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AdS Solutions Through Transgression

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Abstract

We present new classes of explicit supersymmetric AdS_3 solutions of type IIB supergravity with non-vanishing five-form flux and AdS_2 solutions of $D = 11$ supergravity with electric four-form flux. The former are dual to two-dimensional SCFTs with $(0, 2)$ supersymmetry and the latter to supersymmetric quantum mechanics with two supercharges. We also investigate more general classes of AdS_3 solutions of type IIB supergravity and AdS_2 solutions of $D = 11$ supergravity which in addition have non-vanishing three-form flux and magnetic four-form flux, respectively. The construction of these more general solutions makes essential use of the Chern-Simons or "transgression" terms in the Bianchi identity or the equation of motion of the field strengths in the supergravity theories. We construct infinite new classes of explicit examples and for some of the type IIB solutions determine the central charge of the dual SCFTs. The type IIB solutions with non-vanishing three-form flux that we construct include a two-torus, and after two T-dualities and an S-duality, we obtain new AdS_3 solutions with only the NS fields being non-trivial.

1 Introduction

An interesting class of supersymmetric AdS_3 solutions of type IIB supergravity with non-vanishing five-form flux and dual to $(0,2)$ SCFTs in $d=2$ were analysed in [1]. Similarly, a class of AdS_2 solutions of $D=11$ supergravity with electric four-form flux and dual to superconformal quantum mechanics with two supercharges were analysed in [2]. It is remarkable that the geometries of the corresponding internal seven and nine-dimensional spaces have a similar structure. In particular, they both have a Killing vector (dual to an R-symmetry in the corresponding SCFT) which locally defines a foliation, and the metrics are completely determined by a Kähler metric on the corresponding six or eight-dimensional leaves. In both cases, the local Kähler metric satisfies the same differential equation

$$\Box R - \frac{1}{2}R^2 + R^{ij}R_{ij} = 0 \tag{1.1}$$

where R_{ij} and R are the Ricci tensor and Ricci scalar for the Kähler metric. These $2n+1$ dimensional geometries, with $n=3,4$, were further investigated in [3], which also generalised them to all n. It was shown that the $2n+2$ dimensional cone geometries over these spaces admit certain Killing spinors that define an $SU(n+1)$ structure with particular intrinsic torsion that was determined in [3].

This geometry has striking similarities with Sasaki-Einstein (SE) geometry. Recall that a five-dimensional SE manifold SE_5 gives rise to a supersymmetric type IIB $AdS_5 \times SE_5$ solution with non-vanishing five-form flux, while a seven-dimensional SE manifold SE_7 gives rise to a $AdS_4 \times SE_7$ solution of $D=11$ supergravity with electric four-form flux. All SE spaces have a Killing vector, which locally defines a foliation, and the SE metric is completely determined by a Kähler-Einstein metric on the corresponding leaves. Furthermore, the $2n+2$ dimensional cone geometries over the SE spaces are Calabi-Yau i.e. they admit covariantly constant spinors that define an $SU(n+1)$ structure with vanishing intrinsic torsion (i.e. the metric has $SU(n+1)$ holonomy).

The $AdS_5 \times SE_5$ and $AdS_4 \times SE_7$ solutions are the near horizon limits of more general supergravity solutions that describe $D3$-branes and $M2$-branes sitting at the apex of the Calabi-Yau three and four-fold cones, respectively. In these more general solutions, only the five-form flux and electric four-form flux are non-trivial, and the solutions are determined by a harmonic function on the Calabi-Yau space. An interesting further generalisation for the type IIB case, is to consider any Calabi-Yau three fold and to switch on imaginary self-dual harmonic three form flux. One finds that

this solution preserves the same amount of supersymmetry. Furthermore the Bianchi identity for the five-form, modified by Chern-Simons or "transgression" terms,

$$dF_5 = \frac{i}{2} G \wedge G^* \tag{1.2}$$

where G is a complex three-form which contains the NS-NS and R-R three-forms, implies that the solutions are determined by a function that satisfies a Laplace equation with a source term. Similarly, for $D = 11$ supergravity one can consider an arbitrary Calabi-Yau four-fold and switch on a harmonic self-dual four-form. Now it is the equation of motion for the three-form potential with its transgression terms,

$$d *_{11} G_4 + \frac{1}{2} G_4 \wedge G_4 = 0 \tag{1.3}$$

which is playing a key role in the solution. Switching on the additional fluxes in these type IIB and $D = 11$ solutions necessarily breaks the conformal symmetry. A prominent example of such solutions is the Klebanov-Strassler solution of type IIB [4] (see also [5, 6]), which is constructed using the deformed conifold metric. A more general analysis of these kinds of solutions can be found in [7].

One of the main aims of this paper is to show that we can similarly generalise the classes of type IIB solutions considered in [1] and the $D = 11$ solutions considered in [2] to include three-form flux and magnetic four-form flux, respectively. The central idea is to switch on such fluxes on the six and eight dimensional Kähler spaces, respectively. We will show that this can be done in a way that maintains the AdS_3 and AdS_2 factors, and hence the dual conformal symmetry (in contrast to the examples discussed above), and also preserves the same amount of supersymmetry. We find that the solutions are still, locally, specified by a Kähler metric but (1.1) is modified by a term involving the new activated fluxes. We will also construct rich new classes of explicit solutions by following a similar analysis to that of [8].

The plan of the rest of the paper is as follows. We will summarise the general classes of AdS_3 solutions of type IIB and AdS_2 solutions of $D = 11$ in section 2. We have left some details of the derivations, which are very similar to those in [1] and [2], to appendix A. We will also briefly interrupt the main narrative to explain how the solutions can be analytically continued so that the AdS factors are replaced by spheres. This gives rise to new general classes of 1/8 BPS bubble solutions generalising those discussed in [8] (1/2 BPS bubble solutions were first analysed in [9], and other studies of general classes of bubble solutions preserving various amounts of supersymmetry in type IIB and $D = 11$ supergravity have appeared in [10]-[23]).

In section 3 we will construct explicit AdS solutions by taking the six and eight dimensional Kähler metrics to be products of two-dimensional Kähler-Einstein (KE) spaces. For type IIB we will first analyse the global properties of the local solutions with vanishing three-form flux that were found in [8] and calculate the central charge of the dual CFTs. These AdS_3 solutions are labelled by a rational number $s/t \in [-1/2, 0)$ and an integer N fixing the five-form flux. The topology of the internal seven-manifold is a certain $U(1)$ bundle over a product of two two-spheres and a Riemann surface with genus greater than one. We then consider solutions with non-zero three-form flux by taking one of the Kähler-Einstein factors to be a two-torus. We find that the two other KE spaces must be spheres. After two T-dualities we find that the solutions turn out to be the well known $AdS_3 \times S^3 \times S^3 \times S^1$ solutions of type IIB supergravity (see [24, 25, 26, 27]). We conclude section 3 with a similar construction of explicit AdS_2 solutions of $D = 11$ with non-vanishing magnetic four-form flux.

In sections 4 and 5 we will present a different construction of local six and eight dimensional Kähler metrics, using fibrations over KE spaces, generalising the constructions in [8] (see also [28]). We have recorded some details in appendices C and D, respectively. For type IIB we will consider the product of T^2 with a two-dimensional fibration over an S^2. This leads to infinite new explicit examples of AdS_3 solutions of type IIB supergravity with the internal seven dimensional space having topology $S^3 \times S^2 \times T^2$ and the metric labelled by a pair of positive relatively prime integers p, q. When the type IIB three-flux is vanishing we show that demanding that the five-form is properly quantised implies that as solutions of type IIB string theory they depend on two more integers M, N which fix the five-form flux and the size of the T^2. For these solutions we calculate the central charge of the dual CFTs[1]. We also show that after two T-dualities the solutions are mapped to type IIB AdS_3 solutions with non-vanishing dilaton and RR three-form: after a further S-duality only NS fields are non-zero.

Section 5 carries out similar constructions of local eight dimensional Kähler metrics which are the product of T^2 with a two-dimensional fibration over a four dimensional KE space with positive curvature. This gives rise to infinite classes of AdS_2 solutions with non-vanishing magnetic four-form flux. Section 6 briefly concludes.

[1]The corresponding analysis for the case when the three-form flux is non-vanishing will be determined in [29].

2 AdS solutions through transgression

We first consider a general class of supersymmetric AdS_3 solutions of type IIB supergravity that are dual to $(0,2)$ SCFTs in $d=2$. The metric and the self-dual five-form take the form

$$\begin{aligned} ds^2 &= e^{2A}\left[ds^2\left(AdS_3\right)+ds^2(Y_7)\right] \\ F_5 &= \left(1+*_{10}\right)Vol(AdS_3)\wedge F_2 \end{aligned} \tag{2.1}$$

where F_2 is a two-form on Y_7. The dilaton and axion are constant and for simplicity we set them to zero. We also demand that the complex three-form flux, G, which contains the NS-NS and R-R three-form field strengths, is a three-form on Y_7.

As we show in appendix A, by following the analysis of [1], demanding that this is a supersymmetric solution to the equations of motion, preserving supersymmetry as described in the appendix, leads to the following local description. The metric can be written

$$ds^2(Y_7)=\frac{1}{4}\left(dz+P\right)^2+e^{-4A}ds_6^2 \tag{2.2}$$

where ∂_z is a Killing vector, ds_6^2 is a Kähler metric and dP is the Ricci form for ds_6^2. The warp factor is given by

$$e^{-4A}=\frac{1}{8}R \tag{2.3}$$

where R is the Ricci scalar for ds_6^2 and we thus need to demand that $R>0$. The two-form F_2 appearing in the five-form can be written

$$F_2=2J-\frac{1}{2}d\left[e^{4A}\left(dz+P\right)\right] \tag{2.4}$$

where J is the Kähler form for ds_6^2.

So far, this is exactly the same as when the three-form flux vanishes [1]. However, further analysis shows that we can switch on the three-form G, provided that G is a closed, $(1,2)$ and primitive three-form on the Kähler space. In particular G must be imaginary self-dual, $*_6G=iG$, and harmonic. Furthermore, the Bianchi identity for the five-form with its transgression terms (1.2), implies that the Kähler metric ds_6^2 must satisfy

$$\Box R-\frac{1}{2}R^2+R^{ij}R_{ij}+\frac{2}{3}G^{ijk}G^*_{ijk}=0 \tag{2.5}$$

which is the key equation generalising (1.1).

We now consider a general class of supersymmetric AdS_2 solutions of $D=11$ supergravity that are dual to superconformal quantum mechanics with two super-

charges. The metric and the four-form are given by

$$\begin{aligned} ds^2 &= e^{2A}\left[ds^2\left(AdS_2\right) + ds^2(Y_9)\right] \\ G_4 &= \mathrm{Vol}(AdS_2)\wedge F_2 + F_4 \end{aligned} \tag{2.6}$$

where F_2 is a two-form on Y_9 and F_4 is a four-form on Y_9. This generalises the class of solutions studied in [2] which had $F_4 = 0$ i.e. purely electric fluxes.

