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Semi-inclusive deep inelastic scattering at small transverse momentum

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Abstract: We study the cross section for one-particle inclusive deep inelastic scattering off the nucleon for low transverse momentum of the detected hadron. We decompose the cross section in terms of structure functions and calculate them at tree level in terms of transverse-momentum-dependent parton distribution and fragmentation functions. Our results are complete in the one-photon exchange approximation at leading and first subleading twist accuracy, with both beam and target polarization.

Keywords: Deep Inelastic Scattering, QCD, Spin and Polarization Effects.

Contents

1. Introduction

In one-particle-inclusive deep inelastic scattering (DIS) a lepton scatters off a nucleon and one of the hadrons produced in the collision is detected. In the one-photon exchange approximation, the lepton-nucleon interaction proceeds via a photon of virtuality Q. The cross section depends in particular on the azimuthal angle of the final state hadron about the virtual photon axis, as well as on the azimuthal angle of the target polarization. In the kinematic region where the transverse momentum of the outgoing hadron is low compared to Q, the cross section can be expressed in terms of transverse-momentum-dependent parton distribution functions (PDFs) and fragmentation functions (FFs). These partonic functions are generalizations of the distribution and fragmentation functions appearing in standard collinear factorization. They are often referred to as unintegrated functions, as they are not integrated over the transverse momentum.

The most complete treatment to date of one-particle-inclusive deep inelastic scattering at small transverse momentum remains the work of Mulders and Tangerman [1], complemented by Refs. [2, 3]. In the last ten years, however, the subject has seen important theoretical and experimental progress. Initiated by the calculation of a nonvanishing Sivers effect in Ref. [4], unexpected developments arose with the correct treatment of Wilson lines in the definition of transverse-momentum-dependent PDFs and FFs [5, 6]. In particular, the fundamental tenet of universality of PDFs and FFs was revised [7, 8, 9]. New factorization proofs for the process under consideration here were put forward [10, 11], updating past work [12]. Some relations proposed in Ref. [1] turned out to be invalid [13, 14], and three new PDFs were discovered [15, 16]. In the meanwhile, several experimental measurements of azimuthal asymmetries in semi-inclusive DIS were performed [17, 18, 19, 20, 21, 22, 23, 24, 25, 26].

We consider it timely to present in a single, self-contained paper the results for one-particle-inclusive deep inelastic scattering at small transverse momentum, in particular including in the cross section all functions recently introduced. In Section 2 we recall the general form of the cross section for polarized semi-inclusive DIS and parameterize it in terms of suitable structure functions. In Section 3 we give the full parameterization of quark-quark and quark-gluon-quark correlation functions up to twist three and review the relations between these functions which are due to the QCD equations of motion. The structure functions for semi-inclusive DIS at small transverse momentum and twist-three accuracy are given in Section 4, and Section 5 contains our conclusions. The relation of the structure functions in the present paper with the parameterization in Ref. [27] is given in Appendix A, and results for one-jet production in DIS are listed in Appendix B.

2. The cross section in terms of structure functions

We consider the process

$$\ell(l) + N(P) \to \ell(l') + h(P_h) + X, \tag{2.1}$$

where ℓ denotes the beam lepton, N the nucleon target, and h the produced hadron, and where four-momenta are given in parentheses. Throughout this paper we work in the one-photon exchange approximation and neglect the lepton mass. We denote by M and M_h the respective masses of the nucleon and of the hadron h. As usual we define $q = l - l'$ and $Q^2 = -q^2$ and introduce the variables

$$x = \frac{Q^2}{2\,P\cdot q}, \qquad y = \frac{P\cdot q}{P\cdot l}, \qquad z = \frac{P\cdot P_h}{P\cdot q}, \qquad \gamma = \frac{2Mx}{Q}. \tag{2.2}$$

Throughout this section we work in the target rest frame. Following the Trento conventions [28] we define the azimuthal angle ϕ_h of the outgoing hadron by

$$\cos\phi_h = -\frac{l_\mu P_{h\nu}\, g_\perp^{\mu\nu}}{\sqrt{l_\perp^2\, P_{h\perp}^2}}, \qquad \sin\phi_h = -\frac{l_\mu P_{h\nu}\, \epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2\, P_{h\perp}^2}}, \tag{2.3}$$

where $l_\perp^\mu = g_\perp^{\mu\nu} l_\nu$ and $P_{h\perp}^\mu = g_\perp^{\mu\nu} P_{h\nu}$ are the transverse components of l and P_h with respect to the photon momentum. The tensors

$$g_\perp^{\mu\nu} = g^{\mu\nu} - \frac{q^\mu P^\nu + P^\mu q^\nu}{P\cdot q\,(1+\gamma^2)} + \frac{\gamma^2}{1+\gamma^2}\left(\frac{q^\mu q^\nu}{Q^2} - \frac{P^\mu P^\nu}{M^2}\right), \tag{2.4}$$

$$\epsilon_\perp^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}\frac{P_\rho\, q_\sigma}{P\cdot q\,\sqrt{1+\gamma^2}} \tag{2.5}$$

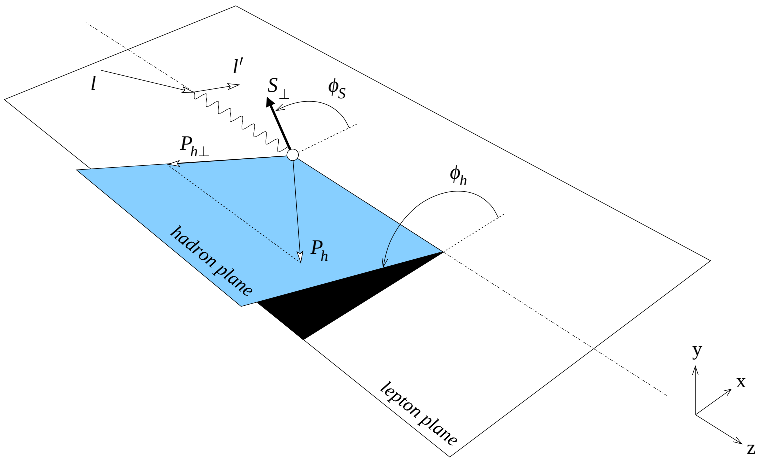

Figure 1: Definition of azimuthal angles for semi-inclusive deep inelastic scattering in the target rest frame [28]. $P_{h\perp}$ and $S_\perp$ are the transverse parts of P_h and S with respect to the photon momentum.

have nonzero components $g_\perp^{11} = g_\perp^{22} = -1$ and $\epsilon_\perp^{12} = -\epsilon_\perp^{21} = 1$ in the coordinate system of Fig. 1, our convention for the totally antisymmetric tensor being $\epsilon^{0123} = 1$. We decompose the covariant spin vector S of the target as

$$S^\mu = S_\parallel \frac{P^\mu - q^\mu M^2/(P\cdot q)}{M\sqrt{1+\gamma^2}} + S_\perp^\mu , \qquad S_\parallel = \frac{S\cdot q}{P\cdot q}\,\frac{M}{\sqrt{1+\gamma^2}} , \qquad S_\perp^\mu = g_\perp^{\mu\nu} S_\nu \tag{2.6}$$

and define its azimuthal angle ϕ_S in analogy to ϕ_h in Eq. (2.3), with P_h replaced by S. Notice that the sign convention for the longitudinal spin component is such that the target spin is parallel to the virtual photon momentum for $S_\parallel = -1$. The helicity of the lepton beam is denoted by λ_e. We consider the case where the detected hadron h has spin zero or where its polarization is not measured.

Assuming single photon exchange, the lepton-hadron cross section can be expressed in a model-independent way by a set of structure functions, see e.g. Refs. [29, 30, 27]. We use here a modified version of the notation in Ref. [27], see App. A, and write[1]

$$\begin{aligned}
&\frac{d\sigma}{dx\, dy\, d\psi\, dz\, d\phi_h\, dP_{h\perp}^2} = \\
&\quad \frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\, F_{UU}^{\cos\phi_h} \\
&\qquad + \varepsilon\cos(2\phi_h)\, F_{UU}^{\cos 2\phi_h} + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\, F_{LU}^{\sin\phi_h} \\
&\qquad + S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\, F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\, F_{UL}^{\sin 2\phi_h}\right]
\end{aligned}$$

[1]The polarizations S_L and S_T in [27] have been renamed to $S_\parallel$ and $|\boldsymbol{S}_\perp|$ here. This is to avoid a clash of notation with Section 3, where subscripts L and T refer to a different z-axis than in Fig. 1.

$$
\begin{aligned}
&+ S_{\|}\lambda_e \Bigg[\sqrt{1-\varepsilon^2}\, F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos\phi_h\, F_{LL}^{\cos\phi_h} \Bigg] \\
&+ |\boldsymbol{S}_\perp| \Bigg[\sin(\phi_h-\phi_S) \Big(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\, F_{UT,L}^{\sin(\phi_h-\phi_S)} \Big) \\
&\qquad + \varepsilon\, \sin(\phi_h+\phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\, \sin(3\phi_h-\phi_S)\, F_{UT}^{\sin(3\phi_h-\phi_S)} \\
&\qquad + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin\phi_S\, F_{UT}^{\sin\phi_S} + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin(2\phi_h-\phi_S)\, F_{UT}^{\sin(2\phi_h-\phi_S)} \Bigg] \\
&+ |\boldsymbol{S}_\perp|\lambda_e \Bigg[\sqrt{1-\varepsilon^2}\, \cos(\phi_h-\phi_S)\, F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos\phi_S\, F_{LT}^{\cos\phi_S} \\
&\qquad + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos(2\phi_h-\phi_S)\, F_{LT}^{\cos(2\phi_h-\phi_S)} \Bigg] \Bigg\}, \qquad (2.7)
\end{aligned}
$$

where α is the fine structure constant and the structure functions on the r.h.s. depend on x, Q^2, z and $P_{h\perp}^2$. The angle ψ is the azimuthal angle of ℓ' around the lepton beam axis with respect to an arbitrary fixed direction, which in case of a transversely polarized target we choose to be the direction of S. The corresponding relation between ψ and ϕ_S is given in Ref. [27]; in deep inelastic kinematics one has $d\psi \approx d\phi_S$. The first and second subscript of the above structure functions indicate the respective polarization of beam and target, whereas the third subscript in $F_{UU,T}$, $F_{UU,L}$ and $F_{UT,T}^{\sin(\phi_h-\phi_S)}$, $F_{UT,L}^{\sin(\phi_h-\phi_S)}$ specifies the polarization of the virtual photon. Note that longitudinal or transverse target polarization refer to the photon direction here. The conversion to the experimentally relevant longitudinal or transverse polarization w.r.t. the lepton beam direction is straightforward and given in [27]. The ratio ε of longitudinal and transverse photon flux in (2.7) is given by

