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Constraints on modular inflation in supergravity and string theory

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We perform a general algebraic analysis on the possibility of realising slow-roll inflation in the moduli sector of string models. This problem turns out to be very closely related to the characterisation of models admitting metastable vacua with non-negative cosmological constant. In fact, we show that the condition for the existence of viable inflationary trajectories is a deformation of the condition for the existence of metastable de Sitter vacua. This condition depends on the ratio between the scale of inflation and the gravitino mass and becomes stronger as this parameter grows. After performing a general study within arbitrary supergravity models, we analyse the implications of our results in several examples. More concretely, in the case of heterotic and orientifold string compactifications on a Calabi-Yau in the large volume limit we show that there may exist fully viable models, allowing both for inflation and stabilisation. Additionally, we show that subleading corrections breaking the no-scale property shared by these models always allow for slow-roll inflation but with an inflationary scale suppressed with respect to the gravitino scale. A scale of inflation larger than the gravitino scale can also be achieved under more restrictive circumstances and only for certain types of compactifications.

1 Introduction

Our current understanding of the very early universe is consistent with a period of dramatic accelerated expansion known as cosmological inflation [1]. The simplest and, so far, most successful way of modelling this stage consists in the 'slow-roll' motion of a single scalar field –the inflaton– behaving as a perfect fluid with negative pressure driving the universe into accelerated expansion [2, 3] (for a recent review, see [4]). A crucial ingredient in these types of models (generically referred to as slow-roll inflation) is the flatness of the scalar field potential characterising the inflaton's dynamics. On the one hand, this feature allows for inflation to last long enough, so that the universe can become flat, homogeneous and isotropic at cosmological scales [1, 2]. On the other, it is required to obtain the correct prediction for the spectrum of primordial density fluctuations [4, 5] as observed in precision measurements of the cosmic microwave background [6] and large scale structure of the universe [7, 8].

Despite of its simplicity, a completely satisfactory realisation of slow-roll inflation in supergravity and string theory has remained elusive, the main reason for this being the difficulty of ensuring the flatness of the inflaton potential [9]. Up to date, the most popular strategy to achieve inflation in string theory has consisted in the search of suitable inflationary trajectories within the vast landscape of string vacua, by studying the class of $\mathcal{N} = 1$ supergravity models arising from string theory. The large amount of freedom available in string compactifications, such as that coming from fluxes, torsion and/or non-perturbative effects, suggests that there should be no obstacles in obtaining a rich variety of scalar potentials, even possessing flat directions. However, in early attempts to achieve inflation, it was already understood that there are actually severe restrictions towards this possibility, particularly for the identification of the inflaton within the moduli sector [10, 11, 12]. In practise most of the successful scenarios of string inflation involve an additional sector beyond the moduli like, for instance, the uplifting sector used in most of the constructions of de Sitter (dS) vacua with fixed moduli. Examples of this type are models of inflation based on the KKLT scenario [13] where the joint contribution of non-perturbative effects and an explicit supersymmetry breaking term induced by anti-D3 branes allows to have dS vacua with a stable volume-modulus. Recently many interesting examples of such models of modular inflation have been proposed [14, 15, 16, 17, 18, 19, 20, 21, 22, 23]. Moreover, with the advent of new uplifting mechanisms replacing the one offered by anti-D3 branes, new string-inspired realisations of inflation with similar characteristics have been constructed [24, 25, 26]. Also some progress has been made recently towards a more general understanding of the origin of the difficulties in realising inflation in the moduli sector of string compactifications [27, 28].

As a matter of fact, the problem of finding viable models of single field slow-roll inflation turns out to be closely related to the problem of finding metastable vacua. A considerable step forward in the last question has been the new understanding of the circumstances under which metastable Minkowski vacua may exist in supergravity models. In ref. [29] for instance, it was shown that positivity of the scalar mass matrix along the direction associated to the scalar partners of the would be Goldstino (sGoldstinos) implies a strong necessary condition on the Kähler potential K independently of the superpotential W. This condition was shown to have strong implications when applied to the moduli sector of the simplest string compactifications available (see also [30, 31, 32, 33, 34]). More recently, in ref. [35], a more comprehensive study of the requirements for the existence of dS vacua in the moduli sector of Calabi-Yau string compactifications was carried out. From these studies it emerged again that the crucial quantity controlling metastability is the value of the mass matrix along a special direction defined by the Goldstino vector, which depends only on K. It was shown, in fact, that the value of the mass matrix along any other direction can be made arbitrarily large by appropriately choosing W, and that, once a suitable choice of K is adopted, it is always possible to construct metastable de Sitter vacua as long as there is enough freedom to tune the superpotential of the model.

