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On the Perturbative Stability of the QCD Predictions for the Ratio $R = F_L/F_T$ in Heavy-Quark Leptoproduction

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We analyze the perturbative and parametric stability of the QCD predictions for the Callan-Gross ratio $R(x,Q^2) = F_L/F_T$ in heavy-quark leptoproduction. We consider the radiative corrections to the dominant photon-gluon fusion mechanism. In various kinematic regions, the following contributions are investigated: exact NLO results at low and moderate $Q^2 \lesssim m^2$, asymptotic NLO predictions at high $Q^2 \gg m^2$, and both NLO and NNLO soft-gluon (or threshold) corrections at large Bjorken x. Our analysis shows that large radiative corrections to the structure functions $F_T(x,Q^2)$ and $F_L(x,Q^2)$ cancel each other in their ratio $R(x,Q^2)$ with good accuracy. As a result, the NLO contributions to the Callan-Gross ratio are less than 10% in a wide region of the variables x and Q^2. We provide compact LO predictions for $R(x,Q^2)$ in the case of low $x \ll 1$. A simple formula connecting the high-energy behavior of the Callan-Gross ratio and low-x asymptotics of the gluon density is derived. It is shown that the obtained hadron-level predictions for $R(x \to 0, Q^2)$ are stable under the DGLAP evolution of the gluon distribution function. Our analytic results simplify the extraction of the structure functions $F_2^c(x,Q^2)$ and $F_2^b(x,Q^2)$ from measurements of the corresponding reduced cross sections, in particular at DESY HERA.

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I. INTRODUCTION

In the framework of perturbative quantum chromodynamics (QCD), the basic spin-averaged characteristics of heavy-flavor hadro- [1], photo- [2] and electro-production [3] are known exactly up to the next-to-leading order (NLO). Although these explicit results are widely used at present for a phenomenological description of available data (for reviews, see Refs. [4, 5]), the key question remains open: How to test the applicability of QCD at fixed order to heavy-quark production? The basic theoretical problem is that the NLO corrections are sizeable; they increase the leading-order (LO) predictions for both charm and bottom production cross sections by approximately a factor of two. Moreover, soft-gluon resummation of the threshold Sudakov logarithms indicates that higher-order contributions can also be substantial. (For reviews, see Refs. [6, 7].) On the other hand, perturbative instability leads to a high sensitivity of the theoretical calculations to standard uncertainties in the input QCD parameters. For this reason, it is difficult to compare pQCD results for spin-averaged cross sections with experimental data directly, without additional assumptions. The total uncertainties associated with the unknown values of the heavy-quark mass, m, the factorization and renormalization scales, μ_F and μ_R, the asymptotic scale parameter $\Lambda_{\rm QCD}$ and the parton distribution functions (PDFs) are so large that one can only estimate the order of magnitude of the pQCD predictions for charm production cross sections in the entire energy range from the fixed-target experiments [8] to the RHIC collider [5].

Since these production cross sections have such slowly converging perturbative expansions, it is of special interest

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to study those observables that are well-defined in pQCD. A nontrivial example of such an observable was proposed in Refs. [9, 10, 11, 12, 13], where the azimuthal $\cos(2\varphi)$ asymmetry in heavy-quark photo- and leptoproduction was analyzed.[1,2] In particular, the Born-level results were considered [9] and the NLO soft-gluon corrections to the basic mechanism, photon-gluon fusion, were calculated [10]. It was shown that, contrary to the production cross sections, the azimuthal asymmetry in heavy-flavor photo- and leptoproduction is quantitatively well defined in pQCD: the contribution of the dominant photon-gluon fusion mechanism to the asymmetry is stable, both parametrically and perturbatively. Therefore, measurements of this asymmetry should provide a useful test of pQCD. As was shown in Ref. [11], the azimuthal asymmetry in open charm photoproduction could be measured with an accuracy of about ten percent in the approved E160/E161 experiments at SLAC [15] using the inclusive spectra of secondary (decay) leptons.

In Ref. [13], the photon-(heavy-)quark scattering contribution to φ-dependent lepton-hadron deep-inelastic scattering (DIS) was investigated. It turned out that, contrary to the basic photon-gluon fusion component, the quark-scattering mechanism is practically $\cos(2\varphi)$-independent. This is due to the fact that the quark-scattering contribution to the $\cos(2\varphi)$ asymmetry is, for kinematic reasons, absent at LO and is negligibly small at NLO, of the order of 1%. This indicates that the azimuthal distributions in charm leptoproduction could be a good probe of the charm PDF in the proton.

In the present paper, we continue the studies of perturbatively stable observables by considering the photon-gluon fusion mechanism in heavy-quark leptoproduction,

$$\ell(l) + N(p) \to \ell(l-q) + Q(p_Q) + X[\bar{Q}](p_X). \tag{1}$$

In the case of unpolarized initial states and neglecting the contribution of Z-boson exchange, the cross section of reaction (1) can be written as

$$\begin{aligned}\frac{\mathrm{d}^2\sigma_{lN}}{\mathrm{d}x\,\mathrm{d}Q^2} &= \frac{4\pi\alpha_{\mathrm{em}}^2}{Q^4}\left\{\left[1+(1-y)^2\right]F_T(x,Q^2) + 2\,(1-y)\,F_L(x,Q^2)\right\} \\ &= \frac{2\pi\alpha_{\mathrm{em}}^2}{xQ^4}\left\{\left[1+(1-y)^2\right]F_2(x,Q^2) - 2xy^2F_L(x,Q^2)\right\},\end{aligned} \tag{2}$$

where α_{em} is Sommerfeld's fine-structure constant, $F_2(x,Q^2) = 2x(F_T+F_L)$ and the kinematic variables are defined by

$$\begin{aligned}&\bar{S} = (\ell+p)^2, \qquad Q^2 = -q^2, \qquad x = \frac{Q^2}{2p\cdot q}, \\ &y = \frac{p\cdot q}{p\cdot \ell}, \qquad Q^2 = xy\bar{S}, \qquad \xi = \frac{Q^2}{m^2}.\end{aligned} \tag{3}$$

In this paper, we investigate radiative corrections to the Callan-Gross ratio in heavy-quark leptoproduction, defined as

$$R(x,Q^2) = \frac{F_L(x,Q^2)}{F_T(x,Q^2)}. \tag{4}$$

First, we consider the exact NLO corrections to the quantity $R(x,Q^2)$ at low and moderate $Q^2 \lesssim m^2$ using explicit results [3, 16]. Then, we analyze the high-Q^2 regime with the help of the asymptotic NLO predictions for the structure functions $F_T(x,Q^2)$ and $F_L(x,Q^2)$ presented in Refs. [17, 18]. Finally, the soft-gluon (or threshold) contributions are investigated in the large-x region in the framework of the formalism developed in Ref. [6]. To next-to-leading logarithmic (NLL) accuracy, we calculate the NLO and NNLO soft-gluon corrections to both structure functions. Our main results can be formulated as follows:

- Exact NLO corrections to the ratio $R(x,Q^2)$ do not exceed 10% in the energy range $x > 10^{-4}$ at low and moderate $Q^2 \lesssim m^2$.

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[1] Well-known examples include the shapes of differential cross sections of heavy-flavor production, which are sufficiently stable under radiative corrections.

[2] Note also the recent paper [14], where the perturbative stability of the QCD predictions for the charge asymmetry in top-quark hadroproduction has been observed.

- At high $Q^2 \gg m^2$, the asymptotic NLO corrections to $R(x,Q^2)$ are less than 10% for $10^{-4} < x < 10^{-1}$.
- At the NLL level, the NLO and NNLO soft-gluon predictions for $R(x,Q^2)$ affect the LO results by less than a few percent at low and moderate Q^2 and $x \gtrsim 10^{-2}$.
- In all the cases mentioned above, the NLO predictions for $R(x,Q^2)$ are sufficiently insensitive, to within ten percent, to standard uncertainties in the QCD input parameters μ_F, μ_R and $\Lambda_{\rm QCD}$, and in the gluon PDF $g(x,\mu_F)$.

We conclude that, in contrast to the production cross sections, the Callan-Gross ratio in heavy-quark leptoproduction is an observable quantitatively well defined in pQCD. Perturbative stability of the photon-gluon fusion results for $R(x,Q^2)$ is mainly due to the cancellation of large radiative corrections to the structure functions $F_T(x,Q^2)$ and $F_L(x,Q^2)$ in their ratio, especially in the large-x region. Measurements of the quantity $R(x,Q^2)$ in charm and bottom leptoproduction should provide a good test of the conventional parton model based on pQCD.

