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Higgs versus Matter in the Heterotic Landscape

W. Buchmüller and J. Schmidt

Deutsches Elektronen-Synchrotron DESY, Hamburg, Germany

Abstract

In supersymmetric extensions of the standard model there is no basic difference between Higgs and matter fields, which leads to the well known problem of potentially large baryon and lepton number violating interactions. Although these unwanted couplings can be forbidden by continuous or discrete global symmetries, a theoretical guiding principle for their choice is missing. We examine this problem for a class of vacua of the heterotic string compactified on an orbifold. As expected, in general there is no difference between Higgs and matter. However, certain vacua happen to possess unbroken matter parity and discrete R-symmetries which single out Higgs fields in the low energy effective field theory. We present a method how to identify maximal vacua in which the perturbative contribution to the μ-term and the expectation value of the superpotential vanish. Two vacua are studied in detail, one with two pairs of Higgs doublets and one with partial gauge-Higgs unification.

1 Introduction

In the standard model there is a clear distinction between Higgs and matter: Quarks and leptons are chiral fermions whereas a scalar field describes the Higgs boson. The most general renormalizable lagrangian consistent with gauge and Lorentz invariance yields a very successful description of strong and electroweak interactions [1]. Furthermore, with appropriate coefficients, the unique dimension-5 operator can account for Majorana neutrino masses, and the baryon number violating dimension-6 operators are consistent with the experimental bounds on proton decay.

In supersymmetric extensions of the standard model the distinction between Higgs and matter is generically lost. Since the lepton doublets and one of the Higgs doublets have the same gauge quantum numbers the most general supersymmetric gauge invariant lagrangian contains unsuppressed R-parity violating terms which lead to rapid proton decay. In grand unified models (GUTs) [1] colour triplet exchange can also generate dangerous baryon number violating dimension-5 operators. These problems can be overcome by introducing continuous or discrete symmetries which distinguish between Higgs and matter fields, such as R-symmetry, Peccei-Quinn type symmetries or matter parity. However, in the context of four-dimensional (4D) field theories the origin and theoretical justification of these symmetries remain unclear.

Higher-dimensional theories provide a promising framework for unified extensions of the supersymmetric standard model [2]. In particular the heterotic string [3] with gauge group $\mathrm{E}_8 \times \mathrm{E}_8$ is the natural candidate for a unified theory including gravity. Its compactifications on orbifolds [4, 5] yield chiral gauge theories in four dimensions including the standard model as well as GUT gauge groups. During the past years some progress has been made in deriving unified field theories from the heterotic string [6–8], separating the GUT scale from the string scale on anisotropic orbifolds [9], and a class of compactifications yielding supersymmetric standard models in four dimensions have been successfully constructed [10–12].

The heterotic string model [10] has a 6D orbifold GUT limit, where two compact dimensions are much larger than the other four, with 6D bulk gauge group SU(6) and unbroken SU(5) symmetry at two fixed points. The corresponding supergravity model has been explicitly constructed in [13], and it has been shown that all bulk and brane anomalies are canceled by the Green-Schwarz mechanism. Furthermore, a class of vacua has been found which have a pair of bulk Higgs fields and two SU(5) bulk families in addition to the two SU(5) brane families. At the SU(5) fixed points these fields form an SU(5) GUT model. In 4D one obtains one quark-lepton 'family' and a pair of Higgs doublets from split bulk multiplets together with the two brane families.

What distinguishes Higgs from matter fields with the same SU(5) quantum numbers in an orbifold GUT? In the vacuum studied in [13] there is no distinction, which leads to unacceptable R-parity violating Yukawa couplings. In [11] interesting 4D vacua with unbroken matter parity were found, which allow to forbid the dangerous R-parity violating couplings. Some of these vacua also have gauge-Higgs unification for which an intriguing relationship exists between μ-term and gravitino mass. Indeed, several vacua with semirealistic Yukawa couplings could be identified where to order six in powers of standard model singlets μ-term and gravitino mass both vanish.

In this paper we further analyse the vacua of the 6D orbifold GUT [13]. Since $M_{\rm GUT} \ll M_{\rm string}$, we consider vacua with expectation values (VEVs) of all 6D zero modes. One then obtains further vacua with unbroken matter parity. The localized Fayet-Iliopoulos terms of anomalous U(1) symmetries may indeed stabilize two compact dimensions at the GUT scale [13,14] but the study of stabilization and profiles of bulk fields [15] is beyond the scope of this paper. In the following we concentrate on local properties of the model at the GUT fixed points, in particular the decoupling of exotics and the generation of superpotential terms.

The existence of a matter parity is not sufficient to distinguish Higgs from matter. One also needs that the μ-term is much smaller than the decoupling mass of exotic states. In principle, there are two obvious solutions: Either a non-zero μ-term is generated at very high powers in standard model singlets, or the perturbative part of the μ-term vanishes exactly and a non-perturbative contribution, possibly related to supersymmetry breaking, yields a correction of the order of the electroweak scale. In Section 4, we shall discuss how to identify 'maximal' vacua with vanishing μ-term, as well as extended vacua with μ-terms generated at high orders. This is the main point of our paper.

The maximal vacua with vanishing μ-term do not include the case of gauge-Higgs unification. Instead, we find a vacuum with two pairs of massless Higgs doublets and one with partial gauge-Higgs unification only for H_u which gives mass to up-type quarks. This is perfectly consistent with the fact that a large top-quark mass is singled out. The original symmetry between $\mathbf{5}$- and $\bar{\mathbf{5}}$-plets is violated by selecting vacua where matter belongs to $\bar{\mathbf{5}}$- and $\mathbf{10}$-plets.

There are also other promising approaches which use elements of unification to find realistic string vacua. This includes compactifications on Calabi-Yau manifolds with vector bundles [16–21], which are related to orbifold constructions whose singularities are blown up [22, 23]. Very recently, also interesting GUT models based on F-theory have been discussed [24–26].

The paper is organized as follows. In Section 2 we recall some symmetry properties of effective SU(5) field theories, which are relevant for the μ-term and baryon number violating interactions. The relevant features of the 6D orbifold GUT model [13] are briefly reviewed in Section 3. New vacua of this model with vanishing μ-term and gravitino mass are analyzed in Section 4, and the corresponding unbroken discrete R-symmetries are determined. Yukawa couplings for these vacua are calculated in Section 5.

2 Effective low energy field theory

The heterotic 6D GUT model [13] has local SU(5) invariance corresponding to Georgi-Glashow unification. Hence, the superpotential of the corresponding low energy 4D field theory has the general form,

$$W = \mu H_u H_d + \mu_i H_u \bar{\mathbf{5}}_{(i)} + C_{ij}^{(u)} \mathbf{10}_{(i)} \mathbf{10}_{(j)} H_u + C_{ij}^{(d)} \bar{\mathbf{5}}_{(i)} \mathbf{10}_{(j)} H_d$$

$$+C^{(R)}_{ijk}\bar{\mathbf{5}}_{(i)}\mathbf{10}_{(j)}\bar{\mathbf{5}}_{(k)} + C^{(L)}_{ij}\bar{\mathbf{5}}_{(i)}H_u\bar{\mathbf{5}}_{(k)}H_u + C^{(B)}_{ijkl}\mathbf{10}_{(i)}\mathbf{10}_{(j)}\mathbf{10}_{(k)}\bar{\mathbf{5}}_{(l)}\ , \tag{2.1}$$

where we have included dimension-5 operators. Here $i, j, ...$ denote generation indices, and for simplicity we have kept the SU(5) notation. Note that the colour triplets contained in the Higgs fields $H_u = \mathbf{5}$ and $H_d = \bar{\mathbf{5}}$ are projected out. μ_i and $C^{(R)}$ yield the well known renormalizable baryon (B) and lepton (L) number violating interactions, and the coefficients $C^{(L)}$ and $C^{(B)}$ of the dimension-5 operators are usually obtained by integrating out states with masses $\mathcal{O}(M_{\mathrm{GUT}})$. In supergravity theories also the expectation value of the superpotential is important since it determines the gravitino mass. One expects

$$\langle W\rangle \sim \mu \sim M_{\mathrm{EW}}, \tag{2.2}$$

if the scale M_{EW} of electroweak symmetry breaking is related to supersymmetry breaking.

Experimental bounds on the proton lifetime and lepton number violating processes imply $\mu_i \ll \mu$, $C^{(R)} \ll 1$ and $C^{(B)} \ll 1/M_{\mathrm{GUT}}$. Furthermore, one has to accommodate the hierarchy between the electroweak scale and the GUT scale, $M_{\mathrm{EW}}/M_{\mathrm{GUT}} = \mathcal{O}(10^{-14})$. On the other hand, lepton number violation should not be too much suppressed, since $C^{(L)} \sim 1/M_{\mathrm{GUT}}$ yields the right order of magnitude for neutrino masses.

These phenomenological requirements can be implemented by means of continuous or discrete symmetries. Imposing an additional U(1) factor with

$$\begin{aligned} SU(5)\times U(1)_X &\subset SO(10)\ ,\\ SU(5)\times U(1)_X &\supset SU(3)\times SU(2)\times U(1)_Y\times U(1)_{B-L}\ , \end{aligned} \tag{2.3}$$

where Y denotes the standard model hypercharge, one has $\mu_i = C^{(R)} = C^{(L)} = 0$, since these operators contain only $B-L$ violating terms. On the other hand, $C^{(B)}$ conserves $B-L$ and is therefore not affected. The canonical $\mathrm{U(1)}_X$ charges read

$$t_X(\mathbf{10}) = \frac{1}{5}\ ,\quad t_X(\bar{\mathbf{5}}) = -\frac{3}{5}\ ,\quad t_X(H_u) = -\frac{2}{5}\ ,\quad t_X(H_d) = \frac{2}{5}\ , \tag{2.4}$$

with

$$t_{B-L} = t_X + \frac{4}{5}\, t_Y\ . \tag{2.5}$$

The wanted result, $\mu_i = C^{(R)} = 0$, $C^{(L)} \neq 0$, can be obtained with a $\mathbb{Z}_2^X$ subgroup of $\mathrm{U(1)}_X$, which contains the 'matter parity' P_X [27],

$$P_X(\mathbf{10}) = P_X(\bar{\mathbf{5}}) = -1\ ,\quad P_X(H_u) = P_X(H_d) = 1. \tag{2.6}$$

Matter parity, however, does not solve the problem $C^{(B)} \neq 0$, and also the hierarchy $M_{\mathrm{EW}}/M_{\mathrm{GUT}} \ll 1$ remains unexplained.

In supersymmetric extensions of the standard model, electroweak symmetry breaking is usually tied to supersymmetry breaking. It is then natural to have $\mu = \mu_i = 0$ for unbroken supersymmetry. One easily verifies that in this case, for $C^{(R)} = C^{(B)} = 0$, the superpotential aquires a unique Peccei-Quinn type $\mathrm{U(1)}_{PQ}$ symmetry with charges

$$t_{PQ}(\mathbf{10}) = \frac{1}{2}\ ,\quad t_{PQ}(\bar{\mathbf{5}}) = 1\ ,\quad t_{PQ}(H_u) = -1\ ,\quad t_{PQ}(H_d) = -\frac{3}{2}\ , \tag{2.7}$$

together with an additional $\mathrm{U}(1)_R$ symmetry with R-charges

$$R(\mathbf{10}) = R(\bar{\mathbf{5}}) = 1\ , \quad R(H_u) = R(H_d) = 0. \tag{2.8}$$

Note that the $\mathrm{U}(1)_R$-symmetry implies the wanted relations $\mu = \mu_i = C^{(R)} = C^{(B)} = 0$, with $C^{(L)}$ unconstrained. On the other hand, the Peccei-Quinn symmetry only yields $\mu = C^{(R)} = C^{(B)} = 0$.

The latter relations can also be obtained by imposing only a discrete $\mathbb{Z}_2^{PQ}$ subgroup with PQ-parities

$$P_{PQ}(\mathbf{10}) = P_{PQ}(H_d) = -1\ , \quad P_{PQ}(\bar{\mathbf{5}}) = P_{PQ}(H_u) = 1\ . \tag{2.9}$$

On the contrary, the familiar R-parity, which is preserved by non-zero gaugino masses,

$$P_R(\mathbf{10}) = P_R(\bar{\mathbf{5}}) = -1\ , \quad P_R(H_u) = P_R(H_d) = 1\ , \tag{2.10}$$

implies $\mu_i = C^{(R)} = 0$, whereas μ, $C^{(L)}$ and $C^{(B)}$ are all allowed.

In summary, the unwanted terms in the lagrangian (2.1) can be forbidden by a continuous global R-symmetry. Supersymmetry breaking will also break $\mathrm{U}(1)_R$ to R-parity, which may lead to an R-axion. The dangerous terms μ and $C^{(B)}$ will then be proportional to the soft supersymmetry breaking terms and therefore strongly suppressed. Alternatively, the unwanted terms in (2.1) can be forbidden by discrete symmetries, such as matter parity, PQ-parity or R-parity.

