DESY 06-159

DESY 06-159
September 2006

A BRIEF ACCOUNT OF $B \to K^* \ell^+ \ell^-$ DECAY IN SOFT-COLLINEAR EFFECTIVE THEORY

Ahmed Ali

Theory Group, Deutsches Elektronen-Synchrotron DESY,
Notkestrasse 85, 22603 Hamburg, Germany

Presented at the XXXIII International Conference on High Energy Physics,
Moscow, Russian Federation, 26th July – 2nd August 2006.
To appear in the Proceedings (Publishers: World Scientific, Singapore)

A BRIEF ACCOUNT OF $B \to K^*\ell^+\ell^-$ DECAY IN SOFT-COLLINEAR EFFECTIVE THEORY

A. ALI*

Theory Group, Deutsches Elektronen-Synchrotron DESY,
Notkestrasse 85, 22603 Hamburg, Germany
**E-mail: ahmed.ali@desy.de*

A brief account of the study of rare B decay $B \to K^*\ell^+\ell^-$ using soft-collinear effective theory (SCET) is presented. Theoretical underpinning of this work is a factorization formula, derived to leading power in $1/m_b$ and valid to all orders in α_s. Partially integrated branching ratio in the dilepton squared mass range 1 GeV$^2 \leq q^2 \leq 7$ GeV^2 and the forward-backward (FB) asymmetry of the leptons are calculated. For the zero-point of the FB asymmetry, we get $q_0^2 = (4.07^{+0.16}_{-0.13})$ GeV^2. The scale-related uncertainty of q_0^2 is improved compared to the earlier estimate of the same.

1. Factorization in SCET

The emergence of an effective theory, called soft-collinear effective theory (SCET) [1], provides a systematic way to deal with the perturbative strong interaction effects in B decays in the heavy-quark expansion. SCET has been used extensively in the so-called heavy-to-light transitions in B decays. In particular, this framework was used to prove the factorization of radiative $B \to K^*\gamma$ decay at leading power in $1/m_b$ and to all orders in α_s [2,3]. In a recent paper [4], summarized below, the related decay $B \to K^*\ell^+\ell^-$ has been studied using the SCET approach.

For the $b \to s$ transitions, the weak effective Hamiltonian can be written as [5]

$$H_{eff} = -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu) Q_i(\mu) \ , \quad (1)$$

neglecting terms proportional to $V_{us}^* V_{ub}$ and using the unitarity of the CKM matrix.

Restricting to the kinematic region where the K^* meson can be viewed approximately as a collinear particle, a factorization formula for the decay amplitude of $B \to K^*\ell^+\ell^-$, to leading power in $1/m_b$ and all orders of α_s, has been derived in SCET [4]:

$$\langle K_a^*\ell^+\ell^-|H_{eff}|B\rangle = T_a^I(q^2)\zeta_a(q^2) + \sum_{\pm}\int_0^\infty \frac{d\omega}{\omega} \times \phi_\pm^B(\omega)\int_0^1 du\ \phi_{K^*}^a(u) T_{a,\pm}^{II}(\omega, u, q^2) \ , \quad (2)$$

where $a = ||, \perp$ denotes the polarization of the K^* meson. The functions T^I and T^{II} are perturbatively calculable; $\zeta_a(q^2)$ are the soft form factors defined in SCET while $\phi_\pm^B(\omega)$ and $\phi_{K^*}^a(u)$ are the light-cone distribution amplitudes (LCDAs) for the B and K^* mesons, respectively. The expression (2) coincides *formally* with the one obtained by Beneke et al. [5] in $O(\alpha_s)$ accuracy, using the QCD factorization approach [6]. We calculate the partial dilepton invariant mass spectrum and the forward-backward (FB) asymmetry, and compare our results with the existing data [7,8] and the earlier theoretical analysis [5].

2. $B \to K^*\ell^+\ell^-$ in SCET

As SCET contains two kinds of collinear fields, called hard-collinear and collinear fields, normally an intermediate effective theory, SCET$_I$, is introduced which contains only soft and hard-collinear fields. While the final effective theory, called SCET$_{II}$, contains only soft and collinear fields. One undertakes a two-step matching from $QCD \to \mathrm{SCET}_I \to \mathrm{SCET}_{II}$ [9].