As we show in appendix A, now following the analysis of [2], demanding that this is a supersymmetric solution to the equations of motion, preserving supersymmetry as described in the appendix, leads to the following local description. The metric can be written

$$ds^2(Y_9) = (dz + P)^2 + e^{-3A}ds_8^2 \tag{2.7}$$

where ∂_z is a Killing vector, ds_8^2 is a Kähler metric and dP is the Ricci form for ds_8^2. The warp factor is given by

$$e^{-3A} = \frac{1}{2}R \tag{2.8}$$

where R is the Ricci scalar for ds_8^2 and so we demand $R > 0$. The two-form F_2 appearing in the four-form can be written

$$F_2 = -J + d\left[e^{3A}\left(dz + P\right)\right] \tag{2.9}$$

where J is the Kähler form for ds_8^2. This is exactly as in the case of purely electric four-form flux [2]. We now find that we can switch on F_4 provided that it is a closed, $(2,2)$ and primitive four-form on the Kähler space. In particular F_4 must be self-dual and harmonic. Furthermore, the equation of motion for the four-form with its transgression terms (1.3) implies that the Kähler metric ds_8^2 must now satisfy

$$\Box R - \frac{1}{2}R^2 + R^{ij}R_{ij} + \frac{1}{4!}F_4^{ijkl}F_{4ijkl} = 0. \tag{2.10}$$

In the special case that the eight-dimensional Kähler metric ds_8^2 contains a T^2 factor, we can dimensionally reduce the $D = 11$ solution on one leg of the T^2 and then T-dualise on the other leg, to obtain a type IIB solution. In the case that $F_4 = 0$, it was shown in [8] that the resulting type IIB solution is in fact the AdS_3 solution with vanishing three-form flux. There is a simple generalisation to non-vanishing F_4. Decompose the eight-dimensional Kähler form as

$$J_8 = J_6 + du^1 \wedge du^2 \tag{2.11}$$

where u^1, u^2 are coordinates on the T^2. Suppose we can write the $(2,2)$ four form as

$$F_4 = iG \wedge \left(du^1 + idu^2\right) - iG^* \wedge \left(du^1 - idu^2\right) \tag{2.12}$$

where G is a closed primitive $(1,2)$ form in six-dimensions (i.e. we are asuuming that there is no term involving the volume form of the two torus, $du^1 \wedge du^2$). If we dimensionally reduce on the u^2 direction and then T-dualise on the u^1 direction we find that the $D = 11$ AdS_2 solution is transformed into the type IIB AdS_3 solution.

2.1 Bubble solutions

In subsequent sections we will find explicit examples of the AdS_3 and AdS_2 solutions just described. Before doing that we pause to briefly comment on how the above classes of solutions can be analytically continued so that the AdS factors are replaced with spheres. These "bubble" solutions preserve 1/8 of the supersymmetry and generalise those discussed in [8].

For the type IIB case, the metric is given by

$$ds^2 = e^{2A}\left[-\frac{1}{4}\left(dt+P\right)^2 + ds^2\left(S^3\right) + e^{-4A}ds_6^2\right] \tag{2.13}$$

where ∂_t is a Killing vector, ds_6^2 is again a Kähler metric and dP is the Ricci form for ds_6^2. The warp factor is given by

$$e^{-4A} = -\frac{1}{8}R \tag{2.14}$$

where R is the Ricci scalar for ds_6^2 and so now we want $R < 0$. The five-form flux is given by

$$F_5 = \left(1 + *_{10}\right) Vol(S^3) \wedge F_2 \tag{2.15}$$

where

$$F_2 = 2J + \frac{1}{2}d\left[e^{4A}\left(dt+P\right)\right] \tag{2.16}$$

and J is the Kähler form for ds_6^2. The three-form G is again a closed, $(1,2)$ and primitive three-form on the Kähler space. Finally the master equation reads

$$\Box R - \frac{1}{2}R^2 + R^{ij}R_{ij} - \frac{2}{3}G^{ijk}G^*_{ijk} = 0. \tag{2.17}$$

For the $D = 11$ case, the metric is given by

$$ds^2 = e^{2A}\left[-\left(dt+P\right)^2 + ds^2(S^2) + e^{-3A}ds_8^2\right] \tag{2.18}$$

where ∂_t is a Killing vector, ds_8^2 is a Kähler metric and dP is the Ricci form for ds_8^2. The warp factor is given by

$$e^{-3A} = -\frac{1}{2}R \tag{2.19}$$

where R is the Ricci scalar for ds_8^2 and we demand $R < 0$. The four-form flux is given by

$$G_4 = Vol(S^2) \wedge F_2 + F_4 \tag{2.20}$$

where

$$F_2 = -J - d\left[e^{3A}\left(dt + P\right)\right] \tag{2.21}$$

and J is the Kähler form for ds_8^2. F_4 is again a closed, $(2,2)$ and primitive four-form on the Kähler space. Finally, the master equation is now

$$\Box R - \frac{1}{2}R^2 + R^{ij}R_{ij} - \frac{1}{4!}F_4^{ijkl}F_{4ijkl} = 0. \tag{2.22}$$

3 Product of KE spaces

In this section we will explore solutions for which the Kähler metrics ds_6^2 and ds_8^2 appearing in (2.2) and (2.7), respectively, are simply the product of a set of two-dimensional Kähler–Einstein metrics

$$ds_{2n}^2 = \sum_{i=1}^{n} ds^2(KE_2^{(i)}) \tag{3.1}$$

where $ds^2(KE_2^{(i)})$ is a two-dimensional Kähler-Einstein metric, i.e. locally proportional to the standard metric on S^2, T^2 or H^2. For the latter case, we can also take a quotient H^2/Γ to get a Riemann surface with genus greater than one. The metric ds_{2n}^2 is normalised so that the Ricci form is given by

$$\mathcal{R} = \sum_{i=1}^{n} \mathcal{R}_i = \sum_{i=1}^{n} l_i J_i \tag{3.2}$$

where $\mathcal{R}_i$ and J_i are the Ricci and Kähler forms of the $ds^2(KE_2^{(i)})$ metrics, respectively, and l_i is zero, positive or negative depending on whether the metric is locally that on T^2, S^2 or H^2, respectively. We also have $P = \sum_i P_i$ with $dP_i = \mathcal{R}_i$ and the Ricci scalar is $R = 2\sum_{i=1}^{n} l_i$. Note that in the special case that two of the l_i are equal, say $l_1 = l_2$, the analysis can be simply extended to cover the case when the product $KE_2^{(1)} \times KE_2^{(2)}$ is replaced with a more general four-dimensional Kähler-Einstein manifold, KE_4. Similar generalisations are possible if more of the l_i are equal. Finally, it will be useful to recall that if the ith KE space, Σ_{g_i}, is a Riemann surface of genus g_i, then

$$\frac{1}{2\pi}\int_{\Sigma_{g_i}} \mathcal{R}_i = 2(1 - g_i)\ . \tag{3.3}$$

3.1 Type IIB

For this case, the metric $ds^2(Y_7)$ appearing in (2.1) is given by

$$\frac{1}{L^2}ds^2(Y_7) = \frac{1}{4}\left(dz+P\right)^2 + e^{-4A}\left[\sum_{i=1}^{3} ds^2(KE_2^{(i)})\right] \tag{3.4}$$

where we have introduced an overall length scale L, and the warp factor is given by

$$e^{-4A} = \frac{1}{4}(l_1+l_2+l_3)\ . \tag{3.5}$$

Writing the five-form flux as

$$F_5 = AdS_3 \wedge F_2 + \omega_5 \tag{3.6}$$

we have

$$\begin{aligned}\frac{1}{L^4}F_2 &= \frac{2}{l_1+l_2+l_3}\left[(l_2+l_3)J_1+(l_1+l_3)J_2+(l_1+l_2)J_3\right]\\ \frac{1}{L^4}\omega_5 &= \frac{1}{4}\left[(l_1+l_2)J_1\wedge J_2+(l_1+l_3)J_1\wedge J_3+(l_2+l_3)J_2\wedge J_3\right](dz+P)\ .\end{aligned} \tag{3.7}$$

3.1.1 $G=0$

We first consider the local solutions with zero three-form flux, $G=0$, that were presented in section 6.1 of [8]. We will show that there are an infinite number of globally defined solutions with appropriately quantised five-form flux and we will calculate the central charges of the dual $d=2$ $(0,2)$ SCFTs.

It was shown in [8] that the master equation (1.1) is solved if $(l_1,l_2,l_3)=(l_1,-\frac{l_1}{1+l_1},1)$ with $l_1\in[-1/2,0]$. When $l_1=0$ we obtain the well known $AdS_3\times S^3\times T^4$ solution. We therefore restrict to $l_1\in[-1/2,0)$ so that the six-dimensional Kähler manifold is $\Sigma_g\times S^2_1\times S^2_2$, where Σ_g is a Riemann surface with genus $g>1$.

We now examine the conditions required for Y_7 to be a well defined $U(1)$ fibration over $\Sigma_g\times S^2_1\times S^2_2$. If we let the period of the coordinate z be $2\pi l$ then we require that $l^{-1}P$ be a bona-fide $U(1)$ connection. This is guaranteed if the integral of $l^{-1}dP/(2\pi)$ over a basis of two cycles on $\Sigma_g\times S^2_1\times S^2_2$ are all integers. Taking the obvious basis, we conclude that we should take z to have period 4π and then the periods are $(1-g,1,1)$.

We now turn to the five-form. We first observe that this is a globally defined five-form on Y_7. To ensure that we have a good solution of type IIB string theory, we demand that the five-form flux is properly quantised:

$$N(D) = \frac{1}{(2\pi l_s)^4 g_s}\int_D F_5 \in \mathbb{Z} \tag{3.8}$$

for any five-cycle $D \in H_5(Y_7, \mathbb{Z})$. A basis for the free part of $H_5(Y_7, \mathbb{Z})$ is obtained by taking the $U(1)$ fibration over a basis of four-cycles on the base $\Sigma_g \times S^2 \times S^2$. Let D_1, D_2 and D_3 denote the five cycles arising from the four-cycles $\Sigma_g \times S^2_1$, $\Sigma_g \times S^2_2$ and $S^2_1 \times S^2_2$, respectively. Since the $U(1)$ fibration is non-trivial, these five-cycles are not independent in homology and we have $[D_1] + [D_2] + (1-g)[D_3] = 0$. Calculating the $N(D_i)$ we then deduce that for them to be all integers, l_1 must be rational, $l_1 = s/t$ and

$$\frac{L^4}{\pi g_s l_s^4} = \frac{s}{h} N \tag{3.9}$$

where $h = hcf(t, (g-1))$. Indeed, we then find that

$$\begin{aligned} N(D_1) &= -\frac{s(1-g)}{h} N \\ N(D_2) &= \frac{(s+t)(1-g)}{h} N \\ N(D_3) &= -\frac{t}{h} N \ . \end{aligned} \tag{3.10}$$

Clearly we have $N(D_1)+N(D_2)+(1-g)N(D_3) = 0$ which corresponds to the relation amongst the five-cycles mentioned above.