$$
\varepsilon = \frac{1-y-\frac{1}{4}\gamma^2 y^2}{1-y+\frac{1}{2}y^2+\frac{1}{4}\gamma^2 y^2}, \qquad (2.8)
$$

so that the depolarization factors can be written as

$$
\begin{aligned}
\frac{y^2}{2\,(1-\varepsilon)} &= \frac{1}{1+\gamma^2}\left(1-y+\tfrac{1}{2}y^2+\tfrac{1}{4}\gamma^2y^2\right) &&\approx \left(1-y+\tfrac{1}{2}y^2\right), &(2.9)\\
\frac{y^2}{2\,(1-\varepsilon)}\,\varepsilon &= \frac{1}{1+\gamma^2}\left(1-y-\tfrac{1}{4}\gamma^2y^2\right) &&\approx (1-y), &(2.10)\\
\frac{y^2}{2\,(1-\varepsilon)}\sqrt{2\,\varepsilon(1+\varepsilon)} &= \frac{1}{1+\gamma^2}\,(2-y)\sqrt{1-y-\tfrac{1}{4}\gamma^2y^2} &&\approx (2-y)\sqrt{1-y}, &(2.11)\\
\frac{y^2}{2\,(1-\varepsilon)}\sqrt{2\,\varepsilon(1-\varepsilon)} &= \frac{1}{\sqrt{1+\gamma^2}}\,y\sqrt{1-y-\tfrac{1}{4}\gamma^2y^2} &&\approx y\sqrt{1-y}, &(2.12)\\
\frac{y^2}{2\,(1-\varepsilon)}\sqrt{1-\varepsilon^2} &= \frac{1}{\sqrt{1+\gamma^2}}\,y\left(1-\tfrac{1}{2}y\right) &&\approx y\left(1-\tfrac{1}{2}y\right). &(2.13)
\end{aligned}
$$

Integration of Eq. (2.7) over the transverse momentum $\boldsymbol{P}_{h\perp}$ of the outgoing hadron gives the semi-inclusive deep inelastic scattering cross section

$$\begin{aligned}\frac{d\sigma}{dx\,dy\,d\psi\,dz} = \frac{2\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{ & F_{UU,T} + \varepsilon F_{UU,L} + S_{\parallel}\lambda_e\,\sqrt{1-\varepsilon^2}\;F_{LL} \\ & + |\boldsymbol{S}_\perp|\,\sqrt{2\,\varepsilon(1+\varepsilon)}\;\sin\phi_S\,F_{UT}^{\sin\phi_S} + |\boldsymbol{S}_\perp|\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\;\cos\phi_S\,F_{LT}^{\cos\phi_S}\Big\}, \end{aligned} \tag{2.14}$$

where the structure functions on the r.h.s. are integrated versions of the previous ones, i.e.

$$F_{UU,T}(x,z,Q^2) = \int d^2\boldsymbol{P}_{h\perp}\,F_{UU,T}(x,z,P_{h\perp}^2,Q^2) \tag{2.15}$$

and similarly for the other functions.

We can finally connect the semi-inclusive structure functions to those for inclusive deep inelastic scattering. With the energies $E_h = (P\cdot P_h)/M$ and $\nu = (P\cdot q)/M$ of the detected hadron and the virtual photon, we have

$$\sum_h \int dz\,z\,\frac{d\sigma(\ell N\to \ell h X)}{dz\,dx\,dy\,d\psi} = \frac{1}{\nu}\sum_h\int dE_h\,E_h\frac{d\sigma(\ell N\to \ell h X)}{dE_h\,dx\,dy\,d\psi} = \frac{\nu+M}{\nu}\,\frac{d\sigma(\ell N\to \ell X)}{dx\,dy\,d\psi}, \tag{2.16}$$

where we have summed over all hadrons in the final state, whose total energy is $\nu + M$. Using $(\nu+M)/\nu = 1+\gamma^2/(2x)$ we have

$$\begin{aligned}\frac{d\sigma}{dx\,dy\,d\psi} = \frac{2\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\Big\{ & F_T + \varepsilon F_L + S_{\parallel}\lambda_e\,\sqrt{1-\varepsilon^2}\;2x\,(g_1-\gamma^2 g_2) \\ & - |\boldsymbol{S}_\perp|\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\;\cos\phi_S\,2x\gamma\,(g_1+g_2)\Big\}, \end{aligned} \tag{2.17}$$

where

$$\sum_h\int dz\,z\,F_{UU,T}(x,z,Q^2) = 2xF_1(x,Q^2) = F_T(x,Q^2), \tag{2.18}$$

$$\sum_h\int dz\,z\,F_{UU,L}(x,z,Q^2) = (1+\gamma^2)F_2(x,Q^2) - 2xF_1(x,Q^2) = F_L(x,Q^2), \tag{2.19}$$

$$\sum_h\int dz\,z\,F_{LL}(x,z,Q^2) = 2x\,\big(g_1(x,Q^2) - \gamma^2 g_2(x,Q^2)\big), \tag{2.20}$$

$$\sum_h\int dz\,z\,F_{LT}^{\cos\phi_S}(x,z,Q^2) = -2x\gamma\,\big(g_1(x,Q^2) + g_2(x,Q^2)\big) \tag{2.21}$$

in terms of the conventional deep inelastic structure functions. For the relation with the more common expression for target polarization along or transverse to the lepton beam direction see Refs. [31, 32, 27]. Finally, time-reversal invariance requires (see, e.g., Ref. [27])

$$\sum_h\int dz\,z\,F_{UT}^{\sin\phi_S}(x,z,Q^2) = 0. \tag{2.22}$$

3. Transverse-momentum dependent distribution and fragmentation functions

3.1 Light-cone coordinates

Manipulations with parton distribution and fragmentation functions are conveniently done using light-cone coordinates. For an arbitrary four-vector v we write $v^{\pm} = (v^0 \pm v^3)/\sqrt{2}$ and $\boldsymbol{v}_T = (v^1, v^2)$ in a specified reference frame and give all components as $[v^-, v^+, \boldsymbol{v}_T]$. We will use the transverse tensors $g_T^{\alpha\beta}$ and $\epsilon_T^{\alpha\beta}$, whose only nonzero components are $g_T^{11} = g_T^{22} = -1$ and $\epsilon_T^{12} = -\epsilon_T^{21} = 1$. The light-cone decomposition of a vector can be written in a Lorentz covariant fashion using two light-like vectors $n_+ = [0, 1, \mathbf{0}_T]$ and $n_- = [1, 0, \mathbf{0}_T]$ and promoting $\boldsymbol{v}_T$ to a four-vector $v_T = [0, 0, \boldsymbol{v}_T]$. One then has

$$v^\mu = v^+ n_+^\mu + v^- n_-^\mu + v_T^\mu, \tag{3.1}$$

where $v^+ = v \cdot n_-$, $v^- = v \cdot n_+$ and $v_T \cdot n_+ = v_T \cdot n_- = 0$. We further have

$$g_T^{\alpha\beta} = g^{\alpha\beta} - n_+^{\alpha} n_-^{\beta} - n_-^{\alpha} n_+^{\beta}, \qquad \epsilon_T^{\alpha\beta} = \epsilon^{\alpha\beta\rho\sigma}\, n_{+\rho} n_{-\sigma}. \tag{3.2}$$

Note that scalar products with transverse four-vectors are in Minkowski space, so that $v_T \cdot w_T = -\boldsymbol{v}_T \cdot \boldsymbol{w}_T$.

For the discussion of distribution functions we will choose light-cone coordinates such that P has no transverse component, i.e.

$$P^\mu = P^+ n_+^\mu + \frac{M^2}{2P^+}\, n_-^\mu\,. \tag{3.3}$$

The spin vector of the target can then be decomposed as

$$S^\mu = S_L\, \frac{(P \cdot n_-)\, n_+^\mu - (P \cdot n_+)\, n_-^\mu}{M} + S_T^\mu\,, \tag{3.4}$$

which implies $S_L = M\,(S \cdot n_-)/(P \cdot n_-)$. Similarly, for the discussion of fragmentation functions, we will assume a coordinate choice with

$$P_h^\mu = P_h^- n_-^\mu + \frac{M_h^2}{2P_h^-}\, n_+^\mu\,. \tag{3.5}$$

3.2 Calculation of the hadronic tensor

We consider semi-inclusive DIS in the kinematical limit where Q^2 becomes large while x, z and $P_{h\perp}^2$ remain fixed, and will perform an expansion in powers of $1/Q$. For the calculation we use a frame where both (3.3) and (3.5) are satisfied, and where $xP^+ = P_h^-/z = Q/\sqrt{2}$. Notice that this differs from the choice in Section 2, where the transverse direction was defined with respect to the momenta of the target and the virtual photon, instead of the momenta of the target and the produced hadron. Details on the relation between the two choices can be found in [1, 33]. In particular, S_L and S_T defined by (3.4) with (3.3) and (3.5) differ from $S_{\|}$ and $S_\perp$ in (2.6) by terms of order $1/Q^2$ and $1/Q$, respectively.

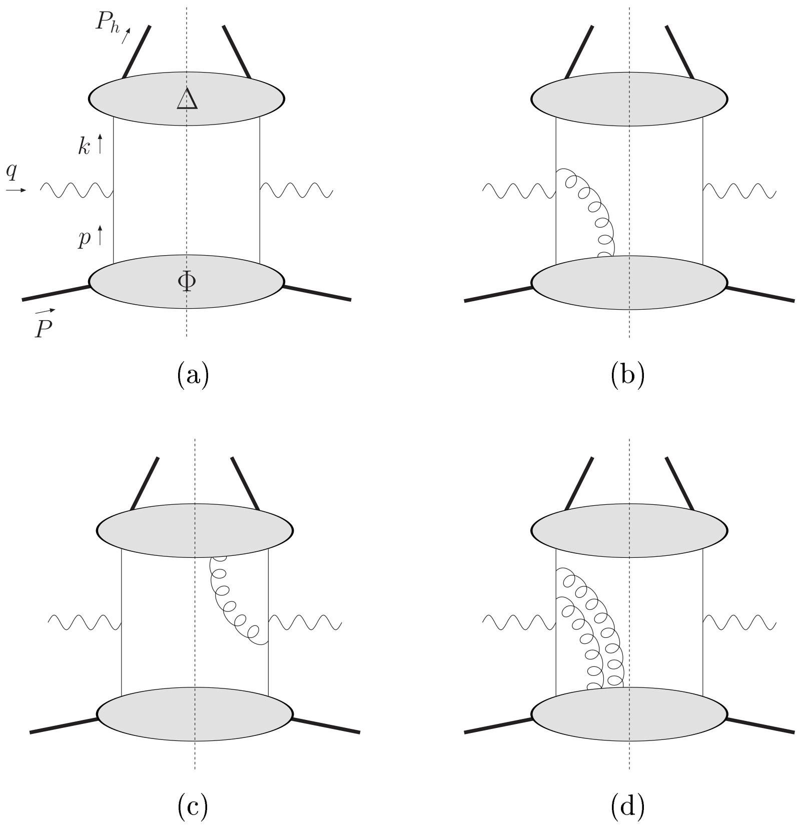

Figure 2: Examples of graphs contributing to semi-inclusive DIS at low transverse momentum of the produced hadron.