2.1. *QCD to SCET$_I$ matching*

In SCET$_I$, the K^* meson is taken as a hard-collinear particle. The matching from QCD to SCET$_I$ at leading power is expressed as

$$H_{eff} \to -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \left(\sum_{i=1}^{4} \int ds \; \widetilde{C}_i^A(s) J_i^A(s) + \sum_{j=1}^{4} \int ds \int dr \; \widetilde{C}_j^B(s,r) J_j^B(s,r) + \int ds \int dr \int dt \; \widetilde{C}^C(s,r,t) J^C(s,r,t) \right), \quad (3)$$

where $\widetilde{C}_i^{(A,B)}$ and $\widetilde{C}^C$ are Wilson coefficients in the position space. The operators J_i^A and J_i^B represent the cases that the lepton pair is emitted from the $b \to s$ transition currents, while J^C represents the diagrams in which the lepton pair is emitted from the spectator quark of the B meson. Their explicit expressions are given in our paper [4]. It is more convenient to define the Wilson coefficients in the momentum space. The corresponding coefficient functions are called $C_i^A(E)$, $C_j^B(E,u)$, and $C^C(E,u)$, with $E \equiv n \cdot v \bar{n} \cdot P/2$ and the velocity of the B meson is defined as $v = P_B/m_B$. To get the order α_s corrections to the decay amplitude, we need the Wilson coefficients C_i^A to one-loop level and C_j^B and C^C to tree level.

2.2. *SCET$_I$ → SCET$_{II}$ matching and SCET matrix elements*

One may define the matrix elements of the A-type SCET$_I$ currents as non-perturbative input since the non-factorizable parts of the form factors are all contained in such matrix elements [9,10]. Thus [2]:

$$\begin{aligned}
&\langle K^* \ell^+ \ell^- | J_1^A | B \rangle \\
&\quad = -2E\zeta_\perp (g_\perp^{\mu\nu} - i\epsilon_\perp^{\mu\nu}) \varepsilon_{\perp\nu}^* \bar{\ell}\gamma_\mu \ell \; , \\
&\langle K^* \ell^+ \ell^- | J_2^A | B \rangle \\
&\quad = -2E\zeta_{||} \frac{n^\mu}{n \cdot v} \bar{\ell}\gamma_\mu \ell \; ,
\end{aligned} \quad (4)$$

where $g_\perp^{\mu\nu} \equiv g^{\mu\nu} - (n^\mu \bar{n}^\nu + \bar{n}^\mu n^\nu)/2$ and $\epsilon_\perp^{\mu\nu} \equiv \epsilon^{\mu\nu\rho\sigma} v_\rho n_\sigma/(n \cdot v)$, and we use the convention $\epsilon^{0123} = +1$. The matrix elements of the other two A-type currents, $\langle K^* \ell^+ \ell^- | J_3^A | B \rangle$ and $\langle K^* \ell^+ \ell^- | J_4^A | B \rangle$, are obtained from the above matrix elements by the replacement $\bar{\ell}\gamma_\mu \ell \to \bar{\ell}\gamma_\mu\gamma_5 \ell$, respectively.

The B-type SCET$_I$ operators are matched onto the SCET$_{II}$ operators O_i^B ($i = 1, ..., 4$). Their matrix elements involve the meson LCDAs, and two different K^*-distribution amplitudes ($\phi_{K^*}^{||}(u,\mu)$ for $\Gamma = 1$ and $\phi_{K^*}^{\perp}(u,\mu)$ for $\Gamma = \gamma_\perp$) with their corresponding decay constants $f_{K^*}^{||}$ and $f_{K^*}^{\perp}(\mu)$, respectively, are required. With the above LCDAs, one has [4]

$$\begin{aligned}
&\langle K^* \ell^+ \ell^- | C_1^B O_1^B | B \rangle \\
&= -\frac{F(\mu) m_B^{3/2}}{4} (1 - \hat{s}) (g_\perp^{\mu\nu} - i\epsilon_\perp^{\mu\nu}) \\
&\times \varepsilon_{\perp\nu}^* \bar{\ell}\gamma_\mu \ell \; \phi_+^B \otimes f_{K_\perp^*} \phi_{K_\perp^*} \otimes \mathcal{J}_\perp \otimes C_1^B \; ,
\end{aligned} \quad (5)$$

