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Gravity and Quantum Fields in Discrete Space-Times

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Abstract. In a 6D model, where the extra dimensions form a discretised curved disk, we investigate the mass spectra and profiles of gravitons and Dirac fermions. The discretisation is performed in detail leading to a star-like geometry. In addition, we use the curvature of the disk to obtain the mass scales of this model in a more flexible way. We also discuss some applications of this setup like generating small fermion masses.

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1. Introduction

We study a six-dimensional (6D) space-time consisting of a flat four-dimensional (4D) subspace and a disk of constant curvature for the extra dimensions. Furthermore, we discretise the disk in a way that N equidistant lattice sites are situated on the boundary and a single site in the centre of the disk (section 2). Such star-like geometries have been proven useful in various contexts, see for example [1, 2, 3, 4, 5, 6, 7]. In this work we investigate the mass spectra and profiles of 4D gravitons and fermions in this setup (sections 3 and 4), which has some nice applications. For instance, we discuss a possibility to hide extra dimensions similar to [8], and we generate small fermion masses via a discrete version of the wavefunction suppression mechanism [9].

2. Curved Disk Geometry

We consider a 6D model where both extra dimensions form a discretised curved disk (for example a part of a 2-sphere) while the 4D subspace remains continuous. Before the discretisation the space-time is described by the line element

$$\mathrm{d}s^2 = g_{\mu\nu}(x^M)\mathrm{d}x^\mu \mathrm{d}x^\nu - (1-er^2)^{-1}\mathrm{d}r^2 - r^2\mathrm{d}\varphi^2, \tag{1}$$

where x^μ and x^M denote 4D and 6D coordinates, respectively. The position on the disk is fixed by the polar coordinate $\varphi := x^6 \in [0, 2\pi]$ and the radial coordinate $r := x^5 \in [0, L]$ with L being the coordinate radius of the disk. From (1) we read off the metric components $g_{\mu\nu}(x^M)$ of the 4D subspace as well as $g_{55} = -(1-er^2)^{-1}$ and $g_{66} = -r^2$. Note that the parameter e controls the curvature of the disk. For $e > 0$ the disk is spherically curved, whereas $e < 0$ leads to a hyperbolic disk, and $e = 0$ corresponds to a flat disk. Following [1, 10] we now decompose the 6D Einstein-Hilbert action

$$S = M_6^4 \int \mathrm{d}^6x\, \sqrt{|g|}\, R = S_{4\mathrm{D}} + S_{\mathrm{surface}} + S_{\mathrm{mass}}, \tag{2}$$

into three parts, where R, g and M_6 denote the 6D curvature scalar, the determinant of the metric g_{MN} and the 6D Planck scale, respectively. We find that the 4D curvature scalar $R_{\rm 4D}$ in $S_{\rm 4D} := M_6^4 \int \mathrm{d}^6x\sqrt{|g|}\,(R_{\rm 4D}+2e)$ contains only the 4D metric $g_{\mu\nu}$ and derivatives with respect to x^μ. The surface terms in $S_{\rm 4D}$ vanish by choosing suitable boundary conditions on the disk, and $S_{\rm mass}$ is given by

$$S_{\rm mass} = M_6^4 \int \mathrm{d}^6x\sqrt{|g|}\sum_{c=5,6}\Big[-\frac{1}{4}g^{cc}g_{\mu\nu,c}(g^{\mu\nu}g^{\alpha\beta}-g^{\mu\alpha}g^{\nu\beta})g_{\alpha\beta,c}\Big]. \quad (3)$$

Let us now introduce 4D graviton fields $h_{\mu\nu}$ on a flat Minkowski metric $\eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$ by the expansion $g_{\mu\nu}\to\eta_{\mu\nu}+h_{\mu\nu}$. As in [11] we choose a gauge, where we ignore graviphoton and radion excitations, which could result from the g_{5M} and g_{6M} components of the metric. Since $\eta_{\mu\nu}$ is constant we have $g_{\mu\nu,A}\to h_{\mu\nu,A}$ and by expanding $S_{\rm mass}$ in second order in $h_{\mu\nu}$ we find