The leptoproduction cross section can be expressed as the contraction of a hadronic and a leptonic tensor,

$$\frac{d\sigma}{dx\, dy\, d\psi\, dz\, d\phi_h\, dP_{h\perp}^2} = \frac{\alpha^2\, y}{8z\, Q^4}\, 2MW^{\mu\nu}\, L_{\mu\nu}\,, \tag{3.6}$$

where the leptonic tensor is given by

$$L_{\mu\nu} = 2\left(l_\mu l'_\nu + l'_\mu l_\nu - l\cdot l'\, g_{\mu\nu}\right) + 2i\lambda_e\, \epsilon_{\mu\nu\rho\sigma}\, l^\rho l'^\sigma. \tag{3.7}$$

The hadronic tensor is defined as

$$2MW^{\mu\nu} = \frac{1}{(2\pi)^3} \sum_X \int \frac{d^3\boldsymbol{P}_X}{2P_X^0}\, \delta^{(4)}\!\left(q+P-P_X-P_h\right) \langle P|J^\mu(0)|h,X\rangle\langle h,X|J^\nu(0)|P\rangle, \tag{3.8}$$

where $J^\mu(\xi)$ is the electromagnetic current divided by the elementary charge and a sum is implied over the polarizations of all hadrons in the final state.

The calculations in this paper are based on the factorization of the cross section into a hard photon-quark scattering process and nonperturbative functions describing the distribution of quarks in the target or the fragmentation of a quark into the observed hadron. We limit ourselves to the leading and first subleading term in the $1/Q$ expansion of the cross section and to graphs with the hard scattering at tree level. Loops can then only occur as shown in Fig. 2b, c, d, with gluons as external legs of the nonperturbative functions. The corresponding expression of the hadronic tensor is [1, 33]

$$2MW^{\mu\nu} = 2z \sum_a e_a^2 \int d^2\boldsymbol{p}_T \, d^2\boldsymbol{k}_T \, \delta^2(\boldsymbol{p}_T + \boldsymbol{q}_T - \boldsymbol{k}_T) \, \mathrm{Tr}\Big\{ \Phi^a(x, p_T)\gamma^\mu \Delta^a(z, k_T)\gamma^\nu - \frac{1}{Q\sqrt{2}} \Big[\gamma^\alpha \, \not{n}_+ \gamma^\nu \, \tilde{\Phi}^a_{A\alpha}(x, p_T)\gamma^\mu \Delta^a(z, k_T) + \gamma^\alpha \, \not{n}_- \gamma^\mu \, \tilde{\Delta}^a_{A\alpha}(z, k_T)\gamma^\nu \Phi^a(x, p_T) + \text{h.c.} \Big] \Big\}, \tag{3.9}$$

with corrections of order $1/Q^2$, where the sum runs over the quark and antiquark flavors a, and e_a denotes the fractional charge of the struck quark or antiquark. In the next subsections we discuss in detail the correlation functions Φ for quark distributions, Δ for quark fragmentation, and their analogs $\tilde{\Phi}_A$ and $\tilde{\Delta}_A$ with an additional gluon leg. The first, second and third term in Eq. (3.9) respectively correspond to the graphs in Fig. 2a, b and c, with gluons having transverse polarization. The analogs of Fig. 2b and c with the gluon on the other side of the final-state cut correspond to the "h.c." terms in Eq. (3.9).

The Wilson lines needed in the color gauge invariant soft functions come from graphs with additional gluons exchanged between the hard scattering and either the distribution or the fragmentation function (as in Fig. 2b,c,d). The corresponding gluons have polarization vectors proportional to n_+ in the first and to n_- in the second case, except for contributions from the gluon potential at infinity. The importance of the latter has been discussed in [5, 6], and a detailed derivation of the Wilson lines appearing in semi-inclusive DIS was given in Ref. [33] for the leading terms in the $1/Q$ expansion. For the contributions subleading in $1/Q$ only the cross section integrated over $\boldsymbol{P}_{h\perp}$ has been analyzed in the same reference.

Going beyond the tree graphs just discussed requires modifications in the factorization formula (3.9). In particular, radiative corrections involving low-frequency gluons introduce Sudakov logarithms and so-called soft factors. A proof of factorization to all orders in α_s for the (similar but simpler) case of two-hadron production in e^+e^- collisions was given long ago [12]. Recent work on all-order factorization in semi-inclusive DIS can be found in Refs. [10, 11] and in Ref. [9]. Whether and how the tree-level factorization used in the present paper extends to subleading level in $1/Q$ is presently not known.

3.3 The quark-quark correlators

The quark-quark distribution correlation function is defined as[2]

$$\Phi_{ij}(x, p_T) = \int \frac{d\xi^- d^2\boldsymbol{\xi}_T}{(2\pi)^3} \, e^{ip\cdot\xi} \, \langle P|\bar{\psi}_j(0) \, \mathcal{U}^{n_-}_{(0,+\infty)} \, \mathcal{U}^{n_-}_{(+\infty,\xi)} \, \psi_i(\xi)|P\rangle \Big|_{\xi^+=0} \tag{3.10}$$

with $p^+ = xP^+$, where here and in the following we omit the flavor index a. The corresponding correlator for antiquarks is obtained by replacing the quark field by its transform

[2] To be precise, one should distinguish the momentum fraction x in the definition of distribution functions from the Bjorken variable defined in Eq. (2.2). They coincide however in the process we consider, so that we drop this distinction for simplicity. An analogous remark holds for the argument z in the fragmentation functions below.

under charge conjugation, see Ref. [1]. In the correlator (3.10) we have gauge links (Wilson lines)

$$\mathcal{U}^{n_-}_{(0,+\infty)} = \mathcal{U}^{n_-}(0^-,\infty^-;\mathbf{0}_T)\,\mathcal{U}^T(\mathbf{0}_T,\boldsymbol{\infty}_T;\infty^-), \tag{3.11}$$

$$\mathcal{U}^{n_-}_{(+\infty,\xi)} = \mathcal{U}^T(\boldsymbol{\infty}_T,\boldsymbol{\xi}_T;\infty^-)\,\mathcal{U}^{n_-}(\infty^-,\xi^-,\boldsymbol{\xi}_T). \tag{3.12}$$

Here $\mathcal{U}^{n_-}(a^-,b^-;\boldsymbol{c}_T)$ indicates a Wilson line running along the minus direction from $[a^-,0^+,\boldsymbol{c}_T]$ to $[b^-,0^+,\boldsymbol{c}_T]$, while $\mathcal{U}^T(\boldsymbol{a}_T,\boldsymbol{b}_T;c^-)$ indicates a Wilson line running in the transverse direction from $[c^-,0^+,\boldsymbol{a}_T]$ to $[c^-,0^+,\boldsymbol{b}_T]$, i.e.

$$\mathcal{U}^{n_-}(a^-,b^-;\boldsymbol{c}_T) = \mathcal{P}\exp\left[-ig\int_{a^-}^{b^-} d\zeta^-\,A^+(\zeta^-,0^+,\boldsymbol{c}_T)\right], \tag{3.13}$$

$$\mathcal{U}^T(\boldsymbol{a}_T,\boldsymbol{b}_T;c^-) = \mathcal{P}\exp\left[-ig\int_{\boldsymbol{a}_T}^{\boldsymbol{b}_T} d\boldsymbol{\zeta}_T\cdot\boldsymbol{A}_T(c^-,0^+,\boldsymbol{\zeta}_T)\right]. \tag{3.14}$$

The correlator in Eq. (3.10) is the one appearing in semi-inclusive DIS. In different processes the structure of the gauge link can change [7, 34, 35, 36]. For instance, in Drell-Yan lepton pair production all occurrences of ∞^- in the gauge links should be replaced by $-\infty^-$. In particular, this reverses the sign of all T-odd distribution functions appearing in the correlator (see below). In partonic processes with colored states in both the initial and final state, the gauge link contains contributions running to ∞^- as well as $-\infty^-$, and T-odd terms differ by more than a simple sign change. Alternatively, it is possible to work always with the correlator for semi-inclusive DIS when convoluting T-odd functions with so-called gluonic-pole cross sections instead of the normal partonic cross sections [35, 37]. We note that beyond tree-level the Wilson lines in Eqs. (3.11) and (3.12) lead to logarithmic divergences in the correlator (3.10). These are due to gluons with vanishing momentum component along n_+ and need to be regularized. The twist-two part of the correlator can be regularized in ways consistent with factorization to leading power in $1/Q$ [12, 38, 10, 9]. It is presently not known how to extend this to the twist-three sector, see Ref. [39] for an investigation of this case.

A complete parameterization of the quark-quark correlation function has been given in Ref. [16]. Here we limit ourselves to the twist-three level, where we have

$$\begin{aligned}
\Phi(x,p_T) = \frac{1}{2}\Bigg\{ & f_1\,\not{n}_+ - f_{1T}^\perp\,\frac{\epsilon_T^{\rho\sigma}p_{T\rho}S_{T\sigma}}{M}\,\not{n}_+ + g_{1s}\gamma_5\,\not{n}_+ \\
& + h_{1T}\,\frac{[\not{S}_T,\not{n}_+]\gamma_5}{2} + h_{1s}^\perp\,\frac{[\not{p}_T,\not{n}_+]\gamma_5}{2M} + i\,h_1^\perp\frac{[\not{p}_T,\not{n}_+]}{2M}\Bigg\} \\
+ \frac{M}{2P^+}\Bigg\{ & e - i\,e_s\,\gamma_5 - e_T^\perp\,\frac{\epsilon_T^{\rho\sigma}p_{T\rho}S_{T\sigma}}{M} \\
& + f^\perp\,\frac{\not{p}_T}{M} - f_T'\,\epsilon_T^{\rho\sigma}\gamma_\rho S_{T\sigma} - f_s^\perp\,\frac{\epsilon_T^{\rho\sigma}\gamma_\rho p_{T\sigma}}{M} \\
& + g_T'\,\gamma_5\,\not{S}_T + g_s^\perp\gamma_5\frac{\not{p}_T}{M} - g^\perp\gamma_5\,\frac{\epsilon_T^{\rho\sigma}\gamma_\rho p_{T\sigma}}{M} \\
& + h_s\,\frac{[\not{n}_+,\not{n}_-]\gamma_5}{2} + h_T^\perp\,\frac{[\not{S}_T,\not{p}_T]\gamma_5}{2M} + i\,h\frac{[\not{n}_+,\not{n}_-]}{2}\Bigg\}.
\end{aligned} \tag{3.15}$$

