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Some News about Generalised Parton Distributions *

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I briefly discuss some recent developments (and recall some old news) in the theory and phenomenology of generalised parton distributions.

1 Parametrising generalised parton distributions

As experimental data on deeply virtual Compton scattering (DVCS) and on exclusive meson production is becoming more and more precise [2], the demands on an adequate theory description are rising. An important part of any theoretical analysis using the framework of generalised parton distributions (GPDs) is to devise a parametrisation of these functions. This is a complex task since GPDs depend on two independent momentum fractions x and ξ and on the invariant momentum transfer t (see Fig. 1) and since the dependence on these variables is subject to a number of consistency constraints. Understanding this dependence is, however, not only a practical necessity but can improve our very understanding of how partons are distributed in the nucleon.

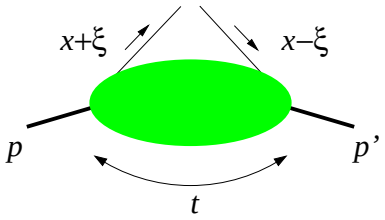

Figure 1: Basic variables describing a GPD.

An important constraint on GPDs is the so-called polynomiality property: the nth Mellin moment in x is a polynomial in ξ with degree n or $n-1$, depending on the particular GPD in question. Being a direct consequence of Lorentz invariance, this has turned out to be essential even at the practical level, since it is required for the consistency of dispersion relations for the processes where GPDs appear (see below). Several methods that ensure this property in the construction of GPD parametrisations have been used in phenomenology:

- the double distribution representation of GPDs. The Radyushkin-Musatov ansatz [3, 4] has been used in a large number of applications—including the VGG code [5]—but it should be emphasized that this is only one particular ansatz for the form of double distributions. A Polyakov-Weiss D-term [6] is typically added to this ansatz so as to provide the term of order ξ^n in the nth Mellin moment of the distributions H and E. In recent work [7] the question was raised whether such a D-term can indeed be freely chosen or whether it is already fixed by the double distribution piece due to further consistency requirements.
  Recall also that there are several alternative double distribution representations of GPDs which generate the term of order ξ^n in the Mellin moments by themselves and hence do not require a D-term from the start [8].

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- the so-called dual parametrisation [9] starts with a partial-wave decomposition in the t-channel and resums these partial waves into a series of two-variable functions (in contrast to the double distribution case, where one deals with a single three-variable function). In practical applications, only the first or the first two functions in this series have been retained.
  Let me note that, in analogy to the magnetic and electric combinations of the Dirac and Pauli form factors, the t-channel partial waves are described by the combinations $H + E$ and $H + \frac{t}{4m^2} E$ of GPDs [10]. This feature is not incorporated in the parametrisations [9].
- an ansatz for the Gegenbauer, or conformal moments of the GPDs [7]. These moments depend on ξ, t and a moment index j, which is to be considered as a variable in the complex plane in order to retrieve the GPDs. This method generalises the representation of usual parton densities in terms of their Mellin moments, and it has the advantage that evolution and α_s corrections can be implemented at NLO and partially at NNLO in a technically simple and numerically efficient way.

The lowest Mellin moments of the quark GPDs H^q and E^q are related to the electromagnetic Dirac and Paul form factors, which are well measured experimentally and are typically used as constraints in GPD parametrisations. Similarly, the axial form factor gives the lowest moment of $\tilde{H}^u - \tilde{H}^d$. Significant progress is being made in calculating higher Mellin moments of quark GPDs on the lattice [11]. Qualitative features found in these calculations provide guidance for parametrising GPDs already now, and eventually one can hope to use lattice results as a quantitative input.