We have thus established that there is an infinite class of solutions labelled by rational $l_1 = s/t \in [-1/2, 0)$, each of which gives rise to a $d = 2$ $(0,2)$ SCFT. The central charge of the SCFTs is given by

$$c = \frac{3 R_{AdS_3}}{2 G_{(3)}} \tag{3.11}$$

where $G_{(3)}$ is the three-dimensional Newton's constant and R_{AdS_3} is radius of the AdS_3 space. In our conventions the type IIB supergravity Lagrangian has the form

$$\frac{1}{(2\pi)^7 g_s^2 l_s^8} \sqrt{-detg} R + \ldots \tag{3.12}$$

and we calculate that

$$c = 6 \frac{(g-1)(s^2 + st + t^2)}{h^2} N^2 \ . \tag{3.13}$$

Note that for the special case of $s = 1, t = -2$ we have $(l_1, l_2, l_3) = (-1/2, 1, 1)$: this is a case whose central charge was already calculated in [30] (substitute $M = 8$, $m = 2$ into equation (6.14) of that reference).

3.1.2 $G \neq 0$

We now turn to the construction of solutions with non-vanishing three-form flux. In order to find a suitable three-form flux G we will demand that the product of the KE

spaces includes a T^2 factor, $l_3 = 0$. We then take the three-form to be given by

$$\frac{1}{L^2}G = d\bar{u} \wedge [m_1 J_1 + m_2 J_2] \tag{3.14}$$

where u is a complex coordinate on the T^2 and m_1, m_2 are constant. This is closed and is also a $(1,2)$ form on the Kähler space. In order that it is primitive we must set $m_1 = -m_2$. Without loss of generality we take $m_1 > 0$. It just remains to solve the master equation (2.5) which gives

$$l_1 l_2 = 4m_1^2 \ . \tag{3.15}$$

Recalling the expression for the warp factor, (3.5) (with $l_3 = 0$), which must be positive, we deduce that $l_i > 0$ and in particular our six-dimensional Kähler space must be $S^2_1 \times S^2_2 \times T^2$. After a possible rescaling we can take $l_2 = 1$. The five-form flux is given by (3.6) and (3.7) with $l_3 = 0$.

To analyse this solution further, it is convenient to perform successive T-dualities on the two legs of the T^2 (which we take to be square). Using the formulae in appendix B, we are led to the following type IIB solution[2]

$$\begin{aligned}
\frac{1}{L^2}ds^2 &= ds^2(AdS_3) + \frac{a+1}{4a}ds^2(S^2_1) + \frac{a+1}{4}ds^2(S^2_2) \\
&\quad + \frac{1}{4}(dz + P_1 + P_2)^2 + \frac{a}{4}(du^1 - \frac{1}{a}P_1 + P_2)^2 + (du^2)^2 \\
\frac{1}{L^2}F_3 &= 2Vol(AdS_3) + \frac{1}{4}(\mathcal{R}_1 + \mathcal{R}_2)(dz + P_1 + P_2) - \frac{a}{4}(\frac{1}{a}\mathcal{R}_1 - \mathcal{R}_2)(du^1 - \frac{1}{a}P_1 + P_2) \\
e^{2\phi} &= 1 \ .
\end{aligned} \tag{3.16}$$

Note that here (unlike above) the metrics on the two-spheres have unit radius and $a = l_1/l_2$. Introducing the coordinates $\psi_1 = (a/(1+a))(z-y)$ and $\psi_2 = (1/(1+a))(z+ay)$ and then completing the squares using the ψ_i we are led to

$$\begin{aligned}
\frac{1}{L^2}ds^2 &= ds^2(AdS_3) + \frac{a+1}{a}ds^2(S^3_1) + (a+1)ds^2(S^3_2) + (du^2)^2 \\
\frac{1}{L^2}F_3 &= 2Vol(AdS_3) + \frac{2(a+1)}{a}Vol(S^3_1) + 2(a+1)Vol(S^3_2)
\end{aligned} \tag{3.17}$$

where $ds^2(S^3_i)$ are the round metrics on unit radius three spheres. This is the well known $AdS_3 \times S^3 \times S^3 \times S^1$ solution of type IIB supergravity (see [24, 25, 26, 27]). Note that this solution is dual to a $d = 2$ SCFT with $(4,4)$ supersymmetry: when we T-dualise back the configuration with $G \neq 0$ we will possibly break some of the supersymmetry: our construction guarantees that there is at least $(0,2)$ supersymmetry, but we haven't checked if more supersymmetry is preserved.

[2]To obtain the solution in this form, we rescaled the u^1 coordinate, $u^1 \to u^1(m_1/l_2)$, we set the dilaton to zero by shifting the dilaton and rescaling F_3, and we also absorbed the warp factor into L^2.

3.2 $D = 11$

We briefly consider similar constructions of AdS_2 solutions of $D = 11$ supergravity. The metric $ds^2(Y_9)$ appearing in (2.6) is given by

$$ds^2(Y_9) = (dz + P)^2 + e^{-3A} \sum_{i=1}^{4} ds^2(KE_2^{(i)}) \tag{3.18}$$

and the warp factor is given by

$$e^{-3A} = \sum_{i=1}^{4} l_i \ . \tag{3.19}$$

The four form flux is

$$G_4 = Vol(AdS_2) \wedge F_2 + F_4 \tag{3.20}$$

with

$$\begin{aligned} F_2 &= \frac{2}{\sum_{i=1}^{4} l_i} [(l_1 + l_2 + l_3)J_1 + (l_1 + l_3 + l_4)J_2 + (l_1 + l_2 + l_4)J_3 + (l_1 + l_2 + l_3)J_4] \\ F_4 &= \sum_{i,j} m_{ij} \, J^i \wedge J^j \end{aligned} \tag{3.21}$$

where the entries of the symmetric matrix m are constants and the diagonal entries are zero. Clearly F_4 is a $(2,2)$ form. Demanding that it is primitive implies that

$$m_{12} = m_{34}, \qquad m_{13} = m_{24}, \qquad m_{14} = m_{23} \ , \tag{3.22}$$

and hence F is self dual, and

$$m_{12} + m_{13} + m_{14} = 0 \ . \tag{3.23}$$

Finally, the master equation (2.10) now implies that

$$l_1 l_2 + l_1 l_3 + l_1 l_4 + l_2 l_3 + l_2 l_4 + l_3 l_4 = 2[(m_{12})^2 + (m_{13})^2 + (m_{14})^2] \ . \tag{3.24}$$

In the special case that one has a T^2 factor, say $l_4 = 0$, one might wonder if one can get a type IIB AdS_3 solution after dimensional reduction and T-duality. Following the discussion at the end of section 2, in order to get an AdS_3 factor one needs that $m_{i4} = 0$ for all i. This implies all the $m_{ij} = 0$ and one returns to the cases analysed in [8].

4 Fibration Constructions using KE spaces: type IIB solutions

In this section we will construct new AdS_3 solutions of type IIB supergravity both with $G = 0$ and $G \neq 0$. For both cases we will take the local six-dimensional dimensional Kähler metric, ds_6^2, to be the product of T^2 with a four dimensional local Kähler metric which is constructed using the line bundle over a two dimensional Kähler Einstein space, which we take to be an S^2. The construction of such Kähler spaces is very similar to the construction in section 3 of [8] which in turn was inspired by [31]. Using this construction we take G to be the wedge product of a $(0,1)$ form on the T^2 with a $(1,1)$ form on the four-dimensional Kähler space. We have presented a few details of the derivation of these solutions in appendix C.

The metric of type IIB supergravity is given by

$$\frac{1}{L^2}ds^2 = \frac{\beta}{y^{1/2}}[ds^2(AdS_3) + ds^2(Y_7)] \tag{4.1}$$

where L is an arbitrary length scale,

$$\begin{aligned} ds^2(Y_7) &= \frac{\beta^2 - 1 + 2y - Q^2y^2}{4\beta^2}Dz^2 + \frac{U(y)}{4(\beta^2 - 1 + 2y - Q^2y^2)}D\psi^2 + \frac{dy^2}{4\beta^2 y^2 U(y)} \\ &\quad + \frac{1}{\beta^2}ds^2(S^2) + \frac{y}{\beta^2}ds^2(T^2) \end{aligned} \tag{4.2}$$

with $D\psi = d\psi + 2V$, $dV = 2J_{S^2}$ and the round metric on S^2, $ds^2(S^2)$, is normalised so that $\mathcal{R}_{S^2} = 4J_{S^2}$. We also have

$$Dz = dz - g(y)D\psi \tag{4.3}$$

with

$$g(y) = \frac{y(1 - Q^2y)}{\beta^2 - 1 + 2y - Q^2y^2} \tag{4.4}$$

and

$$U(y) = 1 - \frac{1}{\beta^2}(1 - y)^2 - Q^2y^2 \tag{4.5}$$

where β, Q are positive constants.

The self-dual five-form can be written

$$F_5 = AdS_3 \wedge F_2 + \omega_5 \tag{4.6}$$

with

$$\frac{1}{L^4}F_2 = \frac{\beta^2(1 - Q^2y)}{2y(\beta^2 - 1 + 2y - Q^2y^2)}dy \wedge D\psi + \frac{\beta^2}{2y^2}dy \wedge Dz + 2J_{S^2} + 2Vol(T^2) \tag{4.7}$$

and

$$\frac{1}{L^4}\omega_5 = -\frac{y(1-Q^2y)}{\beta^2}Vol(T^2)\wedge J_{S^2}\wedge Dz + \frac{U(y)}{(\beta^2-1+2y-Q^2y^2)}Vol(T^2)\wedge J_{S^2}\wedge D\psi$$
$$-\frac{1}{4\beta^2y^2}dy\wedge D\psi\wedge J_{S^2}\wedge Dz - \frac{1}{4\beta^2}Vol(T^2)\wedge dy\wedge D\psi\wedge Dz\ . \qquad (4.8)$$

If we introduce a complex coordinate $u = u^1 + iu^2$ on the T^2 with $ds^2(T^2) = dud\bar{u}$, we can write the three-form flux as

$$\frac{1}{L^2}G = \frac{Q}{\beta}d\bar{u}\wedge\left[\frac{(1-g)}{2}dy\wedge D\psi - \frac{1}{2}dy\wedge Dz + 2yJ_{S^2}\right]\ . \qquad (4.9)$$

We now investigate how to restrict the parameters (β, Q) and choose suitable ranges of the coordinates so that these local solutions can be extended to provide good globally defined solutions. In section 4.1, for $G = 0$, we show that there are an infinite number of solutions of type IIB string theory, labelled by a pair of positive relatively prime integers, p, q, and two integers M, N where Y_7 has topology $S^3\times S^2\times T^2$. The five-form flux is properly quantised an we also calculate the central charge of the corresponding dual CFTs. In section 4.2, for $G\neq 0$, we show that there is a similar infinite class of AdS_3 solutions of type IIB supergravity, but the analysis of the flux quantisation will be studied in [29]. In section 4.3 we show that after two T-dualites and an S-duality all of these solutions get transformed into type IIB solutions with only NS fields being non-trivial.

