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Gauge Symmetry, T-Duality and Doubled Geometry

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Abstract

String compactifications with T-duality twists are revisited and the gauge algebra of the dimensionally reduced theories calculated. These reductions can be viewed as string theory on T-fold backgrounds, and can be formulated in a 'doubled space' in which each circle is supplemented by a T-dual circle to construct a geometry which is a doubled torus bundle over a circle. We discuss a conjectured extension to include T-duality on the base circle, and propose the introduction of a dual base coordinate, to give a doubled space which is locally the group manifold of the gauge group. Special cases include those in which the doubled group is a Drinfel'd double. This gives a framework to discuss backgrounds that are not even locally geometric.

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1 Introduction

Two kinds of dimensional reduction of supergravities were proposed in the seminal paper of Scherk and Schwarz [1], each involving a twist by a group. Each gives a lower dimensional supergravity, which typically is gauged, i.e. has a non-Abelian Yang-Mills group. Recently it was understood how to lift these dimensional reductions to the full supergravity, string theory or M-theory [2, 3, 4, 5]. The key is to show that each can arise from a compactification, so that the full massive spectrum is defined, including Kaluza-Klein modes, massive string modes, wrapped branes etc. Understanding the compactification geometry is important in understanding the structure of the theory, and it turns out that this is intimately related to the structure of the gauge group. Some of the reductions, those with T-duality twists, do not lift to any compactification of supergravity. However, they can lift to non-geometric reductions of string theory on T-folds. Much remains to be understood about such non-geometric backgrounds, and the aim here is to use the gauge algebra of the dimensionally reduced theory to gain some insight into such reductions. In [6, 7], it was shown that string theory on a T-fold that looks like a T^d bundle locally has a natural formulation on a bundle in which the torus fibres are doubled to become T^{2d}. Our considerations here lead to a natural geometry in which all the dimensions are doubled, not just the fibres.

The first class of Scherk-Schwarz reductions look superficially like reductions on an n-torus, but twisted with the action of an n-dimensional group G. For this reason, they have become known, misleadingly, as twisted torus reductions. The reduction can be thought of as choosing an internal space that is the group manifold for G, which is typically non-compact, and then consistently truncating to fields independent of the 'internal' coordinates. In [3], it was shown that in most cases the same theory can be obtained from compactification on a compact manifold which looks like the group manifold locally. This requires the existence of a discrete subgroup $\Gamma \subset G$ such that $\mathcal{X} = G/\Gamma$ is compact (so that Γ is then a cocompact subgroup of G), in which case the theory is simply compactified on $\mathcal{X} = G/\Gamma$. The Scherk-Schwarz ansatz involves the expansion of the higher dimensional fields in terms of a basis of globally defined one-forms $\{\sigma\}$. In order for the one-forms σ to be globally defined on $\mathcal{X}$, it is necessary that they are invariant under the action of Γ, so that the reduction ansatz is invariant under Γ. However, one of the consequences of the reduction ansatz being invariant under Γ is that the gauged supergravity contains little information about the global structure of $\mathcal{X}$.

If we include a constant flux for the H-field so that $H \sim K_{mnp}\sigma^m \wedge \sigma^n \wedge \sigma^p + \ldots$, the supergravity, resulting from such a Scherk-Schwarz compactification on $\mathcal{X}$, has gauge algebra

$$[Z_m, Z_n] = f_{mn}{}^p Z_p + K_{mnp}X^p \qquad [X^m, Z_n] = f_{np}{}^m X^p \qquad [X^m, X^n] = 0 \tag{1.1}$$

where Z_m generate isometries of $\mathcal{X}$ and X^m generate antisymmetric tensor transformations for the B-field (see [3, 8] for details). Here $f_{mn}{}^p$ are the structure constants of the group G.

The other type of Scherk-Schwarz reduction starts with reduction on a torus T^d, so that the dimensionally reduced and truncated theory has a continuous duality symmetry K. This is followed by a further reduction on a circle with a duality twist, so that on going round the extra circle, the theory comes back to itself transformed by a duality transformation. In the full string theory or M-theory, the duality symmetry is broken to a discrete subgroup $K(\mathbb{Z})$ [9] and for the reduction to lift to string or M-theory, the monodromy must be in $K(\mathbb{Z})$ [10]. A subgroup $GL(d;\mathbb{Z})$ of $K(\mathbb{Z})$ acts geometrically as diffeomorphisms of the d-torus, and if the monodromy M is in $GL(d;\mathbb{Z})$, the reduction can be thought of as compactification on a $d+1$ dimensional space $\mathcal{X}$ which is a T^d bundle over a circle with monodromy M [3, 10]. Moreover, as we shall review in section 2, such a space $\mathcal{X}$ is in fact a twisted torus in the previous sense, i.e. it is locally a group manifold, and is of the form $\mathcal{X} = G/\Gamma$ [3]. In the case of reduction of pure gravity, the group G is precisely the gauge group of the reduced theory.

Here we will focus on the T-duality subgroup $O(d,d;\mathbb{Z}) \subseteq K(\mathbb{Z})$. There is a geometric subgroup of $O(d,d;\mathbb{Z})$ acting through torus diffeomorphisms and integral shifts of the B-field. If the monodromy is not in this subgroup, it is not a compactification, but can be thought of as string theory with a non-geometric internal space, known as a T-fold [6]. However, as $O(d,d;\mathbb{Z}) \subset GL(2d;\mathbb{Z})$, it has a natural action as diffeomorphisms of a 'doubled torus' T^{2d} and there is a T^{2d} bundle over a circle with such a monodromy. A formulation using a circle coordinate and its dual arises naturally in the string field theory for toroidal backgrounds [11]. String theory reduced in this way with a duality twist can be formulated as a sigma-model on a bundle over a circle whose fibres are the doubled torus T^{2d} [6]. The doubled formalism has the virtue that it provides a geometric interpretation to many nongeometric backgrounds. The doubled torus has coordinates conjugate to both the d momenta and the d winding numbers. Different dual backgrounds arise from choosing different polarisations or choices of $T^d \subset T^{2d}$, specifying the 'real' spacetime slice of the doubled space. T-duality acts to change the choice of polarisation, and T-folds arise when there is no global polarisation.