An indirect constraint on the nucleon-helicity flip distribution E^g for gluons is provided by the sum rule $\int dx\, E^g + \sum_q \int dx\, x E^q = 0$ at $t = 0$ and $\xi = 0$. Lattice calculations give a very small result for $\sum_q \int dx\, x E^q$, with large cancellations between u and d quark contributions. This is in line with the small value of the lowest moment $\sum_q \int dx\, E^q$ inferred from the magnetic moments of proton and neutron. Under the assumption that E^g does not change unusually fast when going to finite t and ξ, the above sum rule hence implies that the distribution E^g is itself small—in contrast to its counterpart H^g—unless it has one or more nodes in x. Current models [12, 13] do not allow for such nodes (for which there is no indication from data) and consequently have E^g much smaller than E^u or E^d. Going back to the point $t = 0$ and $\xi = 0$, one should note that if $\int dx\, E^g$ is small then the total angular momentum $\frac{1}{2} \int dx\, (H^g + E^g)$ carried by gluons in the nucleon according to Ji's sum rule is rather large, because $\int dx\, H^g = \int dx\, x g(x)$ is about one half.

Further nontrivial constraints on GPDs are positivity conditions, which ensure the interpretation of GPDs as densities or interference terms in impact parameter space. As in the case of the usual parton densities, positivity conditions are preserved by leading-order evolution to higher scales but need not hold at NLO or beyond. It seems reasonable to demand some explanation in cases where they break down (e.g. in the region of very small x, where NLO corrections are known to become important). The most general form of the positivity conditions for GPDs is quite involved [14]. Simplified versions concern the skewness effect and give e.g. an upper limit on $H^q(x, \xi, t)$ for $x > \xi$ in terms of the geometric mean of the quark densities evaluated at $(x \pm \xi)/(1 \pm \xi)$ [15, 3]. To implement or even check these constraints in GPD parametrisations is difficult and most often not done. An exception are approaches based on calculating double distributions in spectator models [16], which allow both polynomiality and positivity to be implemented automatically, but so far have barely been used in phenomenological studies. A different type of simplified positivity condition

involves the GPDs at $\xi = 0$ and limits for instance E in terms of H and $\tilde{H}$ [17]. For simple analytic forms of the GPDs these conditions can readily be checked—unfortunately this is not always done in practice.

An approximate power-law dependence at small x, as suggested by simple Regge phenomenology, works well in phenomenological parametrisations of the usual parton densities, and it is natural to extend this to a behaviour proportional to $x^{-(\alpha+\alpha' t)}$ for GPDs. It is important to note that DGLAP evolution changes the value of the effective shrinkage parameter α' with the factorization scale [18], just as it changes the value of the effective power α at $t = 0$. This change can be rather slow, so that the mixing of gluons and sea quarks under evolution does not by itself guarantee that α' is similar for these partons at moderate scales [18].

2 From exclusive processes to GPDs

It is well known that, to leading order in α_s, the imaginary part of the amplitude for DVCS or light meson production involves only GPDs at the special points $x = \xi$ and $x = -\xi$. While the impact parameter representation is somewhat involved for GPDs at nonzero ξ [19], it simply gives the distance between the struck parton and the *spectator system* in this case [20].

Whereas the factorization formula for the real part of the DVCS or meson production amplitude requires the GPDs in the full x-region, a representation based on dispersion relations [21] involves, at leading order, only the GPDs at $|x| = \xi$ plus a subtraction term which depends on the coefficients of ξ^n in $\int dx\, x^{n-1} H(x, \xi, t)$ and can hence be expressed through the D-term in the Polyakov-Weiss representation. This implies that at tree-level accuracy, exclusive processes are only sensitive to GPDs at $|x| = \xi$ and to one t-dependent subtraction term for each parton species. Both the evolution of the GPDs and explicit NLO corrections to the amplitude involve the region $|x| \geq \xi$, in addition to a more complicated subtraction constant [10, 7]. To access this information experimentally, one must be sensitive to violations of Bjorken scaling and thus needs a sufficient lever arm in Q^2 at given x_B.

The polynomiality of Mellin moments, which ties together the GPDs at $|x| < \xi$ and at $|x| \geq \xi$, is crucial for the consistency of the dispersion relations just mentioned. In [7] an explicit construction is given that allows one to reconstruct a GPD in the region $|x| < \xi$ from its knowledge at $|x| \geq \xi$, apart from a possible D-term ambiguity.