Concerning the experimental aspects, perturbative stability of the QCD predictions for $R(x,Q^2)$ observed in our studies is very useful for the extraction of the structure functions $F_2^c(x,Q^2)$ and $F_2^b(x,Q^2)$ from the data. Usually, it is the so-called "reduced cross section", $\tilde{\sigma}(x,Q^2)$, that can directly be measured in DIS experiments:

$$\tilde{\sigma}(x,Q^2) = \frac{1}{1+(1-y)^2}\frac{xQ^4}{2\pi\alpha_{\rm em}^2}\frac{{\rm d}^2\sigma_{lN}}{{\rm d}x{\rm d}Q^2} = F_2(x,Q^2) - \frac{2xy^2}{1+(1-y)^2}F_L(x,Q^2) \tag{5}$$

$$= F_2(x,Q^2)\left[1-\frac{y^2}{1+(1-y)^2}R_2(x,Q^2)\right], \tag{6}$$

where

$$R_2(x,Q^2) = 2x\frac{F_L(x,Q^2)}{F_2(x,Q^2)} = \frac{R(x,Q^2)}{1+R(x,Q^2)}. \tag{7}$$

In earlier HERA analyses of charm and bottom electroproduction [19], the corresponding longitudinal structure functions were taken to be zero for simplicity. In this case, $\tilde{\sigma}(x,Q^2) = F_2(x,Q^2)$. In recent papers [20, 21], the structure function $F_2(x,Q^2)$ is evaluated from the reduced cross section (5) where the longitudinal structure function $F_L(x,Q^2)$ is estimated from the NLO QCD expectations. Instead of this rather cumbersome procedure, we propose to use the expression (6) with the quantity $R_2(x,Q^2)$ calculated in LO approximation. This simplifies the extraction of $F_2(x,Q^2)$ from measurements of $\tilde{\sigma}(x,Q^2)$ but does not affect the accuracy of the result in practice.

Indeed, the LO corrections to the extracted function $F_2(x,Q^2)$ due to the non-zero value of $R_2(x,Q^2)$ cannot exceed 30% because the ratio $R_2(x,Q^2)$ is itself less than 0.3 practically in the entire region of the variables x and Q^2. For this reason, the NLO corrections to $R_2(x,Q^2)$, having a relative size of the order of 10%, cannot affect the value of $F_2(x,Q^2)$ by more than 3%. In reality, the effect of radiative corrections to $R_2(x,Q^2)$ on the extracted values of $F_2(x,Q^2)$ is less than 1% since $y \ll 1$ in most of the experimentally accessible kinematic range.

In the present paper, we derive compact hadron-level LO predictions for the ratio $R_2(x,Q^2)$ in the limit of low $x \to 0$. Assuming the low-x asymptotic behavior of the gluon PDF to be of the type $g(x,Q^2) \propto 1/x^{1+\delta}$, we provide analytic result for the ratio $R_2(x\to 0,Q^2) \equiv R_2^{(\delta)}(Q^2)$ for arbitrary values of the parameter δ in terms of the Gauss hypergeometric function. Furthermore, we consider compact formulae for $R_2^{(\delta)}(Q^2)$ in two particular cases: $\delta = 1/2$ and $\delta = 0$. The simplest case, $\delta = 0$, which has already been studied recently in Ref. [22], leads to a non-singular behavior of the structure functions for $x \to 0$. The second choice, $\delta = 1/2$, historically originates from the BFKL resummation of the leading powers of $\ln(1/x)$ [23].

In principle, the parameter δ is a function of Q^2 and this dependence is calculated using the DGLAP evolution equations [24]. However, our analysis shows that hadron-level predictions for $R_2(x\to 0,Q^2)$ depend weakly on δ practically in the entire region of Q^2 for $\delta > 0.2$. In particular, the relative difference between $R_2^{(0.5)}(Q^2)$ and $R_2^{(0.3)}(Q^2)$ is less than few percent at $Q^2 \gtrsim m^2$. For this reason, our simple formula for $R_2^{(\delta)}(Q^2)$ with $\delta = 1/2$ (i.e., without any evolution) describes with good accuracy the low-x predictions for $R_2(x,Q^2)$ of the CTEQ PDF versions [25, 26]. We see that the hadron-level predictions for $R_2(x\to 0,Q^2)$ are stable not only under the NLO corrections to the partonic cross sections, but also under the DGLAP evolution of the gluon PDF.

Finally, we show that our compact LO formulae for $R_2^{(\delta)}(Q^2)$ conveniently reproduce the HERA results for $F_2^c(x,Q^2)$ and $F_2^b(x,Q^2)$ obtained by H1 Collaboration [20, 21] with the help of more cumbersome NLO estimations of $F_L(x,Q^2)$.

This paper is organized as follows. In Section II, we analyze the exact NLO results for the Callan-Gross ratio at low and moderate $Q^2 \lesssim m^2$ and the asymptotic NLO predictions at high $Q^2 \gg m^2$. The soft-gluon contributions to $R(x,Q^2)$ are investigated in Section III. To NLL accuracy, we calculate the threshold NLO and NNLO corrections to both structure functions $F_T(x,Q^2)$ and $F_L(x,Q^2)$. The analytic LO results for the ratio $R_2(x,Q^2)$ at low x are discussed in Section IV.

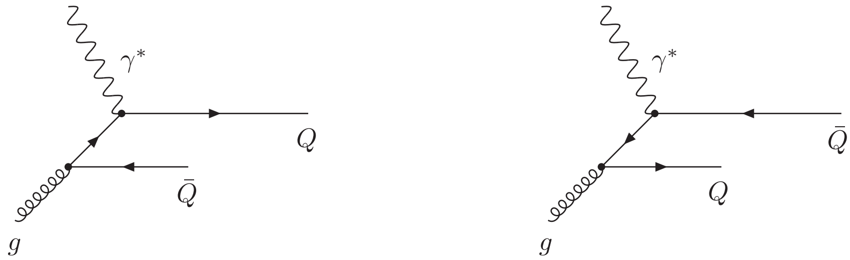

FIG. 1: Feynman diagrams of photon-gluon fusion at LO.

II. NLO PREDICTIONS FOR THE CALLAN-GROSS RATIO $R(x, Q^2)$

A. Born-Level Cross Sections

At LO, $\mathcal{O}(\alpha_{\rm em}\alpha_s)$, leptoproduction of heavy flavors proceeds through the photon-gluon fusion mechanism,

$$\gamma^*(q) + g(k_g) \to Q(p_Q) + \bar{Q}(p_{\bar{Q}}). \tag{8}$$

The relevant Feynman diagrams are depicted in Fig. 1. The $\gamma^* g$ cross sections, $\hat{\sigma}_k^{(0)}(z,\lambda)$ $(k = 2, T, L)$, have the form [27]:

$$\hat{\sigma}_2^{(0)}(z,\lambda) = \frac{\alpha_s(\mu_R^2)}{2\pi}\hat{\sigma}_B(z)\left\{\left[(1-z)^2 + z^2 + 4\lambda z(1-3z) - 8\lambda^2 z^2\right]\ln\frac{1+\beta_z}{1-\beta_z} - \left[1 + 4z(1-z)(\lambda-2)\right]\beta_z\right\}, \tag{9}$$

$$\hat{\sigma}_L^{(0)}(z,\lambda) = \frac{2\alpha_s(\mu_R^2)}{\pi}\hat{\sigma}_B(z) z\left\{-2\lambda z\ln\frac{1+\beta_z}{1-\beta_z} + (1-z)\beta_z\right\}, \tag{10}$$

$$\hat{\sigma}_T^{(0)}(z,\lambda) = \hat{\sigma}_2^{(0)}(z,\lambda) - \hat{\sigma}_L^{(0)}(z,\lambda), \tag{11}$$

with

$$\hat{\sigma}_B(z) = \frac{(2\pi)^2 e_Q^2 \alpha_{\rm em}}{Q^2} z, \tag{12}$$

where e_Q is the electric charge of quark Q in units of the positron charge and $\alpha_s(\mu_R^2)$ is the strong-coupling constant. In Eqs. (9)–(12), we use the following definition of partonic kinematic variables:

$$z = \frac{Q^2}{2q\cdot k_g}, \qquad \lambda = \frac{m^2}{Q^2}, \qquad \beta_z = \sqrt{1 - \frac{4\lambda z}{1-z}}. \tag{13}$$

The hadron-level cross sections, $\sigma_k(x, Q^2)$ $(k = 2, T, L)$, have the form

$$\sigma_k(x, Q^2) = \int\limits_{x(1+4\lambda)}^{1} {\rm d}z\, g(z,\mu_F)\hat{\sigma}_k\left(\frac{x}{z}, \lambda, \mu_F, \mu_R\right), \tag{14}$$

where $g(z,\mu_F)$ is the gluon PDF of the proton. The leptoproduction cross sections $\sigma_k(x, Q^2)$ are related to the structure functions $F_k(x, Q^2)$ as follows:

$$F_k(x, Q^2) = \frac{Q^2}{8\pi^2\alpha_{\rm em} x}\sigma_k(x, Q^2) \qquad (k = T, L), \tag{15}$$

$$F_2(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_{\rm em}}\sigma_2(x, Q^2). \tag{16}$$