The purpose of this work is to show that a similar analysis can be performed to determine if a supergravity model may possess flat-enough directions allowing for inflation. We find that the resulting condition is a generalisation of the constraint granting the existence of metastable dS vacua, and that, as much as for the realisation of these types of vacua, the main obstruction towards the realisation of inflation comes from the choice of the Kähler potential K. There exists, nevertheless, a significant difference between these two situations. At the point where the final stabilisation of the moduli occurs, the value of V is related to the cosmological constant Λ by $V = \Lambda$, which is tiny. In particular, one certainly has phenomenologically $V \ll m_{3/2}^2 M_{\rm Pl}^2$. During inflation, on the other hand, the value of V is related to the Hubble constant by $H^2 \simeq V/(3M_{\rm Pl}^2)$, and the gravitino mass generically differs from the one at the stabilised vacuum. We expect, however, the order of magnitude of that mass to remain the same in most cases, unless some extra tuning is enforced in the model. We therefore consider in the following a single scale for the gravitino mass, and such situation is surely realised for example in models where inflation is driven by the large F-term of a field whose contribution to W is nevertheless suppressed.

In general, it is then desirable to have $H \gg m_{3/2}$, since the scale of inflation should be much higher than the electroweak scale and particle phenomenology calls for $m_{3/2}$ comparable to, or lower than, that scale. As already noticed in [35], the condition for achieving a massless sGoldstino becomes stronger as the ratio $V/(m_{3/2}^2 M_{\rm Pl}^2)$ increases.

This means in particular that, in generic supergravity models, the condition for getting slow roll inflation is stronger than the condition for realising moduli stabilisation. The difference between these two situations can be conveniently parametrised in terms of the following quantity

$$\gamma = \frac{1}{3}\frac{V}{m_{3/2}^2 M_{\rm Pl}^2} \simeq \frac{H^2}{m_{3/2}^2}\,. \tag{1.1}$$

Let us emphasise however that the results presented in this work are also valid for models in which the gravitino mass changes strongly between inflation and the present vacuum. In such cases, the parameter γ has to be rescaled accordingly, and the comparison between the values of γ during and after inflation is still a useful indication of the difficulty in realising the scenarios.

The outline of this paper is as follows. In Section 2 we derive the condition that a generic supergravity theory must fulfil in order to allow for slow-roll inflation. In Section 3 we illustrate our results through some simple examples. In Section 4 we apply it to the case of heterotic and orientifold string compactifications on a Calabi-Yau in the large volume limit, and see what kind of information can be extracted. In Section 5 we study the effect of subleading corrections to the Kähler potential breaking the no-scale property shared by the models presented in Section 4 and its implications on the inflationary analysis. Finally in Section 6 we present our conclusions.

2 Slow-roll inflation in supergravity

To begin with, let us consider a generic model involving several complex neutral scalar fields ϕ^i, with a Lagrangian of the type (in Planck units $M_{\rm Pl} = 1$):

$$\mathcal{L} = \frac{1}{2}R - g_{i\bar{\jmath}}\,\partial\phi^i\partial\bar{\phi}^{\bar{\jmath}} - V(\phi^i, \bar{\phi}^{\bar{\imath}})\,. \tag{2.1}$$

The metric $g_{i\bar{\jmath}}$ must be Hermitian and positive definite, but is otherwise arbitrary. The realisation of a successful and viable stage of slow-roll inflation in such a model requires the existence of a region in field space where the potential in the canonical basis is sufficiently flat. In the case of a single real field, this corresponds to the requirement of having small slow-roll parameters ϵ, $|\eta| \ll 1$, where