Deeply virtual Compton scattering not only offers a large number of observables that can be evaluated in the GPD framework [22] but is also under very good theoretical control, with radiative corrections known at NLO and in part at NNLO. In numerical studies these corrections were found to be of moderate size [23], except for important effects in scaling violation at small x, which are of the same nature as those in inclusive DIS and stem from the behavior of the singlet evolution kernels. Nevertheless, NLO corrections of the order of 20% are not uncommon in collider or fixed-target kinematics. While a leading-order analysis of DVCS data should be adequate to reveal basic features and to provide a starting point, I see no good reason why analyses should be limited to leading order (including the neglect of evolution) as experimental data is becoming more precise. The NLO scattering amplitudes for DVCS are reasonably simple for practical use, and the LO evolution of GPDs has been implemented in a fast numerical code [24].

Meson production is harder to analyse quantitatively, as both power corrections [13] and higher orders in α_s [25, 12] are larger than for DVCS according to the estimates in

the literature. The often substantial size of NLO corrections should not surprise us, given that $\alpha_s \sim 0.4$ for scales relevant to a large part of the experimental data. Cross section ratios are often assumed to be less sensitive to corrections. The study [12] found that this is sometimes true but not always. The ratio of ω and ρ production cross sections at moderate x_B is indeed quite stable w.r.t. NLO corrections. For the cross section ratio of ϕ and ρ production this is, however not the case, since the former is dominated by gluon exchange and the latter by a mixture of quark and gluon exchange, which are affected in different ways by NLO corrections. For the GPD models considered in [12], the transverse target spin asymmetries for ρ, ω, and ϕ production are very small at tree level, due to cancellations between different contributions and to a small relative phase between the amplitudes that have to interfere in order to make the spin asymmetry nonzero. The NLO corrections on these small asymmetries were found to be large. A careful case-by-case analysis is hence required before one can assert the stability of cross section ratios or asymmetries.

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References

[1] Slides: `http://indico.cern.ch/contributionDisplay.py?contribId=287&sessionId=31&confId=24657`

[2] Talks at this conference by W. Augustyniak, L. Favart, I. Hristova, X. Janssen, J. Malka, A. Miller, S. Niccolai, J. Ukleja, H. Ye and D. Zeiler.

[3] A. V. Radyushkin, Phys. Rev. D **59** (1999) 014030 [hep-ph/9805342].

[4] I. V. Musatov and A. V. Radyushkin, Phys. Rev. D **61** (2000) 074027 [hep-ph/9905376].

[5] M. Vanderhaeghen, P. A. M. Guichon and M. Guidal, Phys. Rev. D **60** (1999) 094017 [hep-ph/9905372].

[6] M. V. Polyakov and C. Weiss, Phys. Rev. D **60** (1999) 114017 [hep-ph/9902451].

[7] K. Kumerički, D. Müller and K. Passek-Kumerički, arXiv:0805.0152 [hep-ph];
K. Kumerički, D. Müller and K. Passek-Kumerički, Nucl. Phys. B **794** (2008) 244 [hep-ph/0703179];
talks at this conference by K. Kumerički and D. Müller, arXiv:0807.0159 and arXiv:0807.0170.

[8] O. V. Teryaev, Phys. Lett. B **510** (2001) 125 [hep-ph/0102303];
A. V. Belitsky, D. Müller, A. Kirchner and A. Schäfer, Phys. Rev. D **64** (2001) 116002 [hep-ph/0011314];
B. C. Tiburzi, Phys. Rev. D **70** (2004) 057504 [hep-ph/0405211].