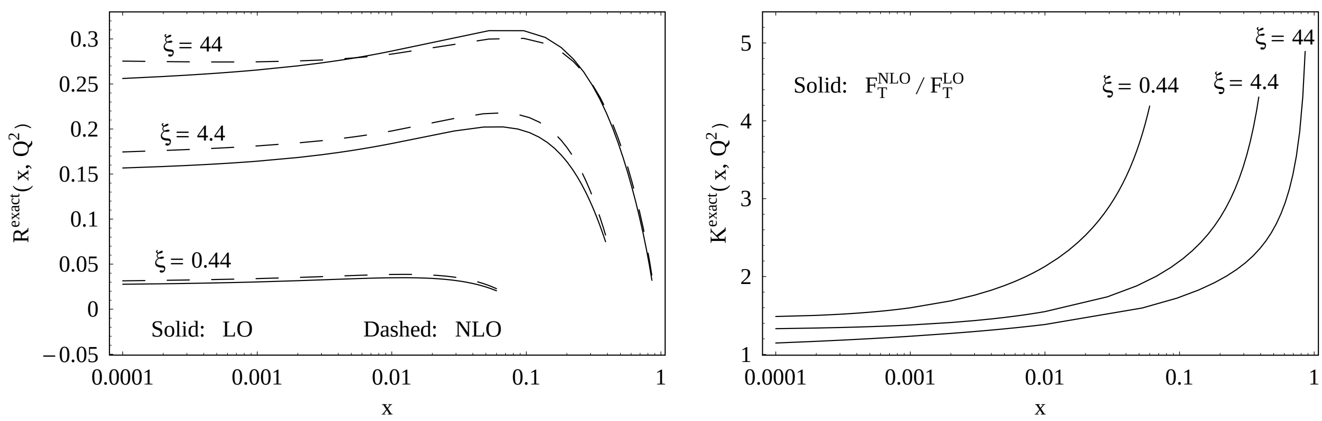

FIG. 2: *Left panel:* x dependence of the Callan-Gross ratio, $R(x,Q^2) = F_L/F_T$, in charm leptoproduction for $\xi = 0.44$, 4.4 and 44 at LO (solid lines) and NLO (dashed lines). *Right panel:* x dependence of the K factor for the transverse structure function, $K(x,Q^2) = F_T^{\rm NLO}/F_T^{\rm LO}$, at the same values of ξ.

B. Exact NLO Predictions at Low and Moderate Q^2

At NLO, $\mathcal{O}(\alpha_{\rm em}\alpha_s^2)$, the contribution of the photon-gluon component is usually presented in terms of the dimensionless coefficient functions $c_k^{(n,l)}(z,\lambda)$ $(k = T, L)$, as

$$\hat{\sigma}_k(z,\lambda,m^2,\mu^2) = \frac{e_Q^2\alpha_{\rm em}\alpha_s(\mu^2)}{m^2}\left\{c_k^{(0,0)}(z,\lambda) + 4\pi\alpha_s(\mu^2)\left[c_k^{(1,0)}(z,\lambda) + c_k^{(1,1)}(z,\lambda)\ln\frac{\mu^2}{m^2}\right]\right\} + \mathcal{O}(\alpha_s^2). \tag{17}$$

where we identify $\mu = \mu_F = \mu_R$.

In this paper, we neglect the $\gamma^* q(\bar{q})$ fusion subprocesses. This is justified as their contributions to heavy-quark leptoproduction vanish at LO and are small at NLO [3]. To be precise, the light-quark-initiated corrections to both F_T and F_L structure functions are negative and less than 10% in a wide kinematic range [3]. Our estimates show that these contributions cancel in the ratio $R(x,Q^2) = F_L/F_T$ with an accuracy less than few percent.

The coefficients $c_T^{(1,1)}(z,\lambda)$ and $c_L^{(1,1)}(z,\lambda)$ of the μ-dependent logarithms can be evaluated explicitly using renormalization group arguments [3, 6]. The results of direct calculations of the coefficient functions $c_k^{(1,0)}(z,\lambda)$ $(k = T, L)$ are presented in Refs. [3, 16]. Using these NLO predictions, we compute the x dependence of the ratio $R(x,Q^2) = F_L/F_T$ at several values of $\xi = 1/\lambda = Q^2/m^2$.

The left panel of Fig. 2 shows the Callan-Gross ratio $R(x,Q^2)$ as a function of x for $\xi = 0.44$, 4.4 and 44. In our calculations, we use the CTEQ5M parametrization of the gluon PDF together with the values $m_c = 1.3$ GeV and $\Lambda_3 = 373$ MeV [26]. Unless otherwise stated, we use $\mu = \sqrt{4m_c^2 + Q^2}$ throughout this paper.

For comparison, the right panel of Fig. 2 shows the x dependence of the QCD correction factor for the transverse structure function, $K(x,Q^2) = F_T^{\rm NLO}/F_T^{\rm LO}$. One can see that large radiative corrections to the structure functions $F_T(x,Q^2)$ and $F_L(x,Q^2)$, especially at non-small x, cancel each other in their ratio $R(x,Q^2) = F_L/F_T$ with good accuracy. As a result, the NLO contributions to the ratio $R(x,Q^2)$ are less than 10% for $x \gtrsim 10^{-4}$ at low and moderate $Q^2 \lesssim m_c^2$.

Another remarkable property of the Callan-Gross ratio closely related to fast perturbative convergence is its parametric stability.[3] Our analysis shows that the fixed-order predictions for the ratio $R(x,Q^2)$ are less sensitive to standard uncertainties in the QCD input parameters than the corresponding ones for the production cross sections. For instance, sufficiently above the production threshold, changes of μ in the range $(1/2)\sqrt{4m_c^2 + Q^2} < \mu < 2\sqrt{4m_c^2 + Q^2}$ only lead to 10% variations of $R(x,Q^2)$ at NLO. For comparison, at $x = 0.1$ and $\xi = 4.4$, such changes of μ affect the NLO predictions for the quantities $F_T(x,Q^2)$ and $R(x,Q^2)$ in charm leptoproduction by more than 100% and less than 10%, respectively.

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[3] Of course, parametric stability of the fixed-order results does not imply a fast convergence of the corresponding series. However, a fast convergent series must be parametrically stable. In particular, it must exhibit feeble μ_F and μ_R dependences.

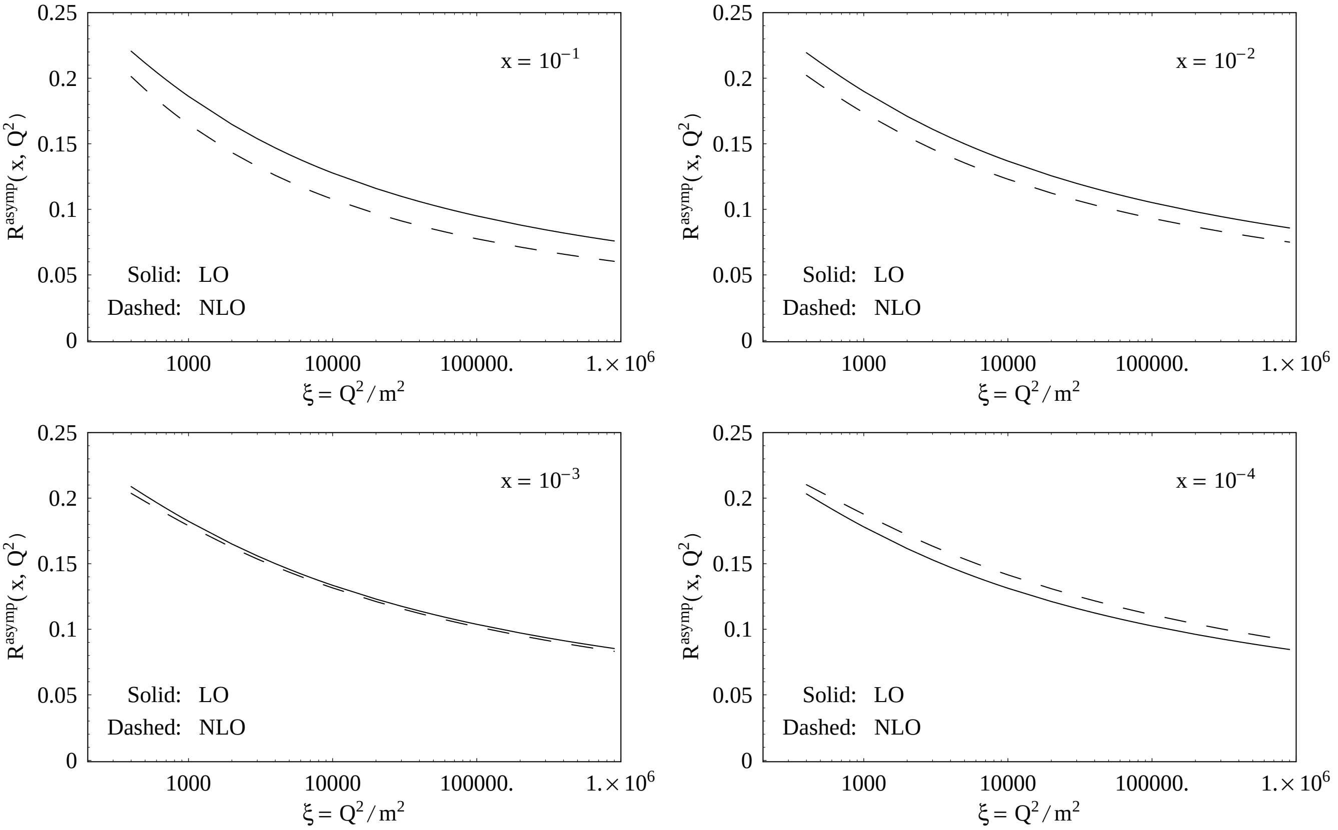

FIG. 3: Q^2 dependence of the asymptotic high-Q^2 ($Q^2 \gg m^2$) predictions for the Callan-Gross ratio, $R(x,Q^2) = F_L/F_T$, in charm leptoproduction at $x = 10^{-1}$, 10^{-2}, 10^{-3} and 10^{-4} in LO (solid lines) and NLO (dashed lines).