In ordinary 4D GUT models continuous or discrete symmetries can be introduced by hand. It is interesting to see how protecting global symmetries arise in higher-dimensional theories. The global $U(1)_R$ symmetry (2.8) indeed occurs naturally [28], and it has been used in 5D and 6D orbifold GUTs [29]. However, as we shall see in the following sections, orbifold compactifications of the heterotic string single out discrete symmetries, which may or may not commute with supersymmetry.

3 Heterotic SU(6) model in six dimensions

Let us now briefly describe the main ingredients of the 6D orbifold GUT model derived in [13]. The starting point is the $\mathrm{E}_8 \times \mathrm{E}_8$ heterotic string propagating in the space-time background $(X_4 \times Y_2)/\mathbb{Z}_2 \times M_4$. Here $X_4 = (\mathbb{R}^4/\Lambda_{\mathrm{G}_2\times\mathrm{SU}(3)})/\mathbb{Z}_3$, $Y_2 = (\mathbb{R}^2/\Lambda_{\mathrm{SO}(4)})$ and M_4 represents four-dimensional Minkowski space; $\mathbb{R}^4/\Lambda_{\mathrm{G}_2\times\mathrm{SU}(3)}$ and $\mathbb{R}^2/\Lambda_{\mathrm{SO}(4)}$ are the tori associated with the root lattices of the Lie groups $\mathrm{G}_2 \times \mathrm{SU}(3)$ and $\mathrm{SO}(4)$, respectively. By construction the $\mathbb{Z}_{6-\mathrm{II}} = \mathbb{Z}_3 \times \mathbb{Z}_2$ twist yielding the orbifold has $\mathbb{Z}_3$ and $\mathbb{Z}_2$ subtwists which act trivially on the SO(4) and the SU(3) plane, respectively. As a consequence, the model has bulk fields living in ten dimensions and fields from twisted sectors, which are confined to six or four dimensions.

The model has twelve fixed points[1] where the $\mathrm{E}_8 \times \mathrm{E}_8$ symmetry is broken to different subgroups whose intersection is the standard model gauge group up to U(1) factors.

[1] In the following we shall use the terms 'brane' and 'fixed point' interchangeably. Furthermore, we follow the notations and conventions of [13].

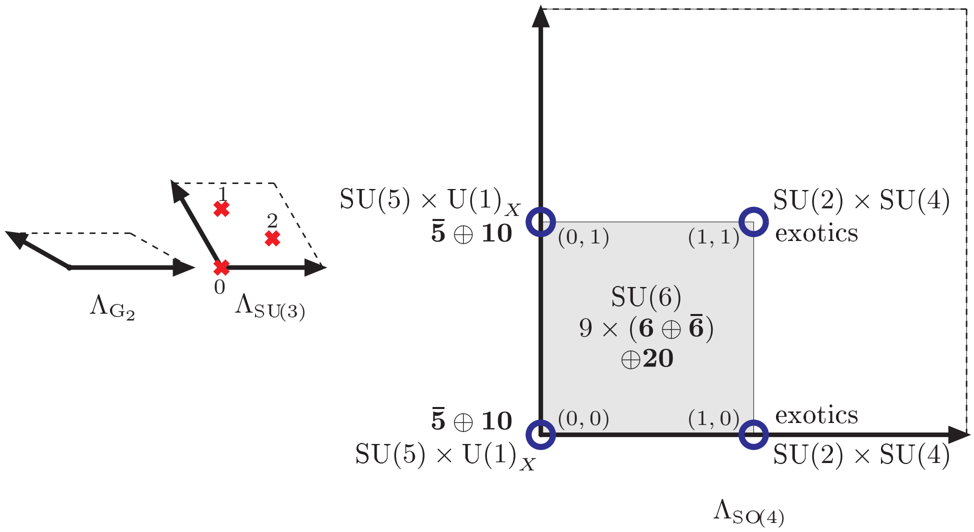

Figure 1: *The six-dimensional orbifold GUT model with the unbroken non-Abelian subgroups of the 'visible' E_8 and the corresponding non-singlet hyper- and chiral multiplets in the bulk and at the SU(5) GUT fixed points, respectively. Fixed points under the $\mathbb{Z}_2$ subtwist in the SO(4) plane are labelled by tupels (n_2, n_2'), those under the $\mathbb{Z}_3$ subtwist in the SU(3) plane carry the label $n_3 = 0, 1, 2$. The $\mathbb{Z}_6$ fixed point in the G_2 plane is located at the origin.*

The geometry has an interesting six-dimensional orbifold GUT limit which is obtained by shrinking the relative size of X_4 as compared to Y_2. Such an anisotropy can account geometrically for the hierachy between the string scale and the GUT scale. The space group embedding [10] includes one Wilson line along a one-cycle in X_4, and a second one as a non-trivial representation of a lattice shift within Y_2. This leads to the MSSM in the effective 4D theory [10, 11] with the 6D orbifold GUT shown in Figure 1 as intermediate step [13]. At two equivalent fixed points, labelled as $(n_2, n_2') = (0, 0), (0, 1)$, the unbroken group contains SU(5); at the two other fixed points, $(n_2, n_2') = (1, 0), (1, 1)$, the unbroken group contains SU(2) × SU(4).[2]

The 6D orbifold GUT has $\mathcal{N} = 2$ supersymmetry and unbroken gauge group

$$\mathrm{G}_6 \;=\; \mathrm{SU}(6) \times \mathrm{U}(1)^3 \times \left[\mathrm{SU}(3) \times \mathrm{SO}(8) \times \mathrm{U}(1)^2\right], \tag{3.1}$$

with the corresponding massless vector multiplets

$$(\mathbf{35}; 1, 1) + (1; \mathbf{8}, 1) + (1; 1, \mathbf{28}) + 5 \times (1; 1, 1) \ . \tag{3.2}$$

In addition one finds the bulk hypermultiplets

$$(\mathbf{20}; 1, 1) + (1; 1, \mathbf{8}) + (1; 1, \mathbf{8}_s) + (1; 1, \mathbf{8}_c) + 4 \times (1; 1, 1)\,, \tag{3.3}$$

where we have dropped the U(1) charges. It is convenient to decompose all $\mathcal{N} = 2$ 6D multiplets in terms of $\mathcal{N} = 1$ 4D multiplets. The 6D vector multiplet splits into a pair of

[2] A 5D orbifold GUT model with the same bulk and brane gauge symmetries and gauge-Higgs unification has been constructed in [30]; the matter and Higgs sector, however, is very different from the model [13].

4D vector and chiral multiplets, $A = (V, \phi)$, whereas a hypermultiplet contains of a pair of chiral multiplets, $H = (H_L, H_R)$; here ϕ and H_L are left-handed, H_R is right-handed. It is often convenient to use the charge conjugate field H_R^c instead of H_R so that all degrees of freedom are contained in left-handed chiral multiplets. In the following we use the same symbol for a hypermultiplet and its left-handed chiral multiplet; the superscript 'c' indicates that the field is the charge conjugate of a right-handed chiral multiplet contained in a hypermultiplet. As an example, the chiral multiplets $\mathbf{5}$ and $\bar{\mathbf{5}}^c$ are both $\mathbf{5}$-plets of SU(5), but they belong to different hypermultiplets which transform as $\mathbf{5}$ and $\bar{\mathbf{5}}$, respectively.

As we shall see, the four non-Abelian singlets, denoted as $U_1 ... U_4$, play a crucial role in vacua with unbroken matter parity; the SU(6) $\mathbf{20}$-plet contains part of one quark-lepton generation. At the SU(5) fixed points one has

$$\mathbf{35} = \mathbf{24} + \mathbf{5} + \bar{\mathbf{5}} + \mathbf{1} \;, \quad \mathbf{20} = \mathbf{10} + \overline{\mathbf{10}} \;. \tag{3.4}$$

In addition to the vector and hypermultiplets from the untwisted sector of the string, there are 6D bulk fields which originate from the twisted sectors T_2 and T_4 of the $\mathbb{Z}_{6-\mathrm{II}}$ orbifold model. They are localized at the fixed points of the $\mathbb{Z}_3$ subtwist in the SU(3) plane, but bulk fields in the SO(4) plane which is left invariant by this subtwist. In contrast, fields of the twisted sectors T_1, T_5 and T_3 are localized at fixed points in the SO(4) plane. For simplicity, we shall list in the following only the states of the 'visible' sector, the complete set of fields can be found in [13]. For each of the three fixed points in the SU(3) plane, one finds

$$3 \times (\mathbf{6}_{n_3} + \bar{\mathbf{6}}_{n_3} + Y_{n_3} + \bar{Y}_{n_3}), \quad n_3 = 0, 1, 2, \tag{3.5}$$

where the omitted $U(1)$ charges depend on n_3. The multiplicity factor 3 is related to three different localizations in the G_2 plane; Y_{n_3} and $\bar{Y}_{n_3}$ denote singlets under the non-Abelian part of G_6. At the SU(5) fixed points $n_2 = 0$, Eq. (3.5) reads

$$3 \times (\mathbf{5}_{n_3} + \bar{\mathbf{5}}_{n_3} + X_{n_3} + \bar{X}_{n_3} + Y_{n_3} + \bar{Y}_{n_3}) \;, \quad n_3 = 0, 1, 2 \;, \tag{3.6}$$

where $X_{n_3}, \bar{X}_{n_3}$ denote SU(5) singlets. Note that each $\mathcal{N} = 2$ hypermultiplet H contains two $\mathcal{N} = 1$ chiral multiplets H and H^c with opposite gauge quantum numbers.

At the two inequivalent fixed points in the SO(4) plane the bulk gauge group G_6 is broken to the subgroups $G_{n_2=0}$ and $G_{n_2=1}$, respectively,

$$G_{n_2=0} = \mathrm{SU}(5) \times \mathrm{U}(1)^4 \times \left[\mathrm{SU}(3) \times \mathrm{SO}(8) \times \mathrm{U}(1)^2\right] , \tag{3.7}$$

$$G_{n_2=1} = \mathrm{SU}(2) \times \mathrm{SU}(4) \times \mathrm{U}(1)^4 \times \left[\mathrm{SU}(2)' \times \mathrm{SU}(4)' \times \mathrm{U}(1)^4\right] . \tag{3.8}$$

At these fixed points $\mathcal{N} = 1$ chiral multiplets from the twisted sectors T_1/T_5 and T_3 are localized. At each SU(5) fixed point one has

$$\bar{\mathbf{5}} + \mathbf{10} + N^c + S_1 + \ldots + S_8 \;. \tag{3.9}$$

This provides two quark-lepton families and additional singlets whose vacuum expectation values, together with those of X_{n_3} and Y_{n_3} can break unwanted U(1) symmetries. Note that

$\bar{\mathbf{5}}$, $\mathbf{10}$ and N^c form together a $\mathbf{16}$-plet of SO(10) which is unbroken at two equivalent fixed points of the 6D orbifold $T^6/\mathbb{Z}_{6-\mathrm{II}}$ [10]. Hence N^c is one of the 'right-handed' neutrinos in the theory.

According to Eqs. (3.4) and (3.6), the 6D theory dimensionally reduced to 4D is vectorlike. In terms of $\mathcal{N}=1$ chiral multiplets there are two $\mathbf{10}$'s, two $\overline{\mathbf{10}}$'s, 19 $\mathbf{5}$'s and 19 $\bar{\mathbf{5}}$'s. The chiral spectrum in 4D is a consequence of the further orbifold compactification. At the fixed points of the SO(4) plane two chiral families, $\bar{\mathbf{5}}+\mathbf{10}$, occur. Furthermore, the boundary conditions for the 6D bulk fields at the fixed points lead to a chiral massless spectrum. Zero modes require positive 'parities' for bulk fields at all fixed points. As shown in [13], positive parities at the SU(5) fixed points reduce the 18 $\bar{\mathbf{5}}$'s and 18 $\mathbf{5}$'s in Eq. (3.6) to 10 $\bar{\mathbf{5}}$'s and 8 $\mathbf{5}$'s, i.e., to a chiral spectrum.