Such reductions with duality twists give theories with a gauge group of dimension $2(d+1)$, the same as it would be for reduction on an untwisted $S^1 \times T^d$. In that case, the group is $U(1)^{2(d+1)}$ with $U(1)^{d+1}$ from the natural geometric action on $S^1 \times T^d$ and another $U(1)^{d+1}$ from B-field gauge transformations. One might expect that reductions with a duality twist would give a gauge group containing $U(1)^{2d}$ associated with the T^d fibres. As it happens, this is not the case, and we give a careful derivation of the gauge algebra here. One of the aims of this paper is to explore the implications of the structure of the gauge algebra.

In the doubled formalism, the T^d fibre is doubled, and this raises the question of whether the base S^1 might also be doubled. This would be relevant for the issue of whether one can T-dualise over the base circle. As the geometry has non-trivial dependence on the S^1 coordinate x, there is no isometry on the circle so the usual formulations of T-duality do

not apply. However, in [12] a generalisation of T-duality to such situations was proposed, in which dependence on the circle coordinate x is transformed under T-duality to dependence on the coordinate of a dual circle, $\tilde{x}$. In this context it is natural to consider more general reductions involving independent duality twists over x and $\tilde{x}$. Such backgrounds would not admit a geometric description even locally.

Conventional considerations are insufficient to discuss the situation with non-trivial dependence on $\tilde{x}$. Here, we identify a $2(d+1)$ dimensional doubled geometry that extends the doubled torus bundle to include one other dimension, with coordinate $\tilde{x}$, and which is the group manifold of the gauge group, identified under a discrete subgroup to give a doubled twisted torus. This is the natural space to include all possible dual backgrounds, including the ones involving the conjectured generalised T-duality on the base S^1. The full gauge group contains generators acting geometrically on the original space and ones acting as B-field gauge transformations, while in this doubled picture, all arise geometrically.

The canonical example of the types of string background discussed above is given by a sequence of T-dualities starting from the three-dimensional nilmanifold. This nilmanifold is a T^2 bundle over S^1 with monodromy given by a parabolic element of $SL(2;\mathbb{Z})$. T-duality interchanges various quantities referred to as generalised fluxes in [13] and called f-flux, Q-flux and R-flux in [13]. As will be reviewed in the next section, the nilmanifold may be thought of as a twisted torus characterized by the structure constant (or 'geometric flux') $f_{xz}{}^{y} = m \in \mathbb{Z}$ [10, 3]. The Buscher rules can be applied fibrewise to dualise along the T^2 fibre directions. Dualising along the y direction gives a T^3 with H-flux $K_{xyz} = m$, whilst dualising along the z direction gives a T-fold, characterized by the 'flux' $Q_x{}^{zy} = m$. It was conjectured in [12] that a further T-duality along the x-direction gives rise to a background constructed as a T^2 fibration over the dual coordinate $\tilde{x}$ with a T-duality twist. This conjectured background is characterized by the nongeometric flux $R^{xyz} = m$ (or 'R-flux'). The duality sequence may be summarized as [13]

$$K_{xyz} \to f_{xz}{}^{y} \to Q_x{}^{yz} \to R^{xyz} \tag{1.2}$$

Whilst the doubled formalism has been successfully employed to give a geometric description of the T-fold, such an understanding of the backgrounds with R-flux has not been forthcoming. It is the aim of this paper to shed some light on the group theoretic and geometric structures which underly the duality sequence above. In particular we shall see that a knowledge of the gauge algebra of the compactified theory suggests a natural local structure for a doubled internal space.

A key objective is to understand how to lift a general gauged supergravity to superstring theory. The structure constants of the gauge algebra can be thought of as arising from the various types of flux, and so this is a question of understanding backgrounds with f, H, Q or R fluxes, and in particular the non-geometric ones with Q or R flux.

The plan of this paper is as follows: In the next section we review the relationship

between duality twist backgrounds with geometric monodromy and twisted tori. Section 3 will consider the general $O(d,d)$-twisted reduction. In section 4 the Yang-Mills gauge symmetries of this theory will be studied. Section 5 describes the doubled geometry of the backgrounds considered here.

2 Reductions with a Geometric Duality Twist

In this section we review the Scherk-Schwarz reduction with a geometric twist [3] and its relation to a reduction on a twisted torus of the form G/Γ.

Consider a $D+d+1$ dimensional field theory. We reduce the theory on a d-dimensional torus T^d, with real coordinates $z^a \sim z^a + 1$ where $a = 1, 2...d$. This produces a theory in $D+1$ dimensions with scalar fields that include those in the coset $GL(d;\mathbb{R})/SO(d)$ arising from the torus moduli. Truncating to the z^a independent zero mode sector gives a theory that has a rigid $GL(d;\mathbb{R})$ symmetry, while in the full Kaluza-Klein theory this is broken to $GL(d;\mathbb{Z})$ – the mapping class group of the T^d. Let

$$ds^2 = \hat{G}_{ab} dz^a dz^b \tag{2.1}$$

be the metric on the d-torus. The symmetric matrix $\hat{G}_{ab}$ parameterises the moduli space $GL(d;\mathbb{R})/SO(d)$. There is a natural action of $GL(d;\mathbb{R})$ on the metric and coordinates z^a in which

$$\hat{G}_{ab} \to (U^t)_a{}^c \hat{G}_{cd} U^d{}_b \qquad z^a \to (U^{-1})^a{}_b z^b \tag{2.2}$$

where $U^b{}_a \in GL(d,\mathbb{R})$. We now truncate to a massless $D+1$ dimensional field theory and consider reduction on a further circle. In the twisted reduction, dependence on the circle coordinate x is introduced through a $GL(d;\mathbb{R})$ transformation $U = \gamma(x)$ where $\gamma(x) = \exp(Nx)$ and $N^a{}_b$ is some matrix in the Lie algebra of $GL(d;\mathbb{R})$. This defines the x-dependence of the torus moduli through

$$G(x)_{ab} = (\gamma(x)^t)_a{}^c \hat{G}_{cd} \gamma(x)^d{}_a \tag{2.3}$$

for some arbitrary choice $\hat{G}_{ab}$. The monodromy round the circle $x \sim x+1$ is $e^N \in GL(d;\mathbb{R})$. The truncation of all Kaluza-Klein modes gives the Scherk-Schwarz reduction [3]. A necessary condition for this to lift to a compactification of the original $D+d+1$ dimensional theory, keeping all Kaluza-Klein modes, is that the monodromy is in $GL(d;\mathbb{Z})$, which puts strong constraints on the choice of N [14]. Assuming $e^N \in GL(d;\mathbb{Z})$, the twisted reduction is equivalent to the reduction on a T^d bundle over S^1 with metric

$$ds^2_{d+1} = dx^2 + G(x)_{ab} dz^a dz^b = (\sigma^x)^2 + \hat{G}_{ab} \sigma^a \sigma^b \tag{2.4}$$

where

$$\sigma^x = dx \qquad \sigma(x)^a = \gamma(x)^a{}_b dz^b \tag{2.5}$$

We now consider the group structure of this space. The forms (2.5) are globally defined on the torus bundle, and satisfy

$$d\sigma^x = 0 \qquad d\sigma^a - N^a{}_b \sigma^x \wedge \sigma^b = 0 \tag{2.6}$$