Keeping the value of the variable Q^2 fixed, we analyze the dependence of the pQCD predictions on the uncertainties in the heavy-quark mass. Sufficiently above the production threshold, i.e. in the plateau regions of the x distributions of $R(x,Q^2)$ in Fig. 2, changes of the charm-quark mass in the interval 1.3 GeV $< m_c <$ 1.7 GeV affect the Callan-Gross ratio by 2%–3% at $Q^2 = 10$ GeV^2. The corresponding variations of the structure functions $F_T(x,Q^2)$ and $F_L(x,Q^2)$ are about 20%. We also verify that all the CTEQ versions [25, 26] of the gluon PDF lead to NLO predictions for $R(x,Q^2)$ that coincide with each other with an accuracy of about 5% practically at all $x \gtrsim 10^{-4}$.

C. Asymptotic NLO Results at High $Q^2 \gg m^2$

The analytic form of the heavy-quark coefficient functions for lepton-hadron DIS in the kinematical regime $Q^2 \gg m^2$ is presented in Refs. [17, 18]. The calculations were performed up to NLO in α_s using operator product expansion techniques.[4] In the asymptotic regime $\xi \to \infty$, the production cross sections have the following decomposition in terms of the coefficient functions $a_k^{l,(n,m)}(z)$ $(k = 2, L)$:

$$\hat{\sigma}_k(z,Q^2,m^2,\mu^2) = \frac{e_Q^2 \alpha_{\rm em}}{4\pi m^2} \sum_{l=1}^{\infty} \left[4\pi\alpha_s(\mu^2)\right]^l \sum_{m+n<l}^{n} a_k^{l,(n,m)}(z) \ln^n \frac{\mu^2}{m^2} \ln^m \frac{Q^2}{m^2} + \mathcal{O}\left(\frac{m^2}{Q^2}\right). \qquad (18)$$

It was found in Refs. [17, 18] that the hadron-level structure function $F_2^{\rm asymp}(x,Q^2)$ approaches, to within ten percent, the corresponding exact value $F_2^{\rm exact}(x,Q^2)$ for $\xi \gtrsim 10$ and $x < 10^{-1}$ both at LO and NLO. In the case

[4] For the longitudinal cross section $\hat{\sigma}_L(z,Q^2,m^2,\mu^2)$, the asymptotic heavy-quark coefficient functions $a_L^{l,(n,m)}(z)$ are known up to NNLO in α_s [28].

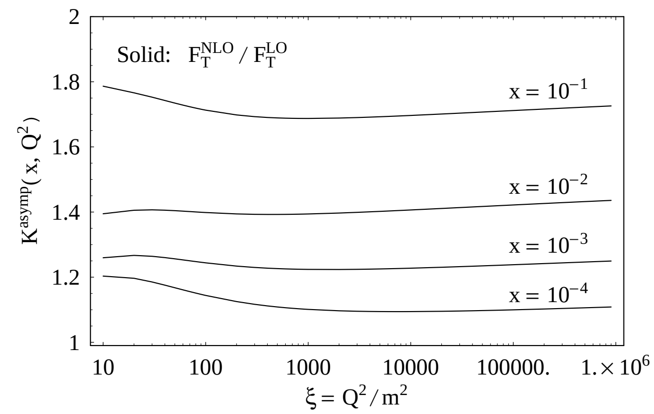

FIG. 4: Q^2-dependence of the asymptotic high-Q^2 ($Q^2 \gg m^2$) predictions for the K factor, $K(x,Q^2) = F_T^{\rm NLO}/F_T^{\rm LO}$, at $x = 10^{-1}$, 10^{-2}, 10^{-3} and 10^{-4}.

of the longitudinal structure function $F_L^{\rm asymp}(x,Q^2)$, the approach to $F_L^{\rm exact}(x,Q^2)$ starts at much larger values of $\xi \gtrsim 4\times 10^2$.

Using the analytic NLO results for the coefficient functions presented in Ref. [17], we calculate the asymptotic high-Q^2 behavior of the ratio $R(x,Q^2) = F_L/F_T$ at several values of x. Figure 3 shows $R^{\rm asymp}(x,Q^2)$ in charm leptoproduction as a function of ξ for $x = 10^{-1}$, 10^{-2}, 10^{-3} and 10^{-4}. In Fig. 4, we show the Q^2 dependence of the asymptotic predictions for the K factor $K(x,Q^2) = F_T^{\rm NLO}/F_T^{\rm LO}$ at the same values of x. One can see that the quantity $K(x,Q^2)$ is practically independent of Q^2 at fixed values of x and tends to unity at low x. This implies that perturbative stability of the Callan-Gross ratio at low x is due to the smallness of the radiative corrections to both structure functions. At non-small x, the radiative corrections to $F_T(x,Q^2)$ and $F_L(x,Q^2)$ are large but cancel each other in their ratio $R(x,Q^2) = F_L/F_T$ with good accuracy.

III. SOFT-GLUON CORRECTIONS AT NLO AND NNLO

In this Section, we consider the NLO and NNLO predictions for the Callan-Gross ratio due to the contribution of the photon-gluon fusion mechanism in the soft-gluon approximation and propose an improvement. For the reader's convenience, we collect the final results for the parton-level cross sections to NLL accuracy. More details may be found in Refs. [6, 10, 12].

At NLO, photon-gluon fusion receives contributions from the virtual $\mathcal{O}(\alpha_{\rm em}\alpha_s^2)$ corrections to the Born process (8) and from real-gluon emission,

$$\gamma^*(q) + g(k_g) \to Q(p_Q) + \bar{Q}(p_{\bar{Q}}) + g(p_g). \tag{19}$$

The partonic invariants describing the single-particle inclusive (1PI) kinematics are

$$\begin{aligned} s' &= 2q\cdot k_g = s + Q^2 = \zeta S', & t_1 &= (k_g - p_Q)^2 - m^2 = \zeta T_1, \\ s_4 &= s' + t_1 + u_1, & u_1 &= (q - p_Q)^2 - m^2 = U_1, \end{aligned} \tag{20}$$

where ζ is defined through $\vec{k}_g = \zeta\vec{p}$ and s_4 measures the inelasticity of the reaction (19). The corresponding 1PI hadron-level variables describing the reaction (1) are

$$\begin{aligned} S' &= 2q\cdot p = S + Q^2, & T_1 &= (p - p_Q)^2 - m^2, \\ S_4 &= S' + T_1 + U_1, & U_1 &= (q - p_Q)^2 - m^2. \end{aligned} \tag{21}$$

The exact NLO calculations of unpolarized heavy-quark production in γg [2], $\gamma^* g$ [3], and gg [1] collisions show that, near the partonic threshold, a strong logarithmic enhancement of the cross sections takes place in the collinear, $|\vec{p}_{g,T}| \to 0$, and soft, $|\vec{p}_g| \to 0$, limits. This threshold (or soft-gluon) enhancement is of universal nature in perturbation theory and originates from an incomplete cancellation of the soft and collinear singularities between the loop and the bremsstrahlung contributions. Large leading and next-to-leading threshold logarithms can be resummed to all orders of the perturbative expansion using the appropriate evolution equations [29]. The analytic results for the resummed cross sections are ill-defined due to the Landau pole in the coupling constant α_s. However, if one considers the obtained expressions as generating functionals and re-expands them at fixed order in α_s, no divergences associated with the Landau pole are encountered.

Soft-gluon resummation for the photon-gluon fusion was performed in Ref. [6] and confirmed in Refs. [10, 12]. To NLL accuracy, the perturbative expansion for the partonic cross sections, $\mathrm{d}^2\hat{\sigma}_k(s',t_1,u_1)/(\mathrm{d}t_1\,\mathrm{d}u_1)$ $(k=T,L)$, can be written in factorized form as

$$s'^2\frac{\mathrm{d}^2\hat{\sigma}_k}{\mathrm{d}t_1\mathrm{d}u_1}(s',t_1,u_1)=B_k^{\mathrm{Born}}(s',t_1,u_1)\left[\delta(s'+t_1+u_1)+\sum_{n=1}^{\infty}\left(\frac{\alpha_sC_A}{\pi}\right)^nK^{(n)}(s',t_1,u_1)\right],\tag{22}$$

with the Born-level distributions $B_k^{\mathrm{Born}}(s',t_1,u_1)$ given by

$$B_T^{\mathrm{Born}}(s',t_1,u_1)=\pi e_Q^2\alpha_{\mathrm{em}}\alpha_s\left\{\frac{t_1}{u_1}+\frac{u_1}{t_1}+4\left(\frac{s}{s'}-\frac{m^2s'}{t_1u_1}\right)\left[\frac{s'(m^2-Q^2/2)}{t_1u_1}+\frac{Q^2}{s'}\right]\right\},\tag{23}$$

$$B_L^{\mathrm{Born}}(s',t_1,u_1)=\pi e_Q^2\alpha_{\mathrm{em}}\alpha_s\frac{8Q^2}{s'}\left(\frac{s}{s'}-\frac{m^2s'}{t_1u_1}\right).\tag{24}$$