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2, \qquad \eta = \frac{V''}{V}\,, \tag{2.2}$$

with $'$ denoting derivatives with respect to the canonically normalised field. These conditions get modified in the multi-field case. At lowest order in the slow-roll approximation

the trajectory in field-space along which inflation is realised is given by the direction $|\nabla V|^{-1}\nabla_i V$, where $|\nabla V| = \sqrt{\nabla^j V \nabla_j V}$, whereas deviations from this direction are controlled by the tensor:

$$N^I{}_J = \frac{1}{V}\begin{pmatrix} \nabla^i\nabla_j V & \nabla^i\nabla_{\bar{\jmath}} V \\ \nabla^{\bar{\imath}}\nabla_j V & \nabla^{\bar{\imath}}\nabla_{\bar{\jmath}} V \end{pmatrix} , \tag{2.3}$$

where $I = (i, \bar{\imath})$ and $J = (j, \bar{\jmath})$, and ∇_i denotes a derivative which is covariant with respect to the metric $g_{i\bar{\jmath}}$. Then, the generalised version of the slow-roll parameters (2.2) can be defined in the following way [36]:

$$\epsilon = \frac{\nabla^i V \nabla_i V}{V^2} , \tag{2.4}$$

$$\eta = \text{min eigenvalue}\,\{N\} \ . \tag{2.5}$$

Let us mention here that a strict characterisation of the slow-roll conditions would require us to distinguish between dynamical effects parallel and perpendicular to the inflaton's trajectory [37]. In particular, η given in eq. (2.2) would have to be generalised in such a way that it coincides with the projection $\eta_{||}$ of N along the direction $|\nabla V|^{-1}\nabla_i V$.[1] Notice from the definition (2.5), however, that for any given unit vector $u^I = (u^i, u^{\bar{\imath}})$ the following inequality is always satisfied:

$$\eta \leq u_I N^I{}_J u^J . \tag{2.6}$$

Indeed, one can always decompose u^I as $u^I = \sum_k c_{(k)}\omega^I_{(k)}$, where the $\omega^I_{(k)}$'s represent a basis of orthogonal and normalised eigenvectors of N with eigenvalues $\lambda_{(k)}$. Since the u^I's are unit vectors, the coefficients $c_{(k)}$ satisfy $\sum_k |c_{(k)}|^2 = 1$ and so it immediately follows that $u_I N^I{}_J u^J = \sum_k |c_{(k)}|^2 \lambda_{(k)} \geq \min\{\lambda_{(k)}\} = \eta$. In particular, one finds that $\eta \leq \eta_{||}$. Nevertheless, in order to avoid significant levels of isocurvature perturbations, a phenomenologically successful model of inflation requires the projection of N along directions perpendicular to $|\nabla V|^{-1}\nabla_i V$ to be much larger than $\eta_{||}$. This means that $\eta \simeq \eta_{||}$, as contributions to η coming from projecting N along directions perpendicular to $|\nabla V|^{-1}\nabla_i V$ have to be suppressed.

Let us consider now the situation in a generic supergravity theory involving only chiral multiplets. Recall that in supergravity the two-derivative Lagrangian can be written in terms of the real function $G = K + \log|W|^2$ and its derivatives with respect to the chiral multiplets Φ^i (and their conjugates $\bar{\Phi}^{\bar{\jmath}}$) which are denoted by lower indices i (and $\bar{\jmath}$). The

[1] Also a second slow roll-parameter $\eta_\perp$, depending on N, may be defined [37]. Loosely speaking, $\eta_\perp$ depends only on those elements of N mixing the tangent vector $|\nabla V|^{-1}\nabla_i V$ with the normal vector relative to the inflaton trajectory.

kinetic term of the scalar fields involves the Kähler metric $g_{i\bar{\jmath}} = G_{i\bar{\jmath}}$, which can be used to raise and lower indices and depends only on K. The Kähler metric is assumed to be positive definite and defines a Kähler geometry for the manifold spanned by the scalar fields. The scalar potential for this kind of theories takes the following simple form:

$$V = e^G (G^i G_i - 3)\,. \tag{2.7}$$

The auxiliary fields of the chiral multiplets are fixed by their equations of motion to be $F^i = m_{3/2} G^i$ with a scale set by the gravitino mass $m_{3/2} = e^{G/2}$. Whenever $F^i \neq 0$ at the vacuum supersymmetry is spontaneously broken, and the direction G^i in the space of chiral fermions defines the Goldstino which is absorbed by the gravitino in the process of supersymmetry breaking. The unit vector defining this direction is given by:

$$f_i = \frac{G_i}{\sqrt{G^j G_j}}\,. \tag{2.8}$$

Note that such direction can be different during and after inflation.