This $d+1$ dimensional space is then parallelisable, and locally looks like a group manifold G with left-invariant Maurer-Cartan forms σ associated with the Lie algebra

$$[t_x, t_a] = -N_a{}^b t_b, \qquad [t_a, t_b] = 0 \tag{2.7}$$

This algebra can be represented by the $(d+1) \times (d+1)$ matrices

$$t_x = \begin{pmatrix} -N^a{}_b & 0 \\ 0 & 0 \end{pmatrix} \qquad t_a = \begin{pmatrix} 0 & e_a \\ 0 & 0 \end{pmatrix} \tag{2.8}$$

where e_a is the d-dimensional column vector with a 1 in the a'th position and zeros everywhere else. A representation of this Lie algebra is given by

$$Z_x = \partial_x - N_a{}^b z^a \partial_b \qquad Z_a = \partial_a \tag{2.9}$$

These vector fields are invariant under the left action of the group and are dual to the one forms σ. Coordinates x, z^a can be introduced locally for the group manifold, with the group element $g = g(x, z^a) \in G$ given by

$$g = \begin{pmatrix} \gamma^{-1}(x) & z \\ 0 & 1 \end{pmatrix} \tag{2.10}$$

Then the left-invariant Maurer-Cartan forms are given by

$$g^{-1}dg = \begin{pmatrix} -N^a{}_b \sigma^x & \sigma^a \\ 0 & 0 \end{pmatrix} = \sigma^m t_m \tag{2.11}$$

in agreement with (2.5), where $m = 1, 2, ..d+1$. The T^d bundle over S^1 with metric (2.4) has the same local geometry as this group manifold.

The torus bundle over a circle is obtained from the compactification of this non-compact group manifold under the identification by a discrete subgroup Γ, acting from the left. The left action of

$$h(\alpha, \beta^a) = \begin{pmatrix} \gamma^{-1}(\alpha)^a{}_b & \beta^a \\ 0 & 1 \end{pmatrix} \tag{2.12}$$

is

$$g(x, z^a) \to h(\alpha, \beta^a) \cdot g(x, z^a) \tag{2.13}$$

and acts on the coordinates through

$$x \to x + \alpha \qquad z^a \to (e^{-N\alpha})^a{}_b z^b + \beta^a \tag{2.14}$$

with $\alpha, \beta^a \in \mathbb{Z}$ and form a discrete subgroup $\Gamma = \{h(\alpha, \beta^a) \in G \mid \alpha, \beta^a \in \mathbb{Z}\}$ and we can identify the group manifold G under Γ. This gives a compact space G/Γ, and is identical to the torus bundle over a circle with metric (2.4) [3].

3 Reduction with an $O(d,d)$ twist

We now turn to the duality-twisted reduction of theories with a metric and B-field, and we will be particularly interested in the cases that arise from string theory. Consider the theory in $D+d+1$ dimensional spacetime with Lagrangian

$$\mathcal{L}_{D+d+1} = e^{-\widehat{\Phi}}\left(\widehat{R}*1 - d\widehat{\Phi}\wedge *d\widehat{\Phi} - \frac{1}{2}\widehat{G}_{(3)}\wedge *\widehat{G}_{(3)}\right) \tag{3.1}$$

where $\widehat{G}_{(3)} = d\widehat{B}_{(2)}$. The compactification on T^d, using the standard Kaluza-Klein ansatz gives [15] a massless field theory with gauge group $U(1)^{2d} \subset O(d,d)$ and a manifestly $O(d,d)$ invariant Lagrangian

$$\begin{aligned}\mathcal{L}_{D+1} &= e^{-\phi}\left(R*1 + *d\phi\wedge d\phi + \frac{1}{2}*G_{(3)}\wedge G_{(3)} + \frac{1}{4}*d\mathcal{M}^{AB}\wedge d\mathcal{M}_{AB}\right.\\ &\quad \left. -\frac{1}{2}\mathcal{M}_{AB}*\mathcal{F}^A\wedge\mathcal{F}^B\right)\end{aligned} \tag{3.2}$$

The details of this reduction are given in [3, 8, 15] and the conventions of [3] have been used. The scalar coset space $O(d,d)/O(d)\times O(d)$ is parameterised by a symmetric metric on this coset $\mathcal{M}_{AB}$, satisfying the constraint

$$\mathcal{M}_{AB} = L_{AC}(\mathcal{M}^{-1})^{CD}L_{BD} \tag{3.3}$$

where L_{AB} is the constant $O(d,d)$ invariant metric, which is used to raise and lower the indices $A,B = 1,...,2d$.

We then reduce on a further circle, with coordinate $x \sim x+1$, with an $O(d,d)$ duality twist. The twist is specified by $N^A{}_B$, a matrix representation of an element of the Lie algebra of $O(d,d)$, and the x-dependence is given in terms of an $O(d,d)$ transformation $\exp(Nx)$. The theory has a Yang-Mills sector with a gauge group with structure constants $t_{MN}{}^P$ that will be discussed in the next subsection. The reduced theory may be written in a manifestly $O(d+1,d+1)$ covariant way

$$\begin{aligned}\mathcal{L}_D &= e^{-\varphi}\left(R*1 + *d\varphi\wedge d\varphi + \frac{1}{2}*\mathcal{H}_{(3)}\wedge\mathcal{H}_{(3)} + \frac{1}{4}*D\mathcal{M}_{MN}\wedge D\mathcal{M}^{MN}\right.\\ &\quad \left. -\frac{1}{2}\mathcal{M}_{MN}*\mathcal{F}^M\wedge\mathcal{F}^N\right) + V*1\end{aligned} \tag{3.4}$$