Note that the functions $K^{(n)}(s',t_1,u_1)$ in Eq. (22) originate from the collinear and soft limits and are the same for both cross sections $\hat{\sigma}_T$ and $\hat{\sigma}_L$. At NLO and NNLO, the soft-gluon corrections to NLL accuracy in the $\overline{\mathrm{MS}}$ scheme read

$$\begin{aligned}K^{(1)}(s',t_1,u_1)&=2\left[\frac{\ln\left(s_4/m^2\right)}{s_4}\right]_+-\left[\frac{1}{s_4}\right]_+\left[1+\ln\frac{u_1}{t_1}-\left(1-\frac{2C_F}{C_A}\right)(1+\mathrm{Re}L_\beta)+\ln\frac{\mu^2}{m^2}\right]\\&\quad+\delta(s_4)\ln\frac{-u_1}{m^2}\ln\frac{\mu^2}{m^2},\end{aligned}\tag{25}$$

$$\begin{aligned}K^{(2)}(s',t_1,u_1)&=2\left[\frac{\ln^3\left(s_4/m^2\right)}{s_4}\right]_+\\&\quad-3\left[\frac{\ln^2\left(s_4/m^2\right)}{s_4}\right]_+\left[1+\ln\frac{u_1}{t_1}-\left(1-\frac{2C_F}{C_A}\right)(1+\mathrm{Re}L_\beta)+\frac{2}{3}\frac{b_2}{C_A}+\ln\frac{\mu^2}{m^2}\right]\\&\quad+2\left[\frac{\ln\left(s_4/m^2\right)}{s_4}\right]_+\left[1+\ln\frac{u_1}{t_1}-\left(1-\frac{2C_F}{C_A}\right)(1+\mathrm{Re}L_\beta)+\ln\frac{-u_1}{m^2}+\frac{b_2}{C_A}+\frac{1}{2}\ln\frac{\mu^2}{m^2}\right]\\&\quad\times\ln\frac{\mu^2}{m^2}-\left[\frac{1}{s_4}\right]_+\ln^2\frac{\mu^2}{m^2}\left[\ln\frac{-u_1}{m^2}+\frac{b_2}{2C_A}\right],\end{aligned}\tag{26}$$

where $b_2=(11C_A-2n_f)/12$ is the first coefficient of the beta function,

$$\beta(\alpha_s)=\frac{\mathrm{d}\ln\alpha_s\left(\mu^2\right)}{\mathrm{d}\ln\mu^2}=-\sum_{k=1}^{\infty}b_{k+1}\left(\frac{\alpha_s}{\pi}\right)^k.\tag{27}$$

In Eqs. (25) and (26), $C_A=N_c$, $C_F=(N_c^2-1)/(2N_c)$, n_f is the number of active quark flavors, N_c is the number of quark colors, and $L_\beta=(1-2m^2/s)\{\ln[(1-\beta_z)/(1+\beta_z)]+\mathrm{i}\pi\}$ with $\beta_z=\sqrt{1-4m^2/s}$. The single-particle inclusive "plus" distributions are defined by

$$\left[\frac{\ln^l\left(s_4/m^2\right)}{s_4}\right]_+=\lim_{\epsilon\to0}\left[\frac{\ln^l\left(s_4/m^2\right)}{s_4}\theta(s_4-\epsilon)+\frac{1}{l+1}\ln^{l+1}\frac{\epsilon}{m^2}\delta(s_4)\right].\tag{28}$$

For any sufficiently regular test function $h(s_4)$, Eq. (28) implies that

$$\int_0^{s_4^{\max}}\mathrm{d}s_4\,h(s_4)\left[\frac{\ln^l\left(s_4/m^2\right)}{s_4}\right]_+=\int_0^{s_4^{\max}}\mathrm{d}s_4\left[h(s_4)-h(0)\right]\frac{\ln^l\left(s_4/m^2\right)}{s_4}+\frac{1}{l+1}h(0)\ln^{l+1}\frac{s_4^{\max}}{m^2}.\tag{29}$$

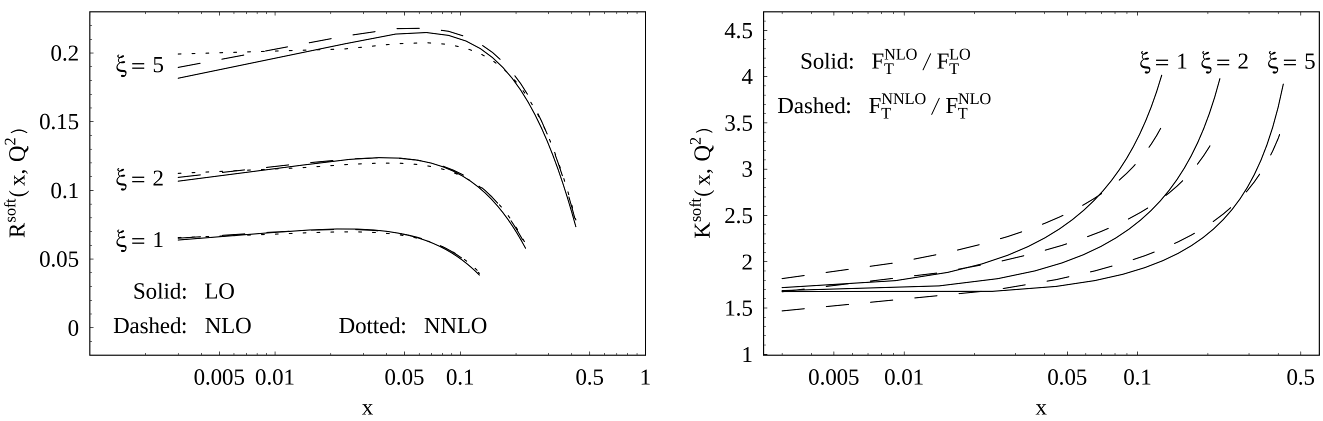

FIG. 5: *Left panel:* LO (solid lines), NLO (dashed lines) and NNLO (dotted lines) soft-gluon predictions for the x dependence of the Callan-Gross ratio, $R(x,Q^2)=F_L/F_T$, in charm leptoproduction at $\xi=1$, 2 and 5. *Right panel:* x dependence of the K factors $K^{(1)}(x,Q^2)=F_T^{\mathrm{NLO}}/F_T^{\mathrm{LO}}$ (solid line) and $K^{(2)}(x,Q^2)=F_T^{\mathrm{NNLO}}/F_T^{\mathrm{NLO}}$ (dashed curve) for the transverse structure function at the same values of ξ.

In Eqs. (25) and (26), we have also preserved the NLL terms for the scale-dependent logarithms. Note that Eqs. (23)–(25) agree to NLL accuracy with the exact $\mathcal{O}(\alpha_{\mathrm{em}}\alpha_s^2)$ calculations of the photon-gluon cross sections $\hat{\sigma}_T$ and $\hat{\sigma}_L$ given in Ref. [3].

Numerical investigation of the results (23)–(26) was performed in Refs. [6, 12]. It was shown that soft-gluon corrections reproduce satisfactorily the threshold behavior of the available exact results for the partonic cross section $\hat{\sigma}_2=\hat{\sigma}_T+\hat{\sigma}_L$ at $\xi\lesssim 1$. Since the gluon PDF supports just the threshold region, the soft-gluon contribution dominates the hadron-level structure function F_2 at energies not so far from the production threshold. It was shown in Ref. [6] that Eqs. (23) and (25) render it possible to describe with good accuracy the exact NLO predictions [3] for the function $F_2(x,Q^2)$ at $x\gtrsim 10^{-3}$ and relatively low virtuality $Q^2\sim m^2$.

In the present paper, we analyze separately the partonic cross sections $\hat{\sigma}_T$ and $\hat{\sigma}_L$. It turns out that the quality of the adopted soft-gluon approximation is worse for $\hat{\sigma}_L$ than for $\hat{\sigma}_T$. To clarify the situation, let us remember that the NLL approximation allows us to determine unambiguously only the singular s_4 behavior of the cross sections defined by Eq. (28). This implies that the s_4 dependence of the Born-level distributions $B_{T,L}^{\mathrm{Born}}(s',t_1,u_1)\big|_{u_1=s_4-s'-t_1}$ is chosen quite arbitrarily in Eqs. (23) and (24). To improve the situation, we propose the following procedure to determine the s_4 dependence of the differential cross sections based on a comparison of the soft-gluon predictions with the exact NLO results. First, we define the on-shell Born-level distributions in the LO kinematics, i.e. at $s_4=0$, as $\tilde{B}_{T,L}^{\mathrm{Born}}(s',t_1)=B_{T,L}^{\mathrm{Born}}(s',t_1,u_1)\big|_{u_1=-s'-t_1}$. Then we introduce new quantities, $\hat{B}_{T,L}^{\mathrm{Born}}(s',t_1,u_1)$, with the following s_4 dependence: $\hat{B}_{T,L}^{\mathrm{Born}}(s',t_1,u_1)\equiv\tilde{B}_{T,L}^{\mathrm{Born}}(x_4s',x_4t_1)$, where $x_4=-u_1/(s'+t_1)=1-s_4/(s'+t_1)$. Comparison with the exact NLO results given by Eqs. (4.7) and (4.8) in Ref. [3] indicates that the usage of the distributions $\hat{B}_{T,L}^{\mathrm{Born}}(s',t_1,u_1)$ instead of $B_{T,L}^{\mathrm{Born}}(s',t_1,u_1)$ leads to a more accurate account of the leading-logarithmic (LL) and NLL contributions originating from collinear gluon emission. Our numerical analysis shows that the new quantities $\hat{B}_{T,L}^{\mathrm{Born}}(s',t_1,u_1)$ improve essentially the quality of the soft-gluon approximation for both $\hat{\sigma}_T$ and $\hat{\sigma}_L$. More details can be found in Ref. [30]. In our further studies, we use the improved Born-level distributions, $\hat{B}_{T,L}^{\mathrm{Born}}(s',t_1,u_1)$, instead of old ones given by Eqs. (23) and (24).