$$\begin{aligned} S_{\rm mass} \to M_6^4 \int \mathrm{d}^4x\,\mathrm{d}\varphi\,\mathrm{d}r\,\Big[&+\frac{1}{4}\sqrt{\frac{g_{66}}{g_{55}}}\partial_r h_{\mu\nu}(\eta^{\mu\nu}\eta^{\alpha\beta}-\eta^{\mu\alpha}\eta^{\nu\beta})\partial_r h_{\alpha\beta}\\ &+\frac{1}{4}\sqrt{\frac{g_{55}}{g_{66}}}\partial_\varphi h_{\mu\nu}(\eta^{\mu\nu}\eta^{\alpha\beta}-\eta^{\mu\alpha}\eta^{\nu\beta})\partial_\varphi h_{\alpha\beta}\Big]. \end{aligned} \quad (4)$$

Next, we discretise the disk by putting N lattice sites on the boundary and a single site in centre of the disk so that only two points are lying in radial direction. The coordinate distance between the centre and each point on the boundary is given by the radius L of the disk, which in general differs from its proper radius. On the boundary, the graviton fields are denoted by $h^i_{\mu\nu}$ with $i=1\ldots N$, and the position is given by $\varphi^i = i\cdot\Delta\varphi$, where $\Delta\varphi = 2\pi/N$ is the angular lattice spacing. The graviton field $h^0_{\mu\nu}$ in the centre carries the index 0, and the lattice spacing in radial direction is just $\Delta r = L$. Formally, we apply the following discretisation prescription to $S_{\rm mass}$

$$\begin{aligned} &\partial_r h(\varphi^i) \to \frac{(h^i-h^0)}{\Delta r}, &&\partial_\varphi h(\varphi^i)\to\frac{(h^{i+1}-h^i)}{\Delta\varphi},\\ &\int \mathrm{d}r\, f(r)\to\Delta r\cdot f(L), &&\int\mathrm{d}\varphi\, f(\varphi)\to\sum_{i=1}^N\Delta\varphi\cdot f(\varphi^i), \end{aligned} \quad (5)$$

where the integral $\int \mathrm{d}r$ is replaced by just one summation interval of length L and the summand is evaluated at the position $r=L$, which avoids problems with the derivative ∂_φ at $r=0$. Thus we obtain (still non-diagonal) Fierz-Pauli mass terms [12] for the gravitons on the discretised disk, that read (with $h^{N+1}_{\mu\nu}\equiv h^1_{\mu\nu}$)

$$\begin{aligned} S_{\rm mass}\to M_4^2\int\mathrm{d}^4x\sum_{i=1}^N\big[&m_\star^2\cdot(h^i_{\mu\nu}-h^0_{\mu\nu})(\eta^{\mu\nu}\eta^{\alpha\beta}-\eta^{\mu\alpha}\eta^{\nu\beta})(h^i_{\alpha\beta}-h^0_{\alpha\beta})\\ &+m^2\cdot(h^{i+1}_{\mu\nu}-h^i_{\mu\nu})(\eta^{\mu\nu}\eta^{\alpha\beta}-\eta^{\mu\alpha}\eta^{\nu\beta})(h^{i+1}_{\alpha\beta}-h^i_{\alpha\beta})\big]. \end{aligned} \quad (6)$$

Note that the actual graviton mass scale from the radial derivatives, $m_\star$, and respectively from the angular derivatives, m, depend on the 4D Planck mass M_4 of the observer's site (brane):

$$m_\star^2 := \frac{M_6^4}{4M_4^2}\cdot\frac{2\pi\sqrt{1-eL^2}}{N}, \qquad m^2 := \frac{M_6^4}{4M_4^2}\cdot\frac{N}{2\pi\sqrt{1-eL^2}}. \quad (7)$$

However, the ratio of masses is independent of the Planck scales,

$$\frac{m_\star^2}{m^2} = \frac{(2\pi)^2}{N^2}(1-eL^2), \quad (8)$$

which also shows that arbitrarily large hierarchies between $m_\star$ and m are possible by choosing the disk parameters eL^2 and N appropriately.