The model clearly has a huge vacuum degeneracy. In most vacua the standard model gauge group will be broken. This can be avoided by allowing only VEVs of the SM singlet fields,

$$U_1^c, ..., U_4, X_0, ..., \bar{X}_2^c, Y_0, ..., \bar{Y}_2^c, S_1, ..., S_8\ , \tag{3.10}$$

but most vacua will have a massless spectrum different from the MSSM. An interesting subset of vacua can be identified by observing that the products $\mathbf{5}_{n_3}\mathbf{5}^c_{n_3}$ and $\bar{\mathbf{5}}_{n_3}\bar{\mathbf{5}}^c_{n_3}$ are total gauge singlets for which one can easily generate masses at the SU(5) fixed points. This allows the decoupling of 6 pairs of $\mathbf{5}$'s and $\bar{\mathbf{5}}$'s [13],

$$W \supset M_* \left(\mathbf{5}_0\mathbf{5}_0^c + \bar{\mathbf{5}}_0\bar{\mathbf{5}}_0^c + \mathbf{5}_1\mathbf{5}_1^c + \bar{\mathbf{5}}_1\bar{\mathbf{5}}_1^c + \mathbf{5}_2\mathbf{5}_2^c + \bar{\mathbf{5}}_2\bar{\mathbf{5}}_2^c\right)\ , \tag{3.11}$$

after which one is left with three $\mathbf{5}$-plets, five $\bar{\mathbf{5}}$-plets and two $\mathbf{10}$-plets,

$$\mathbf{5},\ \bar{\mathbf{5}},\ \mathbf{5}_0^c,\ \bar{\mathbf{5}}_0^c,\ \mathbf{5}_1,\ \bar{\mathbf{5}}_1,\ \mathbf{5}_2^c,\ \bar{\mathbf{5}}_2;\ \ \mathbf{10},\ \overline{\mathbf{10}}^c\ . \tag{3.12}$$

The decoupling scale M_* will be discussed in more detail later on. We are now getting rather close to the standard model. The bulk fields, together with the localized fields (3.9), can account for four quark-lepton families, and the additional three pairs of $\mathbf{5}$- and $\bar{\mathbf{5}}$-plets may contain a pair of Higgs fields.

How can one distinguish between Higgs and matter fields and which fields should be decoupled? The discussion in Section 2 suggests to search for the $\mathrm{U}(1)_X$ symmetry among the six U(1) factors at the SU(5) fixed points, so that the extended $\mathrm{SU}(5)\times\mathrm{U}(1)_X$ gauge symmetry contains $\mathrm{U}(1)_{B-L}$,

$$t_X = \sum_{i=1}^{5} a_i t_i + a_6 t_6^0, \quad t_{B-L} = t_X + \frac{4}{5}t_Y. \tag{3.13}$$

Here $t_1, \ldots, t_6^0$ are generators of the six local U(1) factors[3] at $n_2=0$ (cf. [13]), and t_Y is the hypercharge generator in SU(5). For completeness all charges of the remaining SU(5) multiplets and the singlets (3.10) are listed in Tables 3.2 and 3.3, respectively.

[3]Note that the t_i are orthogonal but not normalized, $t_i \cdot t_j = \mathrm{diag}(1,1,6,1,3,30)$, where $t_6 \equiv t_6^0$.

	$\mathbf{5}$	$\bar{\mathbf{5}}_0^c$	$\mathbf{5}_1$	$\bar{\mathbf{5}}$	$\mathbf{5}_0^c$	$\bar{\mathbf{5}}_1$	$\bar{\mathbf{5}}_2$	$\mathbf{5}_2^c$
$\mathrm{U}(1)_X$	$-\frac{2}{5}$	$-\frac{2}{5}$	$-\frac{2}{5}$	$\frac{2}{5}$	$\frac{2}{5}$	$-\frac{3}{5}$	$\frac{2}{5}$	$-\frac{3}{5}$
$\mathrm{SU}(3)\times\mathrm{SU}(2)$	$(1,\mathbf{2})$	$(\mathbf{3},1)$	$(1,\mathbf{2})$	$(1,\mathbf{2})$	$(\bar{\mathbf{3}},1)$	$(1,\mathbf{2})$	$(\bar{\mathbf{3}},1)$	$(1,\mathbf{2})$
$\mathrm{U}(1)_{B-L}$	0	$-\frac{2}{3}$	0	0	$\frac{2}{3}$	0	$-\frac{1}{3}$	-1
MSSM	H_u?		H_u?	H_d?		H_d?	d_3	l_3

Table 3.1: *SU(5) non-singlet chiral multiplets at $n_2 = 0$. SU(3) × SU(2) representations, $B-L$ charges and MSSM identification refer to the zero modes.*

We can now demand the canonical $\mathrm{U}(1)_X$ charges (2.4) for the localized fields and the bulk $\mathbf{10}$- and $\overline{\mathbf{10}}^c$-plets. This fixes four coefficients: $a_1 = a_2 = 2a_4, a_3 = -1/3, a_6 = 1/(15)$. Two $\mathbf{5}$- and two $\bar{\mathbf{5}}$-plets then have the charges of the Higgs multiplets H_u and H_d, respectively,

$$t_X(\mathbf{5}) = t_X(\bar{\mathbf{5}}_0^c) = -\frac{2}{5}, \quad t_X(\bar{\mathbf{5}}) = t_X(\mathbf{5}_0^c) = \frac{2}{5}. \tag{3.14}$$

This leaves $\bar{\mathbf{5}}_1$, $\mathbf{5}_2^c$ and $\bar{\mathbf{5}}_2$ as candiates for matter fields. The requirement to identify two $\bar{\mathbf{5}}$-plets which, together with $\mathbf{10}$ and $\overline{\mathbf{10}}^c$, form two generations, uniquely determines the last two coefficents, $a_1 = 1$ and $a_5 = 1/6$, so that

$$t_X = t_1 + t_2 - \frac{1}{3}t_3 + \frac{1}{2}t_4 + \frac{1}{6}t_5 + \frac{1}{15}t_6^0. \tag{3.15}$$

The remaining charge assignments read

$$t_X(\mathbf{5}_1) = -t_X(\bar{\mathbf{5}}_1) = -\frac{2}{5}\,, \quad t_X(\bar{\mathbf{5}}_2) = t_X(\mathbf{5}_2^c) = -\frac{3}{5}\,. \tag{3.16}$$

One can also embed the $\mathrm{U}(1)_{PQ}$ symmetry (2.7) in the product $\mathrm{U}(1)^6$. One finds

$$t_{PQ} = -\frac{1}{2}(t_1 + t_2) + \frac{1}{6}t_3 - \frac{1}{2}t_4 + \frac{1}{6}t_5 + \frac{1}{15}t_6^0. \tag{3.17}$$

However, in the vacua considered in the next section, this symmetry is completely broken.

To proceed further we now consider the zero modes of the $\mathbf{5}$- and $\bar{\mathbf{5}}$-plets listed in Table 3.1: $\bar{\mathbf{5}}_0^c$ and $\mathbf{5}_0^c$ yield exotic colour triplets and therefore have to be decoupled,

$$W \supset M'_*\, \bar{\mathbf{5}}_0^c \mathbf{5}_0^c\,. \tag{3.18}$$

where the decoupling scale M'_* will be discussed in more detail later on. $\bar{\mathbf{5}}_2$ and $\mathbf{5}_2^c$ contain a canonical colour-triplet and lepton doublet, respectively. Finally, $\mathbf{5}$ and $\mathbf{5}_1$ are candidates for H_u, whereas $\bar{\mathbf{5}}$ and $\bar{\mathbf{5}}_1$ are candidates for H_d.

For the matter fields we now have a clear picture. There are two localized brane families[4],

$$(n_2, n_2') = (0,0): \; \bar{\mathbf{5}}_{(1)}, \mathbf{10}_{(1)}, \quad (n_2, n_2') = (0,1): \; \bar{\mathbf{5}}_{(2)}, \mathbf{10}_{(2)}, \tag{3.19}$$

and two further families of bulk fields,

$$\bar{\mathbf{5}}_{(3)} \equiv \mathbf{5}_2^c, \; \mathbf{10}_{(3)} \equiv \mathbf{10}; \quad \bar{\mathbf{5}}_{(4)} \equiv \bar{\mathbf{5}}_2, \; \mathbf{10}_{(4)} \equiv \overline{\mathbf{10}}^c . \tag{3.20}$$

At the fixed points $n_2 = 0$, these chiral $\mathcal{N} = 1$ multiplets form a local $\mathrm{SU}(5) \times \mathrm{U}(1)_X$ GUT theory. The corresponding Yukawa couplings are 4×4 matrices which are generated locally [13],

$$W_{\mathrm{Yuk}} = C_{ij}^{(u)} \mathbf{10}_{(i)} \mathbf{10}_{(j)} H_u + C_{ij}^{(d)} \bar{\mathbf{5}}_{(i)} \mathbf{10}_{(j)} H_d, \tag{3.21}$$

according to the string selection rules. Projecting the bulk fields to their zero modes,

$$\begin{aligned} \overline{\mathbf{10}}^c &: (\mathbf{3}, \mathbf{2}) = q, \quad \mathbf{10} : (\bar{\mathbf{3}}, 1) = u^c, \; (1,1) = e^c, \\ \bar{\mathbf{5}}_2 &: (\bar{\mathbf{3}}, 1) = d^c, \quad \mathbf{5}_2^c : (1, \mathbf{2}) = l, \end{aligned} \tag{3.22}$$

yields one quark-lepton generation in the effective 4D theory. From (3.21) one deduces the corresponding 3×3 Yukawa matrices,

$$W_{\mathrm{Yuk}} = Y_{ij}^{(u)} u_i^c q_j H_u + Y_{ij}^{(d)} d_i^c q_j H_d + Y_{ij}^{(l)} l_i e_j^c H_d, \tag{3.23}$$

which avoid the unsuccessful SU(5) prediction of 4D GUTs.

Like all U(1) factors at the SU(5) fixed points, the $\mathrm{U}(1)_X$ symmetry has to be spontaneously broken at low energies. As we saw in Section 2, it is then crucial to maintain a $\mathbb{Z}_2$ subgroup, which includes matter parity, to distinguish between Higgs and matter fields. In order to see whether this is possible in the present model one has to examine the $\mathrm{U}(1)_X$ charges of the singlet fields (3.10), which are listed in Table 3.3. In the vacuum selected in [13] fields with $t_X = \pm 1$ obtained a VEV breaking $\mathrm{U}(1)_X$ completely. This led to phenomenologically unacceptable R-parity violating couplings.

Varying the discrete Wilson line in the SO(4) plane, in [11] 4D models with conserved matter parity were found. In these models only SM singlets with even $B - L$ charge aquire VEVs. These fields are zero modes of the 4D theory. In a 6D orbifold GUT model, in principle all 6D zero modes can aquire VEVs, even if they do not contain 4D zero modes, since the negative mass squared induced by the local Fayet-Iliopoulos terms can compensate the positive Kaluza-Klein GUT mass term. Hence, one can include the fields U_2 and U_4, which have $t_{B-L} = \pm 2$ (see Table 3.3), in the set of vacuum fields. Not allowing VEVs of singlets with $t_{B-L} = \pm 1$ then preserves matter parity. Note that not all vacua of the 6D orbifold GUT can be obtained from the 4D zero modes.

[4] Note that subscripts without brackets denote the localization of T_2/T_4 twisted fields, $n_3 = 0, 1, 2$. Subscripts with brackets, (1) ... (4), label the four brane and bulk families defined in (3.17) and (3.18).

Multiplet	t_1	t_2	t_3	t_4	t_5	t_6^0	R_1	R_2	R_3	k	kn_3	t_X	$\tilde{R}_1$	$\tilde{R}_2$
$\mathbf{10}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	0	3	-1	0	0	0	0	$\frac{1}{5}$	-1	$\frac{1}{10}$
$\bar{\mathbf{10}}^c$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	3	0	-1	0	0	0	$\frac{1}{5}$	-1	$\frac{1}{10}$
$\mathbf{5}$	0	0	0	0	0	-6	0	0	-1	0	0	$-\frac{2}{5}$	0	$\frac{4}{5}$
$\bar{\mathbf{5}}$	0	0	0	0	0	6	0	0	-1	0	0	$\frac{2}{5}$	0	$\frac{6}{5}$
$\mathbf{10}_{(1)}, \mathbf{10}_{(2)}$	0	$-\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{3}$	0	$\frac{1}{2}$	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	1	0	$\frac{1}{5}$	-1	$\frac{1}{10}$
$\bar{\mathbf{5}}_{(1)}, \bar{\mathbf{5}}_{(2)}$	0	$-\frac{1}{6}$	$\frac{3}{2}$	$\frac{1}{3}$	0	$-\frac{3}{2}$	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	1	0	$-\frac{3}{5}$	1	$\frac{7}{10}$
$\mathbf{5}_0^c$	0	$\frac{1}{3}$	-1	$-\frac{2}{3}$	0	1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	4	0	$\frac{2}{5}$	1	$\frac{1}{5}$
$\bar{\mathbf{5}}_0^c$	0	$\frac{1}{3}$	1	$-\frac{2}{3}$	0	-1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	4	0	$-\frac{2}{5}$	0	$\frac{4}{5}$
$\mathbf{5}_1$	0	$-\frac{1}{3}$	-1	$-\frac{1}{3}$	-1	-1	$-\frac{1}{3}$	$-\frac{2}{3}$	0	2	2	$-\frac{2}{5}$	0	$\frac{9}{5}$
$\bar{\mathbf{5}}_1$	$\frac{1}{2}$	$\frac{1}{6}$	0	$-\frac{1}{3}$	-1	1	$-\frac{1}{3}$	$-\frac{2}{3}$	0	2	2	$\frac{2}{5}$	0	$\frac{6}{5}$
$\mathbf{5}_2^c$	$-\frac{1}{2}$	$-\frac{1}{6}$	0	$\frac{1}{3}$	-1	1	$-\frac{2}{3}$	$-\frac{1}{3}$	0	4	8	$-\frac{3}{5}$	1	$-\frac{3}{10}$
$\bar{\mathbf{5}}_2$	0	$-\frac{1}{3}$	1	$-\frac{1}{3}$	1	1	$-\frac{1}{3}$	$-\frac{2}{3}$	0	2	4	$-\frac{3}{5}$	-1	$-\frac{3}{10}$

Table 3.2: *SU(5) non-singlet chiral multiplets at $n_2 = 0$. The subscripts (1) and (2) denote localization at $n_2' = 0$ and $n_2' = 1$, respectively. The charges $\frac{1}{2}t_X$ and $\tilde{R}_2$ agree* mod 1.