with $O(d+1,d+1)$ indices $M,N = 1,...,2(d+1)$ that are raised and lowered using the constant $O(d+1,d+1)$ invariant metric L_{MN}. The two-form field strengths $\mathcal{F}^M$ are written in terms of connection one-forms $\mathcal{A}^M$ and the three-form $\mathcal{H}_{(3)}$ is written in terms of the two-form potential $C_{(2)}$

$$\mathcal{F}^M = d\mathcal{A}^M + \frac{1}{2}t_{NP}{}^M\mathcal{A}^N\wedge\mathcal{A}^P \qquad \mathcal{H}_{(3)} = dC_{(2)} + \frac{1}{2}L_{MN}\mathcal{A}^M\wedge d\mathcal{A}^N - \frac{1}{3}t_{MNP}\mathcal{A}^M\wedge\mathcal{A}^N\wedge\mathcal{A}^P \tag{3.5}$$

where $t_{MNP} = t_{MN}{}^{Q} L_{PQ}$. The scalars $\mathcal{M}_{MN}$ take values in the coset space $O(d+1,d+1)/O(d+1)\times O(d+1)$ and satisfy a constraint similar to (3.3). The scalar potential is

$$V = e^{-\varphi}\left(\frac{1}{4}\mathcal{M}^{MQ}L^{NT}L^{PS}t_{MNP}t_{QTS} - \frac{1}{12}\mathcal{M}^{MQ}\mathcal{M}^{NT}\mathcal{M}^{PS}t_{MNP}t_{QTS}\right) \tag{3.6}$$

Details of the reduction and the explicit forms of the potential and scalars in terms of the $D+1$ dimensional fields are given in appendix A.

Gauge Symmetry

The $D+1$ dimensional theory (3.2) obtained from conventional reduction on T^d has $U(1)^{2d} \subset O(d,d)$ gauge symmetry. $U(1)^d$ comes from the isometry group of the internal T^d and a further $U(1)^d$ comes from the antisymmetric tensor transformations of the B-field. The generators of this gauge group T_A $(A = 1,...2d)$ satisfy $[T_A, T_B] = 0$. The duality twist reduction on a further circle with coordinate x to D dimensions gauges a non-Abelian subgroup $\mathcal{G} \subset O(d+1,d+1)$ given by the algebra

$$[Z_x, T_A] \;=\; -N^{B}{}_{A}T_B \qquad\qquad [T_A, T_B] = -N_{AB}X^x \tag{3.7}$$

where Z_x generates shifts in the circle coordinate x and X^x is the generator of antisymmetric tensor transformations of the B-field component with one leg along the x-direction and one leg in the external spacetime. All other commutators vanish. Here

$$N_{AB} = L_{[A|C}N^{C}{}_{|B]} = -N_{BA} \tag{3.8}$$

The antisymmetry of N_{AB} follows from the requirement that $N^{A}{}_{B}$ be a generator of $O(d,d)$. Note that the algebra satisfied by the generators T_A which can be associated with the action on T^d has been deformed and is no longer Abelian.

The generators

$$T_M = \begin{pmatrix} Z_x \\ X^x \\ T_A \end{pmatrix} \tag{3.9}$$

satisfy a Lie algebra $[T_M, T_N] = t_{MN}{}^{P}T_P$ where $t_{MN}{}^{P}$ are the structure constants given by

$$t_{xB}{}^{A} = -N^{A}{}_{B}, \qquad t_{x[AB]} = -N_{AB} \tag{3.10}$$

The derivation of this algebra is given in appendix B.

The gauging introduces a deformation of the ungauged theory involving the $t_{MN}{}^{P}$, which breaks the rigid $O(d+1,d+1)$ symmetry of the ungauged theory to the subgroup preserving the $t_{MN}{}^{P}$. However, the theory becomes formally invariant under $O(d+1,d+1)$ if the structure constants $t_{MN}{}^{P}$ are taken to transform covariantly under $O(d+1,d+1)$ [3, 8].

4 Lifting to String Theory

The discussion so far has used the framework of conventional field theory. In this section we discuss the lift of these results to string theory.

The T_A generators consist of the Z_a which generate the $U(1)^d$ action on the T^d fibre and the X^a which generate antisymmetric tensor transformations for the B-field components with one leg on the T^d and the other in the external spacetime, so that

$$T_A = \begin{pmatrix} Z_a \\ X^a \end{pmatrix} \tag{4.1}$$

The twist matrix then decomposes as (using $N_{AB} = -N_{BA}$)

$$N^A{}_B = \begin{pmatrix} f_{xa}{}^b & Q_x{}^{ab} \\ K_{xab} & -f_{xb}{}^a \end{pmatrix} \tag{4.2}$$

for some antisymmetric $Q_x{}^{ab} = -Q_x{}^{ba}, K_{xab} = -K_{xba}$. The gauge algebra is then

$$[Z_x, Z_a] = f_{xa}{}^b Z_b + K_{xab} X^b \qquad [Z_x, X^a] = -f_{xb}{}^a X^b + Q_x{}^{ab} Z_b \tag{4.3}$$

$$[Z_a, Z_b] = K_{xab} X^x \qquad [X^a, Z_b] = -f_{xb}{}^a X^x \qquad [X^a, X^b] = Q_x{}^{ab} X^x \tag{4.4}$$

with all other commutators vanishing.

If $Q_x{}^{ab} = 0$, then the twist is geometric, consisting of a $GL(d;\mathbb{Z})$ twist with $f_{xa}{}^b = N_a{}^b$ acting as a diffeormorphism of the T^d fibres generated by $N_a{}^b$ together with a B-shift acting on the fibre components of B, $B_{ab} \to B_{ab} + \alpha K_{xab}$. This is equivalent to the compactification with flux K on a T^d torus bundle over a circle and, as reviewed in section 2, this is a twisted torus G/Γ where G is the $d+1$ dimensional group of matrices of the form (2.10). The K_{xab} gives a constant flux $K_{xab}\sigma^x \wedge \sigma^a \wedge \sigma^b$ on the twisted torus. These backgrounds have been studied extensively in [3, 4, 5, 8].

If $Q_x{}^{ab} \neq 0$, the twist is non-geometric and involves T-dualities, so that the resulting background is a T-fold. As was shown in [12, 13, 16] backgrounds with just one of these three structure constants switched on can be related to one another by T-duality so that T-duality is expected to be a symmetry of the full string theory which identifies certain H-flux, twisted torus and T-fold compactifications as equivalent descriptions of the same physics.

