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Gauge Coupling Unification in a 6D $SO(10)$ Orbifold GUT

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Abstract

We consider the gauge coupling running in a six-dimensional $SO(10)$ orbifold GUT model. The bulk gauge symmetry is broken down to the standard model gauge group with an extra $U(1)_X$ by orbifold boundary conditions and the extra $U(1)_X$ is further broken through the $U(1)_{B-L}$ breaking with bulk hyper multiplets. We obtain the corrections of Kaluza-Klein massive modes to the running of the gauge couplings and discuss their implication to the successful gauge coupling unification.

Grand Unified Theories(GUTs) have been revived recently in the models of extra dimensions which are compactified on orbifolds, the so called GUT orbifolds [1,2]. Thanks to orbifold boundary conditions in extra dimensions, a GUT gauge symmetry can be broken down to the Standard Model(SM) gauge group without the need of a GUT Higgs field in the large representation and the doublet-triplet splitting problem can be solved easily.

On an orbifold M/Γ with M a compact manifold and Γ a point group, there are fixed points which transform into themselves under Γ. When the orbifolding breaks the gauge symmetry, there are some of fixed points where the active gauge symmetry is reduced. Although the non-universal gauge kinetic terms localized at the fixed points can be introduced at tree level and generated even by loop corrections [3–5], those effects may be ignored by making the strong coupling assumption at the GUT scale with a large volume of extra dimensions [6]. Thus, due to contributions coming from Kaluza-Klein(KK) massive modes, the GUT orbifolds can provide a minimal setup to predict the QCD coupling for a successful gauge coupling unification.

In this paper, we consider the running of the gauge couplings in the six-dimensional $SO(10)$ orbifold GUT model proposed in Ref. [7]. This is the minimal setup to break $SO(10)$ down to the SM gauge group up to a $U(1)$ factor only by orbifold boundary conditions without obtaining massless modes from the extra component of gauge bosons. We compute the threshold corrections due to KK massive modes to the gauge coupling running for a number of hyper multiplets with arbitrary parities. By taking the 5D limit where the bulk gauge group becomes the Pati-Salam $SU(4)\times SU(2)_L\times SU(2)_R$, we show that the shape dependent term of the KK threshold corrections gives rise to the 5D power-like threshold corrections with non-universal coefficient. In this paper, focusing on the case that the logarithmic threshold corrections are important, we discuss about the possibility of having a large volume of extra dimensions compatible with the success of the gauge coupling unification in specific realizations of the MSSM.

In our case, after the orbifolding, on top of the SM gauge group, there is an extra $U(1)_X$ gauge symmetry which has to be broken by a usual Higgs breaking of the $U(1)_{B-L}$ [8, 9]. In so doing, **16** Higgs multiplets are introduced in the bulk, so one ends up with extra color triplets as zero modes. Although the extra color triplets can get masses of order the $B-L$ breaking scale M_{B-L} at the fixed points, they could give a large threshold correction to the gauge couplings. We show that the KK threshold corrections can come with opposite sign to the threshold corrections of the color triplets. Thus, even if M_{B-L} is much smaller than the GUT scale, we can get the successful gauge coupling unification due to the cancellation between the large threshold corrections. In this case, the volume of extra dimensions can be large enough for satisfying the strong coupling assumption. We take some specific examples of embedding hyper multiplets to show explicitly that this is the case for M_{B-L} being smaller than the compactification scale. There are an extensive list of references [10] where related discussions on the gauge coupling unification have been done mainly in the context of a 5D $SO(10)$ orbifold GUT.

Two extra dimensions are compactified on a torus and they are identified by a Z_2 reflection symmetry to make up a T^2/Z_2 orbifold. For the extra coordinates $z=x^5+ix^6$, there are double periodicities in extra dimensions such as $z\sim z+2\pi R_5\sim z+2i\pi R_6$. Due to the orbifold action, there are four fixed points or branes, $z_0 = 0$, $z_1 = \pi R_5$, $z_2 = i\pi R_6$ and $z_3 = \pi R_5 + i\pi R_6$.

