superfields may be generated by a 126-dimensional Higgs field though the detailed mechanism actually needs not be specified as long as the characteristic scale of the mechanism is connatural to the unification scale.

In this set-up the Yukawa couplings in the neutrino sector coincide with the Yukawa couplings in the up-type quark matrix. Assuming the normal hierarchy for the light neutrino masses and approximate tri-bimaximal mixing, as suggested experimentally, the texture of the heavy Majorana mass matrix is predicted within the seesaw mechanism. In this framework the evolution of the soft scalar slepton mass parameters can be predicted from the unification scale down to the electroweak scale. Since the right-handed neutrino fields are neutral under the SM gauge group, they merely affect the evolution by Yukawa interactions which are sufficiently large only in the L-sector of the third generation as well as the H_u Higgs sector.

The scalar mass parameters m_{16} and the Higgs parameters m_{10} and $m_{10'}$ at the unification scale will be assumed universal, *cf.* Eq. (1). However, the breaking of the rank-5 SO(10) symmetry group to the lower rank-4 SM group generates GUT D-terms $D_{\mathcal{U}}$ such that the boundary conditions at the GUT scale finally read [5, 25] for the matter fields,

$$\begin{aligned} m_L^2 &= M_0^2 - 3D_{\mathcal{U}} \\ m_E^2 &= M_0^2 + D_{\mathcal{U}} \\ m_R^2 &= M_0^2 + 5D_{\mathcal{U}}\,, \end{aligned} \tag{2}$$

and for the Higgs fields,

$$\begin{aligned} m_{H_d}^2 &= M_0^2 + 2D_{\mathcal{U}} \\ m_{H_u}^2 &= M_0^2 - 2D_{\mathcal{U}}\,. \end{aligned} \tag{3}$$

The L-isodoublet, the charged R-isosinglet, the neutral R-isosinglet and the two Higgs scalar mass parameters, are denoted by $m_L, m_E, m_R, m_{H_{d,u}}$, respectively. It can be shown on general grounds that the D-term is of the order of the soft SUSY breaking masses of the fields responsible for the spontaneous breaking times the gauge coupling squared [14],

$$D_{\mathcal{U}} \sim g^2_{SO(10)}\, \mathcal{O}(M_0^2)\,, \tag{4}$$

while the detailed form depends on the specific SO(10) breaking mechanism. Not specifying the structure of this component of the Higgs sector, we will treat $D_{\mathcal{U}}$ as a free parameter. While the coefficients of the D-terms for the matter parameters are fixed uniquely, they are in general model-dependent for the Higgs fields [9], which however play a minor role in our analysis. The evolution of the scalar masses $m^2_{L,E}$ from the unification scale down to the Terascale scale is determined by lepton-gaugino and slepton-gauge loops, complemented by R-neutrino-higgsino loops *etc.* in the third generation.

a) The neutrino sector:

Neglecting minor higher-order effects in the calculation of the Majorana neutrino mass matrix, it follows from the Higgs-10 SO(10) relation

$$Y_\nu = Y_u \tag{5}$$

between the neutrino and up-type quark Yukawa matrices that

$$Y_\nu \approx \mathrm{diag}(m_u, m_c, m_t)/v_u \tag{6}$$

holds approximately for the neutrino Yukawa matrix; $v_u = v \sin\beta$, with v and $\tan\beta$ being the familiar vacuum and mixing parameters in the Higgs sector. The quark masses are defined at the scale $\Lambda_{\mathcal{U}}$. The identity Eq. (5) is assumed for the third generation while the more complex mass pattern in the first two generations is essentially ineffective in the numerical analysis. Quark mixing and RG running effects in the neutrino sector are neglected in the analytical approach but properly taken into account in the numerical analysis.

The effective mass matrix of the light neutrinos is constrained by the results of the neutrino oscillation experiments:

$$m_\nu = U^*_{\mathrm{MNS}} \cdot \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) \cdot U^\dagger_{\mathrm{MNS}}. \tag{7}$$

We will assume the normal hierarchy for the light neutrino masses m_{ν_i}, and for the MNS mixing matrix the tri-bimaximal form

$$U_{\mathrm{MNS}} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \end{pmatrix}. \tag{8}$$

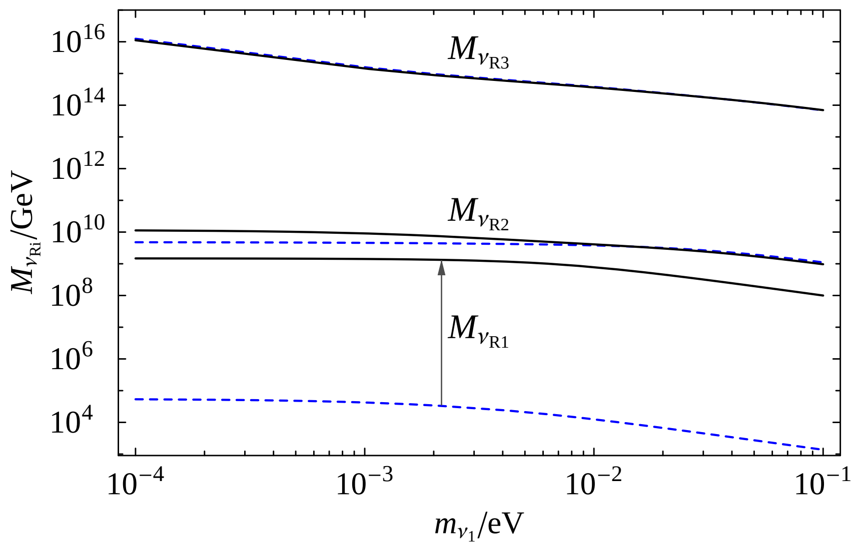

Figure 2: *Masses of right-handed neutrinos $M_{\nu_{Ri}}$ as functions of the lightest neutrino mass m_{ν_1}. The dashed (blue) lines assume perfect Yukawa unification, Eq. (5). The solid (black) lines indicate the shifts of the ν_R masses in the first and and second generation if the Yukawa identity Eq. (5) is modified ad-hoc by a term κ/v with $\kappa = 100$ MeV.*

From the seesaw relation

$$M_{\nu_R} = Y_\nu m_\nu^{-1} Y_\nu^T \cdot v_u^2, \tag{9}$$

the heavy Majorana R-neutrino mass matrix M_{ν_R} can finally be derived as

$$M_{\nu_R} \approx \mathrm{diag}(m_u, m_c, m_t) m_\nu^{-1}\, \mathrm{diag}(m_u, m_c, m_t). \tag{10}$$

Solving Eq. (10) for the eigenvalues $M_{\nu_{Ri}}$ $(i = 1, 2, 3)$, the heavy Majorana masses are determined by the up-quark masses $m_{u,c,t}$ at the GUT scale and the light neutrino masses $m_{\nu_1}, m_{\nu_2}, m_{\nu_3}$. For normal hierarchy, m_{ν_3} and m_{ν_2} are given by $m_{\nu_3} = \sqrt{m_{\nu_1}^2 + \Delta m_{13}^2}$ and $m_{\nu_2} \approx \sqrt{m_{\nu_1}^2 + \Delta m_{12}^2}$, with the mass squared differences $\Delta m_{13}^2 \approx 2 \cdot 10^{-3}\,\mathrm{eV}^2$ and $\Delta m_{12}^2 \approx 8 \cdot 10^{-5}\,\mathrm{eV}^2$, measured in neutrino oscillation experiments. The $M_{\nu_{Ri}}$ spectrum is then predicted as a function of the masses of the light neutrinos. Quite generally, the solution for the eigenvalues can be approximated to a high level of accuracy by the relations [see also Ref. [26]]:

$$\begin{aligned} M_{\nu_{R1}} &\approx \frac{m_{\nu_1} + 2m_{\nu_2} + 3m_{\nu_3}}{3m_{\nu_1}m_{\nu_2} + 2m_{\nu_1}m_{\nu_3} + m_{\nu_2}m_{\nu_3}}\, m_u^2 \\ M_{\nu_{R2}} &\approx \frac{4\, m_{\nu_1} + 2\, m_{\nu_2} + 0\, m_{\nu_3}}{3m_{\nu_1}m_{\nu_2} + 2m_{\nu_1}m_{\nu_3} + m_{\nu_2}m_{\nu_3}}\, m_c^2 \\ M_{\nu_{R3}} &\approx \frac{3m_{\nu_1}m_{\nu_2} + 2m_{\nu_1}m_{\nu_3} + m_{\nu_2}m_{\nu_3}}{6m_{\nu_1}m_{\nu_2}m_{\nu_3}}\, m_t^2\,. \end{aligned} \tag{11}$$

Thus the mass spectrum of the R-neutrinos is strongly ordered in SO(10) with minimal Higgs content, $M_{\nu_{R3}} : M_{\nu_{R2}} : M_{\nu_{R1}} \sim m_t^2 : m_c^2 : m_u^2$.