The twist means that there is no isometry on the final circle acting to shift the coordinate x. Nonetheless, there is some evidence that there should still be a T-duality on this circle [12] that exchanges Z_x with X^x and would act on the structure constants as

$$K_{xab} \to f_{ab}{}^x \qquad f_{xa}{}^b \to Q_a{}^{xb} \qquad Q_x{}^{ab} \to R^{xab} \tag{4.5}$$

to give the algebra

$$[X^x, Z_a] = Q_a{}^{xb} Z_b + f_{ab}{}^x X^b \qquad [X^x, X^a] = -Q^{xa}{}_b X^b + R^{xab} Z_b \tag{4.6}$$

$$[Z_a, Z_b] = f_{ab}{}^x Z_x \qquad [X^a, Z_b] = -Q^{xa}{}_b Z_x \qquad [X^a, X^b] = R^{xab} Z_x \tag{4.7}$$

It was conjectured in [12] that the structure constant R^{xab} ('R-flux') corresponds to a background constructed with a twist over a dual circle $\widetilde{S}^1$ (with coordinate $\tilde{x}$ conjugate to the winding number). In the next section we propose a geometric interpretation for all of these backgrounds and show that it supports this interpretation of the R-flux.

5 Doubled Geometry

In section 2 we considered the case of a twisted reduction which has a simple geometric interpretation as a compactification on a T^d bundle over S^1 in which the torus moduli have monodromy in $GL(d;\mathbb{Z})$ round the base circle. The internal space is a twisted torus, or group manifold identified under a discrete subgroup. Including a monodromy that shifts the B-field corresponds to adding an H-flux to the twisted torus. We now turn to the geometric interpretation of the T-duality twisted reductions of section 3.

The Doubled Torus

For the general (nongeometric) case a geometric approach has been given by the doubled torus formalism of [6]. The $O(d,d;\mathbb{Z})$ duality twist acts non-geometricaly on the torus T^d (mixing the metric and B-field, for example) but as $O(d,d;\mathbb{Z}) \subset GL(2d;\mathbb{Z})$, it has a natural action as diffeormorphisms of a doubled torus T^{2d}. There is then a T^{2d} bundle over a circle with twist generated by $N^A{}_B$ constructed as in section 2. Such a doubled torus arises naturally in string theory, with the original d coordinates z^a on T^d conjugate to the momenta and an additional d coordinates $\tilde{z}_a$ conjugate to the winding numbers on the original T^d. The $O(d,d;\mathbb{Z})$ duality group acts naturally on the periodic doubled coordinates $\mathbb{X}^A = (z^a, \tilde{z}_a)$. It was shown in [6, 7] that string theory compactified in this way could be formulated in terms of a sigma model with target given by this doubled torus bundle. In this formalism, T-duality is a manifest symmetry, and the conventional formalism is recovered on choosing a polarisation, i.e. a $T^d \subset T^{2d}$ which is to be regarded as the real spacetime torus. T-duality can be viewed as acting to change the choice of $T^d \subset T^{2d}$, changing the geometry to a dual one. All dual geometries are encoded in the doubled torus bundle. For a geometric background, a global polarisation can be chosen, but for T-folds the best one can do is choose a polarisation locally. The T-duality transition functions then give the changes in polarisation from patch to patch.

A doubled torus bundle over a circle is a twisted torus G'/Γ', as in section 2. Simply applying the construction of section 2 to the doubled torus gives a background in which the

group G' has generators T_A, Z_x satisfying the algebra

$$[Z_x, T_A] = -N^B{}_A T_B \qquad [T_A, T_B] = 0 \tag{5.1}$$

acting on the coordinates $(x, \mathbb{X}^A)$ as

$$Z_x = \partial_x + N^A{}_B \mathbb{X}^B \partial_A \qquad T_A = \partial_A \tag{5.2}$$

This algebra does not capture the full gauge algebra (3.7). It is not a subalgebra, but it is the algebra acting on the sector in which X^x acts trivially. In order to give a full geometric interpretation to the gauge algebra (3.7) we need to extend the doubled torus construction.

The Doubled Group

The doubled torus formalism in which the fibres are doubled is useful for discussing T-duality on the fibres and the various T-dual spaces arise as different polarisations of the doubled torus bundle. If, as suggested in [12], one can also T-dualise on the base circle with coordinate x, it is natural to ask whether there is a doubled space that would include a T-dual circle to the base circle so that all T-dual spaces are incorporated as different $d+1$ dimensional polarisations of a $2(d+1)$ dimensional space $\mathcal{X}$. In each polarisation, half of the gauge group generators (the ones we have denoted Z) might be expected to act geometrically on the $d+1$ dimensional space (in the simplest cases, these generate diffeomorphisms of the space). For this to apply for any polarisation, it is natural to expect that the full gauge group (generated by the Z's and X's with Lie algebra (3.7)) should act on the doubled space. Comparison with the twisted torus construction suggests then that the doubled space should be locally a group manifold $\mathcal{G}$, with Lie algebra (3.7), identified under a discrete subgroup.

As in the discussion of the twisted torus geometry, one can represent the Lie algebra (3.7) in terms of the $2(d+1)$ coordinates $(x, \tilde{x}, \mathbb{X}^A)$ of $\mathcal{G}$, where $\mathbb{X}^A$ are the coordinates on the doubled torus fibre T^{2d}, as

$$Z_x = \partial_x + N^A{}_B \mathbb{X}^B \partial_A \qquad X^x = \partial_{\tilde{x}} \qquad T_A = \partial_A - \tfrac{1}{2} N_{AB} \mathbb{X}^B \partial_{\tilde{x}} \tag{5.3}$$

Then X^x acts as translation in the new coordinate $\tilde{x}$ and so acts trivially on fields that are independent of $\tilde{x}$, so on such fields the algebra (5.1) is realised and in this sector the doubled torus bundle gives a full geometric representation of the structure. However, the doubled group gives a non-trivial extension to the general case with $\tilde{x}$ dependence.