A bulk vector multiplet is composed of a vector multiplet V and an adjoint chiral multiplet Σ in 4D $\mathcal{N} = 1$ language. In order to break the bulk gauge symmetry down to the SM gauge group, we introduce a nontrivial boundary condition at each fixed point for a bulk vector multiplet by the parity matrices [7],

$$\begin{aligned} P_i V(-z + z_i) P_i^{-1} &= V(z + z_i), & (1)\\ P_i \Sigma(-z + z_i) P_i^{-1} &= -\Sigma(z + z_i), \quad i = 0, 1, 2, 3, & (2) \end{aligned}$$

where

$$\begin{aligned} P_0 &= I_{10\times 10}, & (3)\\ P_1 &= \mathrm{diag}(-1, -1, -1, 1, 1) \times \sigma^0, & (4)\\ P_2 &= \mathrm{diag}(1, 1, 1, 1, 1) \times \sigma^2, & (5) \end{aligned}$$

and $P_3 = P_1 P_2$ from the consistency condition on the orbifold. Then, the parity operations P_1, P_2 break $SO(10)$ down to its maximal subgroups, Pati-Salam group $SU(4) \times SU(2)_L \times SU(2)_R$ and Georgi-Glashow group $SU(5) \times U(1)_X$, respectively. The parity operation P_3 also breaks $SO(10)$ down to flipped $SU(5)$ but it is not an independent breaking. Thus, the intersection of two maximal surviving subgroups leads to $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ as the remaining gauge group. This can be seen from the gauge bosons with positive parities: $\mathbf{45}$ is decomposed into $(\mathbf{15}, \mathbf{1}, \mathbf{1})_+ + (\mathbf{6}, \mathbf{2}, \mathbf{2})_- + (\mathbf{1}, \mathbf{3}, \mathbf{1})_+ + (\mathbf{1}, \mathbf{1}, \mathbf{3})_+$ under P_1 (where $\pm$ indicate the parities) and $\mathbf{24}_{\mathbf{0},+} + \mathbf{10}_{-\mathbf{4},-} + \overline{\mathbf{10}}_{\mathbf{4},-} + \mathbf{1}_{\mathbf{0},+}$ under P_2. Then, finally, the extra $U(1)_X$ or $U(1)_{B-L}$ has to be broken further by the VEV of bulk or brane Higgs fields.

A bulk hyper multiplet is composed of two chiral multiplets with opposite charges (H, H') and it satisfies the orbifold boundary conditions

$$\begin{aligned} \eta_i P_i H(-z + z_i) &= H(z + z_i), & (6)\\ \eta_i P_i H'(-z + z_i) &= -H'(z + z_i), \quad i = 0, 1, 2, 3, & (7) \end{aligned}$$

with $\eta_i^2 = 1$. Here $\eta_0 = 1$ and $\eta_3 = \eta_1\eta_2$, independent of the representation of the hyper multiplet. We consider a set of hyper multiplets, N_{10} $\mathbf{10}$'s and N_{16} $\mathbf{16}$'s satisfying $N_{10} = 2 + N_{16}$ for no irreducible anomalies [11,12]. We also note that both N_{10} and N_{16} have to be even for the absence of localized anomalies unless there are split multiplets at the fixed points [12]. $\mathbf{10} = (H, G, H^c, G^c)$ is decomposed into $(\mathbf{6}, \mathbf{1}, \mathbf{1})_- + (\mathbf{1}, \mathbf{2}, \mathbf{2})_+$ under P_1 and $\mathbf{5}_{-\mathbf{2},-} + \bar{\mathbf{5}}_{\mathbf{2},+}$ under P_2. On the other hand, $\mathbf{16} = (Q, L, U, E, D^c, N^c)$ is decomposed into $(\mathbf{4}, \mathbf{2}, \mathbf{1})_+ + (\bar{\mathbf{4}}, \mathbf{2}, \mathbf{1})_-$ under P_1 and $\mathbf{10}_{\mathbf{1},-} + \bar{\mathbf{5}}_{-\mathbf{3},+} + \mathbf{1}_{\mathbf{5},+}$ under P_2.