The pairwise decoupling (3.11), the decoupling of the exotic $\mathbf{5}$- and $\bar{\mathbf{5}}$-plets, and the matter parity preserving breaking of $\mathrm{U}(1)_{B-L}$ can be achieved with the minimal vacuum

$$\mathcal{S}_0 = \left\{X_0, \bar{X}_0^c, U_2, U_4, S_2, S_5\right\} . \tag{3.24}$$

For the decoupling masses in Eqs. (3.11) and (3.18) one obtains,

$$M_* = \langle \bar{X}_0^c S_2 S_5 \rangle \ , \quad M_*' = \langle X_0^c S_2 S_5 \rangle \ . \tag{3.25}$$

As we shall discuss in detail in the following section, the couplings needed to decouple the $\mathbf{5}\bar{\mathbf{5}}$-pairs satisfy all string selection rules. Note that no exotic matter is located at the fixed points $n_2 = 0$. Most of the exotic matter at $n_2 = 1$ can be decoupled by VEVs of just a few singlet fields (cf. [13]). This decoupling takes place locally at one of the fixed points, which is a crucial difference compared to previous discussions of decoupling in four dimensions [10,11]. The unification of gauge couplings yields important constraints on the decoupling masses M_* and the GUT scale M_{GUT}. This question goes beyond the scope of our paper. Detailed studies have recently been carried out for the 6D model [29] in [31] and for a heterotic 6D model similar to the one described here in [32].

The minimal vacuum $\mathcal{S}_0$ has two pairs of Higgs doublets. In order to have gauge coupling unification, one pair has to be decoupled. This can be done in various ways by enlarging the minimal vacuum. For the decoupling the 6D gauge couplings are important.

Singlet	t_1	t_2	t_3	t_4	t_5	t_6^0	R_1	R_2	R_3	k	kn_3	t_X
U_1^c	$-\frac{1}{2}$	$-\frac{1}{2}$	-3	0	0	0	0	-1	0	0	0	0
U_2	$\frac{1}{2}$	$\frac{1}{2}$	-3	0	0	0	-1	0	0	0	0	2
U_3	1	-1	0	0	0	0	-1	0	0	0	0	0
U_4	-1	-1	0	0	0	0	-1	0	0	0	0	-2
S_1, S_1'	$-\frac{1}{2}$	$-\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	0	$\frac{5}{2}$	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	1	0	-1
S_2, S_2'	$\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{1}{2}$	$\frac{1}{3}$	0	$-\frac{5}{2}$	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	1	0	0
S_3, S_3'	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	0	$\frac{5}{2}$	$-\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{2}$	1	0	1
S_4, S_4'	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	0	$\frac{5}{2}$	$\frac{11}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	1	0	1
S_5, S_5'	$-\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{3}$	0	$-\frac{5}{2}$	$-\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{2}$	1	0	0
S_6, S_6'	$-\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{2}$	$\frac{1}{3}$	0	$-\frac{5}{2}$	$\frac{11}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	1	0	0
S_7, S_7'	0	$-\frac{1}{6}$	$-\frac{1}{2}$	$-\frac{2}{3}$	1	$\frac{5}{2}$	$\frac{5}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	1	1	0
S_8, S_8'	0	$-\frac{1}{6}$	$\frac{3}{2}$	$-\frac{2}{3}$	-1	$\frac{5}{2}$	$-\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{2}$	1	2	-1
X_0	0	$-\frac{1}{3}$	1	$\frac{2}{3}$	0	5	$-\frac{1}{3}$	$-\frac{2}{3}$	0	2	0	0
X_0^c	0	$\frac{1}{3}$	-1	$-\frac{2}{3}$	0	-5	$-\frac{2}{3}$	$-\frac{1}{3}$	0	4	0	0
$\bar{X}_0$	0	$-\frac{1}{3}$	-1	$\frac{2}{3}$	0	-5	$-\frac{1}{3}$	$-\frac{2}{3}$	0	2	0	0
$\bar{X}_0^c$	0	$\frac{1}{3}$	1	$-\frac{2}{3}$	0	5	$-\frac{2}{3}$	$-\frac{1}{3}$	0	4	0	0
X_1	0	$-\frac{1}{3}$	-1	$-\frac{1}{3}$	-1	5	$-\frac{1}{3}$	$-\frac{2}{3}$	0	2	2	0
X_1^c	0	$\frac{1}{3}$	1	$\frac{1}{3}$	1	-5	$-\frac{2}{3}$	$-\frac{1}{3}$	0	4	4	0
$\bar{X}_1$	$\frac{1}{2}$	$\frac{1}{6}$	0	$-\frac{1}{3}$	-1	-5	$-\frac{1}{3}$	$-\frac{2}{3}$	0	2	2	0
$\bar{X}_1^c$	$-\frac{1}{2}$	$-\frac{1}{6}$	0	$\frac{1}{3}$	1	5	$-\frac{2}{3}$	$-\frac{1}{3}$	0	4	4	0
X_2	$\frac{1}{2}$	$\frac{1}{6}$	0	$-\frac{1}{3}$	1	5	$-\frac{1}{3}$	$-\frac{2}{3}$	0	2	4	1
X_2^c	$-\frac{1}{2}$	$-\frac{1}{6}$	0	$\frac{1}{3}$	-1	-5	$-\frac{2}{3}$	$-\frac{1}{3}$	0	4	8	-1
$\bar{X}_2$	0	$-\frac{1}{3}$	1	$-\frac{1}{3}$	1	-5	$-\frac{1}{3}$	$-\frac{2}{3}$	0	2	4	-1
$\bar{X}_2^c$	0	$\frac{1}{3}$	-1	$\frac{1}{3}$	-1	5	$-\frac{2}{3}$	$-\frac{1}{3}$	0	4	8	1
Y_0	1	$-\frac{1}{3}$	0	$\frac{2}{3}$	0	0	$-\frac{1}{3}$	$-\frac{2}{3}$	0	2	0	1
Y_0^c	-1	$\frac{1}{3}$	0	$-\frac{2}{3}$	0	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	4	0	-1
$\bar{Y}_0$	-1	$-\frac{1}{3}$	0	$\frac{2}{3}$	0	0	$-\frac{1}{3}$	$-\frac{2}{3}$	0	2	0	-1
$\bar{Y}_0^c$	1	$\frac{1}{3}$	0	$-\frac{2}{3}$	0	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	4	0	1
Y_1	0	$\frac{2}{3}$	-2	$-\frac{1}{3}$	-1	0	$-\frac{1}{3}$	$-\frac{2}{3}$	0	2	2	1
Y_1^c	0	$-\frac{2}{3}$	2	$\frac{1}{3}$	1	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	4	4	-1
$\bar{Y}_1$	$\frac{1}{2}$	$-\frac{5}{6}$	1	$-\frac{1}{3}$	-1	0	$-\frac{1}{3}$	$-\frac{2}{3}$	0	2	2	-1
$\bar{Y}_1^c$	$-\frac{1}{2}$	$\frac{5}{6}$	-1	$\frac{1}{3}$	1	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	4	4	1
Y_2	0	$\frac{2}{3}$	2	$-\frac{1}{3}$	1	0	$-\frac{1}{3}$	$-\frac{2}{3}$	0	2	4	0
Y_2^c	0	$-\frac{2}{3}$	-2	$\frac{1}{3}$	-1	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	4	8	0
$\bar{Y}_2$	$\frac{1}{2}$	$-\frac{5}{6}$	-1	$-\frac{1}{3}$	1	0	$-\frac{1}{3}$	$-\frac{2}{3}$	0	2	4	0
$\bar{Y}_2^c$	$-\frac{1}{2}$	$\frac{5}{6}$	1	$\frac{1}{3}$	-1	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	4	8	0

Table 3.3: *Non-Abelian singlets at $n_2 = 0$. $S_1, ..., S_8$ and $S_1', ..., S_8'$ are localized at $n_2' = 0$ and $n_2' = 1$, respectively.*

For the bulk fields from the untwisted sector one has

$$\begin{aligned}\mathcal{L}_H &\supset \sqrt{2}g \int d^2\theta\; H_R^c(\mathbf{20}) H_L(\mathbf{20}) \phi(\mathbf{35}) \;+\; \text{h.c.} \\ &\supset \sqrt{2}g \int d^2\theta\; \overline{\mathbf{10}}^c \mathbf{10}\; \mathbf{5} \;+\; \text{h.c.}\;. \end{aligned} \tag{3.26}$$

Identifying the $\mathbf{5}$-plet from the gauge multiplet with one Higgs multiplet, $H_u = \mathbf{5}$, therefore yields the wanted large top-quark Yukawa coupling [10, 11, 13].

For the Higgs field H_d we shall consider both options, $H_d = \bar{\mathbf{5}}_1$ and $H_d = \bar{\mathbf{5}}$, to which we refer as partial and full gauge-Higgs unification, respectively. In the first case, the 6D gauge interactions,

$$\begin{aligned}\mathcal{L}_H &\supset \sqrt{2}g \int d^2\theta\, (H_R^c(\mathbf{6})\phi(\mathbf{35})H_L(\mathbf{6}) + H_R^c(\bar{\mathbf{6}})\phi(\mathbf{35})H_L(\bar{\mathbf{6}})) \;+\; \text{h.c.} \\ &\supset \sqrt{2}g \int d^2\theta \left(X_0 \mathbf{5}\mathbf{5}_0^c + \bar{X}_0 \bar{\mathbf{5}}\bar{\mathbf{5}}_0^c + X_1^c \mathbf{5}_1 \bar{\mathbf{5}} + \bar{X}_1^c \bar{\mathbf{5}}_1 \mathbf{5} + X_2 \mathbf{5}\mathbf{5}_2^c + \bar{X}_2^c \bar{\mathbf{5}}_2 \mathbf{5}\right), \end{aligned} \tag{3.27}$$

can be used to decouple the pair $\bar{\mathbf{5}}\mathbf{5}_1$. The VEV $\langle X_1^c \rangle \neq 0$ yields the needed mass term. On the other hand, $\langle \bar{X}_1^c \rangle = 0$ is required to keep the field $\mathbf{5}$ massless. Full gauge-Higgs unification needs $\langle X_1^c \rangle = \bar{X}_1^c = 0$. Note that VEVs of X_0, $\bar{X}_0$ and X_2^c do not lead to mass terms for zero modes of $\mathbf{5}$ and $\bar{\mathbf{5}}$.

The decoupling terms (3.25) require VEVs of both bulk and localized fields. Note that the localized singlets S_2 and S_5 correspond to oscillator modes. As we will see in Section 5, bulk and brane field backgrounds are typically induced by local FI terms. The non-vanishing VEVs of localized fields are related to a resolution of the orbifold singularities [22, 23]. The study of the blow-up of the 6D orbifold model to a smooth manifold, and the geometrical interpretation of the localized VEVs is beyond the scope of this work.

4 Vanishing couplings and discrete symmetries

The heterotic landscape has a tremendous number of vacua. Orbifold compactifications correspond to a subset of vacua with enhanced symmetries. For 'non-standard' embeddings of the space group into the $E_8 \times E_8$ lattice, to which our $\mathbb{Z}_{6-\text{II}}$ model belongs, Fayet-Iliopoulos terms related to anomalous U(1)'s imply that the orbifold point in moduli space is a 'false vacuum'. In 'true vacua' some scalar fields aquire a non-zero VEV, which spontaneously breaks the large symmetry G_{tot} at the orbifold point to a sbgroup G_{vac}. For a given orbifold compactification with typically $\mathcal{O}(100)$ massless chiral superfields a huge vacuum degeneracy exists. The identification of standard model like vacua and their stabilization still is a major problem.