In order to determine the 4D Planck mass M_4 on the sites we need the proper area of the curved disk, which is given by

$$A := \int_0^{2\pi} \mathrm{d}\varphi \int_0^L \mathrm{d}r \sqrt{|g_{55}g_{66}|} = \frac{2\pi}{e}(1 - \sqrt{1 - eL^2}). \tag{9}$$

We now proceed to discretise $S_{\rm 4D}$. Since the extra-dimensional disk has a constant curvature it is well motivated that M_4 should be constant and universal on all sites, too. Thus the Einstein-Hilbert terms of all $N+1$ sites must have the form

$$M_4^2 \sum_{i=0}^{N} \int \mathrm{d}^4x\, R_{\rm 4D}, \tag{10}$$

which does not depend on r and φ anymore. By comparing this term with $S_{\rm 4D} = M_6^4 \int \mathrm{d}^6x \sqrt{|g|} R_{\rm 4D}$ we find that the 4D Planck scale on the sites is fixed by $M_4^2 = M_6^4 A/(N+1)$, where we used (9) in $S_{\rm 4D}$ and evaluated the sum in (10). However, we remark that M_4 is not the (reduced) Planck scale $M_{\rm Pl} = 1/(8\pi G) \sim 10^{18}$ GeV that couples gravity to 4D matter. But $M_{\rm Pl}$ is determined by integrating out the extra dimensions in the continuum, which means $M_6^4 \int \mathrm{d}^6x \sqrt{|g|} R_{\rm 4D} = M_{\rm Pl}^2 \int \mathrm{d}^4x\, R_{\rm 4D}$ and thus $M_{\rm Pl}^2 = M_6^4 A = (N+1) M_4^2$.

3. Graviton Mass Spectrum

We omit to show the kinetic terms for the 4D gravitons $h^i_{\mu\nu}$. They just follow from applying the graviton expansion to the Einstein-Hilbert terms in (10), see e.g. [13]. To determine the graviton mass spectrum we have to diagonalise $S_{\rm mass}$ in (6) by a unitary transformation. If we denote the graviton mass eigenstates by $H^n_{\mu\nu}$, corresponding to the masses M_n, we find the following relations for the eigenvectors:

$$H^0_{\mu\nu} = \frac{1}{\sqrt{N+1}} \sum_{i=0}^{N} h^i_{\mu\nu}, \tag{11}$$

$$H^p_{\mu\nu} = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \left[\sin(2\pi \tfrac{p}{N}) + \cos(2\pi \tfrac{p}{N})\right] \cdot h^i_{\mu\nu}, \tag{12}$$

$$H^N_{\mu\nu} = \frac{1}{\sqrt{N(N+1)}} \left[-N \cdot h^0_{\mu\nu} + \sum_{i=1}^{N} h^i_{\mu\nu} \right], \tag{13}$$

where $p = 1, \ldots, N-1$. The eigenvalues M_n are respectively given by

$$M_0^2 = 0, \qquad M_p^2 = m_\star^2 + 4m^2 \sin^2 \frac{\pi p}{N}, \qquad M_N^2 = (N+1) m_\star^2. \tag{14}$$

From these results we observe that the zero-mode $H^0_{\mu\nu}$ has a flat profile and is equally located on all sites, whereas the mode $H^N_{\mu\nu}$ with squared mass $(N+1)m_\star^2$ is peaked on the centre site with equal support on the boundary sites. The modes $H^p_{\mu\nu}$ with $p = 1, \ldots, N-1$ are located only on the boundary with a typical finite Kaluza-Klein (KK) mass spectrum that has been shifted by $m_\star^2$. In the limit $m \ll m_\star$ the masses of the states $H^p_{\mu\nu}$ in (12) become degenerate, and for $N \gg 1$ the mode $H^N_{\mu\nu}$ becomes very heavy. Note that the latter case can be realised by a sufficiently large negative curvature of the disk, which is a clear advantage over a flat disk model.

Finally, we mention that a scenario related to ours has been discussed recently in the context of multi-throat geometries [8]. It was shown that large extra dimensions can be hidden in the sense that the occurrence of massive KK modes is shifted to energies much higher than the compactification scale of the extra dimension, which helps evading limits on KK particles. In our model this behaviour can be observed for the modes $H_{\mu\nu}^{n>0}$ in the limit $m_\star \gg m$, too.