The distribution functions on the r.h.s. depend on x and p_T^2, except for the functions with subscript s, where we use the shorthand notation [1]

$$g_{1s}(x, p_T) = S_L\, g_{1L}(x, p_T^2) - \frac{p_T \cdot S_T}{M}\, g_{1T}(x, p_T^2) \tag{3.16}$$

and so forth for the other functions. The first eight distributions of Eq. (3.15) are referred to as twist two, and the next 16 distributions are referred to as twist three, where we use the notion of "dynamical twist" as explained in Ref. [40]. The remaining eight functions of twist four have been omitted here and can be found in Ref. [16]. The 10 functions $f_{1T}^{\perp}$, $h_1^{\perp}$, e_L, e_T, $e_T^{\perp}$, f_T', $f_L^{\perp}$, $f_T^{\perp}$, $g^{\perp}$, h are T-odd [2, 16], i.e. they change sign under "naive time reversal", which is defined as usual time reversal, but without the interchange of initial and final states. The functions $g^{\perp}$ [15], $e_T^{\perp}$ and $f_T^{\perp}$ [16] exist because the direction of the Wilson line provides a vector independent of P and S for a Lorentz invariant decomposition of the correlation function $\Phi(p)$, which is defined as in (3.10) but with ξ^+ integrated over instead of being set to zero [14]. The notation used in (3.15) is consistent with Refs. [1, 2, 15, 16], except for three points of discrepancy: (*i*) the sign of $g^{\perp}$ is opposite to that in Ref. [15], where the function was originally introduced, and consistent with Ref. [16]; (*ii*) the sign of the function $f_L^{\perp}$ is opposite to that in Ref. [16] and consistent with the other articles; (*iii*) the function $f_T^{\perp}$ here is different from Ref. [16], where it was first introduced, in order to maintain the symmetry with the other functions and to have simpler expressions in the following results. The relation between the two notations is the following (recall that $p_T^2 = -\boldsymbol{p}_T^2$):

$$f_T'\Big|_{\text{here}} = \frac{p_T^2}{M^2}\, f_T^{\perp\prime}\Big|_{\text{Ref. [16]}}, \qquad\qquad f_T^{\perp}\Big|_{\text{here}} = f_T^{\perp\prime} - f_T^{\perp}\Big|_{\text{Ref. [16]}}. \tag{3.17}$$

The nomenclature of the distribution functions follows closely that of Ref. [1], sometimes referred to as "Amsterdam notation." We remark that a number of other notations exist for some of the distribution functions, see e.g. Refs. [41, 42, 43]. In particular, transverse-momentum-dependent functions at leading twist have been widely discussed by Anselmino et al. [44, 45, 46]. The connection between the notation in these papers and the one used here is discussed in App. C of Ref. [46].

We also list here the expressions for the traces of the correlator $\Phi(x, p_T)$ from Ref. [16]. With $\Phi^{[\Gamma]} = \frac{1}{2}\,\mathrm{Tr}[\Phi\,\Gamma]$ we have

$$\Phi^{[\gamma^+]} = f_1 - \frac{\epsilon_T^{\rho\sigma} p_{T\rho} S_{T\sigma}}{M}\, f_{1T}^{\perp}, \tag{3.18}$$

$$\Phi^{[\gamma^+\gamma_5]} = S_L\, g_{1L} - \frac{p_T \cdot S_T}{M}\, g_{1T}, \tag{3.19}$$

$$\begin{aligned}\Phi^{[i\sigma^{\alpha+}\gamma_5]} = {} & S_T^{\alpha}\, h_1 + S_L\, \frac{p_T^{\alpha}}{M}\, h_{1L}^{\perp} \\ & - \frac{p_T^{\alpha} p_T^{\rho} - \frac{1}{2}\, p_T^2\, g_T^{\alpha\rho}}{M^2}\, S_{T\rho}\, h_{1T}^{\perp} - \frac{\epsilon_T^{\alpha\rho} p_{T\rho}}{M}\, h_1^{\perp},\end{aligned} \tag{3.20}$$

$$\Phi^{[1]} = \frac{M}{P^+}\left[e - \frac{\epsilon_T^{\rho\sigma} p_{T\rho} S_{T\sigma}}{M}\, e_T^{\perp}\right], \tag{3.21}$$

$$\Phi^{[i\gamma_5]} = \frac{M}{P^+}\left[S_L\, e_L - \frac{p_T\cdot S_T}{M}\, e_T\right], \tag{3.22}$$

$$\Phi^{[\gamma^\alpha]} = \frac{M}{P^+}\bigg[-\epsilon_T^{\alpha\rho} S_{T\rho}\, f_T - S_L\,\frac{\epsilon_T^{\alpha\rho} p_{T\rho}}{M}\, f_L^\perp - \frac{p_T^\alpha p_T^\rho - \frac{1}{2}\, p_T^2\, g_T^{\alpha\rho}}{M^2}\,\epsilon_{T\rho\sigma} S_T^\sigma\, f_T^\perp + \frac{p_T^\alpha}{M} f^\perp\bigg], \tag{3.23}$$

$$\Phi^{[\gamma^\alpha\gamma_5]} = \frac{M}{P^+}\bigg[S_T^\alpha\, g_T + S_L\,\frac{p_T^\alpha}{M}\, g_L^\perp - \frac{p_T^\alpha p_T^\rho - \frac{1}{2}\, p_T^2\, g_T^{\alpha\rho}}{M^2}\, S_{T\rho}\, g_T^\perp - \frac{\epsilon_T^{\alpha\rho} p_{T\rho}}{M}\, g^\perp\bigg], \tag{3.24}$$

$$\Phi^{[i\sigma^{\alpha\beta}\gamma_5]} = \frac{M}{P^+}\left[\frac{S_T^\alpha p_T^\beta - p_T^\alpha S_T^\beta}{M}\, h_T^\perp - \epsilon_T^{\alpha\beta}\, h\right], \tag{3.25}$$

$$\Phi^{[i\sigma^{+-}\gamma_5]} = \frac{M}{P^+}\left[S_L\, h_L - \frac{p_T\cdot S_T}{M}\, h_T\right], \tag{3.26}$$

where α and β are restricted to be transverse indices. Here we made use of the combinations

$$f_T(x,p_T^2) = f_T'(x,p_T^2) - \frac{p_T^2}{2M^2}\, f_T^\perp(x,p_T^2), \tag{3.27}$$

$$g_T(x,p_T^2) = g_T'(x,p_T^2) - \frac{p_T^2}{2M^2}\, g_T^\perp(x,p_T^2), \tag{3.28}$$

$$h_1(x,p_T^2) = h_{1T}(x,p_T^2) - \frac{p_T^2}{2M^2}\, h_{1T}^\perp(x,p_T^2), \tag{3.29}$$

to separate off terms that vanish upon integration of the correlator over transverse momentum due to rotational symmetry. The conversion between the expression in Eq. (3.23) and that in Eq. (19) of Ref. [16] can be carried out using the identity

$$p_T^2\,\epsilon_T^{\alpha\rho} S_{T\rho} = p_T^\alpha\,\epsilon_T^{\rho\sigma} p_{T\rho} S_{T\sigma} + (p_T\cdot S_T)\,\epsilon_T^{\alpha\rho} p_{T\rho}\,, \tag{3.30}$$

which follows from the fact that there is no completely antisymmetric tensor of rank three in two dimensions.

Integrating the correlator over the transverse momentum p_T yields

$$\begin{aligned}\Phi(x) = \int d^2\boldsymbol{p}_T\,\Phi(x,p_T) = \frac{1}{2}\bigg\{&f_1\, \not{n}_+ + S_L g_1\,\gamma_5\,\not{n}_+ + h_1\,\frac{[\not{S}_T, \not{n}_+]\gamma_5}{2}\bigg\}\\ &+ \frac{M}{2P^+}\bigg\{e - i\,S_L e_L\,\gamma_5 + f_T\,\epsilon_T^{\rho\sigma} S_{T\rho}\gamma_\sigma + g_T\,\gamma_5\,\not{S}_T\\ &+ S_L\, h_L\,\frac{[\not{n}_+, \not{n}_-]\gamma_5}{2} + i\,h\frac{[\not{n}_+, \not{n}_-]}{2}\bigg\},\end{aligned} \tag{3.31}$$

where the functions on the r.h.s. depend only on x and are given by

$$f_1(x) = \int d^2\boldsymbol{p}_T\; f_1(x,p_T^2) \tag{3.32}$$

and so forth for the other functions. We have retained here one common exception of notation, namely

$$g_1(x) = \int d^2\boldsymbol{p}_T\, g_{1L}(x, p_T^2). \tag{3.33}$$

Other notations are also in use for the leading-twist integrated functions, in particular $f_1^q = q$ (unpolarized distribution function), $g_1^q = \Delta q$ (helicity distribution function), $h_1^q = \delta q = \Delta_T q$ (transversity distribution function). The T-odd functions vanish due to time-reversal invariance [16]

$$\int d^2\boldsymbol{p}_T\, f_T(x, p_T^2) = 0, \qquad \int d^2\boldsymbol{p}_T\, e_L(x, p_T^2) = 0, \qquad \int d^2\boldsymbol{p}_T\, h(x, p_T^2) = 0. \tag{3.34}$$

We have kept them in Eq. (3.31) so that one can readily obtain the analogous fragmentation correlator, where such functions do not necessarily vanish.

The fragmentation correlation function is defined as

$$\Delta_{ij}(z, k_T) = \frac{1}{2z}\sum_X \int \frac{d\xi^+ d^2\boldsymbol{\xi}_T}{(2\pi)^3}\, e^{ik\cdot\xi}\, \langle 0|\, \mathcal{U}^{n_+}_{(+\infty,\xi)}\, \psi_i(\xi)|h, X\rangle\langle h, X|\bar{\psi}_j(0)\, \mathcal{U}^{n_+}_{(0,+\infty)}|0\rangle\Big|_{\xi^-=0}, \tag{3.35}$$

with $k^- = P_h^-/z$ and the Wilson lines

$$\mathcal{U}^{n_+}_{(+\infty,\xi)} \equiv \mathcal{U}^T(\boldsymbol{\infty}_T, \boldsymbol{\xi}_T; +\infty^+)\, \mathcal{U}^{n_+}(+\infty^+, \xi^+; \boldsymbol{\xi}_T), \tag{3.36}$$

$$\mathcal{U}^{n_+}_{(0,+\infty)} \equiv \mathcal{U}^{n_+}(0^+, +\infty^+; \boldsymbol{0}_T)\, \mathcal{U}^T(\boldsymbol{0}_T, \boldsymbol{\infty}_T; +\infty^+). \tag{3.37}$$

The notation $\mathcal{U}^{n_+}(a^+, b^+; \boldsymbol{c}_T)$ indicates a Wilson line running along the plus direction from $[0^-, a^+, \boldsymbol{c}_T]$ to $[0^-, b^+, \boldsymbol{c}_T]$, while $\mathcal{U}^T(\boldsymbol{a}_T, \boldsymbol{b}_T; \boldsymbol{c}^+)$ indicates a gauge link running in the transverse direction from $[0^-, c^+, \boldsymbol{a}_T]$ to $[0^-, c^+, \boldsymbol{b}_T]$. The definition written above naturally applies for the correlation function appearing in e^+e^- annihilation. For semi-inclusive DIS it seems more natural to replace all occurrences of $+\infty^+$ in the gauge links by $-\infty^+$ [33]. However, in Ref. [9] it was shown that factorization can be derived in such a way that the fragmentation correlators in both semi-inclusive DIS and e^+e^- annihilation have gauge links pointing to $+\infty^+$.