The numerical evaluation, including refinements like RG running effects, is displayed in Fig. 2 for a wide range of m_{ν_1} values. The analytical approximation, Eq. (11), is very accurate across the entire range, from small m_{ν_1} with strong ordering of the hierarchical light neutrino masses, $m_{\nu_1} \ll m_{\nu_2} \ll m_{\nu_3}$, up to nearly degenerate light neutrino masses, $m_{\nu_i} \to m_{\nu_1}$. The heavy R-neutrino masses read

$$M_{\nu_{Ri}} = \begin{cases} (3m_u^2/m_{\nu_2},\ 2m_c^2/m_{\nu_3},\ \frac{1}{6}m_t^2/m_{\nu_1}), & m_{\nu_1} \ll \sqrt{\Delta m_{12}^2} \\ (\ m_u^2/m_{\nu_1},\ \ m_c^2/m_{\nu_1},\ \ m_t^2/m_{\nu_1}), & m_{\nu_1} \gg \sqrt{\Delta m_{13}^2} \end{cases} \tag{12}$$

in these two limits.

It should be noted that the prediction for the third-generation R-neutrino mass $M_{\nu_{R3}}$ is quite robust, contrary to the second and the first generation in particular. Modifying the relation between the neutrino and up-type quark Yukawa couplings, Eqs. (5), *ad-hoc* by a small additional term, $Y_\nu = Y_u + \kappa/v$, with $\kappa \sim$ a few hundred MeV associated potentially with a more complex Higgs scenario, Planck-scale suppressed contributions or non-perturbative effects at small mass scales, the first generation R-neutrino mass is lifted to $\sim 10^9$ GeV. This just illustrates that a small modification of the *ν-up quark* Yukawa identity is sufficient to reconcile the mass estimate with limits suggested within leptogenesis scenarios for the matter-antimatter asymmetry in the Universe [27].

The small mixing of the right-handed neutrinos,

$$U_R \approx \begin{pmatrix} 1 & a & \mathcal{O}(\frac{m_u}{m_t}) \\ -a & 1 & b \\ \mathcal{O}(\frac{m_u}{m_t}) & -b & 1 \end{pmatrix} \qquad \begin{aligned} a &= \frac{-2(m_{\nu_1} - m_{\nu_2})m_{\nu_3}}{3m_{\nu_1}m_{\nu_2} + \ 2m_{\nu_1}m_{\nu_3} + m_{\nu_2}m_{\nu_3}}\frac{m_u}{m_c} = \mathcal{O}\left(\frac{m_u}{m_c}\right) \\ b &= \frac{3m_{\nu_1}m_{\nu_2} - 2m_{\nu_1}m_{\nu_3} - 2m_{\nu_2}m_{\nu_3}}{(3m_{\nu_1} + m_{\nu_3})m_{\nu_2}}\frac{m_c}{m_t} = \mathcal{O}\left(\frac{m_c}{m_t}\right), \end{aligned} \tag{13}$$

hardly affects the analysis, since it enters only indirectly into the RG evolution of the slepton mass parameters when the right-handed neutrinos are decoupled at their own mass scale.

The Yukawa mass matrix squared, which determines the connection of the slepton masses in the third generation at low and high scales, is dominated by the 33 element,

$$\left(Y_\nu^\dagger Y_\nu\right)_{33} \approx m_t^2(\Lambda_{\mathcal{U}})/v_u^2 \approx 0.3\,, \tag{14}$$

while the other elements are suppressed to a level of 10^{-2} down to 10^{-5}.

b) Scalar mass parameters:

The slepton systems of the *first two generations* depend, to leading order, on four parameters: the two fundamental scalar and gaugino mass parameters, the D-term and the unification scale. They can be uniquely determined from the unification scale of the gauge couplings, the L- and R-slepton masses and the masses of the charginos and neutralinos. The complementary analysis of squarks provides an independent cross-check of the underlying picture.

Analytic relations, for the sake of clarity, are presented only to leading logarithmic order while the subsequent numerical analyses include the non-logarithmic higher order contributions. To leading order, the solutions of the RG equations, the masses of the scalar selectrons and the L-type e-sneutrino, can be expressed in terms of the high scale parameter $m_{16} = M_0$, the universal gaugino mass parameter $M_{1/2}$ and the GUT and electroweak D-terms, $D_{\mathcal{U}}$ and $D_{EW} = M_Z^2/2 \cos 2\beta$, respectively:

$$\begin{aligned} m^2_{\tilde{e}_R} &= M_0^2 + D_{\mathcal{U}} + \alpha_R M^2_{1/2} - \tfrac{6}{5} S' - 2 s^2_W D_{EW} \\ m^2_{\tilde{e}_L} &= M_0^2 - 3 D_{\mathcal{U}} + \alpha_L M^2_{1/2} + \tfrac{3}{5} S' - c_{2W} D_{EW} \\ m^2_{\tilde{\nu}_{eL}} &= M_0^2 - 3 D_{\mathcal{U}} + \alpha_L M^2_{1/2} + \tfrac{3}{5} S' + D_{EW}\,, \end{aligned} \tag{15}$$

[as usual, $s^2_W = \sin^2\theta_W$ *etc.*]. This set of relations is valid under the assumption of small threshold corrections at the grand unification scale. Though the observables are different, the assumption is backed qualitatively nevertheless by the strongly suggested unification of the gauge couplings [20] which does not require sizeable contributions from thresholds or intermediate-scale degrees of freedom. It has been explicitly shown that the threshold effects due to the large SO(10) representations are small and do not destroy gauge coupling unification [28].

The coefficients α_L and α_R are given by the gaugino/gauge boson loops in the RG evolution from the global supersymmetry scale $\tilde{M}$ [16] to the unification scale $\Lambda_{\mathcal{U}}$,

$$\begin{aligned} \alpha_L &= \tfrac{3}{10} f_1 + \tfrac{3}{2} f_2 \\ \alpha_R &= \tfrac{6}{5} f_1 \\ f_i &= \frac{1}{b_i}\left(1 - \left[1 + \frac{\alpha_{\mathcal{U}}}{4\pi} b_i \log\frac{\Lambda^2_{\mathcal{U}}}{\tilde{M}^2}\right]^{-2}\right) \quad \text{with} \quad (b_1, b_2) = (\tfrac{33}{5}, 1)\,, \end{aligned} \tag{16}$$

and the numerical evaluation finally yields $\alpha_R \approx 0.15$ and $\alpha_L \approx 0.5$ for $\tilde{M} = 1$ TeV. The universal gaugino mass parameter $M_{1/2}$ can be pre-determined in the chargino/neutralino sector. The non-universal initial conditions in the evolution due to the D-terms generate the small generation-independent corrections

$$S' = -4D_{\mathcal{U}} \frac{\alpha_1(\tilde{M})}{\alpha_1(\Lambda_{\mathcal{U}})}, \tag{17}$$

cf. Ref. [25].

The Higgs parameter $m_{10} = M_0$ can be calculated analogously,

$$m^2_{H_d} = M_0^2 + 2D_{\mathcal{U}} + \alpha_L M^2_{1/2} + \tfrac{3}{5}S' - \Delta_\tau - 3\Delta_b\,. \tag{18}$$

Δ_τ and Δ_b describe contributions involving τ and b loops, respectively, with correspondingly large Yukawa couplings, see Eqs. (21) and (23) below.