The one forms dual to these vector fields satisfy the Maurer-Cartan equations

$$d\mathcal{P}^A - N^A{}_B P^x \wedge \mathcal{P}^B = 0 \qquad dQ_x - \frac{1}{2} N_{AB} \mathcal{P}^A \wedge \mathcal{P}^B = 0 \qquad dP^x = 0 \tag{5.4}$$

which are solved by[1]

$$\mathcal{P}^A = \left(e^{Nx}\right)^A{}_B d\mathbb{X}^B \qquad Q_x = d\tilde{x} + \frac{1}{2}N_{AB}\mathbb{X}^A d\mathbb{X}^B \qquad P^x = dx \tag{5.6}$$

This is a doubling of the geometry given for the twisted torus in section 2, and the one-forms (5.6) are the doubling of the one-forms (2.5). The $\mathcal{P}^A$ and P^x together describe the doubled torus fibred over S^1, but a fully geometric interpretation of the gauge algebra requires a $2d+2$ dimensional space $\mathcal{G}$ into which the doubled torus fibration is non-trivially embedded. It is useful to define the coordinates $\mathbb{X}^I = \left(x, \tilde{x}, \mathbb{X}^A\right)$ on the doubled group and $\mathcal{P}^M = \mathcal{P}^M{}_I d\mathbb{X}^I$ as the one forms on $\mathcal{G}$ satisfying the Maurer-Cartan equations

$$d\mathcal{P}^M + \frac{1}{2}t_{NP}{}^M \mathcal{P}^N \wedge \mathcal{P}^P = 0 \tag{5.7}$$

where $t_{xB}{}^A = -N^A{}_B$ and $t_{x[AB]} = -N_{AB}$.

T-Duality and R-Flux

In the doubled torus picture, choosing a polarisation corresponded to choosing a maximally isotropic subspace (null with respect to the constant $O(d,d)$ metric L_{AB}) $T^d \subset T^{2d}$ as the geometric space with coordinates z^a (and geometric generators Z_a) and the complement $\widetilde{T}^d$, with coordinates $\tilde{z}_a$ (and generators X^a). As $\mathcal{G} \subset O(d+1,d+1)$, it preserves the $O(d+1,d+1)$ invariant metric L_{MN}, and an isotropic subspace of $\mathcal{G}$ is one which is completely null with respect to this metric. In the doubled group case, a choice of polarisation can be given by choosing a maximally isotropic subgroup $G \subset \mathcal{G}$ (i.e. one whose generators are all null with respect to L_{MN}). The geometry of the conventional sigma-model is given locally by G. In some cases, the complement of G will also be a group $\widetilde{G}$, and this defines a dual polarisation. For example, if the gauge algebra is a Poisson-Lie algebra, it takes the form

$$[Z_m, Z_n] = f_{mn}{}^p Z_p \qquad [Z_m, X^n] = f_{mp}{}^n X^p + Q_m{}^{np} Z_p \qquad [X^m, X^n] = Q_p{}^{mn} X^p \tag{5.8}$$

and has two maximally isotropic sub-algebras; one generated by Z_m and the other by X^m, where $m = (a, x)$. These generate two subgroups G and $\widetilde{G}$ and either can be used to define a physical subspace, giving two, locally geometric, string backgrounds.

For general groups $\mathcal{G}$, however, it may be the case that there is no suitable subgroup that can be used to define the desired polarisation, so that one has to use the doubled picture and cannot eliminate half of the coordinates even locally. This is precisely the situation that leads to the locally nongeometric R-space, which we now discuss.

[1] The one-forms (5.6) are dual to the vectors

$$Z_x = \partial_x \qquad X^x = \partial_{\tilde{x}} \qquad T_A = \left(e^{-Nx}\right)_A{}^B \left(\partial_B - \frac{1}{2}N_{BC}\mathbb{X}^C \partial_{\tilde{x}}\right) \tag{5.5}$$

By a coordinate redefinition $\mathbb{X}^A \to \left(e^{Nx}\right)^A{}_B \mathbb{X}^B$, these vector fields are brought to the simpler form (5.3).

In the doubled group picture, one might expect a generalisation of T-duality which acts on all the coordinates $\mathbb{X}^I$. This allows us to consider the possibility of choosing either x or its dual $\tilde{x}$ as the geometric coordinate in a polarisation. This is to be contrasted with the doubled torus picture which, a priori, fixes x to be the geometric coordinate and only doubles the fibres.

Acting with the conjectured T-duality on the algebra (3.7) which exchanges x and $\tilde{x}$ produces the gauge algebra

$$[X^x, T_A] = -N^B{}_A T_B \qquad [T_A, T_B] = -N_{AB} Z_x \tag{5.9}$$

which has corresponding one-forms

$$\mathcal{P}^A = \left(e^{N\tilde{x}}\right)^A{}_B d\mathbb{X}^B \qquad P^x = dx + \frac{1}{2} N_{AB} \mathbb{X}^A d\mathbb{X}^B \qquad Q_x = d\tilde{x} \tag{5.10}$$

which is an $O(d,d)$ twist over the dual coordinate $\tilde{x}$ as conjectured in [12]. This has non-trivial dependence on the dual coordinates $\tilde{x}$, so cannot be interpreted as a conventional background even locally. This space is a $2d+2$ dimensional twisted torus with coordinates $\mathbb{X}^I$.

Global Issues

As in the twisted torus example of section 2, the gauge algebra only fixes the local structure of the (in this case, doubled) geometry. This can be seen by the fact that the one forms (5.6) are invariant under the rigid left action of $\mathcal{G}$, which acts on the coordinates infinitesimally as

$$\delta x = \alpha \qquad \delta\tilde{x} = \tilde{\alpha} - \frac{1}{2} N_{AB} \xi^A \mathbb{X}^B \qquad \delta\mathbb{X}^A = N^A{}_B \mathbb{X}^B \alpha + \xi^A \tag{5.11}$$

and so the global structure of the doubled group is thus far only determined up to a rigid left action of $\mathcal{G}$. In general, the doubled space will be of the form $\mathcal{X} \simeq \mathcal{G}/\Gamma$ for some discrete subgroup Γ. The gauge algebra fixes the local structure of the doubled group, but the global structure remains undetermined. In particular, the choice of discrete subgroup Γ is not determined by the gauge algebra. However, consistency with the doubled torus picture fixes the identification of most of the coordinates, but not that of $\tilde{x}$. In the case of a trivial bundle $\tilde{x}$ is the coordinate for a dual circle with radius inversely related to that of the x circle [17]. It seems reasonable to expect that Γ should be chosen to be cocompact, so that $\mathcal{G}/\Gamma$ is compact. We will return to the discussion of the doubled geometry $\mathcal{X} \simeq \mathcal{G}/\Gamma$ and its role in the discussion of T-duality elsewhere, and show how Γ is fixed in particular examples.

