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$K \to \pi\pi$ amplitudes from lattice QCD with a light charm quark

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We compute the leading-order low-energy constants of the $\Delta S = 1$ effective weak Hamiltonian in the quenched approximation of QCD with up, down, strange, and charm quarks degenerate and light. They are extracted by comparing the predictions of finite volume chiral perturbation theory with lattice QCD computations of suitable correlation functions carried out with quark masses ranging from a few MeV up to half of the physical strange mass. We observe a $\Delta I = 1/2$ enhancement in this corner of the parameter space of the theory. Although matching with the experimental result is not observed for the $\Delta I = 1/2$ amplitude, our computation suggests large QCD contributions to the physical $\Delta I = 1/2$ rule in the GIM limit, and represents the first step to quantify the rôle of the charm quark-mass in $K \to \pi\pi$ amplitudes. The use of fermions with an exact chiral symmetry is an essential ingredient in our computation.

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INTRODUCTION

The decay of a neutral kaon into a pair of pions in a state with isospin I is described by the transition amplitudes

$$iA_I e^{i\delta_I} = \langle(\pi\pi)_I|H_{\rm w}|K^0\rangle, \qquad I = 0,\, 2, \tag{1}$$

where $H_{\rm w}$ is the $\Delta S = 1$ effective weak Hamiltonian and δ_I is the $\pi\pi$-scattering phase shift. The well-known experimental fact

$$|A_0/A_2| \sim 22 \tag{2}$$

is often called the $\Delta I = 1/2$ rule. Many decades after its experimental discovery, it is embarrassing that the origin of this enhancement is still not known. In the Standard Model (SM) a reliable perturbative computation of short-distance Quantum Chromodynamics (QCD) corrections [1–4], together with a naive order-of-magnitude estimate of long-distance contributions, would suggest comparable values for $|A_0|$ and $|A_2|$ [1, 2]. The bulk of the enhancement is thus expected to come from non-perturbative QCD contributions, which makes the $\Delta I = 1/2$ rule one of the rare cases where an interplay between strong and electro-weak interactions gives an opportunity for a refined test of non-perturbative strong dynamics.

Lattice QCD is the only known technique that allows us to attack the problem from first principles and possibly to reveal the origin of the enhancement [5, 6]. It would be interesting to understand whether it is the result of an accumulation of several effects, each giving a moderate contribution, or if it is driven by a dominant mechanism. Recently we proposed a theoretically well defined strategy to disentangle non-perturbative QCD contributions from the various sources [7], and in particular to reveal the rôle of the charm quark and its associated mass scale (whose relevance in this problem was pointed out in Refs. [8, 9]). The main idea is to compute the leading-order low-energy constants (LECs) of the CP-conserving $\Delta S = 1$ weak Hamiltonian of the chiral low-energy effective theory as a function of the charm quark mass. They can be extracted by comparing finite-volume chiral perturbation theory (ChPT) predictions for suitable two- and three-point correlation functions with the analogous ones computed in lattice QCD at small light-quark masses and momenta. The suggestion of using ChPT in connection with kaon amplitudes was pointed out long ago [10, 11]. It is only now that these ideas can be formulated and integrated in a well defined strategy [7], following significant conceptual advances in the discretization of fermions on the lattice as well as enormous gains in computer power. The main theoretical advance is the discovery of Ginsparg–Wilson (GW) regularizations [12–14], which preserve an exact chiral symmetry on the lattice at finite lattice spacings [15]. By using these fermions the problem of ultraviolet power divergences in the effective Hamiltonian $H_{\rm w}$ [16] is avoided in the case of an active charm [17], and quark masses as low as a few MeV can be simulated. Eventually the full $K \to \pi\pi$ amplitudes can be computed using finite-volume techniques [18, 19].