[9] V. Guzey and M. V. Polyakov, Eur. Phys. J. C **46** (2006) 151 [hep-ph/0507183];
V. Guzey and T. Teckentrup, Phys. Rev. D **74** (2006) 054027 [hep-ph/0607099];
M. V. Polyakov, Phys. Lett. B **659** (2008) 542 [arXiv:0707.2509 [hep-ph]];
M. V. Polyakov and M. Vanderhaeghen, arXiv:0803.1271 [hep-ph];
A. M. Moiseeva and M. V. Polyakov, arXiv:0803.1777 [hep-ph];
K. M. Semenov-Tian-Shansky, Eur. Phys. J. A **36** (2008) 303 [arXiv:0803.2218 [hep-ph]].

[10] M. Diehl and D. Yu. Ivanov, Eur. Phys. J. C **52** (2007) 919 [arXiv:0707.0351 [hep-ph]].

[11] Ph. Hägler *et al.* [LHPC Collab.], Phys. Rev. D **77** (2008) 094502 [arXiv:0705.4295 [hep-lat]];
D. Brömmel *et al.* [QCDSF-UKQCD Collab.], PoS **LAT2007** (2007) 158 [arXiv:0710.1534 [hep-lat]];
Ph. Hägler, talk at this conference.

[12] M. Diehl and W. Kugler, Eur. Phys. J. C **52** (2007) 933 [arXiv:0708.1121 [hep-ph]].

[13] S. V. Goloskokov and P. Kroll, Eur. Phys. J. C **53** (2008) 367 [arXiv:0708.3569 [hep-ph]];
P. Kroll, talk at this conference, arXiv:0806.2536 [hep-ph].

[14] P. V. Pobylitsa, Phys. Rev. D **66** (2002) 094002 [hep-ph/0204337].

[15] B. Pire, J. Soffer and O. Teryaev, Eur. Phys. J. C **8** (1999) 103 [hep-ph/9804284].

[16] P. V. Pobylitsa, Phys. Rev. D **67** (2003) 094012 [hep-ph/0210238];
B. C. Tiburzi, W. Detmold and G. A. Miller, Phys. Rev. D **70** (2004) 093008 [hep-ph/0408365];
D. S. Hwang and D. Müller, Phys. Lett. B **660** (2008) 350 [arXiv:0710.1567 [hep-ph]].

[17] M. Burkardt, Phys. Lett. B **582** (2004) 151 [hep-ph/0309116];
M. Diehl and Ph. Hägler, Eur. Phys. J. C **44** (2005) 87 [hep-ph/0504175].

[18] M. Diehl and W. Kugler, Phys. Lett. B **660** (2008) 202 [arXiv:0711.2184 [hep-ph]].

[19] M. Diehl, Eur. Phys. J. C **25** (2002) 223, Erratum ibid. C **31** (2003) 277 [hep-ph/0205208].

[20] M. Burkardt, arXiv:0711.1881 [hep-ph].

[21] O. V. Teryaev, hep-ph/0510031;
I. V. Anikin and O. V. Teryaev, Phys. Rev. D **76** (2007) 056007 [arXiv:0704.2185 [hep-ph]].

[22] A. V. Belitsky, D. Müller and A. Kirchner, Nucl. Phys. B **629** (2002) 323 [hep-ph/0112108];
M. Diehl and S. Sapeta, Eur. Phys. J. C **41** (2005) 515 [hep-ph/0503023].

[23] D. Müller, Phys. Lett. B **634** (2006) 227 [hep-ph/0510109];
K. Kumerički, D. Müller, K. Passek-Kumerički and A. Schäfer, Phys. Lett. B **648** (2007) 186 [hep-ph/0605237].

[24] A. V. Vinnikov, hep-ph/0604248, `http://durpdg.dur.ac.uk/hepdata/gpd.html`

[25] A. V. Belitsky and D. Müller, Phys. Lett. B **513** (2001) 349 [hep-ph/0105046];
D. Yu. Ivanov, A. Schäfer, L. Szymanowski and G. Krasnikov, Eur. Phys. J. C **34** (2004) 297 [hep-ph/0401131];
D. Yu. Ivanov, L. Szymanowski and G. Krasnikov, JETP Lett. **80** (2004) 226, Pisma Zh. Eksp. Teor. Fiz. **80** (2004) 255 [hep-ph/0407207].