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A $O(d,d)$-Twisted Reduction

The reduction ansatz is

$$\begin{aligned} ds^2_{D+1} &= ds^2_D + \rho\nu^x \otimes \nu^x \\ \mathcal{A}^A(x,y) &= \left(e^{Nx}\right)^A{}_B \left(\mathcal{A}^B_{(1)}(y) + \mathcal{A}^B_{(0)}\nu^x\right) \\ \mathcal{B}_{(2)}(x,y) &= B_{(2)}(y) + B_{(1)}(y) \wedge \nu^x \\ \mathcal{M}^{AB}(x,y) &= \left(e^{Nx}\right)^A{}_C \mathcal{M}^{CD}(y) \left(e^{-Nx}\right)_D{}^B \\ \phi &= \varphi + \frac{1}{2}\ln(\rho) \end{aligned} \tag{A.1}$$

where the vielbein ν^x is

$$\nu^x = dx - V^x_{(1)} \tag{A.2}$$

and we have introduced the connection $V^x_{(1)}$ with field strength $F^x_{(2)} = dV^x_{(1)}$. Using the field redefinitions

$$\begin{aligned} C_{(2)} &= B_{(2)} - \frac{1}{2} C_{(1)} \wedge V^x_{(1)} \\ C_{(1)} &= B_{(1)} - \frac{1}{2} L_{AB} \mathcal{A}^A_{(0)} \mathcal{A}^B_{(1)} \\ C_{(0)} &= \frac{1}{2} L_{AB} \mathcal{A}^A_{(0)} \mathcal{A}^B_{(0)} \end{aligned} \tag{A.3}$$

the reduced theory may be written in a manifestly $O(d+1,d+1)$ covariant way

$$\begin{aligned} \mathcal{L}_D &= e^{-\varphi} \left(R * 1 + *d\varphi \wedge d\varphi + \frac{1}{2} * \mathcal{H}_{(3)} \wedge \mathcal{H}_{(3)} + \frac{1}{4} * D\mathcal{M}_{MN} \wedge D\mathcal{M}^{MN} \right. \\ &\quad \left. - \frac{1}{2} \mathcal{M}_{MN} * \mathcal{F}^M \wedge \mathcal{F}^N \right) + V * 1 \end{aligned} \tag{A.4}$$

The scalar potential is now written in the $O(d+1,d+1)$ covariant form

$$V = e^{-\varphi} \left(\frac{1}{4} \mathcal{M}^{MQ} L^{NT} L^{PS} t_{MNP} t_{QTS} - \frac{1}{12} \mathcal{M}^{MQ} \mathcal{M}^{NT} \mathcal{M}^{PS} t_{MNP} t_{QTS} \right) \tag{A.5}$$

The scalars parameterise the coset $O(d+1,d+1)/O(d+1) \times O(d+1)$

$$\mathcal{M}_{MN} = \begin{pmatrix} \rho + \mathcal{M}_{AB} \mathcal{A}^A_{(0)} \mathcal{A}^B_{(0)} + \rho^{-1} C_{(0)} C_{(0)} & \rho^{-1} C_{(0)} & \rho^{-1} C_{(0)} L_{AC} \mathcal{A}^C_{(0)} + \mathcal{M}_{AC} \mathcal{A}^C_{(0)} \\ \rho^{-1} C_{(0)} & \rho^{-1} & \rho^{-1} L_{AC} \mathcal{A}^C_{(0)} \\ \rho^{-1} C_{(0)} L_{BC} \mathcal{A}^C_{(0)} + \mathcal{M}_{BC} \mathcal{A}^C_{(0)} & \rho^{-1} L_{AC} \mathcal{A}^C_{(0)} & \mathcal{M}_{AB} + \rho^{-1} L_{AC} L_{BD} \mathcal{A}^C_{(0)} \mathcal{A}^D_{(0)} \end{pmatrix}$$

The one, two and three-form field strengths are

$$\begin{aligned} \mathcal{H}_{(3)} &= dC_{(2)} + \frac{1}{2} \left(L_{MN} \mathcal{A}^M \wedge \mathcal{F}^N - \frac{2}{3} t_{MNP} \mathcal{A}^M \wedge \mathcal{A}^N \wedge \mathcal{A}^P \right) \\ D\mathcal{M}^{MN} &= d\mathcal{M}^{MN} + \mathcal{M}^{MP} t_{PQ}{}^N \mathcal{A}^Q + \mathcal{M}^{NP} t_{PQ}{}^M \mathcal{A}^Q \\ \mathcal{F}^M &= d\mathcal{A}^M + \frac{1}{2} t_{NP}{}^M \mathcal{A}^N \wedge \mathcal{A}^P \end{aligned} \tag{A.6}$$

The one-forms form an $O(d+1,d+1)$ vector $\mathcal{A}^M$ with field strength $\mathcal{F}^M$

$$\mathcal{A}^M = \begin{pmatrix} V^x_{(1)} \\ C_{(1)} \\ \mathcal{A}^A_{(1)} \end{pmatrix} \qquad \mathcal{F}^M = \begin{pmatrix} F^x_{(2)} \\ G_{(2)x} \\ \mathcal{F}^A_{(2)} \end{pmatrix} \tag{A.7}$$

where we have defined

$$G_{(2)x} \;=\; dC_{(1)} - \frac{1}{2} N_{AB} \mathcal{A}^A_{(1)} \wedge \mathcal{A}^B_{(1)} \tag{A.8}$$

Defining $t_{MNP} = L_{MQ} t_{NP}{}^Q$ where L_{MN} is the $O(d+1,d+1)$ invariant matrix which takes the block diagonal form

$$L_{MN} = \begin{pmatrix} L_{xx} & 0 \\ 0 & L_{AB} \end{pmatrix} \qquad L_{xx} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{A.9}$$

the structure constants are $t_{xB}{}^A = -N^A{}_B$ and $t_{x[AB]} = -N_{AB}$. The presence of $t_{MN}{}^P$ breaks the rigid $O(d+1,d+1)$ symmetry of the ungauged theory to the subgroup preserving $t_{MN}{}^P$. However, the theory becomes formally invariant under $O(d+1,d+1)$ if the structure constants are taken to transform covariantly under $O(d+1,d+1)$.