The aim of this letter is to report on a computation of the LECs of the CP-conserving $\Delta S = 1$ weak Hamiltonian with up, down, strange, and charm quarks degenerate and chiral (GIM limit), i.e. the implementation of the first step of the strategy proposed in Ref. [7]. We perform the first quenched lattice QCD computation of the relevant three-point functions with quark masses as light as a few MeV, which turns out to be essential for a robust extrapolation to the chiral limit. Our results reveal a clear hierarchy between the low-energy constants, which

in turn implies the presence of a $\Delta I = 1/2$ enhancement in this corner of the parameter space of (quenched) QCD.

Since we are looking for an order-of-magnitude effect, and since simulations with dynamical fermions are very expensive, it is appropriate for us to first perform the computation in quenched QCD. The latter is not a systematic approximation of the full theory[1]. However, when quenched results can be compared with experimental measurements, discrepancies of $\mathcal{O}(10\,\%)$ are found in most cases [21]. In the past there were several attempts to attack the problem by using quenched lattice QCD [22–27]. In particular, in Refs. [25, 26], a fermion action with an approximate chiral symmetry was used and, despite the fact that the charm was integrated out and therefore an ultraviolet power-divergent subtraction was needed, the authors observed a good statistical signal for the subtracted matrix elements in a range of quark masses of about half the physical strange quark-mass. Several computations of A_I which use models to quantify QCD non-perturbative contributions in these amplitudes can also be found in the literature (see Refs. [28, 29] and references therein).

THE $\Delta S = 1$ EFFECTIVE HAMILTONIAN

In the SU(4) degenerate case and with GW fermions, the CP-even $\Delta S = 1$ effective Hamiltonian is [1, 2, 7]

$$H_{\rm w} = \frac{g_w^2}{4M_W^2} V_{us}^* V_{ud} \Big\{ k_1^+ \mathcal{Q}_1^+ + k_1^- \mathcal{Q}_1^- \Big\}\,, \tag{3}$$

where

$$\begin{aligned} \mathcal{Q}_1^\pm &= \mathcal{Z}_{11}^\pm \Big\{ (\bar{s}\gamma_\mu P_- \tilde{u})(\bar{u}\gamma_\mu P_- \tilde{d}) \\ &\quad \pm (\bar{s}\gamma_\mu P_- \tilde{d})(\bar{u}\gamma_\mu P_- \tilde{u}) - [u \to c] \Big\}\,, \end{aligned} \tag{4}$$

and any further unexplained notation in the paper can be found in Ref. [7]. We are interested in the ratios of correlation functions

$$R^\pm(x_0, y_0) = \frac{C_1^\pm(x_0, y_0)}{C(x_0) C(y_0)}\,, \tag{5}$$

where

$$C(x_0) = \sum_{\vec{x}} \langle [J_0(x)]_{\alpha\beta} [J_0(0)]_{\beta\alpha} \rangle\,, \tag{6}$$

$$C_1^\pm(x_0, y_0) = \sum_{\vec{x},\vec{y}} \big\langle [J_0(x)]_{du} \, [\mathcal{Q}_1^\pm(0)] \, [J_0(y)]_{us} \big\rangle\,, \tag{7}$$

$[J_\mu]_{\alpha\beta} = \mathcal{Z}_J (\bar{\psi}_\alpha \gamma_\mu P_- \tilde{\psi}_\beta)$, and $\mathcal{Z}_J$ is the renormalization constant of the local left-handed current.

[1] On the other hand the ambiguity in the definition of the LECs pointed out by the Golterman and Pallante [20] is not present in the GIM limit.