$$\begin{aligned}
&\langle K^* \ell^+ \ell^- | C_2^B O_2^B | B \rangle = -\frac{F(\mu) m_B^{3/2}}{4} (1 - \hat{s}) \\
&\times \frac{n^\mu}{n \cdot v} \bar{\ell}\gamma_\mu \ell \phi_+^B \otimes f_{K_{||}^*} \phi_{K_{||}^*} \otimes \mathcal{J}_{||} \otimes C_2^B \; ,
\end{aligned} \quad (6)$$

where $\otimes$ represent convolution in the appropriate variables, $F(\mu)$ is related to the B meson decay constant f_B up to higher orders in $1/m_b$, and the jet functions $\mathcal{J}_i$ arise from the SCET$_I$ → SCET$_{II}$ matching, with $\mathcal{J}_1 = \mathcal{J}_3 \equiv \mathcal{J}_\perp$ and $\mathcal{J}_2 = \mathcal{J}_4 \equiv \mathcal{J}_{||}$. The matrix element of $C_3^B O_3^B$ ($C_4^B O_4^B$) can be obtained by replacing the lepton current $\bar{\ell}\gamma_\mu \ell$ on the right hand side of the above equations by $\bar{\ell}\gamma_\mu\gamma_5 \ell$ and also replacing $C_1^B \to C_3^B$ ($C_2^B \to C_4^B$).

Finally, the C-type SCET_I current is matched onto the SCET_{II} operator O_C, with its Wilson coefficient defined in the momentum space, $D^C(\omega, u, \hat{s}, \mu)$, and we define an auxiliary function $D^C \equiv \widehat{D}^C/(\omega - q^2/m_b - i\epsilon)$. With this, the matrix element of O^C is obtained in SCET, with the result [4]

$$\langle K^* \ell^+ \ell^- | D^C O^C | B \rangle = -\frac{F(\mu) m_B^{3/2}}{4} (1 - \hat{s}) \times \frac{\bar{n}^\mu}{\bar{n} \cdot v} \bar{\ell} \gamma_\mu \ell \frac{\omega \phi_-^B}{\omega - q^2/m_b - i\epsilon} \otimes f_{K^*_{||}} \phi_{K^*_{||}} \otimes \widehat{D}^C \ . \tag{7}$$

Since $\phi_-^B(\omega)$ does not vanish as ω approaches zero, the integral $\int d\omega\ \phi_-^B(\omega)/(\omega - q^2/m_b)$ would be divergent if $q^2 \to 0$. This endpoint singularity will violate the SCET_{II} factorization, and we should restrict the kinematic region so that the invariant mass of the lepton pair is not too small, say $q^2 \geq 1\ \mathrm{GeV}^2$.

2.3. *Resummation of logarithms in SCET*

The two-step matching procedure QCD $\to$ $\mathrm{SCET}_I \to \mathrm{SCET}_{II}$ introduces two matching scales, $\mu_h \sim m_b$ at which QCD is matched onto SCET_I, and $\mu_l \sim \sqrt{m_b \Lambda_h}$ at which SCET_I is matched onto SCET_{II} (Λ_h represents a typical hadronic scale). The large logarithms due to different scales are resummed using the renormalization-group equations (RGE) of SCET_I to evolve from μ_h to μ_l.

For the A-type SCET currents, only the scale μ_h is involved. For the B-type currents, the RGE of SCET_I can be obtained by calculating the anomalous dimensions of the relevant SCET operators [11], and the matching coefficients at any scale μ can be obtained by an evolution from the matching scale μ_h. The resulting evolution equation has been solved numerically [4].

Finally, for the C-type SCET current J^C, its anomalous dimension just equals the sum of the anomalous dimensions of the K^* meson LCDA ϕ_{K^*} and the B meson LCDA ϕ_-^B. As the evolution equation of ϕ_-^B is still unknown, the perturbative logarithms for the J^C current are not resummed. Numerically the contribution from the J^C current to the decay amplitude in $B \to K^* \ell^+ \ell^-$ is small. Furthermore, the J^C current is irrelevant for the FB asymmetry of the charged leptons.