4.1 Type IIB solutions with $G = 0$

Setting $Q = 0$ so that

$$U(y) = 1 - \frac{1}{\beta^2}(1-y)^2 \qquad (4.10)$$

we choose

$$y_1 \leq y \leq y_2 \qquad (4.11)$$

where y_i are two positive distinct roots of U. The roots of U are given by

$$y_1 = 1-\beta, \qquad y_2 = 1+\beta \qquad (4.12)$$

and we therefore choose $0 < \beta < 1$.

We want to argue, after suitable further restrictions, that $Y_7 = M_5\times T^2$ is the product of a two-torus with a five manifold M_5, parametrised by z, y, ψ and the round S^2. More precisely the manifold M_5 will be a good circle fibration, with the fibre coordinate labelled by z, over a four-dimensional base manifold, B_4, parametrised by

y, ψ and the round S^2. The analysis is very similar to that for the five-dimensional Sasaki-Einstein metrics of [32] (for further dicussion see [33]).

We first observe that if we choose the period of ψ to be 2π, then y, ψ parametrise a smooth two-sphere (in particular, one can check that there are no conical singularities at the poles $y = y_1$ and $y = y_2$) and that B_4 is a smooth manifold which is an S^2 bundle over the round S^2. In fact, topologically, $B_4 = S^2 \times S^2$. To construct M_5 as a circle bundle over B_4, we let z be periodic with period $2\pi l$. We next observe that the norm of the Killing vector ∂_z is non-vanishing and so the size of the S^1 fibre doesn't degenerate. If we write $Dz = dz - A$, we require that $l^{-1}A$ is a connection on a bona fide $U(1)$ fibration. This is guaranteed if the corresponding first Chern class $l^{-1}dA$ lies in the integer cohomology $H^2_{\rm deRahm}(B_4, \mathbb{Z})$. It is straightforward to first check that $l^{-1}dA$ is indeed a globally defined two-form on B_4. We next need to check that periods are integral. A basis for the free part of the homology on B_4 is given by Σ_f, the (y, ψ) two-sphere fibre at a point on the round S^2, and Σ_1, Σ_2, the two-spheres located at the poles $y = y_1$, $y = y_2$, respectively. We note that we have the relation $\Sigma_1 = \Sigma_2 - 2\Sigma_f$ in homology. If we denote the periods for Σ_f and Σ_2 to be integers $-q$ and p, respectively, we conclude that must have

$$\begin{aligned} g(y_2) - g(y_1) &= -lq \\ g(y_2) &= \frac{lp}{2} \,. \end{aligned} \tag{4.13}$$

We note that the period for Σ_1 is then $p + 2q$, consistent with the relation between the two-cycles noted above. These conditions are satisfied if

$$\begin{aligned} \beta &= \frac{q}{p+q} \\ l &= \frac{2(p+q)}{p(p+2q)} \end{aligned} \tag{4.14}$$

with $p, q > 0$. We choose p and q to be relatively prime and then Y_7 is the product of T^2 with a simply connected manifold M_5. By following the argument in [32] we conclude that topologically M_5 is $S^2 \times S^3$.

Recalling that the circle bundle (parametrised by z) is trivial over the two cycle $q\Sigma_2 + p\Sigma_f$ we conclude that setting z to be constant, $q\Sigma_2 + p\Sigma_f$ generates $H_2(M_5, \mathbb{Z})$. We also observe that M_5 has three obvious three-cycles: E_1 and E_2 obtained by fixing $y = y_1$ or $y = y_2$, i.e. the circle bundle over Σ_1 and Σ_2, and the three-cycle E_3 obtained by fixing a point on the round S^2, i.e. the circle bundle over Σ_f. If we let E be the generator of $H_3(M_5, \mathbb{Z})$ we have $E_1 = -pE$, $E_2 = -(p + 2q)E$ and

$E_3 = -qE$. The generator E can be obtained, for example, as the linear combination $E = e_1 E_1 + e_2 E_3$ where e_1 and e_2 are integers satisfying $e_1 p + e_2 q = -1$.

At this stage we have shown that for each pair of relatively prime positive integers, (p, q), we have a regular manifold $Y_7 = M_5 \times T^2$ with $M_5 = S^2 \times S^3$. In order to get a good solution of type IIB string theory we now demand that the five-form flux is properly quantised:

$$N(D) = \frac{1}{(2\pi l_s)^4 g_s} \int_D F_5 \in \mathbb{Z} \tag{4.15}$$

for any five-cycle $D \in H_5(Y_7, \mathbb{Z})$. There are two independent five-cycles, M_5 at a fixed point on T^2 and $S^3 \times T^2$. For the latter, the S^3 factor is the generator E of $H_3(M_5, \mathbb{Z})$, at a fixed point on the T^2. It is illuminating to calculate the flux through the five-cycles $E_i \times T^2$, where the E_i are the three-cycles on M_5 introduced in the last paragraph. After setting

$$\begin{aligned} \frac{L^4}{4\pi g_s l_s^4} &= \frac{q p^2 (p+2q)^2}{(p+q)^4} N \\ Vol(T^2) &= \pi \frac{q(p+q)^2}{p(p+2q)} \frac{M}{N} \end{aligned} \tag{4.16}$$

where M and N are integers, we find that

$$\begin{aligned} \frac{1}{(2\pi l_s)^4 g_s} \int_{M_5} F_5 &= -N \\ \frac{1}{(2\pi l_s)^4 g_s} \int_{E_1 \times T^2} F_5 &= -pM \\ \frac{1}{(2\pi l_s)^4 g_s} \int_{E_2 \times T^2} F_5 &= -(p+2q)M \\ \frac{1}{(2\pi l_s)^4 g_s} \int_{E_3 \times T^2} F_5 &= -qM \ . \end{aligned} \tag{4.17}$$

We see that the results are consistent with the relations in homology between the three-cycles E_i on M_5 that we noted above: in particular the five-form flux through the cycle $E \times T^2$ is M.

We are now in a position to calculate the central charge of the corresponding dual $d = 2$ $(0,2)$ SCFT. Using (3.11) and (3.12) we find that

$$c = 6 \frac{p q^2 (p+2q) N M}{(p+q)^2} \ . \tag{4.18}$$

4.2 Type IIB solutions with $G \neq 0$

Let us now consider the solutions with $Q \neq 0$ and hence non-vanishing G. The roots of U are now given by

$$y_{1,2} = \frac{1 \mp \beta\sqrt{1+Q^2(\beta^2-1)}}{1+Q^2\beta^2} \tag{4.19}$$

and in order that we have two positive distinct roots, $y_2 > y_1 > 0$ we demand that

$$0 < \beta^2 < 1, \qquad 0 \leq Q^2 < \frac{1}{1-\beta^2} \ . \tag{4.20}$$

We will again argue that $Y_7 = M_5 \times T^2$ with M_5 a circle fibration, with the fibre coordinate labelled by z, over a four-dimensional base manifold, B_4, parametrised by y, ψ and the round S^2. To ensure that y, ψ parametrise a two-sphere, remarkably, it is again sufficient to choose ψ to have period 2π. This again leads to a regular B_4, which is again topologically $S^2 \times S^2$. Following the logic of the last subsection, and calculating the periods of $l^{-1}dA/(2\pi)$, to ensure that we have a good circle fibration over B_4 we now impose

$$\begin{aligned} g(y_2) - g(y_1) &= -lq \equiv -(lp)/X \\ g(y_2) &= \frac{lp}{2} \end{aligned} \tag{4.21}$$

for relatively prime integers p and q and we have defined $X = p/q$.

Let us first consider $Q \neq 1$. If $X > 0$ we choose $Q < 1$ and if $-1 < X < 0$ we choose $Q > 1$ (other choices for X lead to the same solutions). We have

$$\begin{aligned} \beta^2 &= \frac{1-Q^2}{(1+X)^2-Q^2} \\ l &= \frac{2((1+X)^2-Q^2)}{p(2+X)(1+X)} \end{aligned} \tag{4.22}$$

and

$$\begin{aligned} y_1 &= \frac{X(1+X+Q^2)}{(1+X)^2-Q^4} \\ y_2 &= \frac{(2+X)(1+X-Q^2)}{(1+X)^2-Q^4} \ . \end{aligned} \tag{4.23}$$

Topologically $M_5 = S^2 \times S^3$. For future reference, we note that as in the last subsection, the generator of $H_2(M\mathbb{Z})$ is given by $q\Sigma_2 + p\Sigma_f$ at fixed z. Also as in the last subsection, M_5 has three natural three-cycles E_i and the generator E of $H_3(M_5, \mathbb{Z})$, is a linear combination of them.

For $Q = 1$ we observe that

$$y_1 = \frac{1-\beta^2}{1+\beta^2}, \qquad y_2 = 1 \ . \tag{4.24}$$

We further observe that $g(y_2) = 0$ and hence we just need to demand that the period of $l^{-1}dA/(2\pi)$ over Σ_f, the two sphere fibre parametrised by y, ψ, is quantised which can be achieved by choosing

$$l = \frac{2}{1-\beta^2} \ . \tag{4.25}$$

For $Q = 1$, the topology of M_5 is again $S^2 \times S^3$, but the details are slightly different, since the z circle is only fibred over Σ_f. For future reference, we can take Σ_2 to generate $H_2(M_5, \mathbb{Z})$ and similarly, we can take the z circle fibred over Σ_f to represent $H_3(M_5, \mathbb{Z})$.

We have now shown that it is possible to switch on the three-form flux and obtain infinite classes of regular geometries. Furthermore, we observe that the five-form and the three-form are globally defined on Y_7.

In order to find good solutions of string theory we need to ensure that the three-form is suitably quantised. Writing $G = -dB - idC^{(2)}$ (since the axion and dilaton are zero), we need to demand that

$$\begin{aligned} \frac{1}{(2\pi l_s)^2 g_s} \int dC^{(2)} &\in \mathbb{Z} \\ \frac{1}{(2\pi l_s)^2} \int dB &\in \mathbb{Z} \ . \end{aligned} \tag{4.26}$$

Due to the Bianchi identity

$$dF_5 = \frac{i}{2} G \wedge G^* \tag{4.27}$$

we also need to ensure that corresponding Page charges (see e.g. [34, 35]) are quantised. We will not carry out this analysis here, but an equivalent analysis will be carried out in [29] using the results of the next subsection.