In a 6D non-Abelian gauge theory on orbifolds, where there is no orbifold breaking of the gauge symmetry, the one-loop effective action for the gauge field has been obtained [4]. The analysis has been extended to 6D GUTs with the orbifold breaking of GUT symmetry [5]. By using the general result in the latter analysis, we study the running of the 4D effective gauge couplings of the SM gauge group much below the compactification scale in 6D $SO(10)$ GUTs. After including all possible contributions, the running of the low-energy gauge couplings are governed in dimensional regularization by

$$\begin{aligned}\frac{4\pi}{g^2_{\mathrm{eff},a}(k^2)} &= \frac{4\pi}{g_u^2}+\frac{1}{4\pi}\tilde{b}_a\ln\frac{M_*^2}{M_{B-L}^2}+\frac{1}{4\pi}b'_a\ln\frac{M_{B-L}^2}{k^2}\\ &-\frac{1}{4\pi}\Big(\sum_{\pm\pm}b_a^{\pm\pm}L_{\pm\pm}+\sum_{\pm\mp}b_a^{\pm\mp}L_{\pm\mp}\Big)+\frac{1}{2\pi}(\Delta_a^l+\Delta_a^{B-L}) \qquad (8)\end{aligned}$$

where M_* is the 6D fundamental scale, M_{B-L} is the $B-L$ breaking scale, g_u is the universal renormalized gauge coupling[1] and Δ_a^l are corrections due to renormalized gauge couplings localized at the Pati-Salam and flipped $SU(5)$ fixed points. Δ_a^{B-L} stands for the effect due to the modification of the KK masses due to the $B-L$ breaking brane-localized mass terms. Note further that $b'_a=(33/5,1,-3)$ is the beta function in the MSSM as given below the $B-L$ breaking scale while $\tilde{b}_a$ is the beta function above the $B-L$ breaking scale. More importantly, $L_{\pm\pm}(L_{\pm\mp})$ are the logarithmic KK threshold corrections with the corresponding beta functions $b_a^{\pm\pm}(b_a^{\pm\mp})$. These are a purely bulk contribution [5].

Here we present the details of the beta functions in eq. (8). We split $\tilde{b}_a$ into $\tilde{b}_a = b_a - c_a + b_a^m$. Here b_a is the contribution from zero modes which are distributed both in the bulk and at the fixed points [4, 5]. It is given by

$$b_a = b_a^V + b_a^{10} + b_a^{16} \qquad (9)$$

with

$$\begin{aligned} b_a^V &= (0,-6,-9), &(10)\\ b_a^{10} &= \frac{1}{4}N_{10}(1,1,1)+\frac{1}{4}\sum_{10}\eta_1^{10}(\frac{1}{5},1,-1), &(11)\\ b_a^{16} &= \frac{1}{4}(2N_{16}-\sum_{16}\eta_2^{16})(1,1,1)+\frac{1}{4}\sum_{16}\eta_1^{16}(-\frac{6}{5},2,0)\\ &+\frac{1}{4}\sum_{16}\eta_1^{16}\eta_2^{16}(\frac{7}{5},-1,-1) &(12)\end{aligned}$$

where η_i^{10} and η_i^{16} with $(\eta_i^{10})^2=(\eta_i^{16})^2=1(i=1,2)$ are the parities for **10** and **16**, respectively. c_a is the beta function for vector-like massless modes which would get tree-level brane masses of order the GUT scale. Moreover, b_a^m is the beta function for the brane-localized fields. Depending on the parities, we get the different logarithms for the KK threshold corrections as

$$L_{++} = \ln\Big[4e^{-2}|\eta(iu)|^4 uVM_*^2\Big], \qquad (13)$$

[1] Although there are also power-like threshold corrections in the cutoff regularization [4, 13], they don't contribute to the differential running of gauge couplings. Nevertheless, the power-like contributions may have the net effect of placing an upper limit on the possible volume of the extra dimensions [14].