The scalar masses of the *third generation* receive additional contributions from $\nu_{R\tau}$-higgsino loops *etc.*, coupled by Yukawa interactions with the L and R fields. The masses of the third generation are shifted relative to the masses of the first two generations by two terms [4,20]:

$$\begin{aligned} m^2_{\tilde{\tau}_R} - m^2_{\tilde{e}_R} &= m^2_\tau - 2\Delta_\tau \\ m^2_{\tilde{\tau}_L} - m^2_{\tilde{e}_L} &= m^2_\tau - \Delta_\tau - \Delta_{\nu_\tau} \\ m^2_{\tilde{\nu}_\tau L} - m^2_{\tilde{\nu}_e L} &= -\Delta_\tau - \Delta_{\nu_\tau}\,. \end{aligned} \tag{19}$$

[It should be emphasized that such differences, which are crucial for controlling the R-neutrino mass of the third generation, are generally more immune to corrections than the individual observables.]

The Higgs parameter $m^2_{H_u}$ is given analogously, in the universality class $m_{H_u} = M_0$, by

$$m^2_{H_u} = M_0^2 - 2D_{\mathcal{U}} - \tfrac{3}{5}S' + \alpha_L M^2_{1/2} - \Delta_{\nu_\tau} - 3\Delta_t\,. \tag{20}$$

The shifts Δ_τ and Δ_{ν_τ}, generated by loops [29] involving charged lepton and neutrino superfields, respectively, are predicted by the renormalization group in the SO(10) scenario,

$$\Delta_\tau \approx \frac{m^2_\tau(\Lambda_{\mathcal{U}})}{8\pi^2 v_d^2}\left(3M_0^2 + A_0^2\right)\log\frac{\Lambda^2_{\mathcal{U}}}{\tilde{M}^2} \tag{21}$$

$$\Delta_{\nu_\tau} \approx \frac{m^2_t(\Lambda_{\mathcal{U}})}{8\pi^2 v_u^2}\left(3M_0^2 + A_0^2\right)\log\frac{\Lambda^2_{\mathcal{U}}}{M^2_{\nu_{R3}}}, \tag{22}$$

including the [universal] trilinear coupling A_0, while

$$\Delta_b \approx \frac{m^2_b(\Lambda_{\mathcal{U}})}{8\pi^2 v_d^2}\left(3M_0^2 + A_0^2\right)\log\frac{\Lambda^2_{\mathcal{U}}}{\tilde{M}^2} \tag{23}$$

$$\Delta_t \approx \frac{m^2_t(\Lambda_{\mathcal{U}})}{8\pi^2 v_u^2}\left(3M_0^2 + A_0^2\right)\log\frac{\Lambda^2_{\mathcal{U}}}{\tilde{M}^2} \tag{24}$$

account similarly for the running from the Tera- to the GUT scale induced by b, t loops. The parameter $m_t(\Lambda_{\mathcal{U}}) = m_t(v)(\Lambda_{\mathcal{U}}/v)^\gamma$ is the running top quark mass with, approximately, $\gamma = (6y_t^2 - 16/3g_s^2)/(16\pi^2)$, the top-quark Yukawa coupling y_t and the QCD coupling g_s evaluated at the electroweak scale v; $v_{d(u)} = v\cos\beta\,(v\sin\beta)$.

While the slepton masses, specifically the sum $m^2_{\tilde{\tau}_L} + m^2_{\tilde{\tau}_R} = m^2_{\tilde{\tau}_1} + m^2_{\tilde{\tau}_2}$ of the third generation, are experimentally accessible directly, the Higgs mass parameters,

$$m^2_{H_{d(u)}} = M_A^2 \sin^2\beta\cos^2\beta - |\mu|^2 \mp \tfrac{1}{2}M_Z^2\cos 2\beta \tag{25}$$

can be determined from the pseudo-scalar Higgs-boson mass M_A and the higgsino mass parameter μ. The effect of the Higgs sector on the GUT parameters is small so that potential modifications of D-terms in Eqs. (3,18) do not have a significant impact.

c) Numerical results:

Anticipating high-precision measurements at future colliders, such an SO(10) scenario can be investigated in central facets. As a concrete example, we study the following LR-extended scenario with MSSM parameters close to SPS1a/a′ [16, 30]:

$$\begin{array}{rcl@{\qquad\qquad}rcl} M_0 &=& 90 \text{ GeV} & \tan\beta &=& 10 \\ M_{1/2} &=& 250 \text{ GeV} & \mathrm{sgn}(\mu) &=& + \\ A_0 &=& -640 \text{ GeV} & & & \end{array} \tag{26}$$

and

$$\begin{array}{rcl} D_{\mathcal{U}} &=& -0.9\cdot 10^3 \text{ GeV}^2 \\ M_{\nu_{R3}} &=& 7.2\cdot 10^{14} \text{ GeV}\,. \end{array} \tag{27}$$

The low-energy and cosmological data of this scenario are compatible with observations: QCD coupling $\alpha_s(M_Z) = 0.119$, electroweak mixing parameter $\sin^2\theta^{lept}_{eff} = 0.23140$, radiative b decay BR$(b \to s\gamma) = 3.32\cdot 10^{-4}$, deviation of the muon anomalous magnetic moment from SM value $\Delta a_\mu = \Delta(g_\mu - 2)/2 = 15.3\cdot 10^{-10}$, and cold-dark-matter density $\Omega h_0^2 = 0.110$. [Introducing just one Higgs-10 field, giving rise to the unification of the $t - b - \tau$ Yukawa couplings, would require a larger value of $\tan\beta$. Extended Higgs sectors at the GUT scale avoiding this constraint can be designed without modifying the present analysis.]

The estimated experimental precision for the measurements of slepton, sneutrino and gaugino masses at LHC and ILC can be extrapolated from the results of Refs. [4,16,31–35], which are based on detailed simulations. The masses of the charged sleptons can be measured

Parameter	Value	Error	Parameter	Value	Error
$m_{\tilde{e}_R}$	140.9 GeV	0.05 GeV	μ	481.1 GeV	4.5 GeV
$m_{\tilde{e}_L}$	190.4 GeV	0.4 GeV	$\tan\beta$	10.0	1.0
$m_{\tilde{\nu}_e}$	173.4 GeV	1.3 GeV	$M_{1/2}$	250.0 GeV	0.4 GeV
$m_{\tilde{\tau}_1}$	104.2 GeV	0.3 GeV	A_0	-640.0 GeV	13 GeV
$m_{\tilde{\tau}_2}$	187.8 GeV	1.1 GeV			
$m_{\tilde{\nu}_\tau}$	154.7 GeV	1.6 GeV			

Table 1: *Expected experimental errors for the determination of sfermion masses and other underlying parameters in the sample one-step SO(10) scenario. [The errors quoted correspond to 1σ.]*

with high precision in slepton pair production at ILC [31], while the sneutrino masses can be determined accurately from the decays of charginos [4]. The Higgs masses in Eq. (25) are expressed in terms of the parameters μ and $\tan\beta$, which can be derived accurately from the analysis of chargino and neutralino observables [32, 33]. The pseudoscalar Higgs mass M_A, on the other hand, can be measured in associated Higgs pair production [36]. The direct determination of the A parameters is difficult, *cf.* Refs. [37–39]. However, taking into account stop/sbottom mass measurements but leaving out the sleptons of the third generation and the Higgs bosons to bypass the (yet to be determined) parameters of the right-handed neutrinos, a global analysis including loop effects[1] leads to an accurate determination of A_0 at the level of 2% [16, 35]. The relevant parameters are summarized in Tab. 1, including the estimated experimental errors.

The measurement of the slepton and sneutrino masses of the first two generations allows us to extract the common sfermion parameter $m_{16} = M_0$ as well as the D-term $D_{\mathcal{U}}$. The relations are given in Eq. (15) in leading logarithmic approximation for the RG running. Including the complete one-loop and the leading two-loop corrections, the evolution of the scalar mass parameters, $(m^2_{L_1}, m^2_{E1})$, is displayed in Fig. 3(left) for the first two generations. The curves describe the evolution of the universal L and R mass parameters with, merely for illustration, the D-term set to zero.