In the 6D orbifold GUT model described in the previous section, we have identified fields which provide the building blocks of a local SU(5) GUT. The couplings of the effective field theory are generated by expectation values of products of SU(5) singlet fields. The singlet fields with non-zero VEVs define a vacuum $\mathcal{S}$ which is restricted by the requirement that states with exotic quantum numbers are decoupled and $\mathcal{N} = 1$ supersymmetry is preserved.

The appearence of a coupling between some SU(5) non-singlets in the effective field theory requires the existence of an operator which involves additional singlets from the vacuum $\mathcal{S}$. Such operators are strongly restricted by string selection rules, which can be expressed as a symmetry $\mathrm{G_{tot}}$ at the orbifold point. A necessary condition for the absence of a certain coupling is then the requirement that for the singlets of the vacuum $\mathcal{S}$ the corresponding operators do not exist. The vacuum $\mathcal{S}$ has unbroken symmetry $\mathrm{G_{vac}} \subset \mathrm{G_{tot}}$. Obviously, a sufficient condition for the absence of a coupling between SU(5) non-singlets is its non-invariance under $\mathrm{G_{vac}}$. Both conditions will be studied in the following.

The main question in this section is the absence of unwanted superpotential terms in the effective theory. We focus on the μ-term, but the discussion can easily be extended to dimension-5 proton decay operators as well as other couplings. We shall provide an algorithm for finding 'maximal vacua' which are 'orthogonal' to unwanted terms, and we present a method which allows to calculate vanishing tree-level couplings to all orders in powers of singlets.

4.1 Orbifold geometry and discrete symmetries

The geometry of the compact space, its invariance under discrete rotations and the localization of fields at fixed points and fixed planes lead to discrete symmetries [33] of the superpotential in 4D as well as in 6D at the orbifold fixed points. The discrete rotations in the $\mathrm{G_2}$, SU(3) and SO(4) planes are associated with three R-charges R_i, $i = 1, 2, 3$, which are conserved modulo the order $l_i = 6, 3, 2$ of the twist in the respective plane,

$$\sum_j R_1^{(j)} = -1 \mod 6 \; , \quad \sum_j R_2^{(j)} = -1 \mod 3 \; , \quad \sum_j R_3^{(j)} = -1 \mod 2 \; , \qquad (4.1)$$

where the sum is over all fields of the particular superpotential term.

Fields from different twisted sectors T_k, $k = 1, ..., 6$ have different localization properties. For $k = 1, 5$ fields are localized at fixed points; $k = 2, 4$ and $k = 3$ correspond to brane fields in the SO(4) and SU(3) planes, respectively. For each superpotential term one has

$$\sum_j k^{(j)} = 0 \mod 6 \; . \qquad (4.2)$$

Furthermore, couplings of fields localized in the SU(3) and SO(4) planes have to satisfy the constraints

$$\text{SU(3) plane} \; : \; \sum_j k^{(j)} n_3^{(j)} = 0 \mod 3 \; , \qquad (4.3)$$

$$\text{SO(4) plane} \; : \; \sum_j k^{(j)} n_2^{(j)} = 0 \mod 2 \; , \quad \sum_j k^{(j)} n_2'^{(j)} = 0 \mod 2 \; . \qquad (4.4)$$

The constraints (4.1) - (4.4) correspond to a discrete symmetry which acts on the 6D brane and bulk fields. From Tables 3.2 and 3.3 one reads off that R-charges of fields from the twisted sector T_k have the form $R_i[\phi_k] = -k/l_i \bmod 1$. This implies that the discrete rotations

$$g_m^{(i)} = e^{2\pi i \frac{m}{l_i} R_i} , \quad m \in \mathbb{Z} , \tag{4.5}$$

which are of order l_i^2, form the group $\mathbb{Z}_{l_i} \times \mathbb{Z}_{l_i}^{(k)}$. The group element lies in the latter factor for $m = 0 \bmod l_i$. The superpotential has to transform as

$$g_m^{(i)} W = e^{-2\pi i \frac{m}{l_i}} W , \quad m \in \mathbb{Z} , \tag{4.6}$$

under this product group. For $i = 1$ one deduces that the selection rule (4.2) is implied by the discrete R-symmetries (4.1) and not an additional independent condition.

We can make the product structure explicit by expressing the actions of the two subgroups as

$$\mathbb{Z}_{l_i} \; : \; h_m^{(i)} = e^{2\pi \mathrm{i} \frac{1}{l_i}(mR_i \bmod k)} , \qquad \mathbb{Z}_{l_i}^{(k)} \; : \; \hat{h}_{m'}^{(i)} = e^{2\pi \mathrm{i} \frac{m' k}{l_i}} , \quad m, m' \in \mathbb{Z} . \tag{4.7}$$

This decomposition applies for all three discrete R-symmetries. The groups $\mathbb{Z}_3^{(k)}$ and $\mathbb{Z}_2^{(k)}$ are subgroups of $\mathbb{Z}_6^{(k)}$ so that the total R-symmetry of the lagrangian is given by

$$\mathrm{G}_R = \mathbb{Z}_6^{R_1} \times \mathbb{Z}_3^{R_2} \times \mathbb{Z}_2^{R_3} \times \mathbb{Z}_6^{(k)} . \tag{4.8}$$

The space selection rules (4.3) and (4.4) correspond to further discrete symmetries $\mathbb{Z}_3$ and $\mathbb{Z}_2$, respectively, which commute with supersymmetry. One then obtains for the full discrete symmetry,

$$\mathrm{G}_{\text{discrete}} = \left[\mathbb{Z}_6^{R_1} \times \mathbb{Z}_3^{R_2} \times \mathbb{Z}_2^{R_3} \times \mathbb{Z}_6^{(k)}\right]_R \times \mathbb{Z}_3^{kn_3} \times \mathbb{Z}_2^{kn_2} \times \mathbb{Z}_2^{kn_2 \prime} . \tag{4.9}$$

Introducing the 'discrete charge vector'

$$\mathcal{K} = (R_1, R_2, R_3, k, kn_3, kn_2, kn_2'), \tag{4.10}$$

all superpotential terms have to obey

$$\mathcal{K}(W) = \mathcal{K}_{\text{vac}}, \tag{4.11}$$

where the 'discrete vacuum charges' are given by

$$\mathcal{K}_{\text{vac}} = (-1 \bmod 6, -1 \bmod 3, -1 \bmod 2, 0 \bmod 6, 0 \bmod 3, 0 \bmod 2, 0 \bmod 2) . \tag{4.12}$$

Covariance of the superpotential W corresponds to invariance of the lagrangian $W|_{\theta\theta}$. Together with the gauge symmetry

$$\mathrm{G}_{\text{gauge}} = \mathrm{SU}(5) \times \mathrm{U}(1)^4 \times \left[\mathrm{SU}(3) \times \mathrm{SO}(8) \times \mathrm{U}(1)^2\right] , \tag{4.13}$$

the full symmetry at the SU(5) fixed points of the 6D orbifold GUT is

$$\mathrm{G}_{\text{tot}} = \mathrm{G}_{\text{gauge}} \times \mathrm{G}_{\text{discrete}} . \tag{4.14}$$

Defining for the U(1) symmetries the charge vector

$$Q = (t_1, ..., t_6^0), \tag{4.15}$$

gauge invariance of the superpotential implies

$$Q(W) = (0,0,0,0,0,0). \tag{4.16}$$

Localized FI-terms, related to anomalous U(1)'s, lead to nonvanishing VEVs of some 6D brane and bulk fields. This breaks the symmetry of the 6D theory spontaneously,

$$\mathrm{G}_{\mathrm{tot}} \to \mathrm{G}_{\mathrm{vac}} \ . \tag{4.17}$$

We are interested in vacua which preserve SU(5). We therefore devide all fields into two sets, SU(5) non-singlets ϕ_i and SU(5) singlets s_i. A set $\mathcal{S}$ of singlets which aquire VEVs,

$$\mathcal{S} = \{s_i |\ t_{\mathrm{SU}(5)}(s_i) = 0, \langle s_i \rangle \neq 0\}, \tag{4.18}$$

defines a vacuum of the 6D orbifold GUT.

4.2 Maximal vacua for vanishing couplings

Consider now a vacuum $\mathcal{S}$ and a superpotential term which can lead to a coupling for the product $\Phi = \prod_j \phi_j^{m_j}$ of SU(5) non-singlet fields,

$$W = \lambda\Phi \ , \qquad \lambda = \prod_i^N s_i^{n_i}, \ s_i \in \mathcal{S}, \ n_i, N \in \mathbb{N} \ . \tag{4.19}$$

The two conditions (4.11) and (4.16) can be evaluated separately. First, we factorize a part of λ which transforms non-trivially under gauge transformations by introducing a 'special monomial' λ_s,

$$\lambda = \lambda_0 \lambda_s \ , \quad Q(\lambda_s \Phi) = 0 \ , \quad Q(\lambda_0) = 0 \ . \tag{4.20}$$

Generically, the set of monomials

$$\ker Q(\mathcal{S}) \equiv \left\{ \lambda_0 \ \middle| \ \lambda_0 = \prod_i^N s_i^{n_i}, \ s_i \in \mathcal{S}, \ n_i \in \mathbb{Z}, \ Q(\lambda_0) = 0 \right\} \tag{4.21}$$

is a space of dimension larger than one. Note that we allow both λ_0 and λ_s to have sub-monomials with negative exponents n_i, in contrast to their product λ.[5] Clearly, results for λ cannot depend on the choice of the special monomial λ_s. Covariance of the superpotential under the discrete symmetries (4.9) requires

$$\mathcal{K}(\lambda_0) = \mathcal{K}_{\mathrm{vac}} - \mathcal{K}(\lambda_s \Phi) \ , \tag{4.22}$$

[5] Negative exponents are allowed in order to promote the set of all possible exponents of monomials $\{(n_1, \ldots, n_N), N \in \mathbb{N}\}$ to a vector space.

which defines the subset of monomials in $\ker Q(\mathcal{S})$ yielding a non-vanishing coupling λ.

In order to identify vacua where the superpotential term (4.19) is forbidden we proceed as follows. The elements of $\ker Q(\mathcal{S})$ are given by the solutions of the equations

$$Q(\lambda_0) = \sum_{i=1}^{N} n_i Q(s_i) = 0 \tag{4.23}$$

for the charge vector Q. The solutions can be represented by vectors $(n_1, ..., n_N)$ which are linear combinations of some basis vectors. These correspond to basis monomials whose products are the elements of $\ker Q(\mathcal{S})$.

We now examine the discrete symmetries. After the choice of a special monomial λ_s, Eq. (4.22) can be evaluated for the basis monomials of $\ker Q(\mathcal{S})$. Starting from a sufficiently small set $\mathcal{S}$ which does not satisfy (4.22), one can subsequently add further singlets until a 'maximal vacuum' is reached for which the term (4.19) is forbidden to all orders in powers of singlets. The generalization of this algorithm to the case of more than one forbidden coupling is straightforward.

4.2.1 Full gauge-Higgs unification

As a first example, consider the μ-term in the context of full gauge-Higgs unification in our model, $H_u = \mathbf{5}$ and $H_d = \bar{\mathbf{5}}$. In that case

$$\Phi \equiv \Phi_{\text{GHU}} = H_u H_d = \mathbf{5}\bar{\mathbf{5}}, \quad Q(\Phi) = 0, \quad \mathcal{K}(\Phi) = 0\ . \tag{4.24}$$

Note that Φ is a complete singlet. This leads to $\lambda_s = 1$ and the condition

$$\mathcal{K}(\lambda_0) = \mathcal{K}_{\text{vac}} \tag{4.25}$$

for an allowed μ-term. Let us now define the vacuum

$$\mathcal{S}_1 = \mathcal{S}_0 \cup \left\{X_1, \bar{X}_1, Y_2, S_7\right\}, \tag{4.26}$$

where S_0 was defined in (3.24). One easily verifies that the dimension of $\ker Q(\mathcal{S}_1)$ is four. Basis monomials Ω_i are listed in Table 4.1 from which one reads off that it is impossible to satisfy $R_1(\Omega_i) = -1 \bmod 6$. Hence, the μ-term is absent in the vacuum $\mathcal{S}_1$ to all orders in the singlets.

The vacuum $\mathcal{S}_1$ is maximal since adding any further singlet respecting matter parity leads to a μ-term. This is demonstrated by Table 4.2 where for each possible additional singlet the order is listed at which a μ-term appears. It is intriguing that for some vacua a μ-term only occurs at very high orders in the singlets.

As discussed in Section 3, there is another candidate for H_d with even matter parity, $\bar{\mathbf{5}}_1$ from the twisted sector T_2. The vacuum $\mathcal{S}_1$ has only full gauge-Higgs unification if the field $\bar{\mathbf{5}}_1$ is decoupled by means of a large mass term together with $\mathbf{5}_1$ which also has even matter parity.