In these theories achieving small values for ϵ and η is not completely trivial. This is due to the fact that the potential V is constrained to be a specific function of K and W, and is therefore not entirely arbitrary. Nevertheless, if K is appropriately chosen, it is always possible to make ϵ and η arbitrarily small by tuning W. To see this, we must first compute the first and second derivatives of V and express them in terms of the parameters of the theory. These are:

$$\nabla_i V = e^G \Big(G_i + G^j \nabla_i G_j \Big) + G_i V\,, \tag{2.9}$$

$$\nabla_i \nabla_{\bar{\jmath}} V = e^G \Big(g_{i\bar{\jmath}} + \nabla_i G_k \nabla_{\bar{\jmath}} G^k - R_{i\bar{\jmath}p\bar{q}} G^p G^{\bar{q}} \Big) + G_i \nabla_{\bar{\jmath}} V + G_{\bar{\jmath}} \nabla_i V + (g_{i\bar{\jmath}} - G_i G_{\bar{\jmath}}) V\,, \tag{2.10}$$

$$\nabla_i \nabla_j V = e^G \Big(2\nabla_i G_j + G^k \nabla_i \nabla_j G_k \Big) + G_i \nabla_j V + G_j \nabla_i V + (\nabla_i G_j - G_i G_j) V\,. \tag{2.11}$$

Notice that G_i, $\nabla_i G_j$ and $\nabla_i \nabla_j G_k$ depend on the superpotential and more precisely on $(\log W)_i$, $(\log W)_{ij}$ and $(\log W)_{ijk}$, which are independent quantities. This means that W may be varied in an arbitrary way in order to adjust $\nabla_i V$ and N. It is clear then that, for a given Kähler potential, it is always possible to make ϵ arbitrarily small, simply by tuning $G^k \nabla_i G_k$ with respect to G_i in eq. (2.9). On the other hand, to achieve a small $|\eta|$, we need to have sufficient control on the entries of the matrix N. Observe that by tuning $\nabla_i \nabla_j G_k$ it is possible to set $\nabla_i \nabla_j V$ to any desired value, and the quantities $\nabla_i G_j$ to make most of the eigenvalues of $\nabla_i \nabla_{\bar{\jmath}} V$ large and positive. The only restriction comes from the fact that the projection of $\nabla_i \nabla_{\bar{\jmath}} V$ along the Goldstino direction (2.8) is actually constrained by eq. (2.9) (which has already been fixed to make ϵ small) and therefore

cannot be adjusted so easily. Nevertheless, if the choice of K allows for it, one can still make this last direction flat enough by tuning the remaining quantities G_i.

From the previous discussion it remains to be learned under which circumstances a given K is suitable to produce such a flat direction. To find this out, let us recall that eq. (2.6) is valid for any unit vector u. We can then derive an upper bound on η for the particular choice $u_I = (e^{-i\alpha} f_i, e^{i\alpha} f_{\bar{\imath}})/\sqrt{2}$, $u^J = (e^{i\alpha} f^j, e^{-i\alpha} f^{\bar{\jmath}})/\sqrt{2}$, which is associated to the Goldstino direction f^i given in eq. (2.8):[2]

$$\eta \leq \frac{\nabla_i \nabla_{\bar{\jmath}} V}{V} f^i f^{\bar{\jmath}} + \mathrm{Re}\left\{ e^{2i\alpha} \frac{\nabla_i \nabla_j V}{V} f^i f^j \right\}. \tag{2.12}$$

Averaging this over the two orthogonal choices $\alpha = 0, \pi/2$ one finally deduces the following simple bound, depending only on the Hermitian block of the Hessian matrix:

$$\eta \leq \frac{\nabla_i \nabla_{\bar{\jmath}} V}{V} f^i f^{\bar{\jmath}}. \tag{2.13}$$