In the chiral effective theory the corresponding effective Hamiltonian reads

$$\mathcal{H}_{\rm w} = \frac{g_w^2}{4M_W^2} V_{us}^* V_{ud} \Big\{ g_1^+ Q_1^+ + g_1^- Q_1^- \Big\}\,, \tag{8}$$

where, at leading order in momentum expansion,

$$\begin{aligned} Q_1^\pm &= \frac{F^4}{4} \Big\{ (U\partial_\mu U^\dagger)_{us} (U\partial_\mu U^\dagger)_{du} \\ &\quad \pm (U\partial_\mu U^\dagger)_{ds} (U\partial_\mu U^\dagger)_{uu} - [u \to c] \Big\}\,. \end{aligned} \tag{9}$$

The complete expressions at the next-to-leading order (NLO) can be found in [30, 31]. In the quenched approximation of QCD, an effective low-energy chiral theory is formally obtained if an additional expansion in $1/N_c$, where N_c is the number of colours, is carried out together with the usual one in quark masses and momenta [32, 33]. Here we adopt the pragmatic assumption that quenched ChPT describes the low-energy regime of quenched QCD in certain ranges of kinematical scales at fixed N_c. Correlation functions can be parametrized in terms of effective coupling constants, the latter being defined as the couplings that appear in the Lagrangian of the effective theory. For quark masses light enough to be in the ϵ-regime of quenched QCD [34–36], the ratios corresponding to Eq. (5) in NLO ChPT, in a volume $V = T \times L^3$ and at fixed topological charge ν, are [7, 37]

$$K_\nu^\pm(x_0, y_0) = 1 \pm \frac{2\,T}{F^2 L^3} \left\{ \beta_1 \left(\frac{L}{T}\right)^{3/2} - k_{00} \right\}, \tag{10}$$

where F is the pseudoscalar decay constant in the chiral limit, and the shape coefficients β_1 and k_{00} can be found in Ref. [7]. Remarkably, the r.h.s. of Eq. (10) is determined once F is known, and it turns out to be independent of ν and the quark mass. When the quark masses are heavier and reach the so-called p-regime of QCD, the corresponding ratios are given by [31]

$$K^\pm(x_0, y_0) = 1 \mp 3 \frac{M^2}{(4\pi F)^2} \log\left(\frac{M^2}{\Lambda_\pm^2}\right) \pm \mathcal{K}(x_0, y_0)\,, \tag{11}$$

where M is the pseudoscalar meson mass at LO in ChPT, and $\mathcal{K}(x_0, y_0)$ accounts for leading-order finite-volume effects and can be found in Ref. [31]. The LECs $g_1^\pm$ can be extracted by requiring that

$$g_1^\pm K^\pm(x_0, y_0) = k_1^\pm R^\pm(x_0, y_0) \tag{12}$$

for values of quark masses, volumes, x_0 and y_0, where quenched ChPT is expected to parametrize well the correlation functions.

LATTICE COMPUTATION

The numerical computation is performed by generating gauge configurations with the Wilson action and

am	aM_P	$R^{+,\,\mathrm{bare}}$	$R^{-,\,\mathrm{bare}}$	$(R^+ \cdot R^-)^{\mathrm{bare}}$
		ϵ-regime		
0.002	-	0.600(43)	2.42(13)	1.45(15)
0.003	-	0.603(41)	2.40(12)	1.44(14)
		p-regime		
0.020	0.1960(28)	0.654(40)	2.20(12)	1.44(12)
0.030	0.2302(25)	0.691(33)	1.93(9)	1.33(9)
0.040	0.2598(24)	0.723(31)	1.75(8)	1.26(8)
0.060	0.3110(24)	0.772(30)	1.51(7)	1.17(8)

TABLE I: Results for aM_P and $R^{\pm,\mathrm{bare}}$ as obtained from 746 and 197 gauge configurations in the ϵ and p regimes, respectively.

periodic boundary conditions by standard Monte Carlo techniques. The topological charge and the quark propagators are computed following Ref. [38]. The statistical variance of the estimates of correlation functions has been reduced by implementing a generalization of the low-mode averaging technique proposed in [39], which turns out to be essential to get a signal for the lighter quark masses. The lattice has a bare coupling constant $\beta \equiv 6/g_0^2 = 5.8485$, which corresponds to a lattice spacing $a \sim 0.12$ fm, and a volume of $Va^{-4} = 16^3 \times 32$. The list of simulated bare quark masses, together with the corresponding results for pion masses and unrenormalized ratios $R^{\pm,\,\mathrm{bare}} = \mathcal{Z}_J^2 R^\pm / \mathcal{Z}_{11}^\pm$, are reported in Table I. Further technical details will be provided in a forthcoming publication.