The fragmentation correlation function (for a spinless or an unpolarized hadron) can be parameterized as

$$\begin{aligned}\Delta(z, k_T) = \frac{1}{2}\Bigg\{ & D_1\, \not{n}_- + iH_1^\perp \frac{[\not{k}_T, \not{n}_-]}{2M_h}\Bigg\} \\ & + \frac{M_h}{2P_h^-}\Bigg\{E + D^\perp \frac{\not{k}_T}{M_h} + iH \frac{[\not{n}_-, \not{n}_+]}{2} + G^\perp \gamma_5\, \frac{\epsilon_T^{\rho\sigma}\gamma_\rho k_{T\sigma}}{M_h}\Bigg\},\end{aligned} \tag{3.38}$$

where the functions on the r.h.s. depend on z and k_T^2. To be complete, they should all carry also a flavor index, and the final hadron type should be specified. The correlation function Δ can be directly obtained from the correlation function Φ by changing[3]

$$n_+ \leftrightarrow n_-, \qquad \epsilon_T \to -\epsilon_T, \qquad P^+ \to P_h^-, \qquad M \to M_h, \qquad x \to 1/z, \tag{3.39}$$

[3]The change of sign of the tensor ϵ_T is due to the exchange $n_+ \leftrightarrow n_-$ in its definition (3.2).

and replacing the distribution functions with the corresponding fragmentation functions (f is replaced with D and all other letters are capitalized).

The correlator integrated over transverse momentum reads

$$\Delta(z) = z^2 \int d^2\boldsymbol{k}_T\, \Delta(z, k_T) = D_1 \frac{\not{n}_-}{2} + \frac{M_h}{2P_h^-} \left\{ E + iH \frac{[\not{n}_-, \not{n}_+]}{2} \right\}, \tag{3.40}$$

where the functions on the r.h.s. are defined as

$$D_1(z) = z^2 \int d^2\boldsymbol{k}_T\, D_1(z, k_T^2) \tag{3.41}$$

and so forth for the other functions. The prefactor z^2 appears because $D_1(z, k_T)$ is a probability density w.r.t. the transverse momentum $k_T' = -zk_T$ of the final-state hadron relative to the fragmenting quark. The fragmentation correlator for polarized spin-half hadrons is parameterized in analogy to Eqs. (3.15) and (3.31). As already remarked, in this case the functions D_T, E_L and H (the analogs of f_T, e_L and h) do not vanish because $|h, X\rangle$ is an interacting state that does not transform into itself under time-reversal. Of course, it should be taken as an outgoing state in the fragmentation correlator.

3.4 The quark-gluon-quark correlators

We now examine the quark-gluon-quark distribution correlation functions [33, 47]

$$\left(\Phi_D^\mu\right)_{ij}(x, p_T) = \int \frac{d\xi^- d^2\boldsymbol{\xi}_T}{(2\pi)^3}\, e^{ip\cdot\xi} \langle P|\bar{\psi}_j(0)\, \mathcal{U}^{n_-}_{(0,+\infty)}\, \mathcal{U}^{n_-}_{(+\infty,\xi)}\, iD^\mu(\xi)\, \psi_i(\xi)|P\rangle \bigg|_{\xi^+=0}, \tag{3.42}$$

which contain the covariant derivative $iD^\mu(\xi) = i\partial^\mu + gA^\mu$. Using $\mathcal{U}^{n_-}_{(+\infty,\xi)}\, iD^+(\xi)\, \psi(\xi) = i\partial^+ \big[\mathcal{U}^{n_-}_{(+\infty,\xi)}\, \psi(\xi)\big]$ we can write

$$\Phi_D^+(x, p_T) = xP^+ \Phi(x, p_T) \tag{3.43}$$

for the plus-component of the correlator. For the transverse components we define a further correlator [33]

$$\tilde{\Phi}_A^\alpha(x, p_T) = \Phi_D^\alpha(x, p_T) - p_T^\alpha\, \Phi(x, p_T), \tag{3.44}$$

which is manifestly gauge invariant. It reduces to a correlator defined as in Eq. (3.42) with the covariant derivative iD^μ replaced by gA_T^α if one has $\mathcal{U}^{n_-}_{(+\infty,\xi)} = 1$, which is the case in a light-cone gauge $A^+ = 0$ with suitable boundary conditions at light-cone infinity [6]. Notice that (3.42) does not have the most general form of a twist-three correlation function since it depends on the kinematics of one instead of two partons (i.e., on a single momentum fraction and a single transverse momentum). Correspondingly, the covariant derivative is taken at the same space-time point as one of the quark fields. The tree-level result (3.9) for semi-inclusive DIS can be expressed in terms of only the quark-quark correlator (3.10), the quark-gluon-quark correlator (3.42), and their fragmentation counterparts. This is not too surprising since the kinematics of our process defines a single plus-momentum fraction x, a single minus-momentum fraction z, and a single transverse momentum $P_{h\perp}$.

No further momentum fraction can for instance be constructed from the ratio $P_{h\perp}^2/Q^2$, which is replaced with zero in the kinematical limit we consider.

The correlation function (3.44) can be decomposed as

$$\begin{aligned}
&\tilde{\Phi}_A^\alpha(x,p_T) = \\
&\frac{xM}{2}\Bigg\{\Big[\big(\tilde{f}^\perp - i\tilde{g}^\perp\big)\frac{p_{T\rho}}{M} - \big(\tilde{f}_T' + i\,\tilde{g}_T'\big)\,\epsilon_{T\rho\sigma}S_T^\sigma - \big(\tilde{f}_s^\perp + i\,\tilde{g}_s^\perp\big)\frac{\epsilon_{T\rho\sigma}p_T^\sigma}{M}\Big]\big(g_T^{\alpha\rho} - i\epsilon_T^{\alpha\rho}\gamma_5\big) \\
&\quad - \big(\tilde{h}_s + i\,\tilde{e}_s\big)\gamma_T^\alpha\gamma_5 + \Big[\big(\tilde{h} + i\,\tilde{e}\big) + \big(\tilde{h}_T^\perp - i\,\tilde{e}_T^\perp\big)\frac{\epsilon_T^{\rho\sigma}p_{T\rho}S_{T\sigma}}{M}\Big]i\gamma_T^\alpha + \ldots\big(g_T^{\alpha\rho} + i\epsilon_T^{\alpha\rho}\gamma_5\big)\Bigg\}\frac{\not{n}_+}{2},
\end{aligned} \tag{3.45}$$

where the index α is restricted to be transverse here and in the following equations. The functions on the r.h.s. depend on x and p_T^2, except for the functions with subscript s, which are defined as in Eq. (3.16). The last term in the curly brackets is irrelevant for the construction of the hadronic tensor of semi-inclusive DIS and has not been parameterized explicitly. The only relevant traces of the quark-gluon-quark correlator are

$$\frac{1}{2Mx}\,\mathrm{Tr}\big[\tilde{\Phi}_{A\alpha}\,\sigma^{\alpha+}\big] = \tilde{h} + i\,\tilde{e} + \frac{\epsilon_T^{\rho\sigma}p_{T\rho}S_{T\sigma}}{M}\,\big(\tilde{h}_T^\perp - i\,\tilde{e}_T^\perp\big), \tag{3.46}$$

$$\frac{1}{2Mx}\,\mathrm{Tr}\big[\tilde{\Phi}_{A\alpha}\,i\sigma^{\alpha+}\gamma_5\big] = S_L\,\big(\tilde{h}_L + i\,\tilde{e}_L\big) - \frac{p_T\cdot S_T}{M}\,\big(\tilde{h}_T + i\,\tilde{e}_T\big), \tag{3.47}$$

$$\begin{aligned}
\frac{1}{2Mx}\,\mathrm{Tr}\big[\tilde{\Phi}_{A\rho}\,(g_T^{\alpha\rho} + i\epsilon_T^{\alpha\rho}\gamma_5)\,\gamma^+\big] &= \frac{p_T^\alpha}{M}\big(\tilde{f}^\perp - i\tilde{g}^\perp\big) - \epsilon_T^{\alpha\rho}\,S_{T\rho}\,\big(\tilde{f}_T + i\tilde{g}_T\big) \\
&\quad - S_L\,\frac{\epsilon_T^{\alpha\rho}p_{T\rho}}{M}\,\big(\tilde{f}_L^\perp + i\,\tilde{g}_L^\perp\big) - \frac{p_T^\alpha p_T^\rho - \frac{1}{2}\,p_T^2\,g_T^{\alpha\rho}}{M^2}\,\epsilon_{T\rho\sigma}\,S_T^\sigma\,\big(\tilde{f}_T^\perp + i\tilde{g}_T^\perp\big),
\end{aligned} \tag{3.48}$$

where again we have used the combinations

$$\tilde{f}_T(x,p_T^2) = \tilde{f}_T'(x,p_T^2) - \frac{p_T^2}{2M^2}\,\tilde{f}_T^\perp(x,p_T^2), \tag{3.49}$$

$$\tilde{g}_T(x,p_T^2) = \tilde{g}_T'(x,p_T^2) - \frac{p_T^2}{2M^2}\,\tilde{g}_T^\perp(x,p_T^2). \tag{3.50}$$

The above traces have been given already in Ref. [15] for the terms without transverse polarization, whereas the terms with transverse polarization were partly discussed in Ref. [1] (the functions $e_T^\perp$ and $f_T^\perp$ introduced in Ref. [16] were missing).