3. Dilepton invariant mass and FB asymmetry

The dilepton invariant mass spectrum and the FB asymmetry in $B \to K^* \ell^+ \ell^-$ have the following expressions in SCET:

$$\frac{dBr}{dq^2} = \tau_B \frac{G_F^2 |V_{ts}^* V_{tb}|^2}{96\pi^3} \left(\frac{\alpha_{em}}{4\pi}\right)^2 m_B^3 |\lambda_{K^*}| \times (1 - \frac{q^2}{m_B^2})^2 \mathcal{N}(q^2, \zeta_\perp^2, \zeta_{||}^2) \ ,$$

$$\frac{dA_{FB}}{dq^2} = \frac{-6(q^2/m_B^2)\zeta_\perp^2 Re(\mathcal{C}_9^\perp)\mathcal{C}_{10}^\perp}{\mathcal{N}(q^2, \zeta_\perp^2, \zeta_{||}^2)} \ . \tag{8}$$

where the function $\mathcal{N}(q^2, \zeta_\perp^2, \zeta_{||}^2)$ is defined as

$$\mathcal{N}(q^2, \zeta_\perp^2, \zeta_{||}^2) \equiv 4\frac{q^2}{m_B^2} \times \zeta_\perp^2 (|\mathcal{C}_9^\perp|^2 + (\mathcal{C}_{10}^\perp)^2) + \zeta_{||}^2 (|\mathcal{C}_9^{||}|^2 + (\mathcal{C}_{10}^{||})^2) \tag{9}$$

The expressions for the "effective" Wilson coefficients $\mathcal{C}_9^{\perp,||}$ and $\mathcal{C}_{10}^{\perp,||}$ in SCET, valid at leading power in $1/m_b$ and to all orders in α_s, can be seen in our paper [4].

3.1. *Numerical results*

We use the radiative $B \to K^* \gamma$ decay rate, which has been measured quite precisely, to normalize the soft form factor at $q^2 = 0$, obtaining $\zeta_\perp(0) = 0.32 \pm 0.02$. The longitudinal soft form factor $\zeta_{||}(q^2)$ is obtained from the full QCD form factor $A_0^{B \to K^*}(q^2)$, estimated using the LCSRs [12], yielding $\zeta_{||}(0) = 0.40 \pm 0.05$. For both the soft form factors, we assume that their q^2-dependence can be reliably obtained from the LCSRs [12]. The rest of the input parameters and the values for the Wilson coefficients can be seen in our paper [4]. We obtain

$$\int_{1\ \mathrm{GeV}^2}^{7\ \mathrm{GeV}^2} dq^2 \frac{dBr(B^+ \to K^{*+}\ell^+\ell^-)}{dq^2} =$$
$$(2.92^{+0.57}_{-0.50}|_{\zeta_{||}}\ {}^{+0.30}_{-0.28}|_{\mathrm{CKM}}\ {}^{+0.18}_{-0.20}) \times 10^{-7}\ , \quad (10)$$

making explicit the uncertainties from the soft form factor $\zeta_{||}$ and the CKM factor $|V_{ts}^* V_{tb}|$. The last error reflects the uncertainty due to the variation of the other input parameters and the residual scale dependence. For B^0 decay, the branching ratio is about 7% lower due to the lifetime difference (ignoring the small isospin-violating corrections from the matrix elements). One of the Belle observations [8] of our interest is

$$\int_{4\ \mathrm{GeV}^2}^{8\ \mathrm{GeV}^2} dq^2 \frac{dBr(B \to K^{*}\ell^+\ell^-)}{dq^2} =$$
$$(4.8^{+1.4}_{-1.2}|_{\mathrm{stat.}} \pm 0.3|_{\mathrm{syst.}} \pm 0.3|_{\mathrm{model}}) \times 10^{-7}, \quad (11)$$

for which we predict [4] $(1.94^{+0.44}_{-0.40}) \times 10^{-7}$, which is smaller than the published Belle data by a factor of about 2.5. However, BaBar collaboration measures the total branching ratio of $B \to K^*\ell^+\ell^-$ to be [7] $(7.8^{+1.9}_{-1.7} \pm 1.2) \times 10^{-7}$, which is about a factor 2 smaller than the Belle measurement of the same [8], $(16.5^{+2.3}_{-2.2} \pm 0.9 \pm 0.4) \times 10^{-7}$. Clearly, more data is required to test the theory precisely.