The values in Table I show that $R^{\pm,\,\mathrm{bare}}$ exhibit a pronounced mass dependence, which is more marked in $R^{-,\,\mathrm{bare}}$. We have explored several fit strategies, attempting to minimize the systematic uncertainties due to neglected higher orders in ChPT. The structure of Eqs. (10) and (11) indeed suggests that it is possible to cancel large NLO ChPT corrections by constructing suitable combinations of $R^{\pm,\,\mathrm{bare}}$. We observe that the product $g_1^+\, g_1^-$ is very robust with respect to the details of the fit strategy. The simplest way to extract this quantity is from a fit to the combination $(R^+R^-)^{\mathrm{bare}}$, where NLO ChPT corrections cancel in the limit $m \to 0$. We obtain

$$(g_1^+\, g_1^-)^{\mathrm{bare}} = 1.47(12)\,. \tag{13}$$

To extract $g_1^{+,\,\mathrm{bare}}$ and $g_1^{-,\,\mathrm{bare}}$ separately we then fit $R^{+,\,\mathrm{bare}}$ to NLO ChPT, taking the value of F from a fit to the two-point functions as in Ref. [39] and the bare Σ from Ref. [40]. Putting the result together with Eq. (13) we get

$$g_1^{+,\,\mathrm{bare}} = 0.63(4)(8)\,, \qquad g_1^{-,\,\mathrm{bare}} = 2.33(11)(30)\,, \tag{14}$$

where the first error is statistical and the second is an estimate of the systematic uncertainty from the spread of the central values obtained from fits to different quantities and/or mass intervals. The physical LECs are given

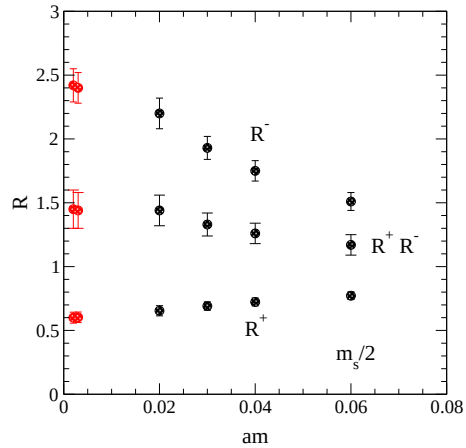

FIG. 1: Mass dependence of $R^{\pm,\mathrm{bare}}$ and $(R^+ \cdot R^-)^{\mathrm{bare}}$.

by

$$g_1^\pm = k_1^\pm \left[\frac{R^{\pm,\mathrm{RGI}}}{R^{\pm,\mathrm{bare}}}\right]_{\mathrm{ref}} g_1^{\pm,\,\mathrm{bare}}\,, \tag{15}$$

where $k_1^\pm$ are the renormalization group-invariant (RGI) Wilson coefficients [1–4, 7]. The RGI quantities

$$R_{\mathrm{ref}}^{\pm,\mathrm{RGI}} \equiv R^{\pm,\mathrm{RGI}}\Big|_{r_0^2 M_P^2 = r_0^2 M_K^2} \tag{16}$$

at the pseudoscalar mass $r_0^2 M_K^2 = 1.5736$ are taken from Refs. [41–44], and r_0 is a low-energy reference scale widely used in quenched QCD computations [45]. This procedure, analogous to the one proposed for the scalar density in Ref. [46], provides values of the LECs that are non-perturbatively renormalized, as explained in detail in Ref. [44].