With the estimated errors in Tab. 1, the high-scale parameters can be calculated, by com-

[1]Based on this reduced set of observables, errors on A_0 of about 6 GeV have been predicted for SPS1a and SPS1a$'$ with A_0 values of -100 and -300 GeV, respectively. Fixing the error on A_0 to 2%, *i.e.* 12 GeV, for the present benchmark point with large $A_0 = -640$ GeV can therefore be considered as a conservative estimate. Thanks go to P. Bechtle and D. Zerwas for running Fittino and Sfitter, respectively, for estimating the increase of the error on A_0 in SPS1a and SPS1a$'$ for the reduced set of experimental observables.

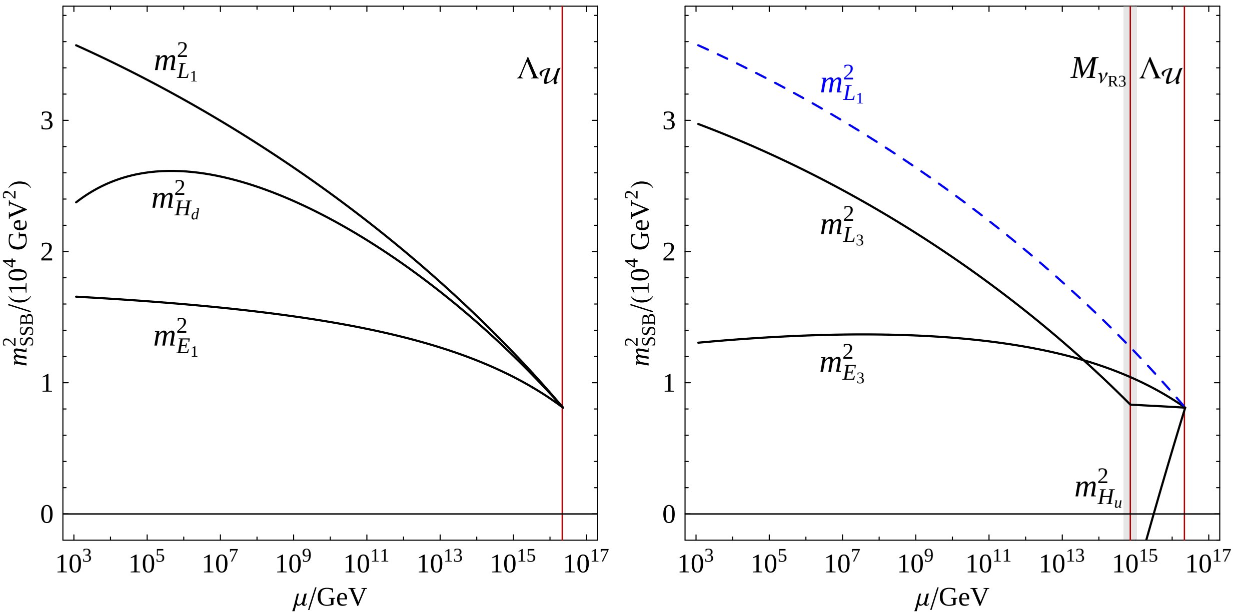

Figure 3: Left: *Evolution of the L,R slepton mass parameters of the first generation for one-step breaking with the D-term, merely for illustration, set to zero.* Right: *Evolution of the first and third generation L,R slepton and Higgs mass parameters [$D_{\mathcal{U}} = 0$] for one-step breaking; the kink in $m^2_{L_3}$ is generated by the right-handed neutrino with mass $M_{\nu_{R3}}$ close to* 10^{15} GeV.

bining the slepton and Higgs sectors, as shown in Tab. 2. The RG evolution equations are evaluated to 2-loop order by means of the SPheno program [40]. The table indicates that the high-scale parameters M_0 and $D_{\mathcal{U}}$, driven by the slepton analysis, can be reconstructed at per-mill to per-cent accuracy.

The right-handed neutrino affects the evolution of the L mass parameter $m^2_{L_3}$ in the third generation and the Higgs parameter $m^2_{H_u}$. [Note that $m^2_{H_u} + \mu^2$ turns negative only at the small scale $Q \simeq 350$ GeV.] The characteristic difference in the evolution between $m^2_{L_3}$ and $m^2_{L_1}$ is exemplified in Fig. 3(right) for a right-handed neutrino mass $M_{\nu_{R3}}$ of $7.2 \cdot 10^{14}$ GeV. From the universality point $\Lambda_{\mathcal{U}}$, where m_{L_3} and m_{L_1} are equal, down to the kink at $M_{\nu_{R3}}$ the evolution of m_{L_3} is affected by the right-handed neutrino, m_{L_1} however is not. Below the kink the difference between m_{L_3} and m_{L_1} is reduced to the standard loop correction Δ_τ and the small τ mass. The position of the kink at $M_{\nu_{R3}}$ can be derived from the measured slepton and sneutrino masses if the scalar mass parameters are universal at the unification scale. Neglecting the small term m^2_τ, the right-handed neutrino mass is fixed by the intersection of the parameter Δ_{ν_τ} as a function of $M_{\nu_{R3}}$ with the measured value extracted from the

Parameters in SO(10) $\to$ SM	Ideal	Error
unification scale $\Lambda_{\mathcal{U}}$	$2.16 \cdot 10^{16}$ GeV	$0.02 \cdot 10^{16}$ GeV
matter scalar mass M_0	90 GeV	0.25 GeV
GUT D-term $\sqrt{-D_{\mathcal{U}}}$	30 GeV	0.9 GeV
heaviest R-neutrino mass $M_{\nu_{R3}}$	$7.2 \cdot 10^{14}$ GeV	$[4.8, 11] \cdot 10^{14}$ GeV
lightest neutrino mass m_{ν_1}	$3.5 \cdot 10^{-3}$ eV	$[1.6, 6.7] \cdot 10^{-3}$ eV

Table 2: *Reconstruction of high-scale SO(10) parameters in one-step SO(10) $\to$ SM breaking, and masses of the heavy 3-generation R-neutrino and the lightest neutrino [The errors quoted correspond to 1σ].*

slepton masses:

$$\begin{aligned}\Delta_{\nu_\tau} &= \frac{1}{2}[m^2_{\tilde{e}_L} + m^2_{\tilde{e}_R} - 3m^2_{\tilde{\nu}_{eL}}] - \frac{1}{2}[m^2_{\tilde{\tau}_1} + m^2_{\tilde{\tau}_2} - 3m^2_{\tilde{\nu}_{\tau L}}] \\ &= \frac{m_t^2(\Lambda_{\mathcal{U}})}{8\pi^2 v_u^2}\left(3M_0^2 + A_0^2\right)\log\frac{\Lambda^2_{\mathcal{U}}}{M^2_{\nu_{R3}}}\,. \end{aligned} \tag{28}$$

In practice the separate evaluation of the charged slepton and sneutrino mass differences,

$$\begin{aligned}\Delta_{\nu_\tau} &= m^2_{\tilde{\nu}_{eL}} - m^2_{\tilde{\nu}_{\tau L}} - \Delta_\tau \\ &= m^2_{\tilde{e}_L} + m^2_{\tilde{e}_R} - m^2_{\tilde{\tau}_L} - m^2_{\tilde{\tau}_R} - 3\Delta_\tau \end{aligned} \tag{29}$$

proves useful to obtain a more precise value, $\Delta^{exp}_{\nu_\tau} = (4.7 \pm 0.5) \cdot 10^3$ GeV^2. As a result, for the parameters of the LR extended point SPS1a$'$ introduced earlier, the right-handed neutrino mass of the third generation is estimated in the margin

$$M_{\nu R_3} = 10^{14.86 \pm 0.19}\ \text{GeV} = 7.2^{+4.0}_{-2.5} \cdot 10^{14}\ \text{GeV} \tag{30}$$

as evident from Fig. 4. Thus, the effect of the heavy ν_{R3} mass can indeed be traced back from measured slepton masses in universal supersymmetric theories.