Relations between correlation functions of different twist are provided by the equation of motion for the quark field

$$\big[i\not{D}(\xi) - m\big]\,\psi(\xi) = \big[\gamma^+ iD^-(\xi) + \gamma^- iD^+(\xi) + \gamma_T^\alpha\,iD_\alpha(\xi) - m\big]\,\psi(\xi) = 0, \tag{3.51}$$

where m is the quark mass. To make their general structure transparent we decompose the correlators into terms of definite twist,

$$\Phi = \Phi_2 + \frac{M}{P^+}\,\Phi_3 + \left(\frac{M}{P^+}\right)^2\Phi_4, \qquad \frac{1}{M}\,\tilde{\Phi}_A^\alpha = \tilde{\Phi}_{A,3}^\alpha + \frac{M}{P^+}\,\tilde{\Phi}_{A,4}^\alpha + \left(\frac{M}{P^+}\right)^2\tilde{\Phi}_{A,5}^\alpha, \tag{3.52}$$

where the twist is indicated in the subscripts and the arguments (x, p_T) are suppressed for ease of writing. One has $\mathcal{P}_+\Phi_4 = \mathcal{P}_-\Phi_2 = 0$ and $\mathcal{P}_+\tilde{\Phi}^\alpha_{A,5} = \mathcal{P}_-\tilde{\Phi}^\alpha_{A,3} = 0$, where $\mathcal{P}_+ = \frac{1}{2}\gamma^-\gamma^+$ and $\mathcal{P}_- = \frac{1}{2}\gamma^+\gamma^-$ are the projectors on good and bad light-cone components, respectively [40]. Projecting Eq. (3.51) on its good components one obtains for the correlators

$$\begin{aligned}\mathcal{P}_+\Big[xM\,\gamma^-\Phi_3 + M\gamma_{T\rho}\tilde{\Phi}^\rho_{A,3} + \not{p}_T\,\Phi_2 - m\Phi_2\Big]\\ + \mathcal{P}_+\Big[xM\,\gamma^-\Phi_4 + M\gamma_{T\rho}\tilde{\Phi}^\rho_{A,4} + \not{p}_T\,\Phi_3 - m\Phi_3\Big]\frac{M}{P^+} = 0,\end{aligned} \tag{3.53}$$

where the term with D^- has disappeared and the terms with D^+ and D^α have been replaced using Eqs. (3.43) and (3.44). Multiplying this relation with one of the matrices $\Gamma^+ = \{\gamma^+, \gamma^+\gamma_5, i\sigma^{\alpha+}\gamma_5\}$, which satisfy $\Gamma^+\mathcal{P}_+ = \Gamma^+(1-\mathcal{P}_-) = \Gamma^+$, and taking the trace gives

$$\mathrm{Tr}\,\Gamma^+\left[\gamma^-x\,\Phi_3 + \gamma_{T\rho}\tilde{\Phi}^\rho_{A,3} + \frac{\not{p}_T}{M}\Phi_2 - \frac{m}{M}\Phi_2\right] = 0, \tag{3.54}$$

where the terms multiplied by M/P^+ in Eq. (3.53) have disappeared because the trace of Dirac matrices cannot produce a term that transforms like P^+ under boosts in the light-cone direction. Inserting the parameterizations (3.15) and (3.45) into (3.54), one finds the following relations between T-even functions:

$$xe = x\tilde{e} + \frac{m}{M}f_1, \tag{3.55}$$

$$xf^\perp = x\tilde{f}^\perp + f_1, \tag{3.56}$$

$$xg'_T = x\tilde{g}'_T + \frac{m}{M}h_{1T}, \tag{3.57}$$

$$xg_T^\perp = x\tilde{g}_T^\perp + g_{1T} + \frac{m}{M}h_{1T}^\perp, \tag{3.58}$$

$$xg_T = x\tilde{g}_T - \frac{p_T^2}{2M^2}g_{1T} + \frac{m}{M}h_1, \tag{3.59}$$

$$xg_L^\perp = x\tilde{g}_L^\perp + g_{1L} + \frac{m}{M}h_{1L}^\perp, \tag{3.60}$$

$$xh_L = x\tilde{h}_L + \frac{p_T^2}{M^2}h_{1L}^\perp + \frac{m}{M}g_{1L}, \tag{3.61}$$

$$xh_T = x\tilde{h}_T - h_1 + \frac{p_T^2}{2M^2}h_{1T}^\perp + \frac{m}{M}g_{1T}, \tag{3.62}$$

$$xh_T^\perp = x\tilde{h}_T^\perp + h_1 + \frac{p_T^2}{2M^2}h_{1T}^\perp. \tag{3.63}$$

These relations can be found in Ref. [1], App. C. Neglecting quark-gluon-quark correlators (often referred to as the Wandzura-Wilczek approximation) is equivalent to setting all functions with a tilde to zero. For T-odd functions we have the following relations:

$$x e_L = x\tilde{e}_L, \tag{3.64}$$

$$x e_T = x\tilde{e}_T, \tag{3.65}$$

$$x e_T^\perp = x\tilde{e}_T^\perp + \frac{m}{M}\, f_{1T}^\perp, \tag{3.66}$$

$$x f_T' = x\tilde{f}_T' + \frac{p_T^2}{M^2}\, f_{1T}^\perp, \tag{3.67}$$

$$x f_T^\perp = x\tilde{f}_T^\perp + f_{1T}^\perp, \tag{3.68}$$

$$x f_T = x\tilde{f}_T + \frac{p_T^2}{2M^2}\, f_{1T}^\perp, \tag{3.69}$$

$$x f_L^\perp = x\tilde{f}_L^\perp, \tag{3.70}$$

$$x g^\perp = x\tilde{g}^\perp + \frac{m}{M}\, h_1^\perp, \tag{3.71}$$

$$x h = x\tilde{h} + \frac{p_T^2}{M^2}\, h_1^\perp. \tag{3.72}$$

Most of these relations can be found in Refs. [48, 49], and Eq. (3.71) can be inferred from Eq. (13) in Ref. [15]. Eqs. (3.66) and (3.68) have not been given before as they require the new functions introduced in Ref. [16]. We emphasize that the constraints due to the equations of motion remain valid in the presence of the appropriate Wilson lines in the correlation functions. All that is required for the gauge link $\mathcal{U}_{(0,\xi)}$ between the quark fields is the relation $\mathcal{U}_{(0,\xi)}\, iD^+(\xi) = i\partial^+\mathcal{U}_{(0,\xi)}$ leading to Eq. (3.43). In contrast, the so-called Lorentz invariance relations used in earlier work are invalidated by the presence of the gauge links [14]. We remark that if quark-gluon-quark correlators are neglected, the time-reversal constraints (3.34) require that $\int d^2p_T\, p_T^2\, f_{1T}^\perp(x,p_T^2) = 0$ and $\int d^2p_T\, p_T^2\, h_1^\perp(x,p_T^2) = 0$.

The quark-gluon-quark fragmentation correlator analogous to Φ_D is defined as

$$\left(\Delta_D^\mu\right)_{ij}(z,k_T) = \\ \frac{1}{2z}\sum_X \int \frac{d\xi^+\, d^2\boldsymbol{\xi}_T}{(2\pi)^3}\, e^{ik\cdot\xi}\, \langle 0|\, \mathcal{U}^{n_+}_{(+\infty,\xi)}\, iD^\mu(\xi)\; \psi_i(\xi)|h,X\rangle\langle h,X|\bar{\psi}_j(0)\, \mathcal{U}^{n_+}_{(0,+\infty)}|0\rangle\bigg|_{\xi^-=0}. \tag{3.73}$$

The transverse correlator

$$\tilde{\Delta}_A^\alpha(z,k_T) = \Delta_D^\alpha(z,k_T) - k_T^\alpha\, \Delta(z,k_T) \tag{3.74}$$

can be decomposed as

$$\tilde{\Delta}_A^\alpha(z,k_T) = \frac{M_h}{2z}\bigg\{\big(\tilde{D}^\perp - i\,\tilde{G}^\perp\big)\frac{k_{T\rho}}{M_h}\left(g_T^{\alpha\rho} + i\epsilon_T^{\alpha\rho}\gamma_5\right) \\ + \big(\tilde{H} + i\,\tilde{E}\big)\, i\gamma_T^\alpha + \ldots\left(g_T^{\alpha\rho} - i\epsilon_T^{\alpha\rho}\gamma_5\right)\bigg\}\frac{\not{n}_-}{2}. \tag{3.75}$$

Using the equation of motion for the quark field, the following relations can be established between the functions appearing in the above correlator and the functions in the quark-quark correlator (3.38):

$$\frac{E}{z} = \frac{\tilde{E}}{z} + \frac{m}{M_h}\, D_1, \tag{3.76}$$

$$\frac{D^{\perp}}{z} = \frac{\tilde{D}^{\perp}}{z} + D_1, \tag{3.77}$$

$$\frac{G^{\perp}}{z} = \frac{\tilde{G}^{\perp}}{z} + \frac{m}{M_h}\, H_1^{\perp}, \tag{3.78}$$

$$\frac{H}{z} = \frac{\tilde{H}}{z} + \frac{k_T^2}{M_h^2}\, H_1^{\perp}. \tag{3.79}$$

4. Results for structure functions

Inserting the parameterizations of the different correlators in the expression (3.9) of the hadronic tensor and using the equation-of-motion constraints just discussed, one can calculate the leptoproduction cross section for semi-inclusive DIS and project out the different structure functions appearing in Eq. (2.7). To have a compact notation for the results, we introduce the unit vector $\hat{\boldsymbol{h}} = \boldsymbol{P}_{h\perp}/|\boldsymbol{P}_{h\perp}|$ and the notation

$$\mathcal{C}\big[w\, f\, D\big] = x \sum_a e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\big)\, w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2), \tag{4.1}$$

where $w(\boldsymbol{p}_T, \boldsymbol{k}_T)$ is an arbitrary function and the summation runs over quarks and anti-quarks. The expressions for the structure functions appearing in Eq. (2.7) are

$$F_{UU,T} = \mathcal{C}\big[f_1 D_1\big], \tag{4.2}$$

$$F_{UU,L} = 0, \tag{4.3}$$

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\, \mathcal{C}\bigg[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\Big(xh\, H_1^{\perp} + \frac{M_h}{M}\, f_1 \frac{\tilde{D}^{\perp}}{z}\Big) - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\Big(xf^{\perp} D_1 + \frac{M_h}{M}\, h_1^{\perp} \frac{\tilde{H}}{z}\Big)\bigg], \tag{4.4}$$

$$F_{UU}^{\cos 2\phi_h} = \mathcal{C}\bigg[-\frac{2\,\big(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\big)\,\big(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\big) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h} h_1^{\perp} H_1^{\perp}\bigg], \tag{4.5}$$

$$F_{LU}^{\sin\phi_h} = \frac{2M}{Q}\, \mathcal{C}\bigg[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\Big(xe\, H_1^{\perp} + \frac{M_h}{M}\, f_1 \frac{\tilde{G}^{\perp}}{z}\Big) + \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\Big(xg^{\perp} D_1 + \frac{M_h}{M}\, h_1^{\perp} \frac{\tilde{E}}{z}\Big)\bigg], \tag{4.6}$$

$$F_{UL}^{\sin\phi_h} = \frac{2M}{Q}\, \mathcal{C}\bigg[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\Big(xh_L H_1^{\perp} + \frac{M_h}{M}\, g_{1L} \frac{\tilde{G}^{\perp}}{z}\Big) + \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\Big(xf_L^{\perp} D_1 - \frac{M_h}{M}\, h_{1L}^{\perp} \frac{\tilde{H}}{z}\Big)\bigg], \tag{4.7}$$

$$F_{UL}^{\sin 2\phi_h} = \mathcal{C}\bigg[-\frac{2\,\big(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\big)\,\big(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\big) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h} h_{1L}^{\perp} H_1^{\perp}\bigg], \tag{4.8}$$

$$F_{LL} = \mathcal{C}\big[g_{1L} D_1\big], \tag{4.9}$$

$$F_{LL}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\biggl[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\biggl(xe_L H_1^{\perp} - \frac{M_h}{M}\,g_{1L}\frac{\tilde{D}^{\perp}}{z}\biggr) - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\biggl(xg_L^{\perp}D_1 + \frac{M_h}{M}\,h_{1L}^{\perp}\frac{\tilde{E}}{z}\biggr)\biggr], \tag{4.10}$$