The zero of the FB asymmetry is determined by $Re(\mathcal{C}_9^{\perp}) = 0$. Including the order α_s corrections, our analysis estimates the zero-point of the FB asymmetry to be [4]

$$q_0^2 = (4.07^{+0.16}_{-0.13})\ \mathrm{GeV}^2\ , \qquad (12)$$

of which the scale-related uncertainty is $\Delta(q_0^2)_{\mathrm{scale}} = ^{+0.08}_{-0.05}\ \mathrm{GeV}^2$ for the range $m_b/2 \le \mu_h \le 2m_b$ together with the jet function scale $\mu_l = \sqrt{\mu_h \times 0.5\ \mathrm{GeV}}$. This is to be compared with the result given in Eq. (74) of Beneke et al. [5], also obtained in the absence of $1/m_b$ corrections: $q_0^2 = (4.39^{+0.38}_{-0.35})\ \mathrm{GeV}^2$. Of this the largest single uncertainty (about $\pm 0.25\ \mathrm{GeV}^2$) is attributed to the scale dependence. The difference in the estimates of the scale dependence of q_0^2 here and by Beneke et al. [5] is mainly due to the incorporation of the SCET logarithmic resummation [4] and to a lesser extent to the different (scheme-dependent) definitions of the effective form factors for the SCET currents. Power corrections in $1/m_b$ are probably comparable to the $O(\alpha_s)$ corrections [5]; it remains to be seen how a model-independent calculation of the same effect the numerical value of q_0^2.

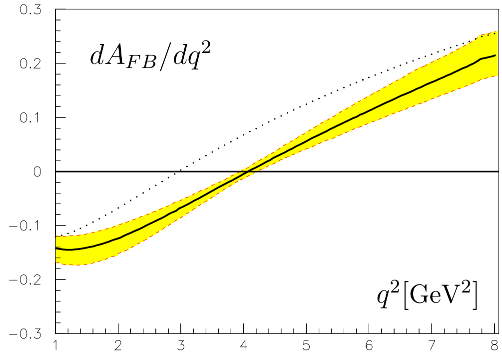

Figure 1. The differential FB asymmetry $dA_{FB}(B \to K^*\ell^+\ell^-)/dq^2$. Solid line corresponds to the input parameters taken at their central values, while the gray band reflects the uncertainties from input parameters and scale dependence. The dotted line represents the LO predictions (from Ref. 4 .)

Bibliography

1. Th. Feldmann (these proceedings).
2. T. Becher, R. J. Hill and M. Neubert, Phys. Rev. D **72**, 094017 (2005).
3. J.g. Chay and C. Kim, Phys. Rev. D **68**, 034013 (2003).
4. A. Ali, G. Kramer and G. -h. Zhu, Euro. Phys. J. C (in press) [hep-ph/0601034]. We refer to this paper for further references.
5. M. Beneke, Th. Feldmann and D. Seidel, Nucl. Phys. B **612**, 25 (2001); Eur. Phys. J. C **41**, 173 (2005).
6. M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999); Nucl. Phys. B **591**, 313 (2000).
7. B. Aubert *et al.* [BaBar], hep-ex/0507005.
8. K. Abe *et al.* [Belle], hep-ex/0410006.
9. C. W. Bauer, D. Pirjol and I. W. Stewart, Phys. Rev. D **67**, 071502 (2003).

10. B. Lange and M. Neubert, Nucl. Phys. B **690**, 249 (2004) [Erratum-ibid B **723**, 201 (2005)].
11. R.J. Hill, T. Becher, S.J. Lee and M. Neubert, JHEP **0407**, 081 (2004).
12. P. Ball and R. Zwicky, Phys. Rev. D **71**, 014029 (2005).