Based on this estimate of $M_{\nu R_3}$, the seesaw mechanism determines the value of the lightest neutrino mass (*cf.* Fig. 2) to

$$m_{\nu_1} = 10^{-2.51 \pm 0.32}\ \text{eV} = 3.1^{+3.5}_{-1.5} \cdot 10^{-3}\ \text{eV}. \tag{31}$$

The second lightest neutrino mass $m_{\nu_2} \simeq (1.0 \pm 0.1) \cdot 10^{-2}$ eV is about three times larger while the third neutrino mass $m_{\nu_3} \simeq 5 \cdot 10^{-2}$ eV coincides with the mass difference Δm_{13} measured in the neutrino oscillation experiments.

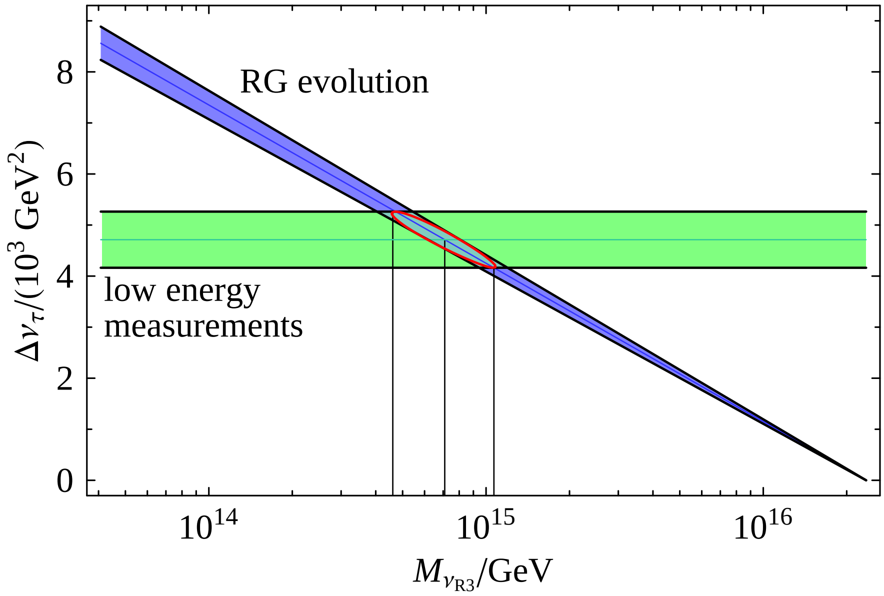

Figure 4: *Shift Δ_{ν_τ} of the third generation L slepton mass parameter generated by loops involving heavy neutrino* R*-superfields. The blue wedge corresponds to the prediction Eq. (22) of the renormalization group [RG], whereas the green band is determined by low-energy mass measurements.*

Thus, the combination of SO(10) symmetry and seesaw mechanism leads, besides the high-scale universal SUSY parameters, to the determination of the heavy Majorana mass $M_{\nu_{R3}}$ of the third generation and, in a consecutive step, to the value of the lightest neutrino mass m_{ν_1} in hierarchical theories. This conclusion has been derived for scenarios in which threshold effects at $\Lambda_{\mathcal{U}}$ and the mechanisms for solving the doublet-triplet splitting problem in the Higgs sector *etc.* do not have a significant impact on the evolution of the scalar mass parameters. The analysis however may serve as a generic paradigm which could be adjusted correspondingly if the structure of the high-scale scenario demanded a more complex extension.

3 Two-Step SO(10) $\to$ SU(5) $\to$ SM Breaking

One possible move in the direction of higher complexity is the extension of the analysis to a simple model of 2-step breaking. If the grand unification symmetry SO(10) is broken in two steps down to the Standard Model, *cf.* Fig. 1(right), the evolution between the Terascale and the SU(5) scale $\Lambda_{\mathcal{U}}$ is determined by measured parameters: gauge couplings, scalar mass parameters and neutrino parameters. However, the evolution between the SU(5) scale

$\Lambda_{\mathcal{U}}$ and the SO(10) scale $\Lambda_{\mathcal{O}}$ depends on the high-scale physics scenario, comprising new matter, gauge and Higgs fields, and potentially effective elements of gravity interactions.

In the present approach we restrict these degrees of freedom to the {24} Higgs field which breaks the SU(5) gauge symmetry. In the same spirit as in the previous section, other degrees of freedom and/or additional mechanisms generating the doublet-triplet splitting and prolonging the proton lifetime, for example, are assumed to be weakly coupled to the scalar mass parameters below $\Lambda_{\mathcal{O}}$. The numerical results in this two-step analysis are less robust than in the previous one-step analysis since they depend explicitly on the physics scenario at the high scales beyond $> \Lambda_U$. Nevertheless this study can serve as an illustration for the potential of parametric analyses if the fundamental structure of a more complex scenario is theoretically pre-determined.

a) Evolution Terascale $\rightarrow \Lambda_{\mathcal{U}}$

Based on the standard SU(5) decomposition, the scalar masses evolve from the electroweak scale to the SU(5) scale $\Lambda_{\mathcal{U}}$ according to the rules

$$\begin{aligned} m^2_{\tilde{e}_R} &= m^2_{10,1} + \alpha_R M^2_{\mathcal{U},1/2} - \tfrac{6}{5}S' - 2s^2_W D_{EW} \\ m^2_{\tilde{e}_L} &= m^2_{\bar{5},1} + \alpha_L M^2_{\mathcal{U},1/2} + \tfrac{3}{5}S' - c_{2W} D_{EW} \\ m^2_{\tilde{\nu}_{eL}} &= m^2_{\bar{5},1} + \alpha_L M^2_{\mathcal{U},1/2} + \tfrac{3}{5}S' + D_{EW} \end{aligned} \tag{32}$$

for the matter fields of the first two generations while the masses of the third generation,

$$\begin{aligned} m^2_{\tilde{\tau}_R} &= m^2_{10,3} + \alpha_R M^2_{\mathcal{U},1/2} - \tfrac{6}{5}S' - 2s^2_W D_{EW} + m^2_\tau - 2\Delta_\tau \\ m^2_{\tilde{\tau}_L} &= m^2_{\bar{5},3} + \alpha_L M^2_{\mathcal{U},1/2} + \tfrac{3}{5}S' - c_{2W} D_{EW} + m^2_\tau - \Delta_\tau - \Delta_{\nu_\tau} \\ m^2_{\tilde{\nu}_{\tau L}} &= m^2_{\bar{5},3} + \alpha_L M^2_{\mathcal{U},1/2} + \tfrac{3}{5}S' + D_{EW} - \Delta_\tau - \Delta_{\nu_\tau}\,, \end{aligned} \tag{33}$$

are affected in addition by the Yukawa interactions in the same way as Eqs. (21,22). The indices $1, 3$ next to the SU(5) multiplet characteristics denote the generation numbers. The evolution of the Higgs mass parameters read correspondingly

$$m^2_{H_d} = m^2_{\bar{5}_1} + \alpha_L M^2_{\mathcal{U},1/2} + \tfrac{3}{5}S' - \Delta_\tau - 3\Delta_b \tag{34}$$

$$m^2_{H_u} = m^2_{5_2} + \alpha_L M^2_{\mathcal{U},1/2} - \tfrac{3}{5}S' - \Delta_{\nu_\tau} - 3\Delta_t\,. \tag{35}$$

Since SU(5) and SU(3)×SU(2)×U(1) are both rank-4 symmetry groups, no GUT D-term is induced in this symmetry breaking step.