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\biggl[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}f_{1T}^{\perp}D_1\biggr], \tag{4.11}$$

$$F_{UT,L}^{\sin(\phi_h-\phi_S)} = 0, \tag{4.12}$$

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\biggl[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}h_1 H_1^{\perp}\biggr], \tag{4.13}$$

$$F_{UT}^{\sin(3\phi_h-\phi_S)} = \mathcal{C}\biggl[\frac{2\,\bigl(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\bigr)\,\bigl(\boldsymbol{p}_T\cdot\boldsymbol{k}_T\bigr) + \boldsymbol{p}_T^2\,\bigl(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\bigr) - 4\,\bigl(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\bigr)^2\,\bigl(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\bigr)}{2M^2M_h}\,h_{1T}^{\perp}H_1^{\perp}\biggr], \tag{4.14}$$

$$\begin{aligned} F_{UT}^{\sin\phi_S} = \frac{2M}{Q}\,\mathcal{C}\biggl\{&\biggl(xf_T D_1 - \frac{M_h}{M}\,h_1\frac{\tilde{H}}{z}\biggr) \\ &- \frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\biggl[\biggl(xh_T H_1^{\perp} + \frac{M_h}{M}g_{1T}\,\frac{\tilde{G}^{\perp}}{z}\biggr) - \biggl(xh_T^{\perp}H_1^{\perp} - \frac{M_h}{M}f_{1T}^{\perp}\,\frac{\tilde{D}^{\perp}}{z}\biggr)\biggr]\biggr\}, \end{aligned} \tag{4.15}$$

$$\begin{aligned} F_{UT}^{\sin(2\phi_h-\phi_S)} = \frac{2M}{Q}\,\mathcal{C}\biggl\{&\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2 - \boldsymbol{p}_T^2}{2M^2}\,\biggl(xf_T^{\perp}D_1 - \frac{M_h}{M}\,h_{1T}^{\perp}\frac{\tilde{H}}{z}\biggr) \\ &- \frac{2\,\bigl(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\bigr)\,\bigl(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\bigr) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\,\biggl[\biggl(xh_T H_1^{\perp} + \frac{M_h}{M}g_{1T}\,\frac{\tilde{G}^{\perp}}{z}\biggr) \\ &+ \biggl(xh_T^{\perp}H_1^{\perp} - \frac{M_h}{M}f_{1T}^{\perp}\,\frac{\tilde{D}^{\perp}}{z}\biggr)\biggr]\biggr\}, \end{aligned} \tag{4.16}$$

$$F_{LT}^{\cos(\phi_h-\phi_S)} = \mathcal{C}\biggl[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}g_{1T}D_1\biggr], \tag{4.17}$$

$$\begin{aligned} F_{LT}^{\cos\phi_S} = \frac{2M}{Q}\,\mathcal{C}\biggl\{&-\biggl(xg_T D_1 + \frac{M_h}{M}\,h_1\frac{\tilde{E}}{z}\biggr) \\ &+ \frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\biggl[\biggl(xe_T H_1^{\perp} - \frac{M_h}{M}g_{1T}\,\frac{\tilde{D}^{\perp}}{z}\biggr) + \biggl(xe_T^{\perp}H_1^{\perp} + \frac{M_h}{M}f_{1T}^{\perp}\,\frac{\tilde{G}^{\perp}}{z}\biggr)\biggr]\biggr\}, \end{aligned} \tag{4.18}$$

$$\begin{aligned} F_{LT}^{\cos(2\phi_h-\phi_S)} = \frac{2M}{Q}\,\mathcal{C}\biggl\{&-\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2 - \boldsymbol{p}_T^2}{2M^2}\,\biggl(xg_T^{\perp}D_1 + \frac{M_h}{M}\,h_{1T}^{\perp}\frac{\tilde{E}}{z}\biggr) \\ &+ \frac{2\,\bigl(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\bigr)\,\bigl(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\bigr) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\,\biggl[\biggl(xe_T H_1^{\perp} - \frac{M_h}{M}g_{1T}\,\frac{\tilde{D}^{\perp}}{z}\biggr) \\ &- \biggl(xe_T^{\perp}H_1^{\perp} + \frac{M_h}{M}f_{1T}^{\perp}\,\frac{\tilde{G}^{\perp}}{z}\biggr)\biggr]\biggr\}. \end{aligned} \tag{4.19}$$

Notice that distribution and fragmentation functions do not appear in a symmetric fashion in these expressions: there are only twist-three fragmentation functions with a tilde and only twist-three distribution functions without tilde. This asymmetry is not surprising because in Eq. (2.7) the structure functions themselves are introduced in an asymmetric way, with azimuthal angles referring to the axis given by the four-momenta of the target nucleon and the photon, rather than of the target nucleon and the detected hadron.

Equations (4.2) to (4.19) are a main result of this paper. A few comments concerning the comparison with the existing literature are in order here. First of all, it has to be stressed that in much of the past literature a different definition of the azimuthal angles has been used, whereas in the present work we adhere to the Trento conventions [28]. To compare with those papers, the signs of ϕ_h and of ϕ_S have to be reversed. The terms with the distribution functions $f_T^\perp$ and $e_T^\perp$ have not been given before. All leading-twist structure functions here are consistent with those given in Eqs. (36) and (37) of Ref. [3] when only photon exchange is taken into consideration. The structure functions $F_{LU}^{\sin\phi_h}$ and $F_{UL}^{\sin\phi_h}$ in our Eqs. (4.6) and (4.7) correspond to Eqs. (16) and (25) in Ref. [15]. The other six twist-three structure functions were partially given in the original work of Mulders and Tangerman [1], but excluding T-odd distribution functions, assuming Gaussian transverse-momentum distributions, and without the contributions from the fragmentation function $G^\perp$.

The structure function $F_{UU}^{\cos\phi_h}$ is associated with the so-called Cahn effect [50, 51]. If one neglects the quark-gluon-quark functions $\tilde{D}^\perp$ in Eq. (4.4) and $\tilde{f}^\perp$ in Eq. (3.56) as well as the T-odd distribution functions h and $h_1^\perp$, our result becomes

$$F_{UU}^{\cos\phi_h} \approx \frac{2M}{Q}\,\mathcal{C}\Big[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\,f_1 D_1\Big]. \tag{4.20}$$

This coincides with the $\cos\phi_h$ term calculated to order $1/Q$ in the parton model with intrinsic transverse momentum included in distribution and fragmentation functions, see e.g. Eqs. (32) and (33) in Ref. [52].

Let us briefly mention some experimental results and phenomenological analyses for the structure functions given above. For simplicity we do not distinguish between measurements of the structure functions and of the associated spin or angular asymmetries, which correspond to the ratio of the appropriate structure functions and $F_{UU,T} + \epsilon F_{UU,L}$.

1. Measurements of the cross-section components containing the structure function $F_{UU}^{\cos\phi_h}$ have been reported in Refs. [53, 54, 17, 26]. A description of the $\cos\phi_h$ modulation by the Cahn effect alone has been given in Ref. [52]. The same analysis can be applied to the structure function $F_{LL}^{\cos\phi_h}$, leading to the results of Ref. [55].

2. $F_{UU}^{\cos 2\phi_h}$ contains the functions $h_1^\perp$ (Boer-Mulders function [2]) and $H_1^\perp$ (Collins function [56]). It has been measured in Refs. [17, 26].

3. The structure function $F_{LU}^{\sin\phi_h}$ has been recently measured by the CLAS collaboration [25].

4. The structure function $F_{UL}^{\sin\phi_h}$ has been measured by HERMES [23]. The precise extraction of this observable requires care because in experiments the target is polarized along the direction of the lepton beam and not of the virtual photon [57, 58, 15, 27]. This implies that the longitudinal target-spin asymmetries measured in Refs. [18, 19, 20] receive contributions not only from $F_{UL}^{\sin\phi_h}$, but at the same order in $1/Q$ also from $F_{UT,T}^{\sin(\phi_h-\phi_S)}$ and $F_{UT}^{\sin(\phi_h+\phi_S)}$ (see also the phenomenological studies of Refs. [59, 60, 61, 62, 63, 64, 65]). In Ref. [23] the HERMES collaboration has separated the different contributions to the experimental $\sin\phi_h$ asymmetry with

longitudinal target polarization and shown that $F_{UL}^{\sin\phi_h}$ is dominant in the kinematics of the measurement.

5. $F_{UT,T}^{\sin(\phi_h-\phi_S)}$ contains the Sivers function [66] and has been recently measured for a proton target at HERMES [22] and for a deuteron target at COMPASS [24]. Extractions of the Sivers function from the experimental data were performed in Refs. [67, 68, 69] (see Ref. [70] for a comparison of the various extractions).

6. The structure function $F_{UT}^{\sin(\phi_h+\phi_S)}$ contains the transversity distribution function [41, 42] and the Collins function. As the previous structure function, it has been measured by HERMES [22] on the proton and by COMPASS [24] on the deuteron. Phenomenological studies have been presented in Ref. [68], where information about the Collins function was extracted, and in Ref. [71], where constraints on the transversity distribution function were obtained by using additional information from a Collins asymmetry measured in e^+e^- annihilation [72].

Integration of Eqs. (4.2) to (4.19) over the transverse momentum $\boldsymbol{P}_{h\perp}$ of the outgoing hadron leads to the following expressions for the integrated structure functions in Eq. (2.14):

$$F_{UU,T} = x \sum_a e_a^2\, f_1^a(x)\, D_1^a(z), \tag{4.21}$$

$$F_{UU,L} = 0, \tag{4.22}$$

$$F_{LL} = x \sum_a e_a^2\, g_1^a(x)\, D_1^a(z), \tag{4.23}$$

$$F_{UT}^{\sin\phi_S} = -x \sum_a e_a^2\, \frac{2M_h}{Q}\, h_1^a(x)\, \frac{\tilde{H}^a(z)}{z}, \tag{4.24}$$

$$F_{LT}^{\cos\phi_S} = -x \sum_a e_a^2\, \frac{2M}{Q} \left(x g_T^a(x) D_1^a(z) + \frac{M_h}{M}\, h_1^a(x)\, \frac{\tilde{E}^a(z)}{z} \right). \tag{4.25}$$

Finally, the structure functions for totally inclusive DIS can be obtained from Eqs. (2.16) and (2.18) to (2.21). This gives the standard results [1]

$$F_1 = \frac{1}{2} \sum_a e_a^2\, f_1^a(x), \tag{4.26}$$

$$F_L = 0, \tag{4.27}$$

$$g_1 = \frac{1}{2} \sum_a e_a^2\, g_1^a(x), \tag{4.28}$$

$$g_1 + g_2 = \frac{1}{2} \sum_a e_a^2\, g_T^a(x), \tag{4.29}$$

where given the accuracy of our calculation we have replaced $g_1 - \gamma^2 g_2$ by g_1 in (4.28), and where we have used

$$\sum_h \int dz\, z\, D_1^a(z) = 1, \qquad \sum_h \int dz\, \tilde{E}^a(z) = 0, \qquad \sum_h \int dz\, \tilde{H}^a(z) = 0. \tag{4.30}$$

The first relation is the well-known momentum sum rule for fragmentation functions. The second relation was already pointed out in Ref. [73]. The sum rule for $\tilde{H}$ follows from Eq. (4.24) and the time-reversal constraint (2.22).