4. Fermions on the Disk

Let us now investigate the incorporation of Dirac fermions into the discretised disk model of section 2. As for the graviton case we start with a 6D Dirac fermion Ψ in the continuum. Using the vielbein formalism, the corresponding action S on the curved disk reads

$$S = \int \mathrm{d}^6 x \, \sqrt{|g|} \left[\frac{1}{2} \mathrm{i} \left(\overline{\Psi} G^A V_A^M \nabla_M \Psi - \overline{\nabla_M \Psi} V_A^M G^A \Psi \right) \right], \tag{15}$$

where we denote 6D Lorentz indices by $A, B, \ldots$ and general coordinate indices by $M, N, \ldots$, respectively. Moreover, G^A are 6D Dirac matrices, and for the barred spinor $\overline{\Psi}$ we use the abbreviation $\overline{\Psi} = \Psi^\dagger G^0$. The vielbein components $V_A^M(x^N)$ follow from the relation $g^{MN} = V_A^M V_B^N \eta^{AB}$, which connects the Lorentz coordinate system with the general coordinate system. For the diagonal metric (1), we find $V_A^M = \delta_A^M$ with the exceptions $V_{A=5}^{M=5} = \sqrt{|g^{55}|} =: V_5$ and $V_{A=6}^{M=6} = \sqrt{|g^{66}|} =: V_6$. On a curved space-time the covariant derivative $\nabla_M = \partial_M + \Gamma_M$ for spinors contains in addition to the usual partial derivative ∂_M also the spin connection $\Gamma_M = \frac{1}{8}[G^A, G^B] V_A^N V_{BN;M}$.

To determine the form of the 6D γ-matrices [14] let us first look at the 4D case, where the γ-matrices are given by

$$\gamma^0 = \begin{bmatrix} 0 & 1_2 \\ 1_2 & 0 \end{bmatrix}, \qquad \gamma^k = \begin{bmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{bmatrix}, \qquad \gamma^5 = \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3 . \tag{16}$$

Here, $k = 1 \ldots 3$ and σ^k denote the Pauli matrices. In five dimensions the number of spinor components is still four and the corresponding γ-matrices are simply given by $\Gamma^0 = \gamma^0$, $\Gamma^k = \gamma^k$ and $\Gamma^5 = \mathrm{i}\gamma^5 = -(\Gamma^5)^\dagger$. In six dimensions, however, the Dirac algebra is 8-dimensional, where we use the following set of γ-matrices

$$G^0 = \begin{bmatrix} 0 & 1_4 \\ 1_4 & 0 \end{bmatrix}, \qquad G^6 = \begin{bmatrix} 0 & \Gamma^0 \\ -\Gamma^0 & 0 \end{bmatrix} = -(G^6)^\dagger, \tag{17}$$

$$G^n = \begin{bmatrix} 0 & \Gamma^0\Gamma^n \\ -\Gamma^0\Gamma^n & 0 \end{bmatrix} = -(G^n)^\dagger, \qquad n = 1, 2, 3, 5, \tag{18}$$

which fulfil the Cifford algebra $\{G^A, G^B\} = 2\eta^{AB} \cdot 1_8$.

From the form of the vielbeins and the γ-matrices it follows that all spin connection components Γ_M vanish except for $\Gamma_6 = \frac{1}{4}[G^5, G^6]\sqrt{1 - er^2}$. And because of $\Gamma_6 = -\Gamma_6^\dagger$ the terms involving Γ_6 in (15) cancel.

Let us now diagonalise the fermion action by the substitution $\Psi = G^6 \Phi$. Then the decomposition of the eight-component spinor $\Phi = (\Phi_a, \Phi_b)^{\mathrm{T}}$ into two four-component spinors Φ_a, Φ_b yields in (15)

$$\begin{aligned} \mathrm{i}\overline{\Psi} G^A V_A^M \nabla_M \Psi = \mathrm{i} \left[\overline{\Phi_a}, \overline{\Phi_b} \right] \times \Bigg[& \begin{pmatrix} \gamma^0 & 0 \\ 0 & \gamma^0 \end{pmatrix} \partial_0 + \begin{pmatrix} -\gamma^k & 0 \\ 0 & \gamma^k \end{pmatrix} \partial_k \\ & + \begin{pmatrix} -\mathrm{i}\gamma^5 & 0 \\ 0 & +\mathrm{i}\gamma^5 \end{pmatrix} V_5 \partial_5 + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} V_6 \partial_6 \Bigg] \times \begin{bmatrix} \Phi_a \\ \Phi_b \end{bmatrix} \end{aligned} \tag{19}$$