Name	Monomial	R_1	R_2	R_3	k	kn_3
Ω_1	$\bar{X}_0^c S_2 S_5$	0	0	-1	6	0
Ω_2	$X_1 Y_2 S_2 S_5$	0	-1	-1	6	6
Ω_3	$X_0 \bar{X}_1 S_5 S_7$	0	-1	-1	6	3
Ω_4	$X_0 \bar{X}_1 Y_2 U_2 U_4$	-3	-2	0	6	6

Table 4.1: *Basis monomials of* $\ker Q(\mathcal{S}_1)$ *and the corresponding discrete charges. All monomials have* $kn_2 = kn_2' = 0$.

Add	Mass term for $\mathbf{5}\bar{\mathbf{5}}$	Order	Mass term for $\mathbf{5}_1\bar{\mathbf{5}}_1$	Order
$\bar{Y}_2$	$(X_0\bar{X}_0^c\bar{X}_1\bar{Y}_2(S_5)^2)^2\Omega_1\Omega_4$	20	$(X_0)^2X_1\bar{X}_1(Y_2)^2(\bar{Y}_2)^2(S_5)^4\Omega_2\Omega_4$	21
$\bar{Y}_2^c$	$(\bar{Y}_2^c S_2 S_7)^2\Omega_1\Omega_4$	14	$X_0Y_2(\bar{Y}_2^c)^2(S_2)^3(S_5)^2(S_7)^3\Omega_2\Omega_4$	21
U_1^c	$(X_0\bar{X}_1Y_2U_1^c)\Omega_2$	8	$X_0(Y_2)^2U_1^cS_2S_5$	6
U_3	$(\bar{X}_0^cU_3(S_5)^2)^2\Omega_2\Omega_4$	17	$X_0\bar{X}_0^c(Y_2)^2U_2(U_3)^2U_4(S_5)^4\Omega_1$	15
S_6	$(X_1Y_2S_2S_6)\Omega_4$	9	$X_0(Y_2)^2U_2U_4S_2S_6$	7

Table 4.2: *Addition of any further field to* $\mathcal{S}_1$ *generates monomials which induce mass terms for* $\mathbf{5}\bar{\mathbf{5}}$ *and* $\mathbf{5}_1\bar{\mathbf{5}}_1$. *Shown are lowest order examples. The monomials* Ω_i *are defined in table 4.1. Singlets which complete pairs* A^cA *are not listed, since they allways allow to form mass terms proportional to* $\Omega_1 A^c A$. *We do only consider singlets which conserve matter parity.*

Using the method described above we can easily study the mass term $\Phi = \mathbf{5}_1\bar{\mathbf{5}}_1$. Choosing as special monomial $\lambda_s = (X_1\bar{X}_1)^{-1}$, which has the convenient feature $Q(\lambda_s\mathbf{5}_1\bar{\mathbf{5}}_1) = 0$, one obtains

$$\mathcal{K}(\lambda_s\mathbf{5}_1\bar{\mathbf{5}}_1) = 0. \tag{4.27}$$

The conditions for the existence of a μ-term then read

$$\mathcal{K}(\lambda_0) = \mathcal{K}_{\text{vac}}, \quad n_s(\lambda) \geq 0, \tag{4.28}$$

where $n_s(\lambda)$ is the exponent of the singlet $s \in \mathcal{S}_1$ in the monomial $\lambda = \lambda_0\lambda_s$. The last condition requires the appearance of at least one factor of Ω_2, and Ω_3 or Ω_4 from Table 4.1 in the monomial λ_0. However, the R-charges of these monomials imply that again it is impossible to satisfy the first condition (4.28) for the vacuum $\mathcal{S}_1$. Hence, also the mass term $\mathbf{5}_1\bar{\mathbf{5}}_1$ vanishes to all orders in the singlets. Analogously, one easily verifies that the mass terms $\mathbf{5}\bar{\mathbf{5}}_1$ and $\mathbf{5}_1\bar{\mathbf{5}}$ vanish as well.

Adding further singlets to the vacuum $\mathcal{S}_1$ leads to a non-zero $\mathbf{5}_1\bar{\mathbf{5}}_1$ mass term as demonstrated in Table 4.2. The mass terms for $\mathbf{5}\bar{\mathbf{5}}$ and $\mathbf{5}_1\bar{\mathbf{5}}_1$ are roughly of the some order in the singlets. It is intriguing that in some cases very high orders occur, which could explain the hierarchy between the electroweak scale and the GUT scale. However, the main result of this section is that the vacuum $\mathcal{S}_1$ does not correspond to gauge-Higgs unification. Instead, it represents a model with two pairs of Higgs doublets. This may be phenomenologically acceptable, but it is inconsistent with gauge coupling unification.

Name	Monomial	R_1	R_2	R_3	k	kn_3
Ω'_1	$\bar{X}_0^c S_2 S_5$	0	0	-1	6	0
Ω'_2	$\bar{X}_0^c X_1^c Y_2^c$	-2	-1	0	12	12
Ω'_3	$\bar{X}_0^c (S_5)^2 U_3$	-2	1	-1	6	0
Ω'_4	$X_0 X_1 S_5 S_7$	0	-1	-1	6	3
Ω'_5	$X_0 \bar{X}_0^c X_1^c \bar{X}_1 U_1^c$	-2	-3	0	12	6
Ω'_6	$X_0 \bar{X}_0^c \bar{X}_1 \bar{Y}_2 (S_5)^2$	-2	-1	-1	12	6
Ω'_7	$X_0 \bar{X}_0^c \bar{X}_1 \bar{Y}_2 (S_6)^2$	2	-3	-1	12	6
Ω'_8	$X_0 \bar{X}_0^c X_1^c \bar{X}_1 U_2 U_4$	-4	-2	0	12	6

Table 4.3: *Basis monomials of* $\ker Q(\mathcal{S}_2)$ *and their discrete charges. All monomials have* $kn_2 = kn'_2 = 0$.

4.2.2 Partial gauge-Higgs unification

Consider now the case of partial gauge-Higgs unification, $H_u = \mathbf{5}$ and $H_d = \bar{\mathbf{5}}_1$,

$$\Phi \equiv \Phi_{\text{PGHU}} = H_u H_d = \mathbf{5}\bar{\mathbf{5}}_1, \tag{4.29}$$

which can be realized with the vacuum

$$\mathcal{S}_2 = \mathcal{S}_0 \cup \left\{X_1^c, \bar{X}_1, Y_2^c, \bar{Y}_2, U_1^c, U_3, S_6, S_7\right\}. \tag{4.30}$$

As discussed in Section 3, the $\mathbf{5}_1\bar{\mathbf{5}}$ pair can be decoupled with the VEV $\langle X_1^c\rangle \neq 0$. For the new vacuum $\ker Q(\mathcal{S}_2)$ is again easily calculated, it has dimension eight. A set of basis monomials is listed in Table 4.3.

For partial gauge-Higgs unification the μ-term is the $\mathbf{5}\bar{\mathbf{5}}_1$ mass term. Choosing as special monomial $\lambda_s = (\bar{X}_1)^{-1}$, with $Q(\lambda_s \mathbf{5}\bar{\mathbf{5}}_1) = 0$, one obtains

$$\mathcal{K}(\lambda_s \mathbf{5}\bar{\mathbf{5}}_1) = (0, 0, -1, 0, 0, 0, 0). \tag{4.31}$$

The conditions for the existence of a μ-term now read

$$\begin{aligned} &\mathcal{K}(\lambda_0) = (-1 \bmod 6, -1 \bmod 3, 0 \bmod 2, 0 \bmod 6, 0 \bmod 3, 0 \bmod 2, 0 \bmod 2)\,, \\ &n_s(\lambda) \geq 0, \end{aligned} \tag{4.32}$$

where $n_s(\lambda)$ is the exponent of the singlet $s \in \mathcal{S}_2$ in the monomial $\lambda = \lambda_0\lambda_s$. The last condition requires the presence of at least one factor of Ω'_4, Ω'_5, Ω'_6, Ω'_7 or Ω'_8. Since all basis monomials have even R_1 charge the first condition (4.32) is always violated by monomials in $\ker Q(\mathcal{S}_2)$. Hence, the μ-term vanishes again to all orders in the singlets.

The vacuum $\mathcal{S}_2$ is also maximal, since the only possibility to enlarge it without breaking matter parity is to add singlets A (A^c) whose $\mathcal{N} = 2$ superpartners A^c (A) already belong to $\mathcal{S}_2$. One then obtains the μ-term

$$\mu = \lambda_0\lambda_s, \quad \lambda_0 = AA^c(\Omega'_5)^3, \tag{4.33}$$

which is of order 16 in the singlets. This power may be sufficiently high to provide an explanation of the hierarchy between the electroweak and the GUT scale.

4.2.3 μ-term and gravitino mass

The method of maximal vacua also allows to relate the existence of different couplings. In particular, one can show for full and partial gauge-Higgs unification that the existence of a μ-term and a singlet contribution W_0 to the superpotential, which determines the gravitino mass $m_{3/2} \propto \langle W_0 \rangle$, are equivalent.

For full gauge-Higgs unification the equivalence follows directly from the fact that μ and W_0 are given by invariant monomials in $\ker Q(\mathcal{S})$ [11],

$$\mu \Phi_{\mathrm{GHU}} \text{ allowed} \quad \Leftrightarrow \quad W_0 = \mu \text{ allowed} . \tag{4.34}$$

For partial gauge-Higgs unification the condition for a μ-term $\mu \equiv \mu_0 \lambda_s$ depends on the quantum numbers of the Higgs fields,

$$\mathcal{K}(\mu_0) = \mathcal{K}_{\mathrm{vac}} - \mathcal{K}(\lambda_s \Phi_{\mathrm{PGHU}}) = \mathcal{K}(W_0) - \mathcal{K}(\lambda_s \Phi_{\mathrm{PGHU}}). \tag{4.35}$$

From Eq. (4.31) and Table 4.3 one reads off

$$\mathcal{K}(\lambda_s \Phi_{\mathrm{PGHU}}) = \mathcal{K}(\Omega_1') = \mathcal{K}((\Omega_4')^3), \tag{4.36}$$

which implies

$$\mu \Phi_{\mathrm{PGHU}} = \mu_0 (\lambda_s \Phi_{\mathrm{PGHU}}) \text{ allowed} \quad \Rightarrow \quad W_0 = \mu_0 \Omega_1' \text{ allowed} , \tag{4.37}$$

$$W_0 \text{ allowed} \quad \Rightarrow \quad \mu \Phi_{\mathrm{PGHU}} = W_0 (\Omega_4')^3 (\lambda_s \Phi_{\mathrm{PGHU}}) \text{ allowed}. \tag{4.38}$$

Note that $\Omega_1' = \bar{X}_0^c S_2 S_5$ is the monomial used for the decoupling of $\mathbf{5\bar{5}}$ pairs in Section 3.

Our analysis demonstrates that the μ-term and the gravitino mass are closely related, in particular for vacua with full and partial gauge-Higgs unification.

4.3 Unbroken symmetries

In a given vacuum $\mathcal{S}$ the symmetry at the SU(5) fixed points

$$\mathrm{G}_{\mathrm{tot}} = \mathrm{G}_{\mathrm{gauge}} \times \mathrm{G}_{\mathrm{discrete}} \tag{4.39}$$

is spontaneously broken to some subgroup,

$$\mathrm{G}_{\mathrm{tot}} \to \mathrm{G}_{\mathrm{vac}}(\mathcal{S}), \tag{4.40}$$

which can be identified in the standard manner. Knowledge of $\mathrm{G}_{\mathrm{vac}}(\mathcal{S})$ is obviously very valuable since it restricts possible terms in the superpotential. Forbidden couplings for Yukawa matrices correspond to 'texture zeros'.

Consider a singlet $s_i \in \mathcal{S}$. Under the symmetry G_{tot} it transforms as

$$s_i \to e^{2\pi \mathrm{i} (\alpha \cdot Q + r \cdot \mathcal{K})} s_i \, . \tag{4.41}$$

Here the vectors α and r,

$$\alpha = (\alpha_1, ..., \alpha_6) \, , \ \alpha_i \in \mathbb{R}, \quad r = \left(\frac{r_1}{6}, \frac{r_2}{3}, \frac{r_3}{2}, \frac{r_4}{6}, \frac{r_5}{3}, \frac{r_6}{2}, \frac{r_7}{2} \right) \, , \ r_i \in \mathbb{Z}, \tag{4.42}$$

parametrize the continuous and discrete symmetries of the theory.