In App. B, we give results for one-jet production at low transverse momentum in DIS.

5. Conclusions

We have analyzed one-particle inclusive deep inelastic scattering off a polarized nucleon for low transverse momentum of the detected hadron, starting from a general decomposition of the cross section in terms of 18 structure functions, given in Eq. (2.7) and expressed in a helicity basis in App. A. Using tree-level factorization as discussed in Sec. 3.2, the structure functions can be calculated up to subleading order in $1/Q$ (twist three) using transverse-momentum-dependent quark-quark and quark-gluon-quark correlators. to subleading order, we need the parameterizations of the correlators up to twist three, involving 24 parton distributions. In particular our treatment includes those twist three transverse momentum dependent functions that appear after the proper inclusion of Wilson lines in the quark-quark correlators. We also give the relations between the quark-quark and quark-gluon-quark correlators that follow from the QCD equations of motion.

Using the parameterization of the correlators we eventually expressed the structure functions appearing in the cross section in terms of transverse-momentum-dependent parton distribution and fragmentation functions, see Eqs. (4.2) to (4.19). Several of these results were already present in the literature, but never collected in a single paper. The complete results for transversely polarized targets, Eqs. (4.11) to (4.19), including all T-odd distribution functions, are presented here for the first time. It is straightforward to generalize the present results to include the production of a polarized hadron.

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A. Structure functions in a helicity basis

Up to a kinematical factor, the structure functions we have introduced in Eq. (2.7) are simple combinations of cross sections or interference terms for the subprocess $\gamma^* N \to hX$ with definite helicities of the nucleon and the virtual photon. Let us define helicity structure functions

$$F_{mn}^{ij}(x, Q^2, z, P_{h\perp}^2) = \frac{Q^2(1-x)}{4\pi^3\alpha} \left(1 + \frac{\gamma^2}{2x}\right)^{-1} \frac{d\sigma_{mn}^{ij}}{dz\, dP_{h\perp}^2}, \tag{A.1}$$

in terms of the $\gamma^* p$ cross sections and interference terms introduced in Ref. [27], where j and i (n and m) are the helicities of the nucleon (virtual photon) in the amplitude and its complex conjugate. We then have

$$
\begin{aligned}
F_{UU,T} &= \frac{1}{2}\left(F_{++}^{++} + F_{++}^{--}\right), & F_{UU,L} &= F_{00}^{++}, \\
F_{UU}^{\cos\phi_h} &= -\frac{1}{\sqrt{2}}\,\mathrm{Re}\left(F_{+0}^{++} + F_{+0}^{--}\right), & F_{UU}^{\cos 2\phi_h} &= -\mathrm{Re}\,F_{+-}^{++},
\end{aligned} \tag{A.2}
$$

$$
F_{LU}^{\sin\phi_h} = -\frac{1}{\sqrt{2}}\,\mathrm{Im}\left(F_{+0}^{++} + F_{+0}^{--}\right), \tag{A.3}
$$

$$
F_{UL}^{\sin\phi_h} = -\frac{1}{\sqrt{2}}\,\mathrm{Im}\left(F_{+0}^{++} - F_{+0}^{--}\right), \qquad F_{UL}^{\sin 2\phi_h} = -\mathrm{Im}\,F_{+-}^{++}, \tag{A.4}
$$

$$
F_{LL} = \frac{1}{2}\left(F_{++}^{++} - F_{++}^{--}\right), \qquad F_{LL}^{\cos\phi_h} = -\frac{1}{\sqrt{2}}\,\mathrm{Re}\left(F_{+0}^{++} - F_{+0}^{--}\right), \tag{A.5}
$$

$$
\begin{aligned}
F_{UT,T}^{\sin(\phi_h-\phi_S)} &= -\mathrm{Im}\,F_{++}^{+-}, & F_{UT,L}^{\sin(\phi_h-\phi_S)} &= -\mathrm{Im}\,F_{00}^{+-}, \\
F_{UT}^{\sin(\phi_h+\phi_S)} &= -\frac{1}{2}\,\mathrm{Im}\,F_{+-}^{+-}, & F_{UT}^{\sin(3\phi_h-\phi_S)} &= -\frac{1}{2}\,\mathrm{Im}\,F_{+-}^{-+}, \\
F_{UT}^{\sin\phi_S} &= -\frac{1}{\sqrt{2}}\,\mathrm{Im}\,F_{+0}^{+-}, & F_{UT}^{\sin(2\phi_h-\phi_S)} &= -\frac{1}{\sqrt{2}}\,\mathrm{Im}\,F_{+0}^{-+},
\end{aligned} \tag{A.6}
$$

$$
\begin{aligned}
F_{LT}^{\cos(\phi_h-\phi_S)} &= \mathrm{Re}\,F_{++}^{+-}, & F_{LT}^{\cos\phi_S} &= -\frac{1}{\sqrt{2}}\,\mathrm{Re}\,F_{+0}^{+-}, \\
F_{LT}^{\cos(2\phi_h-\phi_S)} &= -\frac{1}{\sqrt{2}}\,\mathrm{Re}\,F_{+0}^{-+}.
\end{aligned} \tag{A.7}
$$

Comparing with our results (4.2) to (4.19) we find a number of simple patterns. The leading structure functions in the $1/Q$ expansion are those where the photon is transverse in both the amplitude and its conjugate, and the subleading structure functions correspond to the interference between transverse and longitudinal photon polarizations. The two structure functions $F_{UU,L}$ and $F_{UT,L}^{\sin(\phi_h-\phi_S)}$ involve only longitudinally polarized photons. In the kinematics we consider, they are of order $1/Q^2$ and thus beyond the accuracy to which we have calculated. We finally remark that the number of transverse momentum factors appearing in the different structure functions (4.2) to (4.19) can be related to the mismatch between the helicity differences $(m-i)$ and $(n-j)$ using angular momentum conservation, see Ref. [27].

B. Semi-inclusive jet production

In this appendix, we take into consideration the process

$$
\ell(l) + N(P) \to \ell(l') + \mathrm{jet}(P_j) + X \tag{B.1}
$$

in the kinematical limit of large Q^2 at fixed x and $P_{h\perp}^2$. In the context of our tree-level calculation, we identify the jet with the quark scattered from the virtual photon. We then have $z = 1$ and the cross section formula is identical to Eq. (2.7), except that it is not differential in z. Correspondingly, the structure functions do not depend on this variable. The structure functions for the process (B.1) can be obtained from those of one-particle inclusive DIS in Eqs. (4.2) to (4.19) by replacing $D_1(z, k_T^2)$ with $\delta(1-z)\,\delta^{(2)}(\boldsymbol{k}_T)$, setting all other fragmentation functions to zero and integrating over z. This gives

$$F_{UU,T} = x \sum_a e_a^2\, f_1^a(x, P_{j\perp}^2), \tag{B.2}$$

$$F_{UU}^{\cos\phi_h} = -x \sum_a e_a^2\, \frac{2|\boldsymbol{P}_{j\perp}|}{Q}\, x f^{\perp a}(x, P_{j\perp}^2), \tag{B.3}$$

$$F_{LU}^{\sin\phi_h} = x \sum_a e_a^2\, \frac{2|\boldsymbol{P}_{j\perp}|}{Q}\, x g^{\perp a}(x, P_{j\perp}^2), \tag{B.4}$$

$$F_{UL}^{\sin\phi_h} = x \sum_a e_a^2\, \frac{2|\boldsymbol{P}_{j\perp}|}{Q}\, x f_L^{\perp a}(x, P_{j\perp}^2), \tag{B.5}$$

$$F_{LL} = x \sum_a e_a^2\, g_{1L}^a(x, P_{j\perp}^2), \tag{B.6}$$

$$F_{LL}^{\cos\phi_h} = -x \sum_a e_a^2\, \frac{2|\boldsymbol{P}_{j\perp}|}{Q}\, x g_L^{\perp a}(x, P_{j\perp}^2), \tag{B.7}$$

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = -x \sum_a e_a^2\, \frac{|\boldsymbol{P}_{j\perp}|}{M}\, f_{1T}^{\perp a}(x, P_{j\perp}^2), \tag{B.8}$$

$$F_{UT}^{\sin\phi_S} = x \sum_a e_a^2\, \frac{2M}{Q}\, x f_T^a(x, P_{j\perp}^2), \tag{B.9}$$

$$F_{UT}^{\sin(2\phi_h-\phi_S)} = x \sum_a e_a^2\, \frac{|\boldsymbol{P}_{j\perp}|^2}{MQ}\, x f_T^{\perp a}(x, P_{j\perp}^2), \tag{B.10}$$

$$F_{LT}^{\cos(\phi_h-\phi_S)} = x \sum_a e_a^2\, \frac{|\boldsymbol{P}_{j\perp}|}{M}\, g_{1T}^a(x, P_{j\perp}^2), \tag{B.11}$$

$$F_{LT}^{\cos\phi_S} = -x \sum_a e_a^2\, \frac{2M}{Q}\, x g_T^a(x, P_{j\perp}^2), \tag{B.12}$$

$$F_{LT}^{\cos(2\phi_h-\phi_S)} = -x \sum_a e_a^2\, \frac{|\boldsymbol{P}_{j\perp}|^2}{MQ}\, x g_T^{\perp a}(x, P_{j\perp}^2), \tag{B.13}$$

whereas the remaining 6 structure functions are zero. The results for the terms with indices UU, LL, and LT correspond to those in Ref. [1], Eqs. (119) to (121). The results for the terms with indices LU and UL correspond to those in Ref. [15]. Integration of the cross section over $\boldsymbol{P}_{h\perp}$ leads to the results for inclusive DIS in Eqs. (4.26) to (4.29). Most terms vanish due to the angular integration, and $F_{UT}^{\sin\phi_S}$ in Eq. (B.9) vanishes due to the time-reversal condition (3.34).

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