$$L_{-+} = \ln\Big[\frac{e^{-2}}{4}\Big|\vartheta_1(\frac{1}{2}|iu)\Big|^4 uVM_*^2\Big], \tag{14}$$
$$L_{+-} = \ln\Big[\frac{e^{-2}}{4}\Big|\vartheta_1(-\frac{1}{2}iu|iu)\Big|^4 uVM_*^2\Big], \tag{15}$$
$$L_{--} = \ln\Big[\frac{e^{-2}}{4}\Big|\vartheta_1(\frac{1}{2}-\frac{1}{2}iu|iu)\Big|^4 uVM_*^2\Big] \tag{16}$$

where $u = R_6/R_5$, $V = 4\pi^2 R_5 R_6$, η and ϑ_1 are the Dedekind eta function and the Jacobi theta function, respectively. The beta function for KK massive modes is

$$b_a^{++} = \frac{1}{4}(-8 + N_{10} + 2N_{16})(1,1,1), \tag{17}$$
$$b_a^{-+} = \frac{1}{4}(\frac{12}{5},4,0) + \frac{1}{4}\sum_{10}\eta_1^{10}(\frac{1}{5},1,-1) + \frac{1}{4}\sum_{16}\eta_1^{16}(-\frac{6}{5},2,0), \tag{18}$$
$$b_a^{+-} = \frac{1}{4}(2 + \sum_{16}\eta_2^{16})(-1,-1,-1), \tag{19}$$
$$b_a^{--} = \frac{1}{4}(\frac{38}{5},-2,-2) + \frac{1}{4}\sum_{16}\eta_1^{16}\eta_2^{16}(\frac{7}{5},-1,-1). \tag{20}$$

Compared to eq. (9), we obtain the relation between beta functions as

$$b_a = (0,-4,-6) + b_a^{++} + b_a^{-+} + b_a^{+-} + b_a^{--} \tag{21}$$

where the first term is due to the difference between the beta functions of $\mathcal{N}=1$ vector multiplets and $\mathcal{N}=2$ vector multiplets for the SM gauge group. Consequently, from the beta functions (11), (12), (18) and (20), one can find that the part proportional to η_1^R or $\eta_1^R\eta_2^R$ is non-universal. So, because of the orbifold actions associated with Pati-Salam and flipped $SU(5)$ gauge groups, both massless and massive mode contributions can affect the differential running of the gauge couplings.

For a number of hyper multiplets with arbitrary parities, we assume that both vector-like particles (getting brane masses of order the GUT scale) and brane-localized particles fill GUT multiplets, i.e. c_a and b_a^m are universal. In this case, those particles do not affect the unification of the one-loop gauge couplings. Then, we get the general formula for the differential running of gauge couplings as

$$\frac{1}{g_3^2} - \frac{12}{7}\frac{1}{g_2^2} + \frac{5}{7}\frac{1}{g_1^2} = \frac{1}{8\pi^2}\Big(\tilde{b}\ln\frac{M_*}{M_{B-L}} - \frac{1}{2}b^{-+}L_{-+} - \frac{1}{2}b^{--}L_{--} + \tilde{\Delta}^l + \tilde{\Delta}^{B-L}\Big) \tag{22}$$

where

$$\tilde{b} = \frac{9}{7} - \frac{9}{14}\sum_{10}\eta_1^{10} - \frac{15}{14}\sum_{16}\eta_1^{16} + \frac{3}{7}\sum_{16}\eta_1^{16}\eta_2^{16}, \tag{23}$$
$$b^{-+} = -\frac{9}{7} - \frac{9}{14}\sum_{10}\eta_1^{10} - \frac{15}{14}\sum_{16}\eta_1^{16}, \tag{24}$$
$$b^{--} = \frac{12}{7} + \frac{3}{7}\sum_{16}\eta_1^{16}\eta_2^{16}. \tag{25}$$