Note that the redefinition of the usual Born-level distributions used in the present paper does not affect any previous predictions of the standard resummation approach. The only purpose of our redefinition is to extend the region of applicability of the soft-gluon approximation to higher values of Q^2.

Our results for the x distributions of the Callan-Gross ratio $R(x,Q^2)=F_L/F_T$ in charm leptoproduction are presented at several values of ξ in the left panel of Fig. 5. For comparison, the K factors $K^{(1)}(x,Q^2)=F_T^{\mathrm{NLO}}/F_T^{\mathrm{LO}}$ and $K^{(2)}(x,Q^2)=F_T^{\mathrm{NNLO}}/F_T^{\mathrm{NLO}}$ for the transverse structure function are shown at the same values of ξ in the right panel of Fig. 5. One can see that the sizeable soft-gluon corrections to the production cross sections affect the Born predictions for $R(x,Q^2)$ both at NLO and NNLO very little, by a few percent only.

Let us briefly discuss the origin of the perturbative stability of the Callan-Gross ratio. Note that the mere spin-independent structure of the Sudakov logarithms can not explain our results, since perturbative stability does not take place at the parton level. In fact, the ratios $\frac{c_L^{(1,0)}}{c_T^{(1,0)}}(z,Q^2)$ and $\frac{c_L^{(0,0)}}{c_T^{(0,0)}}(z,Q^2)$ differ essentially from each other, even near the threshold. This is due to the fact that, according to Eq. (22), the soft-gluon corrections are determined by convolutions of the Born cross sections with the Sudakov logarithms, which, apart from factorized $\delta(s_4)$ terms, contain also nonfactorizable ones, see Eq. (29). For instance, values of $z \sim 10^{-1}$ allow $s_4/m^2 \sim 1$ at $Q^2 \sim m^2$, which leads to significant nonfactorizable corrections. In other words, collinear bremsstrahlung carries away a large part of the initial energy. Since the longitudinal and transverse Born-level partonic cross sections have different energy behaviors, the so-called soft-gluon radiation has different impacts on these quantities.

Our analysis shows that two more factors are responsible for perturbative stability of the hadron-level ratio $R(x, Q^2)$. First, for relatively low virtuality $Q^2 \sim m^2$, both $\hat{\sigma}_T(z, Q^2)$ and $\hat{\sigma}_L(z, Q^2)$ take their maximum values practically at the same values of z not far from the threshold. Second, at $x \sim 10^{-2}$–10^{-1}, the gluon distribution function supports just the threshold region contribution. According to the saddle point arguments, both these factors together lead to an approximate factorization of the Sudakov logarithms at the hadron level and essential cancellation of their contributions in the ratio $R(x, Q^2) = F_L/F_T$.

Note also that the situation with perturbative stability of the Callan-Gross ratio is very similar to the corresponding one that takes place for the azimuthal asymmetry in heavy-quark photo- and leptoproduction. In detail, the soft-gluon corrections to the azimuthal asymmetry were considered in Refs. [10, 12].

IV. ANALYTIC LO PREDICTIONS AT LOW x

Since the radiative corrections to the Callan-Gross ratio in heavy-flavor leptoproduction are small, it makes sense to investigate in more detail the corresponding LO predictions. In this Section, we derive compact low-x approximation formulae for the ratio $R_2(x, Q^2) = 2xF_L/F_2$ at LO, which greatly simplify the extraction of the structure function $F_2(x, Q^2)$ from measurements of the reduced cross section, $\tilde{\sigma}(x, Q^2)$, defined by Eqs. (5) and (6). For this purpose, we convolute the LO partonic cross sections given by Eqs. (9) and (10) with the low-x asymptotics of the gluon PDF:

$$g(x, Q^2) \stackrel{x\to 0}{\longrightarrow} \frac{1}{x^{1+\delta}}. \tag{30}$$

The value of δ in Eq. (30) is a matter of discussion. The simplest choice, $\delta = 0$, leads to a non-singular behavior of the structure functions for $x \to 0$. Another extreme value, $\delta = 1/2$, historically originates from the BFKL resummation of the leading powers of $\ln(1/x)$ [23]. In reality, δ is a function of Q^2 (for an experimental review, see Ref. [31]). Theoretically, the Q^2 dependence of δ is calculated using the DGLAP evolution equations [24].

First, we calculate the LO hadron-level cross sections for both extreme cases, $\delta = 0$ and $1/2$. Our predictions for the quantity $R_2(x, Q^2)$ in the limit of $x \to 0$ have the following form:

$$R_2^{(0)}(Q^2) = \frac{2}{1+4\lambda}\,\frac{1+6\lambda-4\lambda(1+3\lambda)J(\lambda)}{1+2(1-\lambda)J(\lambda)}, \tag{31}$$

$$R_2^{(1/2)}(Q^2) = \frac{8}{1+4\lambda}\,\frac{[3+4\lambda\,(13+32\lambda)]\,E(1/(1+4\lambda)) - 4\lambda\,(9+32\lambda)\,K(1/(1+4\lambda))}{(-37+72\lambda)\,E(1/(1+4\lambda)) + 2\,(23-36\lambda)\,K(1/(1+4\lambda))}, \tag{32}$$

where λ is defined in Eq. (13),

$$J(\lambda) = \frac{1}{\sqrt{1+4\lambda}}\ln\frac{\sqrt{1+4\lambda}+1}{\sqrt{1+4\lambda}-1}, \tag{33}$$

and the functions $K(y)$ and $E(y)$ are the complete elliptic integrals of the first and second kinds defined as

$$K(y) = \int_0^1 \frac{\mathrm{d}t}{\sqrt{(1-t^2)(1-yt^2)}}, \qquad E(y) = \int_0^1 \mathrm{d}t\sqrt{\frac{1-yt^2}{1-t^2}}. \tag{34}$$

The result in Eq. (31) was previously found in Ref. [22], where an approximation to its NLO counterpart was also presented.

The left panel of Fig. 6 shows the ratios $R_2^{(0)}(Q^2)$ and $R_2^{(1/2)}(Q^2)$ as functions of ξ. One can see that the difference between these quantities varies slowly from 20% at low Q^2 to 10% at high Q^2. For comparison, also the LO results for $R_2(x, Q^2)$ calculated at several values of x using the CTEQ5L gluon PDF [26] are shown. We observe that, for $x \to 0$, the CTEQ5L predictions converge to the function $R_2^{(1/2)}(Q^2)$ practically in the entire region of Q^2. We have verified that the same situation takes also place for all other LO and NLO CTEQ PDF versions [25, 26].

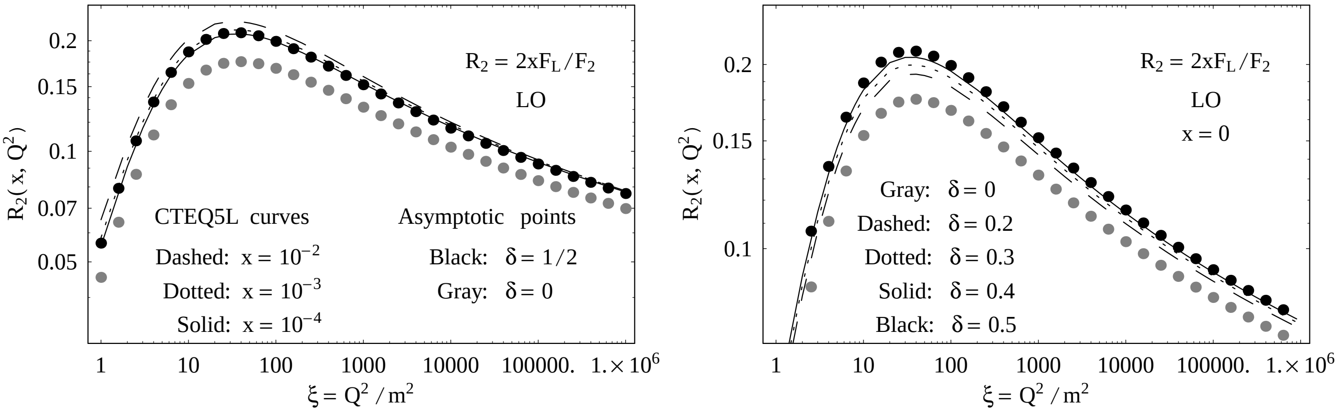

FIG. 6: LO low-x predictions for the ratio $R_2(x,Q^2) = 2xF_L/F_2$ in charm leptoproduction. *Left panel:* Asymptotic ratios $R_2^{(0)}(Q^2)$ (gray points) and $R_2^{(1/2)}(Q^2)$ (black points), as well as CTEQ5L predictions for $R_2(x,Q^2)$ at $x = 10^{-2}$, 10^{-3} and 10^{-4}. *Right panel:* Asymptotic ratio $R_2^{(\delta)}(Q^2)$ at $\delta = 0$, 0.2, 0.3, 0.4 and 0.5.