PHYSICS DISCUSSION

By using the non-perturbative renormalization factors in Ref. [44]

$$\left[\frac{R^{+,\mathrm{RGI}}}{R^{+,\mathrm{bare}}}\right]_{\mathrm{ref}} = 1.15(12)\,, \qquad \left[\frac{R^{-,\mathrm{RGI}}}{R^{-,\mathrm{bare}}}\right]_{\mathrm{ref}} = 0.56(6), \tag{17}$$

and the perturbative values $k_1^+ = 0.708$ and $k_1^- = 1.978$ (see Ref. [7]), we obtain our final results

$$g_1^+ = 0.51(9)\,, \quad g_1^- = 2.6(5)\,, \quad g_1^+ g_1^- = 1.2(2)\,. \tag{18}$$

A solid estimate of discretization effects would require simulations at several lattice spacings, which is beyond the scope of this exploratory study. However, computations of $R^\pm$ at different lattice spacings and for masses close to $m_s/2$ [7, 47] indicate that discretization effects may be smaller than the errors quoted above. It is also interesting to note that quenched computations of various physical quantities carried out with Neuberger fermions

show small discretization effects at the lattice spacing of our simulations [48, 49].

The values of $g_1^{\pm}$ in Eq. (18) are the main results of this paper. They reveal a clear hierarchy between the low-energy constants, $g_1^- \gg g_1^+$, which implies the presence of a $\Delta I = 1/2$ enhancement in the GIM-limit of (quenched) QCD. The strong mass dependence of $R^{\pm,\,\mathrm{bare}}$ in Fig. 1 indicates that an extrapolation of data around or above the physical kaon mass to the chiral limit is probably subject to large systematic uncertainties.

When the charm mass m_c is sufficiently heavier than the three light-quark masses, the chiral effective theory has a three-flavour SU(3) symmetry and the LO $\Delta S = 1$ effective Hamiltonian has two unknown LECs, g_{27} and g_8. In our strategy these LECs are considered functions of the charm mass, and our normalizations are such that [2]

$$g_{27}(0) = g_1^+ \,, \qquad g_8(0) = g_1^- + \frac{g_1^+}{5} \,. \tag{19}$$

The values of $g_{27}(\overline{m}_c)$ and $g_8(\overline{m}_c)$ can be estimated at the physical value of the charm mass $\overline{m}_c$ by matching the LO CHPT expressions with the experimental results for $|A_0|$ and $|A_2|$. The result is

$$|g_{27}^{\mathrm{exp}}(\overline{m}_c)| \sim 0.50 \,, \qquad |g_8^{\mathrm{exp}}(\overline{m}_c)| \sim 10.5 \,. \tag{20}$$

These estimates are, of course, affected by systematic errors due to higher-order ChPT contributions [51]. Keeping this in mind, the value of $g_{27}^{\mathrm{exp}}(\overline{m}_c)$ is in good agreement with our result. Since g_{27} is expected to have a mild dependence on the charm-quark mass (only via the fermion determinant in the effective gluonic action), and barring accidental cancellations among quenching effects and higher-order ChPT corrections, this agreement points to the fact that higher-order ChPT corrections in $|A_2|$ may be relatively small. Our value for $g_8(0)$ differs by roughly a factor of 4 from $g_8^{\mathrm{exp}}(\overline{m}_c)$ given in Eq. (20). Apart from possible large quenching artefacts, our result suggests that the charm mass dependence and/or higher-order effects in ChPT are large for $|A_0|$. Indeed in this case penguin contractions, which are absent in the GIM limit, can be responsible for a large charm-mass dependence in g_8, a dependence that can be studied in the next step of our strategy [7, 50].

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[2] In the literature different normalizations of the LECs are used, e.g. $g_{\underline{27}} = (3/5)\, g_{27}$ and $g_{\underline{8}} = g_8/2$ in Ref. [29, 50].

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