Thus, we find a general relation between coefficients as

$$\tilde{b} = \frac{6}{7} + b^{-+} + b^{--}. \tag{26}$$

Then, from eq. (22) with the relation (26), we find the deviation from the 4D SGUT prediction of the QCD coupling at M_Z, i.e. $\Delta\alpha_s \equiv \alpha_s^{KK} - \alpha_s^{SGUT,0}$ as

$$\begin{aligned}\Delta\alpha_s(M_Z) \approx & -\frac{1}{2\pi}\alpha_s^2(M_Z)\Big\{\tilde{b}\ln\frac{M_*}{M_{B-L}} - (\tilde{b}-\frac{6}{7})\ln(M_*\sqrt{V}) \\ & -\frac{1}{2}b^{-+}\ln\Big[\frac{e^{-2}}{4}\Big|\vartheta_1(\frac{1}{2}|iu)\Big|^4 u\Big] \\ & -\frac{1}{2}(\tilde{b}-\frac{6}{7}-b^{-+})\ln\Big[\frac{e^{-2}}{4}\Big|\vartheta_1(\frac{1}{2}-\frac{1}{2}iu|iu)\Big|^4 u\Big] \\ & +\tilde{\Delta}^l + \tilde{\Delta}^{B-L}\Big\}. \end{aligned} \tag{27}$$

The first term corresponds to the contribution due to the extra particles above the $B-L$ scale. The second term is the volume dependent correction due to the KK massive modes while the third part containing the theta functions is the shape dependent correction. The last two terms $\tilde{\Delta}^l$ and $\tilde{\Delta}^{B-L}$ are the effect of the brane-localized gauge couplings and the $B-L$ breaking brane-localized mass terms, respectively.

Suppose to take the 5D limit with $u = R_6/R_5 \gg 1$, in which case the bulk gauge group becomes the Pati-Salam and there remain only two fixed points with the Pati-Salam group and the SM gauge group enlarged with a $U(1)$ factor. Then, since $|\vartheta_1(z|iu)| \sim 2e^{-\pi u/4}|\sin(\pi z)|$ for $u \gg 1$, the shape dependent terms could give a significant effect on the gauge coupling unification by the non-universal power-like threshold corrections proportional to u as in the case with the bulk VEV of extra components of gauge bosons for a simple gauge group [18]. In this case, the effective 5D gauge coupling($1/g_5^2 = 1/(g_4^2 R_6)$) gets a power-like threshold correction like $u/R_6 \sim 1/R_5$ which is set by the mass scale of heavy gauge bosons belonging to $SO(10)/SU(4)\times SU(2)_L \times SU(2)_R$.

On the other hand, when $u \sim 1$, the shape dependent term is subdominant compared to the other logarithmic terms. As can be shown explicitly in the specific models, the last two terms can be also ignored by making a strong coupling assumption and choosing the $B-L$ breaking scale to be smaller than the compactification, respectively. Then, the first two logarithms become a dominant contribution. For $\tilde{b}(\tilde{b}-\frac{6}{7}) > 0$, we can see that the individual logarithm can be large, being compatible with the gauge coupling unification due to a cancellation. We will focus on this possibility later on. The case with the anisotropic compactification $u \gg 1$ will be discussed in detail elsewhere in Ref. [5].

Now we are in a position to apply our general formula (27) to particular cases for the unification of the SM gauge couplings. To this purpose, we consider some known $SO(10)$ models of embedding the MSSM into the extra dimensions. In the minimal model(: model I) [8] that contains Higgs fields in the bulk for breaking $U(1)_{B-L}$ and the SM gauge group[2],

[2] In order to cancel the bulk anomalies due to one **45**, we need to add in the bulk two **10**'s. So, it is