The Δ's are given by the loop corrections,

$$
\begin{aligned}
\Delta_\tau &= \frac{m_\tau^2(\Lambda_\mathcal{U})}{8\pi^2 v_d^2}\left(m_{10,3}^2 + m_{\bar{5},3}^2 + m_{\bar{5}_1}^2 + A_{5,3}^2\right)\log\frac{\Lambda_\mathcal{U}^2}{\tilde{M}^2} && (36)\\
\Delta_{\nu_\tau} &= \frac{m_t^2(\Lambda_\mathcal{U})}{8\pi^2 v_u^2}\left(m_{1,3}^2 + m_{\bar{5},3}^2 + m_{5_2}^2 + A_{1,3}^2\right)\log\frac{\Lambda_\mathcal{U}^2}{M_{\nu_{R3}}^2} && (37)
\end{aligned}
$$

and

$$
\begin{aligned}
\Delta_b &= \frac{m_b^2(\Lambda_\mathcal{U})}{8\pi^2 v_d^2}\left(m_{10,3}^2 + m_{\bar{5},3}^2 + m_{\bar{5}_1}^2 + A_{5,3}^2\right)\log\frac{\Lambda_\mathcal{U}^2}{\tilde{M}^2} && (38)\\
\Delta_t &= \frac{m_t^2(\Lambda_\mathcal{U})}{8\pi^2 v_u^2}\left(2m_{10,3}^2 + m_{5_2}^2 + A_{10,3}^2\right)\log\frac{\Lambda_\mathcal{U}^2}{\tilde{M}^2}\,, && (39)
\end{aligned}
$$

while the contributions to the S' parameter trace back to the non-universal Higgs mass parameters at the scale $\Lambda_\mathcal{U}$,

$$
S' = \frac{\alpha_1(\tilde{M})}{\alpha_1(\Lambda_\mathcal{U})}\left(m_{5_2}^2 - m_{\bar{5}_1}^2\right). \tag{40}
$$

$M_{\mathcal{U},1/2}$ is the universal gaugino mass parameter at the SU(5) scale $\Lambda_\mathcal{U}$.

b) Evolution $\Lambda_\mathcal{U} \to \Lambda_\mathcal{O}$:

The subsequent evolution from the SU(5) breaking scale $\Lambda_\mathcal{U}$ to the SO(10) breaking scale $\Lambda_\mathcal{O}$ unifies the mass parameters m_{10}, $m_{\bar{5}}$ and m_1 to m_{16},

$$
\begin{aligned}
m_{10,1}^2 &= M_0^2 + D_\mathcal{O} + \alpha_R' M_{1/2}^2\\
m_{\bar{5},1}^2 &= M_0^2 - 3D_\mathcal{O} + \alpha_L' M_{1/2}^2 && (41)
\end{aligned}
$$

and

$$
\begin{aligned}
m_{10,3}^2 &= M_0^2 + D_\mathcal{O} + \alpha_R' M_{1/2}^2 - 3\Delta_t' - 2\Delta_b'\\
m_{\bar{5},3}^2 &= M_0^2 - 3D_\mathcal{O} + \alpha_L' M_{1/2}^2 - \Delta_{\nu_\tau}' - 4\Delta_b'\\
m_{1,3}^2 &= M_0^2 + 5D_\mathcal{O} - 5\Delta_{\nu_\tau}'\,. && (42)
\end{aligned}
$$

The Higgs parameters transform according to

$$
\begin{aligned}
m_{\bar{5}_1}^2 &= M_0^2 + 2D_\mathcal{O} + \alpha_L' M_{1/2}^2 - 4\Delta_b' - \tfrac{24}{5}\Delta_\lambda'\,,\\
m_{5_2}^2 &= M_0^2 - 2D_\mathcal{O} + \alpha_L' M_{1/2}^2 - \Delta_{\nu_\tau}' - 3\Delta_t' - \tfrac{24}{5}\Delta_\lambda'\,. && (43)
\end{aligned}
$$

The D-term associated with the breaking of the rank-5 SO(10) to the rank-4 SU(5) symmetry is given again by a relation analogous to Eq. (4) for a Higgs-16 field, for example, responsible for the symmetry breaking of SO(10) $\to$ SU(5). The universal gaugino mass parameter $M_{1/2}$ at $\Lambda_{\mathcal{O}}$ is related to the universal parameter $M_{\mathcal{U},1/2}$ at $\Lambda_{\mathcal{U}}$ by

$$M_{1/2} = \frac{\alpha_{\mathcal{O}}}{\alpha_{\mathcal{U}}}\, M_{\mathcal{U},1/2}\,. \tag{44}$$

The coefficients $\alpha'_{L,R}$ are given by

$$\alpha'_L \;=\; \frac{3}{2}\alpha'_R = \frac{36}{5b_5}\left(1 - \left[1 + \frac{g^2_{SO(10)}}{16\pi^2} b_5 \log\frac{\Lambda^2_{\mathcal{O}}}{\Lambda^2_{\mathcal{U}}}\right]^{-2}\right), \tag{45}$$

with $b_5 = -3$ in the minimal SU(5) model including the Higgs $\{24\}$ representation, *cf.* Fig. 1(right):

$$\begin{aligned} b_5 &= b_5(\text{matter}) + b_5(\text{gauge}) + b_5(\text{Higgs}) \\ b_5(\text{matter}) &= 6\,;\;\; b_5(\text{gauge}) = -15\,; \\ b_5(\text{Higgs}\;\{24\}) &= 5\,;\;\; b_5(\text{Higgs}\;\{5\}+\{\bar{5}\}) = 1. \end{aligned} \tag{46}$$

The Δ' coefficients,

$$\Delta'_t \;\approx\; \Delta'_{\nu_\tau} \approx \frac{m_t^2(\Lambda_{\mathcal{O}})}{8\pi^2 v_u^2}\left(3m_{16}^2 + A_0^2\right)\log\frac{\Lambda^2_{\mathcal{O}}}{\Lambda^2_{\mathcal{U}}} \tag{47}$$

$$\Delta'_b \;\approx\; \frac{m_b^2(\Lambda_{\mathcal{O}})}{8\pi^2 v_d^2}\left(3m_{16}^2 + A_0^2\right)\log\frac{\Lambda^2_{\mathcal{O}}}{\Lambda^2_{\mathcal{U}}}\,, \tag{48}$$

are (s)quark and (s)lepton loop contributions to the transport from $\Lambda_{\mathcal{U}}$ to $\Lambda_{\mathcal{O}}$. The shift Δ'_λ, accounting for contributions involving the heavy $\{24\}$ Higgs field which couples to the $\{5\}$ and $\{\bar{5}\}$ Higgs fields, is small in general; moreover, it can be neglected since the Higgs sector, in effect, plays a minor role in the analysis.

The evolution of the gauge coupling from $\Lambda_{\mathcal{U}}$ to $\Lambda_{\mathcal{O}}$,

$$\alpha(\Lambda_{\mathcal{O}}) = \frac{\alpha(\Lambda_{\mathcal{U}})}{1 - \frac{\alpha(\Lambda_{\mathcal{U}})}{2\pi} b_5 \log\frac{\Lambda^2_{\mathcal{O}}}{\Lambda^2_{\mathcal{U}}}}\,, \tag{49}$$

is affected by the Higgs-24 field which breaks SU(5) $\to$ SM.

Finally, the shift of the A parameters between $\Lambda_{\mathcal{U}}$ and $\Lambda_{\mathcal{O}}$ is small:

$$A_{5,3} \;\simeq\; 0.97A_0 - 0.15M_{1/2} \tag{50}$$

$$A_{10,3} \;\simeq\; 0.91A_0 - 0.16M_{1/2} \tag{51}$$

$$A_{1,3} \;\simeq\; 0.89A_0 - 0.10M_{1/2} \tag{52}$$

Parameter	Value	Error	Parameter	Value	Error
$m_{\tilde{e}_R}$	192.2 GeV	0.07 GeV	μ	429.5 GeV	5.0 GeV
$m_{\tilde{e}_L}$	217.8 GeV	0.5 GeV	$\tan\beta$	20.0	5.0
$m_{\tilde{\nu}_e}$	202.7 GeV	0.7 GeV	$M_{1/2}$	230.0 GeV	0.4 GeV
$m_{\tilde{\tau}_1}$	101.9 GeV	0.3 GeV	A_0	-425.0 GeV	8.5 GeV
$m_{\tilde{\tau}_2}$	219.4 GeV	1.2 GeV			
$m_{\tilde{\nu}_\tau}$	178.0 GeV	1.0 GeV			

Table 3: *Expected experimental errors for the determination of sfermion masses and other underlying parameters in the two-step SO(10) → SU(5) → SM scenario. [The errors quoted correspond to 1σ.]*

for the reference point defined below.