4.3 T-dual solutions

After carrying out T-dualities along each of the two legs of the T^2, using the formulae in appendix B, we arrive at the following type IIB solutions. The string frame metric is given by

$$\frac{1}{\bar{L}^2} ds^2_\sigma = \frac{\beta}{y^{1/2}} [ds^2(AdS_3) + ds^2(X_7)] \tag{4.28}$$

where

$$\begin{aligned} ds^2(X_7) &= \frac{\beta^2-1+2y-Q^2y^2}{4\beta^2}Dz^2+\frac{U(y)}{4(\beta^2-1+2y-Q^2y^2)}D\psi^2+\frac{dy^2}{4\beta^2y^2U(y)} \\ &+\frac{1}{\beta^2}ds^2(S^2)+(du^1-\frac{Qy}{2\beta}[(1-g)D\psi-Dz])^2+(du^2)^2\ . \end{aligned} \tag{4.29}$$

The dilaton is given by

$$e^{2\phi}=\frac{\beta^2}{y} \tag{4.30}$$

and the RR three-form field strength is

$$\begin{aligned} \frac{1}{\bar{L}^2}dC^{(2)} &= -\frac{1}{4\beta^2}dy\wedge D\psi\wedge Dz-\frac{y}{\beta^2}J\wedge Dz+[\frac{1-yg}{\beta^2}]J\wedge D\psi \\ &+ \frac{Q}{2\beta}du^1\wedge[dy\wedge Dz-4yJ-(1-g)dy\wedge D\psi]+2Vol(AdS_3) \end{aligned} \tag{4.31}$$

Note that $\bar{L}$ is an arbitrary length scale that will be fixed by considering quantisation of the flux.

After a further S-duality transformation we obtain AdS_3 solutions with only NS fields non-vanishing, but we will continue to work with the above solution.

For these solutions to be good solutions of type IIB string theory we need to ensure that the metric extends to a metric on a globally defined manifold X_7 and that both the electric and magnetic RR three-form charges are properly quantised:

$$n_1=\frac{1}{(2\pi l_s)^6g_s}\int_{X_7}*dC^{(2)}\in\mathbb{Z} \tag{4.32}$$

and

$$\frac{1}{(2\pi l_s)^2g_s}\int_T dC^{(2)}\in\mathbb{Z} \tag{4.33}$$

when integrated over any three-cycle $T\in H_3(X_7,\mathbb{Z})$.

It is useful to note that since

$$\frac{1}{\bar{L}^6}*dC^{(2)}=\frac{1}{4\beta^2y^2}J\wedge dy\wedge D\psi\wedge Dz\wedge du^1\wedge du^2+Vol(AdS_3)\wedge(\dots) \tag{4.34}$$

we have

$$n_1=\left(\frac{\bar{L}}{l_s}\right)^6\frac{l}{g_s64\pi^3\beta^2}\Delta u^1\Delta u^2\frac{y_2-y_1}{y_1y_2}\ . \tag{4.35}$$

Thus, for any good solution of type IIB string theory, the central charge can then be written

$$c=6n_1\left(\frac{\bar{L}}{l_s}\right)^2\frac{1}{g_s}\ . \tag{4.36}$$

To get the explicit expression we need the values of Δu^1, Δu^2 and $\bar{L}^2$. In this paper we will only analyse this further for the case of $Q = 0$, recovering results compatible with those of the last subsection. The analysis for the case of $Q \neq 0$ will be carried out in [29].

4.3.1 $Q = 0$

When $Q = 0$, we first observe that $ds^2(X_7)$ is precisely the same as $ds^2(Y_7)$ in (4.2). In section 4.1 we showed that $X_7 = M_5 \times T^2$ where M_5 is a manifold parametrised by z, ψ, y and the round S^2 and the T^2 is parametrised by u^1 and u^2. Further $M_5 = S^2 \times S^3$.

Let us now consider the quantisation of the three-form on X_7. After fixing a point on the torus, the three-cycles E_i on M_5, introduced in section 4.1, all give rise to three cycles on X_7. If we choose the length scale to satisfy

$$\frac{1}{g_s}\left(\frac{\bar{L}}{l_s}\right)^2 = \frac{pq^2(p+2q)M}{(p+q)^2} \tag{4.37}$$

where M is an integer then we calculate

$$\begin{aligned}
\frac{1}{(2\pi l_s)^2 g_s}\int_{E_1} dC^{(2)} &= -pM \\
\frac{1}{(2\pi l_s)^2 g_s}\int_{E_2} dC^{(2)} &= -(p+2q)M \\
\frac{1}{(2\pi l_s)^2 g_s}\int_{E_3} dC^{(2)} &= -qM\ .
\end{aligned} \tag{4.38}$$

In particular we see that the flux through the generator of $H_3(X_7, \mathbb{Z})$, the three-cycle E introduced in section 4.1 at a fixed point on the torus, is M.

The expression (4.35) takes the more explicit form

$$n_1 = \left(\frac{\bar{L}}{l_s}\right)^6 \frac{1}{g_s 16\pi^3} Vol(T^2) \frac{(p+q)^4}{p^2 q(p+2q)^2} \tag{4.39}$$

which, after substituting (4.37), provides a quantisation condition on $Vol(T^2)$. For the central charge, after substituting (4.37) into (4.36), we now recover the previous result (4.18) (with $N = n_1$), as expected.

The fluxes that we have activated, plus the amount of supersymmetry preserved, suggests that the dual SCFT might arise by taking configurations of fundamental strings intersecting NS fivebranes with the other four directions of the NS fivebranes wrapped on a holomorphic four-cycles inside a Calabi-Yau four-fold.

4.3.2 $Q \neq 0$

A careful analysis of the topology of X_7 and the quantisation of the three-form flux when $Q \neq 0$ will be carried out in [29].

5 Fibration Constructions using KE spaces: $D = 11$ solutions

In this section we will present new AdS_2 solutions of $D = 11$ supergravity with magnetic four-form flux switched on. We take the local eight-dimensional dimensional Kähler metric, ds_8^2, to be the product of T^2 with a six-dimensional local Kähler metric which is constructed using the line bundle over a four dimensional Kähler Einstein space with positive curvature. We have presented a few details of the derivation of these solutions in appendix D.

The metric of $D = 11$ supergravity is given by

$$\frac{1}{L^2} ds^2 = \frac{1}{6^{4/3} \beta^{2/3} y^{4/3}} [ds^2(AdS_2) + ds^2(Y_9)] \tag{5.1}$$

where L is an arbitrary length scale,

$$\begin{aligned} ds^2(Y_9) = \quad & (1 - 8\beta y + 12\beta y^2 - 4\beta Q y^4) Dz^2 + \frac{4\beta y U(y)}{(1 - 8\beta y + 12\beta y^2 - 4\beta Q y^4)} D\psi^2 \\ & + \frac{9\beta}{y U(y)} dy^2 + 36\beta y ds^2(KE_4^+) + 36\beta y^2 ds^2(T^2) \end{aligned} \tag{5.2}$$

with $D\psi = d\psi + 2V$, $dV = 2J_{S^2}$ and the metric on the four-dimensional positively curved Kähler-Einstein space, $ds^2(KE_4^+)$, is normalised so that $\mathcal{R}_{KE} = 6 J_{KE}$. We also have

$$Dz = dz - g(y) D\psi \tag{5.3}$$

with

$$g(y) = -\frac{2\beta y (1 - 3y + 2Qy^3)}{1 - 8\beta y + 12\beta y^2 - 4\beta Q y^3} \tag{5.4}$$

and

$$U(y) = 1 - 9\beta y (1 - y)^2 - Q y^3 \tag{5.5}$$

with β, Q constants.

Writing the four-form as

$$G_4 = AdS_2 \wedge F_2 + F_4 \tag{5.6}$$

we have

$$\frac{1}{L^3}F_2 = -J_{KE} - \frac{2}{y^3}dy\wedge Dz + \frac{2g}{y^3}dy\wedge D\psi - \frac{i}{2}du\wedge d\bar{u} \tag{5.7}$$

and

$$\begin{aligned}\frac{1}{L^3}F_4 &= 6\beta^{1/2}Q\Bigg(2J_{KE}\wedge J_{KE} + \frac{1}{3}[(1-g)D\psi - Dz]\wedge J_{KE}\wedge dy \\ &\quad -2iy^2 J_{KE}\wedge du\wedge d\bar{u} - \frac{iy}{3}dy\wedge[(1-g)D\psi - Dz]\wedge du\wedge d\bar{u}\Bigg)\,. \end{aligned}\tag{5.8}$$

We will not carry out a complete analysis of these solutions, but it is clear that there are infinitely many new regular solutions. As in the last section, the task is to choose appropriate values of the constants β, Q and ranges of the coordinates so that Y_9 is a $U(1)$ fibration, with fibre parametrised by z, over an eight dimensional base manifold, parametrised by ψ, y, the KE_4^+ space and the two-torus. By choosing appropriate β, Q we can restrict y to lie between two suitable roots of the cubic $U = 0$. One can then show that if ψ has period 2π, then, remarkably, the eight-dimensional base manifold is a regular S^2 bundle, with S^2 parametrised by y, ψ, over $KE_4^+ \times T^2$. Demanding that the $U(1)$ fibration is well defined, for appropriately chosen period for z, will lead to additional restrictions on the parameters, but it is clear that there will be infinite number of solutions. Finally, there will be additional restrictions imposed by demanding that the four-form flux Page charges are suitably quantised.

We conclude this section by pointing out that when $F_4 = 0$, i.e. when $Q = 0$, if we dimensionally reduce on one leg of the T^2 and T-dualise on the other, we obtain type IIB AdS_3 solutions as constructed in [28] (see appendix A and section 3.1 of [8]). However, when $F_4 \neq 0$, while we still get type IIB solutions, because F_4 has a term proportional to the volume of the torus, the metric will no longer be a warped product of AdS_3 with a seven manifold.

6 Conclusions

We have analysed new general classes of supersymmetric AdS_3 solutions of type IIB supergravity and AdS_2 solutions of $D = 11$ supergravity, which are dual to SCFTs with $(0,2)$ supersymmetry in $d = 2$ and supersymmetric quantum mechanics with two supercharges, respectively. The constructions which generalise those of [1, 2] to allow for additional fluxes, depend crucially on the "transgression terms" appearing in the Bianchi identities.