A parametrization of the unbroken group $\mathrm{G}_{\mathrm{vac}}(\mathcal{S})$ in terms of vectors α' and r' can be found by solving the equations

$$s_i = e^{2\pi\mathrm{i}(\alpha'\cdot Q + r'\cdot\mathcal{K})} s_i, \quad \forall\ s_i \in \mathcal{S}. \tag{4.43}$$

Knowing the allowed vectors α' and r', the group $\mathrm{G}_{\mathrm{vac}}(\mathcal{S})$ can be determined.

One unbroken discrete subgroup in both vacua $\mathcal{S}_1$ and $\mathcal{S}_2$ is easily identified since U_2 and U_4 are the only fields with non-zero $\mathrm{U}(1)_X$ charge,

$$t_X(U_2) = -t_X(U_4) = 2. \tag{4.44}$$

The smallest $\mathrm{U}(1)_X$ charge is $t_X(\mathbf{10}) = 1/5$. Hence, $\mathrm{U}(1)_X$ is broken to the discrete subgroup $\mathbb{Z}_{10}^X$ with elements $g_m^X = \exp\left(2\pi\mathrm{i}\frac{m}{2}t_X\right)$, $m \in \mathbb{Z}$, which contains matter parity,

$$P_X = e^{2\pi\mathrm{i}\left(\frac{5}{2}t_X\right)}. \tag{4.45}$$

The identification of the further unbroken symmetries is more cumbersome. We find that in both vacua no continuous U(1) symmetry survives. Solving explicitly equations (4.43) we find for the vacuum $\mathcal{S}_1$,

$$G_{\mathrm{vac}}(\mathcal{S}_1) = \mathbb{Z}_3^{\tilde{R}_1} \times \mathbb{Z}_{10}^X. \tag{4.46}$$

The elements of the $\mathbb{Z}_3$ R-symmetry are $\tilde{g}_m^{(1)} = \exp\left(2\pi\mathrm{i}\frac{m}{3}\tilde{R}_1\right)$, $m \in \mathbb{Z}$, with

$$\tilde{R}_1 = \alpha_1 \cdot Q + r_1 \cdot \mathcal{K}, \quad \alpha_1 = \left(\frac{5}{2}, \frac{15}{2}, 0, \frac{5}{2}, -\frac{5}{2}, \frac{1}{2}\right), \ r_1 = (5,0,0,0,0,0,0)\,. \tag{4.47}$$

The 'vacuum R-charge' is given by

$$r_1 \cdot \mathcal{K}_{\mathrm{vac}} = 1 \bmod 3\ . \tag{4.48}$$

The $\tilde{R}_1$ charges of the SU(5) non-singlets are listed in Table 3.2. Note that $\tilde{R}_1$ is embedded in the R-symmetry as well as the U(1) symmetries of the theory.

Following the same procedure for the vacuum $\mathcal{S}_2$, one obtains the unbroken group

$$G_{\mathrm{vac}}(\mathcal{S}_2) = \mathbb{Z}_2^{\tilde{R}_2} \times \mathbb{Z}_{10}^X. \tag{4.49}$$

The elements of the $\mathbb{Z}_2$ R-symmetry are $\tilde{g}_m^{(2)} = \exp\left(2\pi\mathrm{i}\frac{1}{2}\left(m\tilde{R}_2 \bmod t_X\right)\right)$, $m \in \mathbb{Z}$, with

$$\tilde{R}_2 = \alpha_2 \cdot Q + r_2 \cdot \mathcal{K}, \quad \alpha_2 = \left(7, 0, -\frac{7}{6}, \frac{35}{4}, \frac{7}{12}, -\frac{7}{15}\right), \ r_2 = (7,0,0,0,0,0,0)\,, \tag{4.50}$$

and vacuum R-charge

$$r_2 \cdot \mathcal{K}_{\mathrm{vac}} = 1 \bmod 2\ . \tag{4.51}$$

$\tilde{R}_2$ is again a non-trivial linear combination of U(1) and discrete R-charges. The $\tilde{R}_2$-charges of the SU(5) non-singlets are listed in Table 3.2.

Once the unbroken subgroups are known one can calculate the corresponding zeros of the superpotential. Consider again a term of the form (4.19), which transforms under the discrete symmetry $\mathbb{Z}_{l_i}$, $l_i = 3, 2$, generated by $\tilde{R}_i$, with $i = 1, 2$, respectively, as

$$W = \lambda\Phi \to \lambda\, \tilde{g}_m^{(i)} g_n^X\, \Phi = e^{2\pi \mathrm{i} \frac{m}{l_i} r_i \cdot \mathcal{K}_{\text{vac}}}\, W\ , \quad m, n \in \mathbb{Z}\ . \tag{4.52}$$

We thus obtain as sufficient condition for the appearance of a vanishing coupling,

$$\tilde{R}_i(\Phi) \neq r_i \cdot \mathcal{K}_{\text{vac}} \mod l_i \quad \vee \quad \frac{1}{2} t_X(\Phi) \neq 0 \bmod 10 \quad \Rightarrow \quad \langle\lambda\rangle = 0. \tag{4.53}$$

Given the $\tilde{R}_i$ charges of the SU(5) non-singlet fields ϕ_j this condition is easily evaluated.

We can now confirm the result from the previous section that the vacuum $\mathcal{S}_1$ has two massless Higgs pairs. From Table 3.2 we read off

$$\begin{aligned} \tilde{R}_1(\mathbf{5}\bar{\mathbf{5}}) = \tilde{R}_1(\mathbf{5}_1\bar{\mathbf{5}}) = \tilde{R}_1(\mathbf{5}\bar{\mathbf{5}}_1) = \tilde{R}_1(\mathbf{5}_1\bar{\mathbf{5}}_1) = 0 \bmod 3 \\ \neq 1 \bmod 3 = r_1 \cdot \mathcal{K}_{\text{vac}}. \end{aligned} \tag{4.54}$$

Extending the vacuum $\mathcal{S}_1$ by one of the singlets listed in Table 4.2 preserves $\mathbb{Z}_{10}^X$ but breaks $\mathbb{Z}_3^{\tilde{R}_1}$. As a consequence, Higgs mass terms are generated.

Likewise we can study the symmetry transformations of the above terms in the vacuum $\mathcal{S}_2$,

$$\tilde{R}_2(\mathbf{5}\bar{\mathbf{5}}) = \tilde{R}_2(\mathbf{5}\bar{\mathbf{5}}_1) = 0 \bmod 2\ , \quad \tilde{R}_2(\mathbf{5}_1\bar{\mathbf{5}}) = \tilde{R}_2(\mathbf{5}_1\bar{\mathbf{5}}_1) = 1 \bmod 2\ . \tag{4.55}$$

Furthermore, all $\mathbb{Z}_{10}^X$ charges vanish. Recalling (4.51), this shows that the unbroken R-symmetry forbids the generation of mass terms for $\mathbf{5}\bar{\mathbf{5}}$ and $\mathbf{5}\bar{\mathbf{5}}_1$, but allows them for the two remaining combinations. Indeed, at lowest order we find the mass term

$$W = \langle X_1^c\rangle\, \mathbf{5}_1(\bar{\mathbf{5}} + \epsilon\, \bar{\mathbf{5}}_1)\ , \quad \epsilon = \langle X_0 \bar{X}_0^c X_1^c Y_2^c S_6 S_7\rangle\ . \tag{4.56}$$

This shows that $\mathbf{5}_1$ decouples together with a linear combination of $\bar{\mathbf{5}}$ and $\bar{\mathbf{5}}_1$. The orthogonal linear combination is the down-type Higgs,

$$H_d = \bar{\mathbf{5}}_1 - \epsilon\, \bar{\mathbf{5}}\ . \tag{4.57}$$

It is interesting that the vacuum $\mathcal{S}_2$ leads to a down-type Higgs with dominant component from a twisted sector. In contrast, the up-type Higgs $H_u = \mathbf{5}$ is a pure gauge field in six dimensions, which is the reason for the large top-quark mass. Compared to the case of full gauge-Higgs unification, where both Higgs fields arise from the untwisted sector, this induces non-trivial discrete R-charges for the product $H_u H_d$.

The discrete R-symmetries $\tilde{R}_1$ and $\tilde{R}_2$ of the vacua S_1 and S_2, respectively, may be anomalous [32]. This question is important since in the case of an anomaly one can expect the generation of μ-term and gravitino mass by nonperturbative effects. These questions will be studied elsewhere.

5 Local Yukawa Couplings

In the previous section we have identified two vacua with conserved matter parity and vanishing μ-terms. The first vacuum $\mathcal{S}_1$ corresponds to a model with two pairs of massless Higgs doublets, and thus without gauge coupling unification. We therefore focus on the second vacuum $\mathcal{S}_2$ with partial gauge-Higgs unification.

The vacuum $\mathcal{S}_2$ contains the brane fields S_2, S_5, S_6, S_7 localized at $(n_2, n_2') = (0,0)$, to which we now add the fields S_2', S_5', S_6', S_7' at the equivalent fixed point $(n_2, n_2') = (0,1)$,

$$\mathcal{S}_0 = \left\{X_0, \bar{X}_0^c, U_2, U_4, S_2, S_5, S_2', S_5'\right\} \ , \tag{5.1}$$

$$\mathcal{S}_2 = \mathcal{S}_0 \cup \left\{X_1^c, \bar{X}_1, Y_2^c, \bar{Y}_2, U_1^c, U_3, S_6, S_7, S_6', S_7'\right\} \ . \tag{5.2}$$

The Higgs fields are $H_u = \mathbf{5}$ and $H_d \simeq \bar{\mathbf{5}}_1$. The vacuum $\mathcal{S}_2$ has the following properties:

- $\mathrm{U}(1)_X$ is spontaneously broken to $\mathbb{Z}_{10}^X$ containing matter parity,
- all vector-like exotics at $n_2 = 0$ decouple,
- all D-terms at $n_2 = 0$ vanish locally,
- the μ-term vanishes to all orders in the singlets,
- $\langle W \rangle$ vanishes to all orders in the singlets.

The remarkable last two features are a consequence of an unbroken discrete R-symmetry. The vacuum $\mathcal{S}_2$ is maximal in the sense that adding more singlets either breaks matter parity or generates a μ-term.

Low-energy supersymmetry requires vanishing F- and D-terms. In the 6D theory with localized FI-terms the corresponding equations have complicated solutions, leading to non-trivial profiles for bulk fields [15]. We do not study the full problem here but focus on the local conditions at the GUT fixed points $n_2 = 0$. We expect that the local VEVs can be extended to full dynamical solutions in six dimensions.

The $\mathcal{N} = 2$ vector multiplet has three auxilliary fields D_1, D_2, D_3 which form a triplet under $\mathrm{SU}(2)_R$ and must all vanish in the bulk. However, at the fixed points half of the supersymmetry is broken and the local $\mathcal{N} = 1$ vector multiplet has an effective D-term $D \equiv -D_3 + F_{56}$, where F_{56} is the associated field strength in the y^5, y^6 direction. Thus the local D-term cancelation condition at $n_2 = 0$ (cf. [15]),

$$D_3^a = F_{56}^a = \frac{g M_P^2}{384\pi^2} \frac{\mathrm{tr}\, t_a}{|t_a|^2} + \sum_i q_i^a |s_i|^2, \tag{5.3}$$

where q_i^a is the $\mathrm{U}(1)_a$ charge of the singlet s_i, has always a solution, even for non-vanishing right-hand-side. This means that in principle localized FI-terms do not necessarily induce singlet VEVs and the corresponding $\mathrm{U}(1)$ can remain unbroken. However, since our model has distinct anomalous $\mathrm{U}(1)$ factors at the inequivalent fixed points $n_2 = 0, 1$ and a non-vanishing net anomalous $\mathrm{U}(1)$ in 4D [13], its global D-flat solution cannot be of that kind. We rather expect a mixture of singlet VEVs and a nontrivial gauge backround $\langle F_{56}^{\mathrm{an}} \rangle$.

For non-anomalous U(1)'s the local field strength in (5.3) in the vacuum $\mathcal{S}_2$ can vanish since each of the singlets appears in one of the gauge invariant basis monomials Ω'_i of $\ker Q(\mathcal{S}_2)$ (cf. Table 4.3). At $n_2 = 0$ the model has an anomalous $\mathrm{U}(1)_{\mathrm{an}}$ [13],

$$t^0_{\mathrm{an}} = -4t_2 + 5t_4 - t_5 + t^0_6, \qquad \mathrm{tr}\, t^0_{\mathrm{an}}/|t^0_{\mathrm{an}}|^2 = 2. \tag{5.4}$$

In fact, also $\langle F^{\mathrm{an}}_{56}\rangle$ can vanish since one can form monomials of singlets with negative anomalous charge, which are gauge invariant otherwise. An example is

$$\bar{X}^c_0 X^c_1 (\bar{X}_1)^2 S_5 S_6 (S_7)^2, \tag{5.5}$$

which has $q_{\mathrm{an}} = -74/3$.