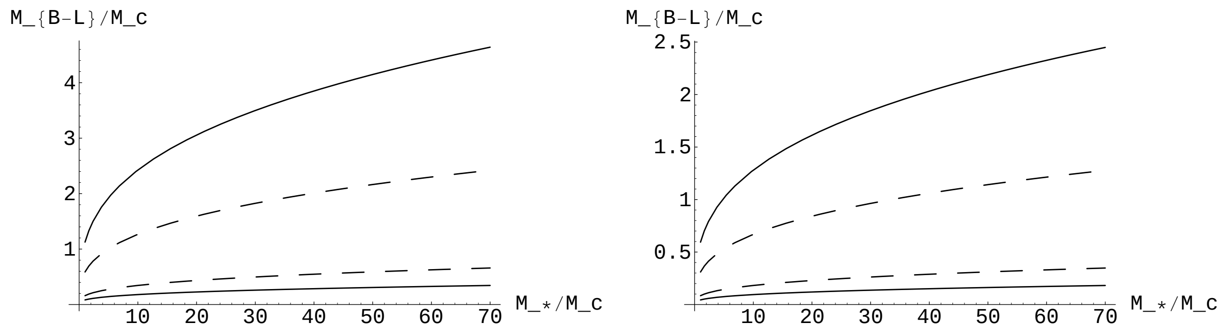

Figure 1: The 1σ and 2σ band of $\Delta\alpha_s$: the model I on the left and the model II on the right for $u = R_6/R_5 \sim 1$. The dashed lines and the thin lines denote 1σ and 2σ bounds of the experimental data, respectively.

there are 4 **10**'s with parities (η_1, η_2) such as $H_1 = (+,+)$, $H_2 = (+,-)$, $H_3 = (-,+)$ and $H_4 = (-,-)$, and one pair of **16** and $\overline{\mathbf{16}}$ with parities $\Phi = (-,+)$, $\Phi^c = (-,+)$. Then, the resulting massless modes are two doublet Higgs fields H_1^c and H_2 from H_1 and H_2, and $G_3^c, G_4, (D^c, N^c), (D, N)$ from H_3, H_4, Φ and Φ^c in order. Moreover, each family of quarks and leptons is introduced as a **16** being localized at the fixed point without $SO(10)$ gauge symmetry. After the $B-L$ breaking via the bulk **16**'s with $\langle N\rangle = \langle N^c\rangle \neq 0$, neutrino masses are generated at the fixed points by a usual see-saw mechanism. Moreover, $G_3^c, G_4, (D^c, N^c), (D, N)$ can acquire masses of order the $B-L$ breaking scale by the brane superpotential [8, 9] $W = \lambda N D G_3^c + \lambda' N^c D^c G_4$ for $\langle N\rangle = \langle N^c\rangle \neq 0$. In this case, since $\sum_{10} \eta_1^{10} = 0$, $\sum_{16} \eta_1^{16} = \sum_{16} \eta_1^{16}\eta_2^{16} = -2$, we get the values $\tilde{b} = \frac{18}{7}, b^{-+} = b^{--} = \frac{6}{7}$ in eq. (27).

We consider another 6D $SO(10)$ GUT model where the realistic flavor structure of the SM was discussed(: model II) [9]. In this case, on top of the minimal model, there are more hyper multiplets: 2 **10**'s such as $H_5 = (-,+)$ and $H_6 = (-,-)$, and one pair of **16** and $\overline{\mathbf{16}}$ with $\phi = (+,+)$ and $\phi^c = (+,+)$. Then, there are additional zero modes G_5^c, G_6, L, L^c from H_5, H_6, ϕ and ϕ^c in order. They are assumed to get brane masses of order the GUT scale. Thus, the running of gauge couplings between the GUT scale and the $B-L$ breaking scale is the same as in the minimal model. In this case, since $\sum_{10} \eta_1^{10} = -2$, $\sum_{16} \eta_1^{16} = \sum_{16} \eta_1^{16}\eta_2^{16} = 0$, we get the values $\tilde{b} = \frac{18}{7}, b^{-+} = 0$ and $b^{--} = \frac{12}{7}$ in eq. (27).

Consequently, in both cases, we can see that logarithmic contributions of zero modes and those of KK massive modes appear with opposite signs so that there is a possibility of having the large volume of extra dimensions consistent with perturbativity and gauge coupling unification. From the data of the electroweak gauge couplings at the scale of the

necessary to have two Higgs doublets of the **10**'s in the bulk unlike in 5D case [2]. Moreover, in order to break the $U(1)_{B-L}$, we need one **16** in the bulk. However, for cancellation of localized and bulk anomalies, one needs one $\overline{\mathbf{16}}$ and two more **10**'s.