with $\overline{\Phi_{a,b}} = \Phi^{\dagger}_{a,b}\gamma^0$. From the last line one can read off that Φ_a corresponds to Φ_b but with negative energy, therefore we will work only with Φ_b in the following. If we now denote the left- and right-handed components of Φ_b by $\Phi_{\mathrm{L,R}} := \frac{1}{2}(1 \mp \gamma^5)\Phi_b$, then the full action for Φ_b can be written in the form

$$\begin{aligned} S = &\int \mathrm{d}^6x\sqrt{|g|}\Big[\frac{1}{2}\mathrm{i}\left(\overline{\Phi_b}\gamma^\mu\partial_\mu\Phi_b - \overline{\partial_\mu\Phi_b}\gamma^\mu\Phi_b\right) \\ &- V_5\frac{1}{2}\left(\overline{\Phi_{\mathrm{L}}}\partial_5\Phi_{\mathrm{R}} + \overline{\partial_5\Phi_{\mathrm{R}}}\Phi_{\mathrm{L}}\right) - \mathrm{i}V_6\frac{1}{2}\left(\overline{\Phi_{\mathrm{L}}}\partial_6\Phi_{\mathrm{R}} - \overline{\partial_6\Phi_{\mathrm{R}}}\Phi_{\mathrm{L}}\right)\Big], \end{aligned} \tag{20}$$

where we have applied the boundary conditions of section 2 after integration by parts.

Since 6D spinors have mass dimension $\frac{5}{2}$ we have to rescale them in order to obtain usual 4D spinors. As in the graviton case we integrate the kinetic terms over the extra dimensions and apply a similar discretisation procedure as in section 2, which means

$$\int \mathrm{d}^6x\sqrt{|g|}\frac{1}{2}\mathrm{i}\overline{\Phi_b}\gamma^\mu\partial_\mu\Phi_b \to \sum_{j=0}^{N}\frac{A}{N+1}\int \mathrm{d}^4x\frac{1}{2}\mathrm{i}\overline{\Phi_b^j}\gamma^\mu\partial_\mu\Phi_b^j, \tag{21}$$

where A is the proper area given in (9). Finally, we absorb the factor $A/(N+1)$ into the fermion fields $\chi := \Phi_b\sqrt{A/(N+1)}$ and subsequently apply the discretisation prescriptions (5) with $h_{\mu\nu}$ replaced by χ. As a result we obtain the action for $N+1$ 4D fermions, $(\chi^{N+1} \equiv \chi^1)$

$$\begin{aligned} S = &\sum_{j=0}^{N}\int \mathrm{d}^4x\frac{1}{2}\mathrm{i}\left(\overline{\chi^j}\gamma^\mu\partial_\mu\chi^j - \overline{\partial_\mu\chi^j}\gamma^\mu\chi^j\right) \\ &- \sum_{j=1}^{N}\int \mathrm{d}^4x\cdot m_\star\left(\overline{\chi_{\mathrm{L}}^j}(\chi_{\mathrm{R}}^j - \chi_{\mathrm{R}}^0) + (\overline{\chi_{\mathrm{R}}^j} - \overline{\chi_{\mathrm{R}}^0})\chi_{\mathrm{L}}^j\right) \\ &- \sum_{j=1}^{N}\int \mathrm{d}^4x\cdot\mathrm{i}m\left(\overline{\chi_{\mathrm{L}}^j}(\chi_{\mathrm{R}}^{j+1} - \chi_{\mathrm{R}}^j) - (\overline{\chi_{\mathrm{R}}^{j+1}} - \overline{\chi_{\mathrm{R}}^j})\chi_{\mathrm{L}}^j\right) \end{aligned} \tag{22}$$

with the mass scales $m_\star := 2\pi L(N+1)/(AN)$ and $m := L(N+1)/(A\sqrt{1-eL^2})$. Hence, the ratio $m_\star^2/m^2$ is the same ratio as in (8) for the gravitons. Next, we apply a bi-unitary transformation relating the states χ to the mass eigenstates ψ:

$$\begin{aligned} &\overline{\chi_{\mathrm{L}}^0} = \overline{\psi_{\mathrm{L}}^0}, \qquad \overline{\chi_{\mathrm{L}}^j} = \frac{1}{\sqrt{N}}\sum_{n=1}^{N}\exp(+2\pi\mathrm{i}\cdot j\tfrac{n}{N})\overline{\psi_{\mathrm{L}}^n}, \\ &\chi_{\mathrm{R}}^0 = \frac{1}{\sqrt{N+1}}\psi_{\mathrm{R}}^0 - \frac{N}{\sqrt{N(N+1)}}\psi_{\mathrm{R}}^N, \\ &\chi_{\mathrm{R}}^j = \frac{1}{\sqrt{N}}\sum_{n=1}^{N-1}\exp(-2\pi\mathrm{i}\cdot j\tfrac{n}{N})\psi_{\mathrm{R}}^n + \frac{1}{\sqrt{N+1}}\psi_{\mathrm{R}}^0 + \frac{1}{\sqrt{N(N+1)}}\psi_{\mathrm{R}}^N. \end{aligned} \tag{23}$$

The corresponding mass spectrum contains one massless fermion ψ^0, one heavy fermion ψ^N with mass $m_\star\sqrt{N+1}$ and $N-1$ fermions $\psi^1,\ldots,\psi^{N-1}$ with squared absolute mass values $m_\star^2 + 4m^2\sin^2(\frac{\pi n}{N}) + 2m_\star m\sin(\frac{2\pi n}{N})$. In contrast to the graviton mass spectrum (14), here we find an additional interference term $\propto m_\star m$, which can be removed by a slightly modified discretisation procedure for the angular direction. Instead of $\partial_6\chi \to (\chi^{j+1} - \chi^j)/\Delta\varphi$ we use the prescription $\partial_6\chi \to \mathrm{i}(\chi^{j+\frac{1}{2}} - \chi^{j-\frac{1}{2}})/\Delta\varphi$. This does not change the zero more or the heavy mode, but the transformations in (23) lead now to the mass spectrum $m_\star^2 + 4m^2 \sin^2(\frac{\pi n}{N})$ for the modes $\psi^1 \dots \psi^{N-1}$, which has exactly the same structure as that of the gravitons in (14).

Our results for the fermions on the discretised disk can be applied directly to generate small fermion masses. For this purpose we put the standard model of particles (SM) on the centre site of our disk. In this place the left-handed SM lepton doublet ℓ may couple to the 4D component $\chi^0_{\rm R}$ of the 6D Dirac field and to the vacuum expectation value $\langle H \rangle$ of the Higgs doublet via an Yukawa interaction term schematically given by $\overline{\ell}\langle H \rangle \chi^0_{\rm R}$. Now, a large number $N \sim 10^{24}$ of lattice sites lets Φ^N decouple due to its large mass $m_\star\sqrt{N+1}$, and (23) shows that the right-handed fermion $\chi^0_{\rm R}$ on the centre site essentially consists only of the zero-mode $\psi^0_{\rm R}$ with a tiny weight factor $1/\sqrt{N+1}$. Thus the Yukawa interaction of ℓ with $\chi^0_{\rm R}$,

$$\overline{\ell}\langle H \rangle \chi^0_{\rm R} \sim \frac{1}{\sqrt{N+1}} \nu_{\rm L} \langle H \rangle \psi^0_{\rm R}, \tag{24}$$

leads to a strong suppression of the SM neutrino ($\nu_{\rm L}$) mass, representing a discrete version [1] of the wave function suppression mechanism in continuous higher dimensions [9].

5. Conclusions

Our 6D model with a discretised extra-dimensional curved disk leads to mass spectra that have the same structure for gravitons and fermions. Moreover, the special discretisation of the disk allows that the ratio of mass scales in the spectra can be adjusted in a flexible manner by the parameters of the disk. It is thus possible to obtain a gap between the zero mode and the first massive mode that is much larger than the gap between the other massive modes. We have also discussed the generation of small SM fermion masses in this setup. Finally, we mention that the strong coupling regime of this model and a more refined scenario including warping were investigated in [1], where some of our results have been applied, too.

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