We note that the extension of the vacuum S_2 to a global solution is not straightforward. As demonstrated in Table 4.3, it does not provide uncharged monomials of bulk fields only, which include $\bar{Y}_2$ or U_3. Thus VEVs of these fields are incompatible with $D^a_3 = 0$. One may reduce the vacuum to $S_2 \setminus \{\bar{Y}_2, U_3\}$, or incorporate profiles of (partly) odd fields. Here we restrain our attention to local properties of the vacuum S_2 at the GUT fixed points, leaving the problem of global solutions to further studies.

The F-terms $F_i = \partial W/\partial s_i$ vanish trivially for all vacuum fields $s_i \in \mathcal{S}_2$, since they only arise from monomials which contain at least one other singlet with zero vacuum expectation value. Thus only monomials of the form $W = (\prod_i s_i)u$, with $s_i \in \mathcal{S}_2$ and $\langle u\rangle = 0$, induce non-trivial F-terms. For the vacuum $\mathcal{S}_2$ there are six such terms, arising from $u \in \{X^c_0, \bar{X}_0, X_1, \bar{X}^c_1, Y_2, \bar{Y}^c_2\}$. Each of these singlets u has a partner u^c which is contained in $\mathcal{S}_2$ and thus has a non-vanishing vev. Note that u cannot be a singlet with odd matter parity since the latter is preserved by $\mathcal{S}_2$. The relevant part of the superpotential is then given by

$$W = \left(a_{u1} + a_{u2}(\Omega'_1)^2 + a_{u3}(\Omega'_2)^3 + \cdots\right)\Omega'_1 u^c u\ , \tag{5.6}$$

where the Ω'_i were introduced in Table 4.3, and a_{uj} are coefficients labeling all completely invariant monomials which can be constructed from vacuum singlets. The F-term conditions become

$$F_u \propto a_{u1} + a_{u2}(\Omega'_1)^2 + a_{u3}(\Omega'_2)^3 + \cdots = 0\ . \tag{5.7}$$

We expect the existence of non-trivial solutions, with VEVs of the singlets $s_i \in \mathcal{S}_2$ determined by the coefficients a_{uj}. Explicit finite order examples for similar models were discussed in [11].

In the framework of heterotic orbifold compactifications, all couplings of SU(5) non-singlet fields arise from higher dimensional operators. In the vacuum $\mathcal{S}_2$, to lowest order in the singlets, we find the SU(5) Yukawa couplings for the two brane and two bulk families,

$$C^{(u)} = (a_{ij}) = \begin{pmatrix} \tilde{s}^4 & \tilde{s}^4 & \tilde{s}^5 & \tilde{s}^5 \\ \tilde{s}^4 & \tilde{s}^4 & \tilde{s}^5 & \tilde{s}^5 \\ \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^6 & g \\ \tilde{s}^5 & \tilde{s}^5 & g & \tilde{s}^6 \end{pmatrix}, \quad C^{(d)} = (b_{ij}) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \tilde{s}^{10} & \tilde{s}^{10} & \tilde{s}^6 & \tilde{s}^6 \\ \tilde{s}^1 & \tilde{s}^1 & \tilde{s}^2 & \tilde{s}^2 \end{pmatrix}. \tag{5.8}$$

Coupling	Order	Monomial
a_{11}	4	$(\bar{X}_0^c)^2\ S_2\ S_5$
a_{12}	4	$(\bar{X}_0^c)^2\ S_2'\ S_5$
a_{13}	5	$(\bar{X}_0^c)^2\ (S_2)^2\ S_5$
a_{14}	5	$(\bar{X}_0^c)^2\ S_2\ (S_5)^2$
a_{22}	4	$(\bar{X}_0^c)^2\ S_2'\ S_5'$
a_{23}	5	$(\bar{X}_0^c)^2\ (S_2')^2\ S_5'$
a_{24}	5	$(\bar{X}_0^c)^2\ S_2'\ (S_5)^2$
a_{33}	6	$(\bar{X}_0^c)^2\ (S_2)^3\ S_5$
a_{34}	0	g
a_{44}	6	$(\bar{X}_0^c)^2\ S_2\ (S_5)^3$

Table 5.1: *Examples of lowest order monomials for $C_{ij}^{(u)} = a_{ij}$ in the vacuum $\mathcal{S}_2$.*

Coupling	Order	Monomial
b_{31}	10	$X_0\ (\bar{X}_0^c)^2\ (X_1^c)^2\ \bar{X}_1\ \bar{Y}_2\ U_2\ U_4\ S_5$
b_{32}	10	$X_0\ (\bar{X}_0^c)^2\ (X_1^c)^2\ \bar{X}_1\ \bar{Y}_2\ U_2\ U_4\ S_5'$
b_{33}	6	$X_0\ X_1^c\ \bar{X}_1\ \bar{Y}_2\ S_6\ S_7$
b_{34}	6	$\bar{X}_0^c\ (X_1^c)^2\ Y_2^c\ S_6\ S_7$
b_{41}	1	S_5
b_{42}	1	S_5'
b_{43}	2	$S_2\ S_5$
b_{44}	2	$(S_5)^2$

Table 5.2: *Examples of lowest order monomials for $C_{ij}^{(d)} = b_{ij}$ in the vacuum $\mathcal{S}_2$.*

Here $\tilde{s}^n$ denotes one or more monomial of order n. Explicit lowest order monomials are given in Tables 5.1 and 5.2. Note that all vanishing terms are texture zeros which are protected by the unbroken discrete R-symmetry to arbitrary order. After orbifold projection to four dimensions the Yukawa couplings for the zero modes read

$$Y^{(u)} = \begin{pmatrix} a_{11} & a_{12} & a_{14} \\ a_{21} & a_{22} & a_{24} \\ a_{31} & a_{32} & a_{34} \end{pmatrix} = \begin{pmatrix} \tilde{s}^4 & \tilde{s}^4 & \tilde{s}^5 \\ \tilde{s}^4 & \tilde{s}^4 & \tilde{s}^5 \\ \tilde{s}^5 & \tilde{s}^5 & g \end{pmatrix}, \tag{5.9}$$

$$Y^{(d)} = \begin{pmatrix} b_{11} & b_{12} & b_{14} \\ b_{21} & b_{22} & b_{24} \\ b_{41} & b_{42} & b_{44} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \tilde{s}^1 & \tilde{s}^1 & \tilde{s}^2 \end{pmatrix}, \tag{5.10}$$

$$Y^{(l)} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \tilde{s}^{10} & \tilde{s}^{10} & \tilde{s}^6 \end{pmatrix}. \tag{5.11}$$

Clearly, these matrices are not fully realistic since $m_e = m_\mu = m_d = m_s = 0$. On the other

Coupling	Order	Monomial
c_{11}	11	$(X_0)^2 (\bar{X}_0^c)^2 \bar{X}_1 Y_2^c U_2 S_5 S_6 (S_7)^2$
c_{12}	11	$(X_0)^2 (\bar{X}_0^c)^2 \bar{X}_1 Y_2^c U_2 S_5' S_6 (S_7)^2$
c_{22}	11	$(X_0)^2 (\bar{X}_0^c)^2 \bar{X}_1 Y_2^c U_2 S_5' S_6' (S_7')^2$
c_{33}	12	$X_0(\bar{X}_0^c)^4(X_1^c)^2U_1^cU_2U_3S_2S_5$
c_{34}	7	$(X_0)^2 \bar{X}_0^c \bar{X}_1 U_2 S_6 S_7$
c_{44}	11	$(X_0)^3 (\bar{X}_0^c)^2 (\bar{X}_1)^2 U_1^c U_2 (S_6)^2$

Table 5.3: *Examples of lowest order monomials for $C_{ij}^{(L)} = c_{ij}$ in the vacuum $\mathcal{S}_2$.*

hand, they show the wanted hierarchical structure with a large top-quark mass singled out. Unsuccessful SU(5) mass predictions are avoided since the third 4D quark-lepton family is a combination of split multiplets from two 6D families.

Since $\mathrm{U}(1)_{B-L}$ is broken the model also predicts Majorana neutrinos. 'Right-handed' neutrinos with $t_{B-L} = 1$ can be inferred from Table 3.3. Via the seesaw mechanism they generate light neutrino masses. We obtain for the coefficients $C^{(L)}$ (cf. (2.1)) of the corresponding dimension-5 operator, which can be calculated directly,

$$C^{(L)} = (c_{ij}) = \begin{pmatrix} \tilde{s}^{11} & \tilde{s}^{11} & 0 & 0 \\ \tilde{s}^{11} & \tilde{s}^{11} & 0 & 0 \\ 0 & 0 & \tilde{s}^{12} & \tilde{s}^{7} \\ 0 & 0 & \tilde{s}^{7} & \tilde{s}^{11} \end{pmatrix} . \tag{5.12}$$

Examples of lowest order monomials are given in Table 5.3. Projection to four dimensions yields for SU(2) doublet zero modes the 3×3 sub-matrix with $i, j = 1, 2, 3$.

By construction, the μ-term vanishes to all orders in the vacuum $\mathcal{S}_2$ since it is protected by an unbroken discrete R-symmetry. However, this symmetry is not sufficient to forbid dangerous dimension-5 proton decay operators. This can be seen from the $\tilde{R}_2$-charges in Table 3.2, e.g.,

$$\tilde{R}_2(\bar{\mathbf{5}}_{(1)}\mathbf{10}_{(1)}\mathbf{10}_{(1)}\mathbf{10}_{(1)}) = 1 \bmod 2 \ , \quad \tilde{R}_2(\mathcal{K}_{\mathrm{vac}}) = 1 \bmod 2 \ . \tag{5.13}$$

Since these charges agree and the total $\mathbb{Z}_{10}^X$ charge vanishes, the proton decay term is not forbidden in the superpotential (2.1). Indeed, we find a lowest order monomial at $\mathcal{O}(7)$, $C_{1111}^{(B)} = (\bar{X}_0^c)^2 X_1^c \bar{X}_1 Y_2^c S_6 S_7$.

Note that the methods presented in Section 4 allow to design vacua with vanishing μ-term and dimension-5 proton decay terms to all orders in the singlets. An example is the vacuum $\mathcal{S}_0$, leading to $\mu = C_{ijkl}^{(B)} = 0$. However, this vacuum has other problems. It is incompatible with local D-term cancelation, has no gauge-coupling unification and vanishing down-type Yukawa couplings, $C_{ij}^{(d)} = 0$. This demonstrates that the various phenomenological properties of a vacuum are closely interrelated.

In summary, the vacuum $\mathcal{S}_2$ leads to too rapid proton decay, and also the quark and lepton mass matrices are not fully realistic. However, they show the correct qualitative features of the standard model, and we are optimistic that a systematic scan of the heterotic 'mini-landscape' can lead to phenomenologically more viable models.

6 Conclusions

How to distinguish between Higgs and matter is a crucial question in supersymmetric extensions of the standard model, in particular in compactifications of the heterotic string. We have analyzed this question for vacua of an anisotropic orbifold compactification which has an effective 6D supergravity theory as intermediate step between the GUT scale and the string scale.

Our main result is that for generic vacua, there is no difference between Higgs and matter, as there is nothing special about the standard model gauge group. However, certain vacua with standard model gauge group and particle content can possess discrete symmetries which single out Higgs fields. They are distinguished from matter fields by a matter parity, and a mass term allowed by gauge symmetries is forbidden by an elusive discrete R-symmetry, a remnant of the large symmetry exhibited by the fundamental theory.

We have identified maximal vacua of a heterotic orbifold model with local SU(5) unification for which the perturbative contribution to the μ-term vanishes. Nonperturbative corrections, possibly related to supersymmetry breaking, may then have the size of the electroweak scale. Alternatively, a non-zero μ-term suppressed by high powers of singlet fields can appear in extensions of the maximal vacua.

We have also determined the unbroken discrete R-symmetries of the maximal vacua. They are judiciously embedded into the large symmetry of the theory, which is a consequence of the large number of singlet fields forming the vacuum. It is intriguing that the maximal vacua do not include gauge-Higgs unification, but rather partial gauge-Higgs unification for the Higgs field H_u which gives mass to the up-type quarks. The original symmetry between **5**- and $\bar{\mathbf{5}}$-plets is broken by selecting vacua where matter belongs to $\bar{\mathbf{5}}$- and **10**-plets.

The method developed to find maximal vacua can be applied to all theories where couplings are generated by higher-dimensional operators. We have focussed on the μ-term, but one can also determine maximal vacua for several couplings, like the μ-term and dimension-5 proton decay operators. In addition to the vanishing of some couplings one may require the appearance of certain couplings, like Yukawa couplings or Majorana neutrino masses.

The features of the standard model imply strong constraints on phenomenolocially allowed vacua. Further important restrictions will follow from supersymmetry breaking and stabilization of the compact dimensions. Given the finite number of heterotic string vacua one may then hope to identify some generic features of standard model vacua, which can eventually be experimentally tested.

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