Z mass, one can compare the predicted value of the QCD coupling in a theory to a measure one [15] $\alpha_s^{exp} = 0.1176 \pm 0.0020$. In the 4D supersymmetric GUTs, the prediction without threshold corrections for the QCD coupling is $\alpha_s^{SGUT,0} = 0.130 \pm 0.004$. Thus, in this case, there is a discrepancy from the experimental data as $\delta\alpha_s = \alpha_s^{SGUT,0} - \alpha_s^{exp} = 0.0124 \pm 0.0045$. For the models that we considered above, ignoring the unknown brane-localized gauge couplings and the $B-L$ breaking effect, we depict in Fig. 1 the parameter space of (M_c, M_{B-L}) with $M_c \equiv 1/\sqrt{V}$ and $u \sim 1$, being compatible with the experimental data. Taking $M_*/M_c \sim 63/\sqrt{C} \sim 22$ with the group theory factor $C = 8$ for strong coupling assumption at the 6D fundamental scale, the correction due to the brane-localized gauge couplings is $\tilde{\Delta}^l = \mathcal{O}(1)$ so it is negligible to the KK threshold corrections which is of order $\ln(M_*/M_c) \sim 3$. For $M_{B-L} \ll M_c$, it has been shown [17] that the KK massive modes of the color triplets are modified to $m^2_{n_5,n_6} \approx (n_5/2R_5)^2 + (n_6/2R_6)^2 + cM^2_{B-L}$ where c is of order unity independent of the KK level for $R_5 \neq R_6$. In this case, the $B-L$ breaking effect to the differential running (22) is estimated as $\tilde{\Delta}^{B-L} \sim M^2_{B-L}/M^2_c$. In the model I(II), for $M_*/M_c \sim 22$, M_{B-L}/M_c can be as small as 0.23(0.12) at the 2σ level so that the $B-L$ breaking can be suppressed compared to the KK threshold corrections. Apart from the two models, we can consider other possibilities of embedding the matter representations into extra dimensions, like in the field-theory limit of a successful string orbifold compactification [16] where there are two families at the fixed points and one family in the bulk. In view of the general formula (27), however, as far as an extra particle contributes to the running of the gauge couplings above the $B-L$ breaking scale, M_{B-L} tends to be close to M_c for the success of the gauge coupling unification, independent of the details of the model.

To conclude, we have obtained the KK massive mode corrections as a dominant contribution to the gauge coupling running in a six-dimensional $SO(10)$ orbifold GUT model. The shape dependent correction of the KK massive modes can be dominant in the anisotropic compactification of the extra dimensions. Compared to the 5D case, the 5D limit of our computation shows that the 5D power-like threshold corrections can be computed to be non-universal for the SM gauge couplings. Focusing on the isotropic compactification of the extra dimensions, we have shown that there is a generic cancellation between the dominant logarithmic corrections to the differential logarithmic running of the SM gauge couplings: one is the contribution of the extra particles above the $B-L$ scale and the other is the KK massive mode contribution. In the models that we considered, extra color triplets contribute to the running of the gauge couplings above the $B-L$ scale but the KK threshold corrections can be large enough to cancel the contribution of the extra color triplets for the large volume of extra dimensions. Therefore, the $B-L$ scale can be much smaller than the GUT scale.

Since the $B-L$ breaking scale tends to be close to or larger than the compactification scale as shown in the allowed parameter space of Fig. 1, it may be also important to see how much the modified KK massive modes of the color triplets due to the brane-localized mass terms can affect the running of the gauge couplings. On the other hand, one can look for a consistent model where the color triplets make up GUT multiplets together with some extra doublets, i.e. $\tilde{b} = 0$. Then, the $B-L$ breaking would not be relevant for the gauge coupling unification any more. In this case, the extra dimensions could be also large enough for the successful gauge coupling unification, independent of the details of the model with hyper multiplets. We leave the relevant issues in a future publication.

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