We also presented a rich set of new explicit examples using some constructions that generalise those of [8]. For the type IIB AdS_3 solutions we found an infinite class of solutions with vanishing three-form flux in section 3.1 and determined the central charge of the dual SCFT. In section 4 we presented a different class of explicit solutions of type IIB, with the three-form flux labelled by Q. The solutions have a two-torus and after two T-dualities and an S-duality we showed that the solutions can be written in terms of NS fields only. For the case when $Q = 0$ we showed that the solutions extend to well defined solutions of type IIB string theory and we calculated the corresponding central charge. The analysis for the case of $Q \neq 0$ will be carried out in [29].

We also constructed analogous AdS_2 solutions of $D = 11$ supergravity. It would worthwhile carefully analysing the conditions required on the local solutions to give rise to properly quantised solutions of M-theory.

Despite the richness of the constructions we have presented, it is clear that they can be generalised still further. For example, the $D = 11$ solutions in section 5 are constructed using a four-dimensional Kähler-Einstein manifold. For the special case when this is $S^2 \times S^2$ there are almost certainly generalisations when we allow the ratio of the curvatures of the two S^2's to vary.

It remains an important outstanding problem to identify the dual SCFTs for all of these examples. For the classes of type IIB AdS_3 solutions that depend on NS fields only, it would also be very interesting to construct the worldsheet CFT describing the type IIB solutions.

We also showed how the general class of AdS solutions can be analytically continued to obtain general classes of 1/8 BPS bubble solutions with additional fluxes to the classes of solutions considered in [8]. It would be interesting to study these further. For example, the constructions of this paper can be used to obtain explicit solutions.

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A AdS solutions

A.1 AdS_3 solutions of type IIB supergravity

We will be interested in bosonic configurations of type IIB supergravity with constant axion and dilaton. For simplicity we will mostly set the axion and dilaton to zero. We will use the conventions for type IIB supergravity that were used in [36]. The conditions for such a configuration to be supersymmetric read:

$$\begin{aligned} \nabla_M \epsilon - \frac{1}{96}\left(\Gamma_M{}^{P_1P_2P_3} G_{P_1P_3P_3} - 9\Gamma^{P_1P_2} G_{MP_1P_2}\right)\epsilon^c & \\ + \frac{i}{16\cdot 5!}\Gamma^{M_1\ldots M_5} F_{M_1\ldots M_5}\Gamma_M\epsilon &= 0, \qquad (A.1) \\ \Gamma^{P_1P_2P_3} G_{P_1P_2P_3}\epsilon &= 0, \qquad (A.2) \end{aligned}$$

where F_5 is self-dual, $F_5 = *_{10}F_5$ and the complex three-form G can be written[3]

$$\begin{aligned} G &= ie^{\phi/2}\left(\tau dB - dC^{(2)}\right), \\ \tau &= C^{(0)} + ie^{-\phi}. \qquad (A.3) \end{aligned}$$

We have also chosen $\Gamma_{11}\epsilon = -\epsilon$ where $\Gamma_{11} = \Gamma_0\ldots\Gamma_9$. and we take $\epsilon_{0\ldots9} = +1$. To obtain a supersymmetric solution to the equations of motion it is sufficient [36] to also impose

$$\begin{aligned} \nabla^P G_{MNP} &= -\frac{i}{6}F_{MNP_1P_2P_3}G^{P_1P_2P_3} \qquad (A.4) \\ G_{P_1P_2P_3}G^{P_1P_2P_3} &= 0 \qquad (A.5) \\ dG &= 0 \qquad (A.6) \\ dF &= \frac{i}{2}G\wedge G^* \qquad (A.7) \end{aligned}$$

and at most one component of the Einstein equations, which is automatically solved for the classes of solutions we consider.

We now introduce the following ansatz

$$\begin{aligned} ds^2 &= e^{2A}ds^2\left(AdS_3\right) + ds_7^2, \\ F_5 &= \left(1 + *_{10}\right)Vol(AdS_3)\wedge F_2, \qquad (A.8) \end{aligned}$$

[3]If one changes the sign of $C^{(2)}$ one gets the conventions used in [38] .

as in [1], but generalised to include a closed three form G defined on the seven dimensional space. We also demand that the Killing spinors are the same as those for the AdS_3 solutions with $G = 0$ that were analysed in [1].

For the gamma matrices we take

$$\begin{aligned} \Gamma_\mu &= \sigma_1 \otimes \boldsymbol{I}_{8\times 8} \otimes \tau_\mu, \quad \mu = 0, 1, 2 \\ \Gamma_a &= \sigma_2 \otimes \gamma_a \otimes \boldsymbol{I}_{2\times 2}, \quad a = 3, \ldots, 9 \end{aligned} \tag{A.9}$$

where σ_i are Pauli matrices and we choose the three-dimensional and seven dimensional gamma matrices τ_μ and γ_a, respectively, to satisfy

$$\begin{aligned} \tau_0\tau_1\tau_2 &= -\boldsymbol{I}_{2\times 2}, \\ \prod_a \gamma_a &= -i\boldsymbol{I}_{8\times 8}. \end{aligned} \tag{A.10}$$

For the Killing spinor ϵ we make the ansatz

$$\epsilon = \chi \otimes \eta \otimes \psi_n^{(i)} \tag{A.11}$$

where χ is a constant spinor satisfying

$$\sigma_3\chi = \chi \tag{A.12}$$

$\psi_n^{(i)}$ are Killing spinors on AdS_3 satisfying

$$\hat{\nabla}_\mu \psi_n^{(i)} = \frac{n}{2}\tau_\mu \psi_n^{(i)}, \quad n = \pm 1,\ i = 1, 2, \tag{A.13}$$

and η is a seven dimensional Dirac spinor. After substituting into (A.1) we find the following system of equations

$$\nabla_a \eta - \frac{1}{16} e^{-3A} \not{F}_2 \gamma_a \eta = 0 \tag{A.14}$$

$$\left(\frac{n}{2} e^{-A} + \frac{i}{2} \not{\partial} A + \frac{i}{16} e^{-3A} \not{F}_2 \right) \eta = 0 \tag{A.15}$$

$$\gamma^{p_2p_3} G^*_{ap_2p_3} \eta = 0 \tag{A.16}$$

$$\gamma^{p_1p_2p_3} G_{p_1p_2p_3} \eta = 0. \tag{A.17}$$

As shown in [1], by just using equations (A.14) and (A.15), the geometry and five form flux are constrained to take the local form

$$\begin{aligned} ds^2 &= e^{2A}\left[ds^2\left(AdS_3\right) + \frac{1}{4}\left(dz + P\right)^2 + e^{-4A} ds_6^2 \right] \\ F_2 &= 2nJ - \frac{1}{2} d\left[e^{4A}\left(dz + P\right) \right] \end{aligned} \tag{A.18}$$

where ∂_z is a Killing vector, ds_6^2 is a six dimensional Kähler metric with Kähler form J, Ricci form given by $\mathcal{R} = n\, dP$, scalar curvature $R = 8e^{-4A}$ and holomorphic three form Ω. This result is obtained by analysing various bilinears in η. In particular we note that $\eta^\dagger\eta = e^A$, $\Omega = e^{2A}e^{inz}\eta^T\gamma_{(3)}\eta$ and $J = -e^A i\eta^\dagger\gamma_{(2)}\eta$. Furthermore, $K \equiv \eta^\dagger\gamma_{(1)}\eta = (e^{2A}/2)(dz + P)$, so that the corresponding dual vector is the Killing vector $2\partial_z$. It is also useful to note that $\Psi \equiv \eta^T\gamma_{(4)}\eta = -e^{-3A}e^{-niz}K \wedge \Omega$.

We next argue that

$$i_K G = 0. \tag{A.19}$$

To see this we first multiply (A.16) by $\eta^T\gamma_k\gamma_a$ and (A.17) by $\eta^T\gamma_k$ to deduce that

$$\Omega_k{}^{p_1p_2}(i_K G)_{p_1p_2} = 0, \qquad \bar{\Omega}_k{}^{p_1p_2}(i_K G)_{p_1p_2} = 0\ . \tag{A.20}$$

This shows that the $(0,2)$ and $(0,2)$ pieces of $i_K G$ vanish. Next multiplying (A.16) by $\eta^T\gamma_{q_1q_2q_3}$ we deduce that

$$\bar{\Omega}^p{}_{[q_1q_2}G_{q_3]pr} = 0\ . \tag{A.21}$$

Letting q_1 be just in the z direction we deduce that

$$\bar{\Omega}^p{}_{q_1q_2}(i_K G)_{pr} = 0 \tag{A.22}$$

showing that the $(1,1)$ piece of $i_K G$ also vanishes.

Since $i_K G = 0$ we can now decompose G in terms of (p,q) forms on $\mathcal{B}_6$

$$G = G^{(1,2)} + G^{(2,1)} + G^{(3,0)} + G^{(0,3)}.$$

From equations (A.16) and (A.17) we obtain

$$\Omega^{p_1p_2p_3}G_{p_1p_2p_3} = 0, \qquad \bar{\Omega}^{p_1p_2}{}_a G_{p_1p_2b} = 0, \tag{A.23}$$

implying that only the $(1,2)$ component of the three form G can be non-zero. From equation (A.16) we have that

$$J \wedge G = 0. \tag{A.24}$$

Thus we conclude that supersymmetry implies that the $(1,2)$ form G is primitive. These two properties when combined give the duality condition on the base $\mathcal{B}_6$

$$*_6\, G = iG, \tag{A.25}$$

where we used the volume form

$$\mathrm{Vol}_6 = \frac{1}{6}\, J \wedge J \wedge J. \tag{A.26}$$

We can now easily check that (A.4) and (A.5) are both satisfied.

Thus to ensure that all equations of motion are satisfied we just need to ensure that (A.7) holds. Using (A.25) we find that (A.7) can be written as

$$\frac{1}{16}J\wedge\mathcal{R}\wedge\mathcal{R}+\frac{1}{32}d*_6 dR=-\frac{1}{8}G\wedge *_6 G^*, \tag{A.27}$$

which may also be written as a scalar equation

$$\Box R-\frac{1}{2}R^2+\mathcal{R}^{ij}\mathcal{R}_{ij}+\frac{2}{3}G^{ijk}G^*_{ijk}=0. \tag{A.28}$$

Note that in the main text we have fixed n to be $+1$. The solution preserves four supersymmetries since i runs from 1 to 2 in the AdS_3 Killing spinors $\psi_n^{(i)}$ appearing in (A.11) and η is a Dirac spinor. Two of these are Poincaré supersymmetries and two are special conformal supersymmetries. Using horospherical coordinates, the Poincaré Killing spinors on AdS_3 are eigenvalues of the gamma matrix along the radial direction, say τ_2 [37]. Observing that $\Gamma_{01}=-\boldsymbol{I}_{2\times2}\otimes\boldsymbol{I}_{8\times8}\otimes\tau_2$ we see that the two Poincaré supersymmetries are eigenvalues of Γ_{01} with the same eigenvalue and hence the solutions are dual to SCFTs with $(0,2)$ supersymmetry.