This 2-step breaking system has been analyzed quantitatively for a parameter set closely related to the previous example:

$$\begin{aligned} M_0 &= 100 \text{ GeV} & \tan\beta &= 20 \\ M_{1/2} &= 230 \text{ GeV} & \text{sgn}(\mu) &= + \\ A_0 &= -425 \text{ GeV} \end{aligned} \tag{53}$$

and

$$\begin{aligned} \Lambda_{\mathcal{O}} &= 5 \cdot 10^{17} \text{ GeV} \\ D_{\mathcal{O}} &= 0.9 \cdot 10^3 \text{ GeV}^2 \\ M_{\nu_{R_3}} &= 6.3 \cdot 10^{14} \text{ GeV}\,. \end{aligned} \tag{54}$$

The scenario[2] is compatible with values of the low-energy and cosmological data specified earlier: $\alpha_s(M_Z) = 0.119$, $\sin^2\theta^{lept}_{eff} = 0.23140$, $\text{BR}(b \to s\gamma) = 3.11 \cdot 10^{-4}$, $\Delta a_\mu = 25.5 \cdot 10^{-10}$, and $\Omega h_0^2 = 0.094$.

The analysis is performed in the same way as before, with the measurement errors extrapolated from existing studies to this scenario as listed in Tab. 3. The error on A_0 can be estimated again to be 2% according to the guideline followed in the one-step analysis, as the evolution from $\Lambda_{\mathcal{U}}$ to $\Lambda_{\mathcal{O}}$ has little impact on the A parameters. Though uncertainties due to the shift of the universality scale are expected to be small, the analysis has been repeated for an error increased to 5%, nevertheless. The final results are affected only slightly.

[2]Note that this high-scale point is outside the parameter range investigated in Ref. [41]; we thank Y. Mambrini for a clarifying comparison.

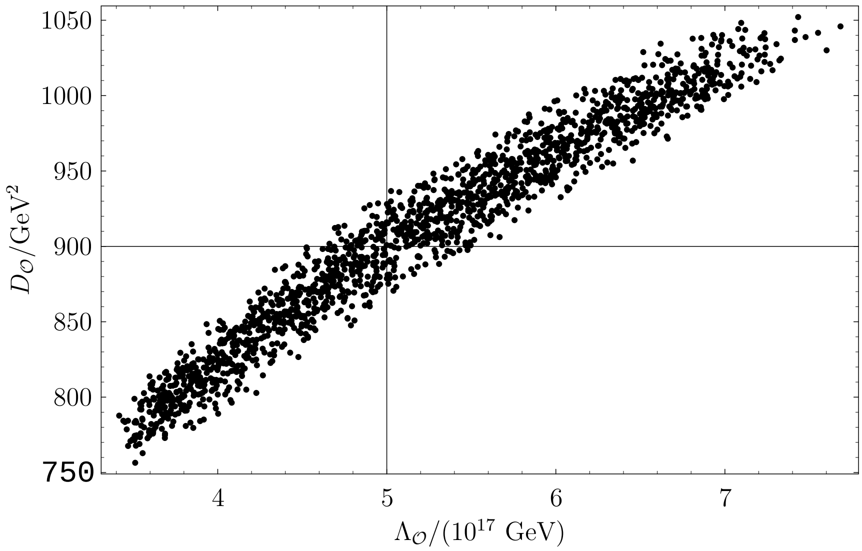

Figure 5: *Correlation between the SO(10) unification scale $\Lambda_{\mathcal{O}}$ and the D-term $D_{\mathcal{O}}$ in the analysis of the two-step SO(10) $\to$ SU(5) $\to$ SM scenario. The scatter plot represents a χ^2 analysis based on the extrapolation of the expected experimental errors from the weak scale to the unification scale.*

As naturally expected, a strong correlation between the D-term $D_{\mathcal{O}}$ and the SO(10) unification scale $\Lambda_{\mathcal{O}}$ is observed, *cf.* Fig. 5. Since $D_{\mathcal{O}}$ is predicted theoretically when the SO(10) $\to$ SU(5) breaking mechanism is devised, $D_{\mathcal{O}}$ is fixed in the following analysis while $\Lambda_{\mathcal{O}}$ is allowed to float. Results of the reverse analysis are briefly summarized for completeness.

Inspecting the solutions of the evolution equations, and Fig. 6, a simple picture emerges for the reconstruction of the high-scale theory:

(i) The SU(5) scale $\Lambda_{\mathcal{U}}$ can be derived from the unification point of the strong, electromagnetic and weak couplings, *cf.* Fig. 6(left), continuing to $\Lambda_{\mathcal{O}}$ by including the $\{24\}$ Higgs representation for SU(5) breaking;

(ii) The SU(5) mass parameters m_{10}, $m_{\bar{5}}$ are determined by the measured slepton masses of the first two generations, and the down-type scalar Higgs mass parameter analogously. Noticeable deviations from universality are observed for the mass parameters at the intermediate SU(5) GUT scale $\Lambda_{\mathcal{U}}$. [The evolution equations, Eqs. (32) and (34), depend on

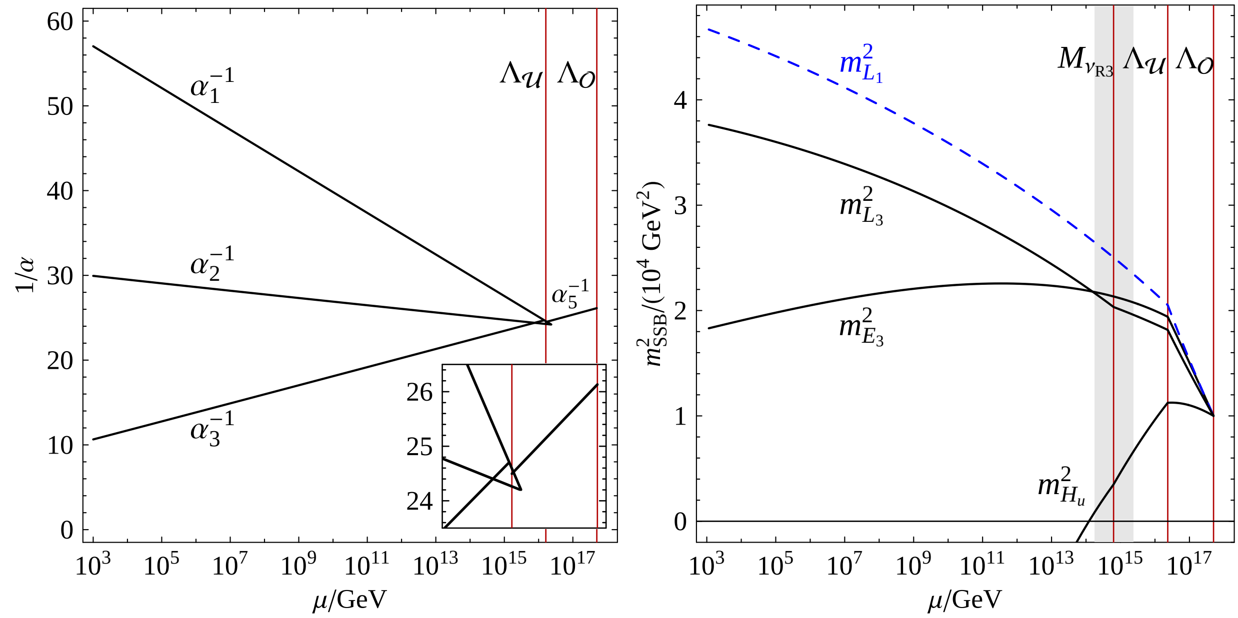

Figure 6: Left: *Evolution of the gauge couplings in 2-step* SO(10) → SU(5) → SM *symmetry breaking for the* SU(5) *Higgs representation* {24}*;* Right: *Evolution of scalar matter and down-type Higgs mass parameters,* $D_{\mathcal{O}} = 0$ *chosen for illustration.*

S', which involves the difference between the down- and up-type Higgs mass parameters, $m_{\bar{5}_1}$ m_{5_2}. The corrections from the small difference of these parameters have been properly taken into account after performing the next step in the analysis based on the universality of the soft supersymmetry breaking parameters at $\Lambda_{\mathcal{O}}$];