Next, we derive an analytic low-x formula for the ratio $R_2^{(\delta)}(x,Q^2)$ with arbitrary values of δ, in terms of the Gauss hypergeometric function. Our result has the following form:

$$R_2^{(\delta)}(Q^2) = 4\frac{\frac{2+\delta}{3+\delta}\Phi\left(1+\delta,\frac{1}{1+4\lambda}\right) - (1+4\lambda)\,\Phi\left(2+\delta,\frac{1}{1+4\lambda}\right)}{\left[1+\frac{\delta(1-\delta^2)}{(2+\delta)(3+\delta)}\right]\Phi\left(\delta,\frac{1}{1+4\lambda}\right) - (1+4\lambda)\left(4-\delta-\frac{10}{3+\delta}\right)\Phi\left(1+\delta,\frac{1}{1+4\lambda}\right)}, \tag{35}$$

where the function $\Phi(r,z)$ is defined as

$$\Phi\left(r,z\right) = \frac{z^{1+r}}{1+r}\frac{\Gamma\left(1/2\right)\Gamma\left(1+r\right)}{\Gamma\left(3/2+r\right)}\,{}_2F_1\left(\frac{1}{2},1+r;\frac{3}{2}+r;z\right). \tag{36}$$

The hypergeometric function ${}_2F_1(a,b;c;z)$ has the following series expansion:

$${}_2F_1\left(a,b;c;z\right) = \frac{\Gamma\left(c\right)}{\Gamma\left(a\right)\Gamma\left(b\right)}\sum_{n=0}^{\infty}\frac{\Gamma\left(a+n\right)\Gamma\left(b+n\right)}{\Gamma\left(c+n\right)}\frac{z^n}{n!}. \tag{37}$$

In the right panel of Fig. 6, the δ dependence of the asymptotic ratio $R_2^{(\delta)}(Q^2)$ is investigated. One can see that the ratio $R_2^{(\delta)}(Q^2)$ rapidly converges to the function $R_2^{(1/2)}(Q^2)$ for $\delta > 0.2$. In particular, the relative difference between $R_2^{(0.5)}(Q^2)$ and $R_2^{(0.3)}(Q^2)$ varies slowly from 6% at low Q^2 to 2% at high Q^2.

As mentioned above, the Q^2 dependence of the parameter δ is determined with the help of the DGLAP evolution. However, our analysis shows that hadron-level predictions for $R_2(x \to 0, Q^2)$ depend weakly on δ practically in the entire region of Q^2 for $0.2 < \delta < 0.9$. For this reason, our simple formula (32) with $\delta = 1/2$ (i.e., without any evolution) describes with good accuracy the low-x CTEQ results for $R_2(x,Q^2)$. We conclude that the hadron-level predictions for $R_2(x \to 0, Q^2)$ are stable not only under the NLO corrections to the partonic cross sections, but also under the DGLAP evolution of the gluon PDF.

Finally, we use the analytic expressions (31), (32) and (35) for the extraction of the structure functions $F_2^c(x,Q^2)$ and $F_2^b(x,Q^2)$ from the HERA measurements of the reduced cross sections $\tilde{\sigma}^c(x,Q^2)$ and $\tilde{\sigma}^b(x,Q^2)$, respectively. The results of our analysis of the HERA data on the charm and bottom electroproduction are collected in Tables I and II, respectively. In our calculations, the values $m_c = 1.3$ GeV and $m_b = 4.3$ GeV for the charm and bottom quark masses are used. The LO predictions, F_2(LO), for the cases of $\delta = 0.5$, 0.3 and 0 are presented and compared with the NLO values, F_2(NLO), obtained in the H1 analysis [20, 21]. One can see that all the considered LO predictions agree with the NLO results with an accuracy better than 1%. This is because the contributions of the longitudinal structure functions, $F_L^c(x,Q^2)$ and $F_L^b(x,Q^2)$, to the reduced cross sections, $\tilde{\sigma}^c(x,Q^2)$ and $\tilde{\sigma}^b(x,Q^2)$, are small, less than 5%, in the kinematic range of the HERA H1 experiment.

TABLE I: Values of $F_2^c(x,Q^2)$ extracted from the HERA measurements of $\tilde{\sigma}^c(x,Q^2)$ at low [21] and high [20] Q^2 (in GeV^2) for various values of x (in units of 10^{-3}). The NLO H1 results [20, 21] are compared with the LO predictions corresponding to the cases of $\delta = 0.5$, 0.3 and 0.

Q^2 (GeV^2)	x ($\times 10^{-3}$)	y	$\tilde{\sigma}^c$	Error (%)	F_2^c(NLO) H1	F_2^c(LO) $\delta = 0.5$	F_2^c(LO) $\delta = 0.3$	F_2^c(LO) $\delta = 0$
12	0.197	0.600	0.412	18	0.435 ± 0.078	0.435 ± 0.078	0.434 ± 0.078	0.431 ± 0.077
12	0.800	0.148	0.185	13	0.186 ± 0.024	0.185 ± 0.024	0.185 ± 0.024	0.185 ± 0.024
25	0.500	0.492	0.318	13	0.331 ± 0.043	0.331 ± 0.043	0.330 ± 0.043	0.328 ± 0.043
25	2.000	0.123	0.212	10	0.212 ± 0.021	0.212 ± 0.021	0.212 ± 0.021	0.212 ± 0.021
60	2.000	0.295	0.364	10	0.369 ± 0.040	0.369 ± 0.040	0.368 ± 0.040	0.368 ± 0.040
60	5.000	0.118	0.200	12	0.201 ± 0.024	0.200 ± 0.024	0.200 ± 0.024	0.200 ± 0.024
200	0.500	0.394	0.197	23	0.202 ± 0.046	0.202 ± 0.046	0.202 ± 0.046	0.201 ± 0.046
200	1.300	0.151	0.130	24	0.131 ± 0.032	0.130 ± 0.031	0.130 ± 0.031	0.130 ± 0.031
650	1.300	0.492	0.206	27	0.213 ± 0.057	0.213 ± 0.057	0.213 ± 0.057	0.212 ± 0.057
650	3.200	0.200	0.091	31	0.092 ± 0.028	0.091 ± 0.028	0.091 ± 0.028	0.091 ± 0.028

TABLE II: Values of $F_2^b(x,Q^2)$ extracted from the HERA measurements of $\tilde{\sigma}^b(x,Q^2)$ at low [21] and high [20] Q^2 (in GeV^2) for various values of x (in units of 10^{-3}). The NLO H1 results [20, 21] are compared with the LO predictions corresponding to the cases of $\delta = 0.5$, 0.3 and 0.

Q^2 (GeV^2)	x ($\times 10^{-3}$)	y	$\tilde{\sigma}^b$	Error (%)	F_2^b(NLO) H1	F_2^b(LO) $\delta = 0.5$	F_2^c(LO) $\delta = 0.3$	F_2^b(LO) $\delta = 0$
12	0.197	0.600	0.0045	60	0.0045 ± 0.0027	0.0046 ± 0.0027	0.0046 ± 0.0027	0.0046 ± 0.0027
12	0.800	0.148	0.0048	45	0.0048 ± 0.0022	0.0048 ± 0.0022	0.0048 ± 0.0022	0.0048 ± 0.0022
25	0.500	0.492	0.0122	31	0.0123 ± 0.0038	0.0124 ± 0.0038	0.0124 ± 0.0038	0.0123 ± 0.0038
25	2.000	0.123	0.0061	39	0.0061 ± 0.0024	0.0061 ± 0.0024	0.0061 ± 0.0024	0.0061 ± 0.0024
60	2.000	0.295	0.0189	29	0.0190 ± 0.0055	0.0190 ± 0.0055	0.0190 ± 0.0055	0.0190 ± 0.0055
60	5.000	0.118	0.0130	36	0.0130 ± 0.0047	0.0130 ± 0.0047	0.0130 ± 0.0047	0.0130 ± 0.0047
200	0.500	0.394	0.0393	31	0.0413 ± 0.0128	0.0402 ± 0.0125	0.0401 ± 0.0125	0.0400 ± 0.0124
200	1.300	0.151	0.0212	38	0.0214 ± 0.0081	0.0213 ± 0.0081	0.0213 ± 0.0081	0.0212 ± 0.0081
650	1.300	0.492	0.0230	51	0.0243 ± 0.0124	0.0240 ± 0.0122	0.0239 ± 0.0122	0.0238 ± 0.0121
650	3.200	0.200	0.0124	44	0.0125 ± 0.0055	0.0125 ± 0.0055	0.0125 ± 0.0055	0.0125 ± 0.0055

V. CONCLUSION

We conclude by summarizing our main observations. In the present paper, we studied the radiative corrections to the Callan-Gross ratio $R(x,Q^2)$ in heavy-quark leptoproduction. We considered the exact NLO results at low and moderate $Q^2 \lesssim m^2$, asymptotic NLO predictions at high $Q^2 \gg m^2$, and both NLO and NNLO soft-gluon (or threshold) corrections at large Bjorken x. It turned out that large (especially, at non-small x) radiative corrections to the structure functions $F_T(x,Q^2)$ and $F_L(x,Q^2)$ cancel each other in their ratio $R(x,Q^2) = F_L/F_T$ with good accuracy. As a result, the NLO contributions to the ratio $R(x,Q^2)$ are less than 10% in a wide region of the variables x and Q^2. Our analysis also shows that the NLO predictions for $R(x,Q^2)$ are sufficiently insensitive (to within ten percent) to standard uncertainties in the QCD input parameters. We conclude that, unlike the production cross sections, the Callan-Gross ratio in heavy-quark leptoproduction is quantitatively well defined in pQCD. Measurements of the quantity $R(x,Q^2)$ in charm and bottom leptoproduction would provide a good test of the conventional parton model based on pQCD.