A.2 AdS_2 solutions of $D=11$ supergravity

The condition for a bosonic configuration of $D=11$ supergravity to be supersymmetric reads

$$\delta\psi_M=\nabla_M\epsilon+\frac{1}{288}\left[\Gamma_M{}^{N_1N_2N_3N_4}-8\delta_M^{N_1}\Gamma^{N_2N_3N_4}\right]G_{4N_1N_2N_3N_4}\epsilon=0, \tag{A.29}$$

where we are using the conventions of [39] and in particular $\Gamma_{0\ldots10}=1$ and $\epsilon_{0\ldots10}=+1$. For the supersymmetric bosonic configurations we will be considering, in order that all equations of motion are satisfied it is sufficient [39] to also just demand that

$$\begin{aligned} dG_4 &= 0,\\ d*_{11}G_4 &= -\frac{1}{2}G_4\wedge G_4. \end{aligned} \tag{A.30}$$

Our AdS_2 ansatz is

$$\begin{aligned} ds^2&=e^{2A}ds^2\left(AdS_2\right)+ds_9^2,\\ G_4&=\mathrm{Vol}(AdS_2)\wedge F_2+F_4, \end{aligned} \tag{A.31}$$

where F_2 and F_4 are closed forms defined on the nine dimensional space. For the gamma matrices we perform the reduction

$$\begin{aligned} \Gamma_\mu&=\tau_\mu\otimes\boldsymbol{I},\quad \mu=0,1\\ \Gamma_a&=\tau_2\otimes\gamma_a,\quad a=2,\ldots,10 \end{aligned} \tag{A.32}$$

with τ and γ being real matrices and we use the conventions

$$\begin{aligned}\tau_0\tau_1\tau_2 &= -1,\\ \prod_a \gamma_a &= -1.\end{aligned} \tag{A.33}$$

In this representation we can make the ansatz for the eleven dimensional Majorana spinor

$$\epsilon = \chi_n^{(i)} \otimes \eta + \text{c.c.} \tag{A.34}$$

where the η is a nine-dimensional Dirac spinors and the real three-dimensional spinor $\chi_n^{(i)}$ satisfies

$$\hat{\nabla}_\mu \chi_n^{(i)} = \frac{in}{2}\tau_\mu\tau_2\chi_n^{(i)}, \quad i = \pm 1,\, n = \pm 1, \tag{A.35}$$

and can be taken to satisfy the orthogonality condition

$$(\chi_n^{(i)})^\dagger \tau_2 \chi_n^{(i)} = 0. \tag{A.36}$$

(which can be checked, for example, by explicitly calculating the spinors).

We now find the following system of equations

$$\left[\nabla_a + \frac{1}{24}e^{-2A}\left(\gamma_a{}^{bc}F_{2bc} - 4F_{2ab}\gamma^b\right)\right]\eta = 0 \tag{A.37}$$

$$\left[ine^{-A} + \gamma^a\partial_a A - \frac{1}{6}e^{-2A}\gamma^{ab}F_{2ab}\right]\eta = 0 \tag{A.38}$$

$$\gamma^{b_1b_2b_3}F_{4ab_1b_2b_3}\eta = 0. \tag{A.39}$$

Using the results of [2] one can show that equations (A.37) and (A.38) imply that the metric and the two form flux are constrained to be of the form

$$ds^2 = e^{2A}\left[ds^2\left(AdS_2\right) + \left(dz + P\right)^2 + e^{-3A}ds_8^2\right], \tag{A.40}$$

$$F_2 = nJ + d\left[e^{3A}\left(dz + P\right)\right], \tag{A.41}$$

where $\mathcal{R} = -ndP$ and ds_8^2 is Kähler with Kähler form J, Ricci potential given by P and scalar curvature given by $R = 2e^{-3A}$.

The constraint (A.39) implies that the only non-zero part of the magnetic component F_4 is a $(2,2)$ and primitive form with no non-zero components along the z direction:

$$J \wedge F_4 = 0, \tag{A.42}$$

$$i_K F_4 = 0. \tag{A.43}$$

Here K is the one-form constructed out of the nine dimensional bilinears $K = \eta^\dagger \gamma_{(1)} \eta = e^{2A}(dz + P)$ whose dual is the Killing vector ∂_z. Note that these conditions imply that the four form is also self-dual with respect to ds_8^2:

$$*_8 F_4 = F_4. \tag{A.44}$$

Using that the $D = 11$ epsilon tensor is given by $\epsilon = -e^{-A} Vol(AdS_2)(dz + P)\frac{J^4}{4!}$, we find that the equation of motion for the four form (A.30) implies that

$$J^2 \wedge \mathcal{R} \wedge \mathcal{R} + d *_8 dR = F_4 \wedge F_4. \tag{A.45}$$

which may also be written as a scalar equation

$$\Box R - \frac{1}{2}R^2 + \mathcal{R}^{ij}\mathcal{R}_{ij} + \frac{1}{4!}F_{ijkl}F^{ijk} = 0. \tag{A.46}$$

B T-duality

We consider a type IIB solution with a square two-torus, parametrised by u^1 and u^2, of the form

$$\begin{aligned} ds^2 &= e^{2A}\left[ds^2(AdS_3) + ds^2(M_5) + \Sigma((du^1)^2 + (du^2)^2)\right] \\ F_5 &= f_5 + f_3 \wedge du^1 \wedge du^2 \\ G &= (du^1 - i du^2) \wedge dv \\ \phi &= 0, \qquad C^{(0)} = 0 \end{aligned} \tag{B.1}$$

where f_5, f_3, v and $ds^2(M_5)$ have no dependence on the coordinates u^i. Using the formulae in, for example, [40] we can T-dualise on the u^1 direction and then the u^2 direction to get the following type IIB solution

$$\begin{aligned} ds_\sigma^2 &= e^{2A}\left[ds^2(AdS_3) + ds^2(M_5)\right] + \frac{1}{\Sigma e^{2A}}\left[(du^1 - v)^2 + (du^2)^2\right] \\ dC^{(2)} &= f_3 - dv \wedge (du^1 - v) \\ e^{2\phi} &= \frac{1}{\Sigma^2 e^{4A}} \end{aligned} \tag{B.2}$$

where the metric, here, is written in the string frame.

C Type IIB solutions from fibrations over $S^2 \times T^2$

Consider the following ansatz for a six dimensional Kähler metric

$$ds_6^2 = \frac{dx^2}{4x^3 U(x)} + \frac{U(x)}{x} D\phi^2 + \frac{1}{x} ds^2\left(S^2\right) + du\, d\bar{u}, \tag{C.1}$$

where $D\phi = d\phi + V$, $dV = 2J_{S^2}$, the S^2 is normalised so that $\mathcal{R}_{S^2} = 4J_{S^2}$ and we have introduced a complex coordinate $u = u^1 + iu^2$ for a T^2 factor. In this case the Kähler form J and the $(3,0)$ form Ω read

$$\begin{aligned} J &= -\frac{1}{2x^2} dx \wedge D\phi + \frac{1}{x} J_{S^2} + \frac{i}{2}\, du \wedge d\bar{u}, \\ \Omega &= e^{2i\phi} \left[-\frac{1}{2x^2\sqrt{U}} dx + i\frac{\sqrt{U}}{x} D\phi \right] \wedge \Omega_{S^2} \wedge du. \end{aligned} \tag{C.2}$$

We have $d\Omega = iP \wedge \Omega$ where P is the Ricci form given by

$$P = fD\phi, \qquad f = 2(1-U) + xU'. \tag{C.3}$$

It is easy to calculate the Ricci form, given by $\mathcal{R} = dP$, and we record that the Ricci scalar is given by

$$R = 4xf - 4x^2 f'. \tag{C.4}$$

For the three form G we make the simple ansatz that it is the wedge product of $d\bar{u}$ with a primitive $(1,1)$ form on the four-dimensional Kähler space parametrised by x, ϕ and the S^2. This leads us to consider

$$G = d\bar{u} \wedge d\left[qxD\phi\right] \ . \tag{C.5}$$

If we now substitute into (2.5), after integrating once, we are led to the following differential equation for U:

$$2f^2 + U\,R' + 8q^2x^2 = constant. \tag{C.6}$$

We look for polynomial solutions to this equation by considering the ansatz $U(x) = 1 + \sum_{i=0}^{2} a_i x^i$. This implies that $R = -8a_0 x$ and in order to have $R > 0$ we choose $a_0 = -1/\beta^2$. A little calculation shows that U takes the form

$$U(x) = 1 - \frac{1}{\beta^2}\left(1 - \frac{a_1\beta^2}{2}x\right)^2 - q^2\beta^2x^2. \tag{C.7}$$

It is now straightforward to assemble the full ten-dimensional solution using (2.1)-(2.4). It is convenient to make the following rescalings

$$y = \frac{a_1\beta^2}{2}x, \qquad Q = \frac{2}{a_1\beta}q, \qquad \tilde{u} = \frac{\sqrt{2}}{\beta\sqrt{a}_1}u \ . \tag{C.8}$$

Furthermore we also perform a simultaneous scaling of the ten-dimensional metric and the three-form by a factor of $\frac{\sqrt{2}}{\beta\sqrt{a}_1}$ and the five-form by a factor of $\frac{2}{\beta^2 a_1}$ (which indeed transforms a solution to another solution). Finally, it is very helpful to perform the coordinate change $\phi = (\psi - z)/2$ and this then leads to the type IIB solutions as recorded in the main text, although we note that we have dropped the tildes form the coordinates on the torus for clarity.