B Gauge Symmetry

In $D+d+1$ dimensions the theory has the antisymmetric tensor transformation symmetry

$$\widehat{B}_{(2)} \to \widehat{B}_{(2)} + d\widehat{\Lambda}_{(1)} \tag{B.1}$$

The reduction ansatz for the parameter $\widehat{\Lambda}_{(1)}$ on T^d is $\widehat{\Lambda}_{(1)} = \Lambda_{(1)} + \Lambda_{(0)a}\nu^a$. The remainder of the $U(1)^{2d}$ gauge symmetry comes from the d isometries of the T^d, $x^a \to x^a - \omega^a$, under which $\delta A^a_{(1)} = -d\omega^a$ and all other fields are invariant. In $D+1$ dimensions this $U(1)^{2d}$ gauge symmetry acts on the fields as

$$\begin{aligned} \delta_T A^A_{(1)} &= d\Lambda^A_{(0)} \\ \delta_T \mathcal{B}_{(2)} &= d\Lambda_{(1)} + \frac{1}{2} L_{AB} \Lambda^A_{(0)} \widehat{\mathcal{F}}^B_{(2)} \end{aligned} \tag{B.2}$$

where we have defined

$$\Lambda^A_{(0)} = \begin{pmatrix} -\omega^a \\ \lambda_{(0)a} \end{pmatrix} \tag{B.3}$$

Antisymmetric tensor transformations

The duality twist reduction ansatz for the $D+1$ dimensional gauge parameters $\lambda_{(1)}$ and $\lambda^A_{(0)}$ is

$$\Lambda^A_{(0)} = \left(e^{Nx}\right)^A{}_B \lambda^B \qquad \Lambda_{(1)} = \lambda_{(1)} + \lambda_x \nu^x \tag{B.4}$$

We denote the infinitesimal variation of the fields under this transformation by δ_T. It is easy to show, by calculating $d\widehat{\lambda}^A_{(0)}$, that the D-dimensional gauge potentials transform as

$$\begin{aligned} \delta_T \mathcal{A}^A_{(1)} &= d\lambda^A + N^A{}_B \lambda^B V^x_{(1)} \\ \delta_T C_{(1)} &= d\lambda_x + N_{AB} \lambda^A \mathcal{A}^B_{(1)} \end{aligned} \tag{B.5}$$

S^1 Diffeomorphisms

The theory must be invariant under reparameterisations of the circle coordinate

$$x \to x - \omega \tag{B.6}$$

The matrix e^{Nx} changes as $\left(e^{Nx}\right)^A{}_B \to \left(e^{Nx}\right)^A{}_C \left(e^{-N\omega}\right)^C{}_B = \left(e^{Nx}\right)^A{}_C \left(\delta^C{}_B - N^C{}_B \omega + ...\right)$. From this is it easy to see how the D-dimensional fields must transform in order for the $D+1$ dimensional ansatz to be invariant. The gauge fields transform as

$$\begin{aligned} \delta_Z \mathcal{A}^A_{(1)} &= N^A{}_B \mathcal{A}^B_{(1)} \omega \\ \delta_Z V^x_{(1)} &= -d\omega \end{aligned} \tag{B.7}$$

Symmetry Algebra

We define

$$\delta_Z = \omega Z_x \qquad \delta_T = \lambda^A T_A \qquad \delta_X = \lambda_x X^x \tag{B.8}$$

where Z_x, X^x and T_A are generators of gauge transformations with parameters ω, λ_x and λ^A respectively. The Lie algebra of the gauge group is

$$[Z_x, T_A] = -N^B{}_A T_B \qquad [T_A, T_B] = -N_{AB} X^x \tag{B.9}$$

with all other commutators vanishing.

References

[1] J. Scherk and J. H. Schwarz, "How To Get Masses From Extra Dimensions," Nucl. Phys. B **153**, 61 (1979).

[2] A. Dabholkar and C. Hull, "Duality twists, orbifolds, and fluxes," JHEP **0309**, 054 (2003) [arXiv:hep-th/0210209].

[3] C. M. Hull and R. A. Reid-Edwards, "Flux compactifications of string theory on twisted tori," arXiv:hep-th/0503114.

[4] C. M. Hull and R. A. Reid-Edwards, "Flux compactifications of M-theory on twisted tori," JHEP **0610** (2006) 086 [arXiv:hep-th/0603094].

[5] R. A. Reid-Edwards, "Geometric and non-geometric compactifications of IIB supergravity," arXiv:hep-th/0610263.

[6] C. M. Hull, "A geometry for non-geometric string backgrounds," JHEP **0510** (2005) 065 [arXiv:hep-th/0406102].

[7] C. M. Hull, "Doubled geometry and T-folds," JHEP **0707** (2007) 080 [arXiv:hep-th/0605149].

[8] N. Kaloper and R. C. Myers, "The O(dd) story of massive supergravity," JHEP **9905**, 010 (1999) [arXiv:hep-th/9901045].

[9] C. M. Hull and P. K. Townsend, "Unity of superstring dualities," Nucl. Phys. B **438** (1995) 109 [arXiv:hep-th/9410167].

[10] C. M. Hull, "Massive string theories from M-theory and F-theory," JHEP **9811** (1998) 027 [arXiv:hep-th/9811021].

[11] T. Kugo and B. Zwiebach, "Target space duality as a symmetry of string field theory," Prog. Theor. Phys. **87** (1992) 801 [arXiv:hep-th/9201040].

[12] A. Dabholkar and C. Hull, "Generalised T-duality and non-geometric backgrounds," JHEP **0605**, 009 (2006) [arXiv:hep-th/0512005].

[13] J. Shelton, W. Taylor and B. Wecht, "Nongeometric flux compactifications," JHEP **0510**, 085 (2005) [arXiv:hep-th/0508133].

[14] C. M. Hull, "Gauged D = 9 supergravities and Scherk-Schwarz reduction," Class. Quant. Grav. **21**, 509 (2004) [arXiv:hep-th/0203146].

[15] J. Maharana and J. H. Schwarz, "Noncompact symmetries in string theory," Nucl. Phys. B **390**, 3 (1993) [arXiv:hep-th/9207016].

[16] C. M. Hull, "Global Aspects of T-Duality, Gauged Sigma Models and T-Folds," arXiv:hep-th/0604178.

[17] T. H. Buscher, "A Symmetry of the String Background Field Equations," Phys. Lett. B **194**, 59 (1987).