Concerning the experimental aspects, we propose to exploit the observed perturbative stability of the Callan-Gross ratio in the extraction of the structure functions $F_2^c(x,Q^2)$ and $F_2^b(x,Q^2)$ from the corresponding reduced cross sections. For this purpose, we provided compact LO hadron-level formulae for the ratio $R_2(x,Q^2) = 2xF_L/F_2 = R/(1+R)$ in the limit $x \to 0$. We demonstrated that these analytic expressions simplify the extraction of $F_2(x,Q^2)$ without affecting the accuracy of the result in practice. In particular, our LO formula for $R_2(x,Q^2)$ with $\delta = 1/2$ usefully reproduces the results for $F_2^c(x,Q^2)$ and $F_2^b(x,Q^2)$ obtained by the H1 Collaboration [20, 21] with the help of the more cumbersome NLO evaluation of $F_L(x,Q^2)$.

In this paper, we investigated the contribution to $R(x,Q^2)$ from the dominant mechanism, photon-gluon fusion, within the fixed-flavor-number scheme. To take into account the photon-heavy-quark scattering component, one should adopt the variable-flavor-number scheme, which allows one to resum potentially large mass logarithms of the type $\alpha_s \ln\left(Q^2/m^2\right)$, whose contribution dominates at $Q^2 \gg m^2$. Some recent developments concerning this scheme may be found in Ref. [32]. The variable-flavor-number-scheme predictions for the Callan-Gross ratio as well as the possibility to discriminate experimentally between photon-gluon fusion and quark-scattering contributions to $R(x,Q^2)$ will be considered in a forthcoming publication.

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[1] P. Nason, S. Dawson, and R. K. Ellis, Nucl. Phys. B **303**, 607 (1988); P. Nason, S. Dawson, and R. K. Ellis, Nucl. Phys. B **327**, 49 (1989); P. Nason, S. Dawson, and R. K. Ellis, Nucl. Phys. B **335**, 260 (1990); W. Beenakker, H. Kuijf, W. L. van Neerven, and J. Smith, Phys. Rev. D **40**, 54 (1989).

[2] R. K. Ellis and P. Nason, Nucl. Phys. B **312**, 551 (1989); J. Smith and W. L. van Neerven, Nucl. Phys. B **374**, 36 (1992).

[3] E. Laenen, S. Riemersma, J. Smith, and W. L. van Neerven, Nucl. Phys. B **392**, 162 (1993).

[4] S. Frixione, M. L. Mangano, P. Nason and G. Ridolfi, published in: Heavy Flavours II, A. J. Buras and M. Lindner (Eds.), Advanced Series on Directions in High Energy Physics, Vol. 15, World Scientific, Singapore, 1998 [hep-ph/9702287].

[5] R. Vogt, Eur. Phys. J. ST **155**, 213 (2008).

[6] E. Laenen and S. -O. Moch, Phys. Rev. D **59**, 034027 (1999).

[7] N. Kidonakis, Phys. Rev. D **64**, 014009 (2001); N. Kidonakis, Phys. Rev. D **73**, 034001 (2006).

[8] M. L. Mangano, P. Nason, and G. Ridolfi, Nucl. Phys. B **373**, 295 (1992); S. Frixione, M. L. Mangano, P. Nason, and G. Ridolfi, Nucl. Phys. B **412**, 225 (1994).

[9] N. Ya. Ivanov, A. Capella, and A. B. Kaidalov, Nucl. Phys. B **586**, 382 (2000).

[10] N. Ya. Ivanov, Nucl. Phys. B **615**, 266 (2001).

[11] N. Ya. Ivanov, P. E. Bosted, K. Griffioen, and S. E. Rock, Nucl. Phys. B **650**, 271 (2003).

[12] N. Ya. Ivanov, Nucl. Phys. B **666**, 88 (2003).

[13] L. N. Ananikyan and N. Ya. Ivanov, Phys. Rev. D **75**, 014010 (2007); L. N. Ananikyan and N. Ya. Ivanov, Nucl. Phys. B **762**, 256 (2007).

[14] L. G. Almeida, G. Sterman, and W. Vogelsang, Report No. BNL-NT-08/13 and YITP-SB-08-15 [arXiv:0805.1885 [hep-ph]].

[15] SLAC E161 (2000), http://www.slac.stanford.edu/exp/e160.

[16] S. Riemersma, J. Smith, and W. L. van Neerven, Phys. Lett. B **347**, 43 (1995); B. W. Harris and J. Smith, Nucl. Phys. B **452**, 109 (1995).

[17] M. Buza, Y. Matiounine, J. Smith, R. Migneron, and W. L. van Neerven, Nucl. Phys. B **472**, 611 (1996); I. Bierenbaum, J. Blumlein, and S. Klein, Nucl. Phys. B **780**, 40 (2007).

[18] M. Buza, Y. Matiounine, J. Smith, and W. L. van Neerven, Eur. Phys. J. C **1**, 301 (1998).

[19] H1 Collaboration, C. Adloff et al., Z. Phys. C **72**, 593 (1996); H1 Collaboration, C. Adloff et al., Phys. Lett. B **393**, 452 (1997); H1 Collaboration, C. Adloff et al., Nucl. Phys. B **545**, 21 (1999); ZEUS Collaboration, J. Breitweg et al., Phys. Lett. B **407**, 402 (1997); ZEUS Collaboration, J. Breitweg et al., Eur. Phys. J. C **12**, 35 (2000).

[20] H1 Collaboration, A. Aktas et al., Eur. Phys. J. C **40**, 349 (2005).

[21] H1 Collaboration, A. Aktas et al., Eur. Phys. J. C **45**, 23 (2006).

[22] A. Yu. Illarionov, B. A. Kniehl and A. V. Kotikov, Phys. Lett. B **663**, 66 (2008).

[23] E. A. Kuraev, L. N. Lipatov and V. S. Fadin, Sov. Phys. JETP **44**, 443 (1976) [Zh. Eksp. Teor. Fiz. **71**, 840 (1976)]; E. A. Kuraev, L. N. Lipatov and V. S. Fadin, Sov. Phys. JETP **45**, 199 (1977) [Zh. Eksp. Teor. Fiz. **72**, 377 (1977)]; I. I.. Balitzki and L. N. Lipatov, Sov. J. Nucl. Phys. **28**, 822 (1978) [Yad. Fiz. **28**, 1597 (1978)]; L. N. Lipatov, Sov. Phys. JETP **63**, 904 (1986) [Zh. Eksp. Teor. Fiz. **90**, 1536 (1986)].

[24] V. N. Gribov and L. N. Lipatov, Sov. J. Nucl. Phys. **15**, 438 (1972); Y. L. Dokshitzer, Sov. Phys. JETP **46**, 641 (1977); G. Altarelli and G. Parisi, Nucl. Phys. B **126**, 298 (1977).

[25] H.L. Lai, J. Huston, S. Kuhlmann, F. Olness, J. Owens, D. Soper, W.K. Tung and H. Weerts, Phys. Rev. D **55**, 1280 (1997); J. Pumplin, D. R. Stump, J. Huston, H.-L. Lai, P. Nadolsky and W.-K. Tung, JHEP **0207**, 012 (2002).

[26] H.L. Lai, J. Huston, S. Kuhlmann, J. Morfin, F. Olness, J.F. Owens, J. Pumplin and W.K. Tung, Eur. Phys. J. C **12**, 375 (2000).

[27] J. P. Leveille and T. Weiler, Phys. Rev. D **24**, 1789 (1981); A. D. Watson, Z. Phys. C **12**, 123 (1982).

[28] J. Blumlein, A. De Freitas, W. L. van Neerven, and S. Klein, Nucl. Phys. B **755**, 272 (2006).

[29] H. Contopanagos, E. Laenen, and G. Sterman, Nucl. Phys. B **484**, 303 (1997); N. Kidonakis, G. Oderda, and G. Sterman, Nucl. Phys. B **531**, 365 (1998); E. Laenen, G. Oderda, and G. Sterman, Phys. Lett. B **438**, 173 (1998).

[30] N. Ya. Ivanov et al., in preparation.

[31] A. Vogt, in: Proceedings of the 15th International Workshop on Deep-Inelastic Scattering and Related Subjects (DIS2007), edited by G. Grindhammer and K. Sachs, DESY, Hamburg, 2007, p. 39 [arXiv:0707.4106 [hep-ph]].

[32] M. Krämer, F. I. Olness and D. E. Soper, Phys. Rev. D **62**, 096007 (2000); W.-K. Tung, S. Kretzer and C. Schmidt, J. Phys. G: Nucl. Part. Phys. **28**, 983 (2002). S. Kretzer, H. L. Lai, F. I. Olness and W. K. Tung, Phys. Rev. D **69**, 114005 (2004); R. S. Thorne, Phys. Rev. D **73**, 054019 (2006); W. K. Tung, H. L. Lai, A. Belyaev, J. Pumplin, D. Stump and C.-P. Yuan, JHEP **0702**, 053 (2007); J. Pumplin, H. L. Lai, and W. K. Tung, Phys. Rev. D **75**, 054029 (2007).