D $D = 11$ solutions from fibrations over $KE_4^+ \times T^2$

Consider the following ansatz for an eight dimensional Kähler metric

$$ds_8^2 = \frac{dx^2}{4x^3U(x)} + \frac{U(x)}{x}D\phi^2 + \frac{1}{x}ds^2\left(KE_4^+\right) + du\, d\bar{u}, \tag{D.1}$$

where $D\phi = d\phi + V$, $dV = 2J_{KE}$, the Kähler-Einstein four metric with positive curvature, $ds^2(KE_4^+)$, is normalised so that $\mathcal{R}_{S^2} = 6J_{S^2}$ and $u = u^1 + iu^2$ is a complex coordinate for a T^2 factor. In this case the Kähler form J and the $(4,0)$ form Ω read

$$\begin{aligned} J &= -\frac{1}{2x^2}dx \wedge D\phi + \frac{1}{x}J_{KE} + \frac{i}{2}\, du \wedge d\bar{u}, \\ \Omega &= e^{3i\phi}\left[-\frac{1}{2x^{5/2}\sqrt{U}}dx + i\frac{\sqrt{U}}{x^{3/2}}D\phi\right] \wedge \Omega_{KE} \wedge du. \end{aligned} \tag{D.2}$$

We have $d\Omega = iP \wedge \Omega$ where P is the Ricci form given by

$$P = fD\phi, \qquad f = 3(1-U) + xU'. \tag{D.3}$$

It is easy to calculate the Ricci form, given by $\mathcal{R} = dP$, and we record that the Ricci scalar is given by

$$R = 8xf - 4x^2f'. \tag{D.4}$$

For the magnetic four form, F_4, we choose the ansatz:

$$\begin{aligned} F_4 &= A_2 \wedge (J_6 - \frac{i}{2}du \wedge d\bar{u}) \\ &= A_2 \wedge (-\frac{1}{2x^2}dx \wedge D\phi + \frac{1}{x}J_{KE} - \frac{i}{2}du \wedge d\bar{u}) \end{aligned} \tag{D.5}$$

where J_6 is the Kähler form on the six space excluding the torus. We clearly have that F_4 is $(2,2)$ and is closed provided that the two-form A_2 is $(1,1)$ and closed. A suitable ansatz is $A_2 = d[\Phi(x)D\phi]$ and we find that F_4 is primitive provided that $\Phi = qx^2$ for an arbitrary constant q. We thus have

$$F_4 = d[qx^2D\phi] \wedge (-\frac{1}{2x^2}dx \wedge D\phi + \frac{1}{x}J_{KE} - \frac{i}{2}du \wedge d\bar{u})\ . \tag{D.6}$$

If we now substitute into (2.10), after integrating once, we are led to the following differential equation for U:

$$4f^2 + U\,R' + 4q^2x^4 = constant \times x. \tag{D.7}$$

We look for polynomial solutions to this equation by considering the ansatz $U(x) = \sum_{i=0}^{3} a_i x^i$. We find two classes of solutions, one with $a_0 = 1$ and the other with $a_0 = 3$. Since we are interested here in AdS_2 solutions, we only consider the solution with $a_0 = 1$ and we have

$$U(x) = 1 + a_1 x \left(1 + \frac{a_2}{2a_1} x\right)^2 + \frac{q^2}{4a_1} x^3 \ . \tag{D.8}$$

Since $R = -8a_1 x^2$, we demand that $a_1 < 0$.

It is now straightforward to assemble the full eleven-dimensional solution using (2.6)-(2.9). It is convenient to make the following rescalings

$$y = \frac{-a_2}{2a_1} x, \qquad Q = \frac{2a_1^2}{a_2^3} q^2, \qquad \tilde{u} = \frac{\sqrt{-2a_1}}{\sqrt{a_2}} u \ . \tag{D.9}$$

We also define $\beta = \frac{2a_1^2}{9a_2}$. Furthermore we also perform a simultaneous scaling of the eleven-dimensional metric by a factor of $\left(\frac{2(-a_1)}{a_2}\right)^{2/3}$ and the four-form by a factor of $\frac{2(-a_1)}{a_2}$ (which indeed transforms a solution to another solution). Finally, it is very helpful to perform the coordinate change $\phi = (\psi - z)/3$ and this then leads to the $D = 11$ solutions as recorded in the main text, although we note that we have dropped the tildes form the coordinates on the torus for clarity.

References

[1] N. Kim, "AdS(3) solutions of IIB supergravity from D3-branes," JHEP **0601**, 094 (2006) [arXiv:hep-th/0511029].

[2] N. Kim and J. D. Park, "Comments on AdS(2) solutions of D = 11 supergravity," JHEP **0609** (2006) 041 [arXiv:hep-th/0607093].

[3] J. P. Gauntlett and N. Kim, "Geometries with Killing Spinors and Supersymmetric AdS Solutions," arXiv:0710.2590 [hep-th].

[4] I. R. Klebanov and M. J. Strassler, "Supergravity and a confining gauge theory: Duality cascades and chiSB-resolution of naked singularities," JHEP **0008** (2000) 052 [arXiv:hep-th/0007191].

[5] I. R. Klebanov and A. A. Tseytlin, "Gravity duals of supersymmetric SU(N) x SU(N+M) gauge theories," Nucl. Phys. B **578** (2000) 123 [arXiv:hep-th/0002159].

[6] M. Grana and J. Polchinski, "Supersymmetric three-form flux perturbations on AdS(5)," Phys. Rev. D **63** (2001) 026001 [arXiv:hep-th/0009211].

[7] M. Cvetic, H. Lu and C. N. Pope, "Brane resolution through transgression," Nucl. Phys. B **600**, 103 (2001) [arXiv:hep-th/0011023].

[8] J. P. Gauntlett, N. Kim and D. Waldram, "Supersymmetric AdS(3), AdS(2) and bubble solutions," JHEP **0704** (2007) 005 [arXiv:hep-th/0612253].

[9] H. Lin, O. Lunin and J. M. Maldacena, "Bubbling AdS space and 1/2 BPS geometries," JHEP **0410** (2004) 025 [arXiv:hep-th/0409174].

[10] S. Yamaguchi, "Bubbling geometries for half BPS Wilson lines," Int. J. Mod. Phys. A **22** (2007) 1353 [arXiv:hep-th/0601089].

[11] O. Lunin, "On gravitational description of Wilson lines," JHEP **0606** (2006) 026 [arXiv:hep-th/0604133].

[12] J. Gomis and C. Romelsberger, "Bubbling defect CFT's," JHEP **0608** (2006) 050 [arXiv:hep-th/0604155].

[13] A. Donos, "A description of 1/4 BPS configurations in minimal type IIB SUGRA," Phys. Rev. D **75** (2007) 025010 [arXiv:hep-th/0606199].

[14] A. Donos, "BPS states in type IIB SUGRA with SO(4) x SO(2)(gauged) symmetry," JHEP **0705** (2007) 072 [arXiv:hep-th/0610259].

[15] E. G, G. Milanesi, K. S. Narain and M. O'Loughlin, "1/8 BPS states in AdS/CFT," JHEP **0705** (2007) 030 [arXiv:hep-th/0611065].

[16] O. A. P. Mac Conamhna and E. O Colgain, "Supersymmetric wrapped membranes, AdS(2) spaces, and bubbling geometries," JHEP **0703** (2007) 115 [arXiv:hep-th/0612196].

[17] B. Chen *et al.*, "Bubbling AdS and droplet descriptions of BPS geometries in IIB supergravity," JHEP **0710** (2007) 003 [arXiv:0704.2233 [hep-th]].

[18] O. Lunin, "1/2-BPS states in M theory and defects in the dual CFTs," JHEP **0710** (2007) 014 [arXiv:0704.3442 [hep-th]].

[19] E. D'Hoker, J. Estes and M. Gutperle, "Gravity duals of half-BPS Wilson loops," JHEP **0706** (2007) 063 [arXiv:0705.1004 [hep-th]].

[20] H. Kim, K. K. Kim and N. Kim, "1/4-BPS M-theory bubbles with SO(3) x SO(4) symmetry," JHEP **0708** (2007) 050 [arXiv:0706.2042 [hep-th]].

[21] J. P. Gauntlett and O. A. P. Mac Conamhna, "AdS spacetimes from wrapped D3-branes," Class. Quant. Grav. **24** (2007) 6267 [arXiv:0707.3105 [hep-th]].

[22] O. Lunin, "Brane webs and 1/4-BPS geometries," arXiv:0802.0735 [hep-th].

[23] E. D'Hoker, J. Estes, M. Gutperle and D. Krym, "Exact Half-BPS Flux Solutions in M-theory I, Local Solutions," arXiv:0806.0605 [hep-th].

[24] P. M. Cowdall and P. K. Townsend, "Gauged supergravity vacua from intersecting branes," Phys. Lett. B **429** (1998) 281 [Erratum-ibid. B **434** (1998) 458] [arXiv:hep-th/9801165].

[25] H. J. Boonstra, B. Peeters and K. Skenderis, Nucl. Phys. B **533** (1998) 127 [arXiv:hep-th/9803231].

[26] J. de Boer, A. Pasquinucci and K. Skenderis, "AdS/CFT dualities involving large 2d N = 4 superconformal symmetry," Adv. Theor. Math. Phys. **3** (1999) 577 [arXiv:hep-th/9904073].

[27] S. Gukov, E. Martinec, G. W. Moore and A. Strominger, Adv. Theor. Math. Phys. **9** (2005) 435 [arXiv:hep-th/0403090].

[28] J. P. Gauntlett, O. A. P. Mac Conamhna, T. Mateos and D. Waldram, "Supersymmetric AdS(3) solutions of type IIB supergravity," Phys. Rev. Lett. **97** (2006) 171601 [arXiv:hep-th/0606221].

[29] A. Donos, J. P. Gauntlett, J. Sparks, to appear.

[30] J. P. Gauntlett, O. A. P. Mac Conamhna, T. Mateos and D. Waldram, "New supersymmetric AdS(3) solutions," Phys. Rev. D **74** (2006) 106007 [arXiv:hep-th/0608055].

[31] D. N. Page and C. N. Pope, "Inhomogeneous Einstein Metrics On Complex Line Bundles," Class. Quant. Grav. **4** (1987) 213.

[32] J. P. Gauntlett, D. Martelli, J. Sparks and D. Waldram, "Sasaki-Einstein metrics on S(2) x S(3)," Adv. Theor. Math. Phys. **8** (2004) 711 [arXiv:hep-th/0403002].

[33] D. Martelli and J. Sparks, "Toric geometry, Sasaki-Einstein manifolds and a new infinite class of AdS/CFT duals," Commun. Math. Phys. **262** (2006) 51 [arXiv:hep-th/0411238].

[34] D. N. Page, "Classical Stability Of Round And Squashed Seven Spheres In Eleven-Dimensional Supergravity," Phys. Rev. D **28** (1983) 2976.

[35] D. Marolf, "Chern-Simons terms and the three notions of charge," arXiv:hep-th/0006117.

[36] J. P. Gauntlett, D. Martelli, J. Sparks and D. Waldram, "Supersymmetric AdS(5) solutions of type IIB supergravity," Class. Quant. Grav. **23** (2006) 4693 [arXiv:hep-th/0510125].

[37] H. Lu, C. N. Pope and P. K. Townsend, "Domain walls from anti-de Sitter spacetime," Phys. Lett. B **391** (1997) 39 [arXiv:hep-th/9607164].

[38] R. C. Myers, "Dielectric-branes," JHEP **9912** (1999) 022 [arXiv:hep-th/9910053].

[39] J. P. Gauntlett and S. Pakis, "The geometry of D = 11 Killing spinors," JHEP **0304** (2003) 039 [arXiv:hep-th/0212008].

[40] S. F. Hassan, "T-duality, space-time spinors and R-R fields in curved backgrounds," Nucl. Phys. B **568** (2000) 145 [arXiv:hep-th/9907152].