(iii) The triple meeting point of the SU(5) parameters of the first two generations and the down-type Higgs in the evolution from the SU(5) to the SO(10) grand unification scale determines the SO(10) mass parameter M_0 and the GUT scale $\Lambda_{\mathcal{O}}$, *cf.* Fig. 6(right);

(iv) By matching the evolution of the slepton mass parameters in the third generation and the up-type Higgs with M_0, the value of the heavy R-neutrino mass $M_{\nu_{R3}}$ in the seesaw mechanism can be estimated. The mass $8.6 \cdot 10^{13}$ GeV $< M_{\nu_{R3}} < 5.0 \cdot 10^{15}$ GeV can only be estimated very roughly in this specific two-step scenario. This is not primarily due to the increased complexity of the 2-step structure, but rather due to the smaller value of A_0 compared to the 1-step scenario; this leads to a reduced slope in the RG evolution of Δ_{ν_τ} (*cf.* Fig. 4) so that the uncertainty rises in $M_{\nu_{R3}}$. [Heavier slepton masses, however, would also reverse this tendency and reduce the error.] The uncertainty is only slightly larger, $8.0 \cdot 10^{13}$ GeV $< M_{\nu_{R3}} < 6.0 \cdot 10^{15}$ GeV, if the error on the A_0 parameter is increased to 5% as indicated before. If the integrated luminosities are raised to 1 ab^{-1} both at (S)LHC and

ILC, the estimate of the R-sneutrino mass narrows down to $1.8 \cdot 10^{14}$ GeV $< M_{\nu_{R3}} < 2.4 \cdot 10^{15}$ GeV, *i.e.* almost by an order of magnitude. These estimates point clearly to a value in the area just below the SU(5) unification scale.

Parameter	Mass/Scale	Error for 500 fb^{-1}	Error for 1000 fb^{-1}
SU(5) unification scale $\Lambda_{\mathcal{U}}$	$2.35 \cdot 10^{16}$ GeV	$0.02 \cdot 10^{16}$ GeV	$0.02 \cdot 10^{16}$ GeV
matter scalar mass $m_{10,1}$	163.3 GeV	0.14 GeV	0.10 GeV
matter scalar mass $m_{\bar{5},1}$	133.5 GeV	0.6 GeV	0.45 GeV
matter scalar mass $m_{10,3}$	142.2 GeV	1.2 GeV	0.85 GeV
matter scalar mass $m_{\bar{5},3}$	124.2 GeV	0.45 GeV	0.4 GeV
Higgs scalar mass $m_{\bar{5}_1}$	127.8 GeV	0.5 GeV	0.35 GeV
Higgs scalar mass m_{5_2}	113.7 GeV	0.75 GeV	0.65 GeV
SO(10) unification scale $\Lambda_{\mathcal{O}}$	$5.0 \cdot 10^{17}$ GeV	$0.55 \cdot 10^{17}$ GeV	$0.35 \cdot 10^{17}$ GeV
D-Term $\sqrt{D_{\mathcal{O}}}$	30.0 GeV	*fixed*	*fixed*
matter scalar mass M_0	100 GeV	2.2 GeV	1.4 GeV
heaviest R-neutrino mass $M_{\nu_{R3}}$	$6.30 \cdot 10^{14}$ GeV	$[0.86, 50] \cdot 10^{14}$ GeV	$[1.8, 24] \cdot 10^{14}$ GeV
lightest neutrino mass m_{ν_1}	$3.5 \cdot 10^{-3}$ eV	$[0.26, 82] \cdot 10^{-3}$ eV	$[0.58, 31] \cdot 10^{-3}$ eV

Table 4: *High-scale SO(10) and SU(5) parameters in 2-step SO(10) $\to$ SU(5) $\to$ SM breaking, and masses of the heavy 3-generation R-neutrino and of the lightest neutrino, as reconstructed from measurements at LHC and ILC assuming a fixed value for the D-term [The errors quoted correspond to 1σ].*

In this way the complete set of couplings and mass parameters could be analyzed. The results, with errors extrapolated from Tab. 3, are shown in Tab. 4. For illustration, Tab. 5 demonstrates how well the parameters can be determined when the SO(10) unification scale $\Lambda_{\mathcal{O}}$ is identified with the string scale while the SO(10) D-term is kept as a free variable. The differences between the two sets of results are small.

The complex structure of the two-step breaking scenario, with strong correlations between some parameters, makes it impossible to derive *all* aspects of a general SO(10) scenario from measurements without making any assumption on the SO(10) breaking mechanism. Nevertheless, introducing assumptions about the SO(10) breaking as exemplified in this section, the Terascale data allow us to determine the two scales $\Lambda_{\mathcal{U}}$, $\Lambda_{\mathcal{O}}$, the universal scalar mass M_0, and the right-handed neutrino mass $M_{\nu_{R3}}$ of the third generation independently. In particular, the two-step breaking scenario can be clearly distinguished from the one-step scenario analyzed in the previous section.

4 Conclusions

If the roots of physics are located near the Planck scale, experimental methods must be devised to explore the high-scale physics scenario including the grand unification of the Standard Model (SM) interactions up to, finally, gravity.

In this report we have studied two examples in which high-scale parameters in supersymmetric SO(10) models have been connected with experimental observations that could be expected in future high-precision Terascale experiments at LHC and e^+e^- linear colliders. For the sake of clarity, sectors that must be constructed for a comprehensive SO(10) theory of all states but that are not essential for the key points of our analysis, have not been specified explicitly. The renormalization group provides the tool for bridging the gap between the Terascale experiments and the underlying high-scale grand unification theory. Even though it depends on the detailed values of the parameters with which resolution the high-scale picture can be reconstructed, an accurate picture could be established in the example for one-step breaking SO(10) $\to$ SM, including the heavy mass of the right-handed neutrino ν_{R3} expected in the seesaw mechanism. As naturally anticipated, the analysis of two-step breaking SO(10) $\to$ SU(5) $\to$ SM turns out to be significantly more difficult, demanding a larger set of additional assumptions before the parametric analysis can be performed.

The two examples have demonstrated nevertheless that renormalization-group extrapola-

Parameter	Mass/Scale	Error for 500 fb^{-1}	Error for 1000 fb^{-1}
SU(5) unification scale $\Lambda_{\mathcal{U}}$	$2.35{\cdot}10^{16}$ GeV	$0.02{\cdot}10^{16}$ GeV	$0.02{\cdot}10^{16}$ GeV
matter scalar mass $m_{10,1}$	163.3 GeV	0.14 GeV	0.10 GeV
matter scalar mass $m_{\bar{5},1}$	133.5 GeV	0.55 GeV	0.4 GeV
matter scalar mass $m_{10,3}$	142.2 GeV	0.85 GeV	0.75 GeV
matter scalar mass $m_{\bar{5},3}$	124.2 GeV	0.65 GeV	0.5 GeV
Higgs scalar mass $m_{\bar{5}_1}$	127.8 GeV	0.22 GeV	0.15 GeV
Higgs scalar mass m_{5_2}	113.7 GeV	0.85 GeV	0.7 GeV
SO(10) unification scale $\Lambda_{\mathcal{O}}$	$5.0 \cdot 10^{17}$ GeV	*fixed*	*fixed*
D-Term $\sqrt{D_{\mathcal{O}}}$	30.0 GeV	0.65 GeV	0.5 GeV
matter scalar mass M_0	100 GeV	0.5 GeV	0.4 GeV
heaviest R-neutrino mass $M_{\nu_{R3}}$	$6.30 \cdot 10^{14}$ GeV	$[0.76, 51] \cdot 10^{14}$ GeV	$[1.4, 27] \cdot 10^{14}$ GeV
lightest neutrino mass m_{ν_1}	$3.5 \cdot 10^{-3}$ eV	$[0.25, 90] \cdot 10^{-3}$ eV	$[0.51, 44] \cdot 10^{-3}$ eV

Table 5: *Same as Tab. 4, but for fixed SO(10) unification scale.*

tions based on high-precision results expected from Terascale experiments can provide essential elements for the reconstruction of the physics scenario near the grand unification if the theoretical frame of a comprehensive SO(10) grand unified theory is specified.

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