Using now eq. (2.10) it is straightforward to find:

$$\frac{\nabla_i \nabla_{\bar{\jmath}} V}{V} f^i f^{\bar{\jmath}} = -\frac{2}{3} + \frac{4}{\sqrt{3}} \frac{1}{\sqrt{1+\gamma}} \mathrm{Re}\left\{ \frac{\nabla_i V}{V} f^i \right\} + \frac{\gamma}{1+\gamma} \frac{\nabla^i V \nabla_i V}{V^2} + \frac{1+\gamma}{\gamma} \hat{\sigma}(f^i), \tag{2.14}$$

where the parameter γ is given by eq. (1.1) and the function $\hat{\sigma}(f^i)$ is defined to be

$$\hat{\sigma}(f^i) = \frac{2}{3} - R(f^i), \tag{2.15}$$

where $R(f^i) = R_{i\bar{\jmath}p\bar{q}}\, f^i f^{\bar{\jmath}} f^p f^{\bar{q}}$ denotes the holomorphic sectional curvature along the Goldstino direction f^i. Note that the quantity $\hat{\sigma}$ is the normalised version of the homogeneous quantity σ that was introduced in ref. [35]:[3] $\hat{\sigma}(f^i) = \sigma(G^i)/(G^k G_k)^2$.

Since f^i is a unit vector, it is clear that $|f^i \nabla_i V/V| \leq \sqrt{\epsilon}$. Using this inequality, the definition of ϵ, and the result given in (2.14) we finally obtain the following simple upper bound on η:

$$\eta \leq \eta_{\max} \equiv -\frac{2}{3} + \frac{4}{\sqrt{3}} \frac{1}{\sqrt{1+\gamma}} \sqrt{\epsilon} + \frac{\gamma}{1+\gamma} \epsilon + \frac{1+\gamma}{\gamma} \hat{\sigma}(f^i). \tag{2.16}$$

Notice now that $\eta_{\max}$ should be either negative and very small or positive ($\eta_{\max} \gtrsim 0$) in order for the bound (2.16) to be compatible with the requirement of having a small $|\eta|$. More precisely, assuming $\epsilon \ll 1$, one needs:

$$\hat{\sigma}(f^i) \gtrsim \frac{2}{3} \frac{\gamma}{1+\gamma}. \tag{2.17}$$

[2] Notice that if $G^k \nabla_i G_k \propto G_i$ then $V_i \propto G_i$ and the inflaton and Goldstino directions in field space are aligned. Then the value of $\eta_{||}$ is equal to the right hand side of (2.12) with $\alpha = 0$.

[3] In the notation of ref. [35] the bound (2.13) takes the form $\eta \leq e^G \lambda/(V G^i G_i)$, where $\lambda = -2/3\, e^{-G} V \big(e^{-G} V + 3\big) + \sigma + 2e^{-G}(G^m V_m + G^{\bar{n}} V_{\bar{n}}) + V^n V_n$.

This condition can also be rewritten in the following alternative form, which has the same structure as the conditions derived in refs. [29, 32, 34]:

$$R(f^i) \lesssim \frac{2}{3}\frac{1}{1+\gamma}\,. \tag{2.18}$$

The condition (2.17), or equivalently (2.18), represents our main result and implies a strong restriction on the Kähler potential, generalising the one obtained in refs. [27, 28] for single fields models. If it is satisfied, one still needs to further tune the superpotential to adjust η to a sufficiently small value compatible with current data.

For $\gamma \ll 1$, this condition reduces to $\hat{\sigma}(f^i) \gtrsim 0$ (or $R(f^i) \lesssim 2/3$), which coincides with the condition for the existence of metastable dS vacua with small cosmological constant. On the other hand, for $\gamma \gg 1$, it tends to the more restrictive condition $\hat{\sigma}(f^i) \gtrsim 2/3$ (or $R(f^i) \lesssim 0$). Since $\gamma = (H/m_{3/2})^2$ parametrises the ratio between the Hubble scale H and the gravitino scale $m_{3/2}$, this means that inflationary scales much smaller than the gravitino scale are as difficult to realise as dS vacua, whereas higher inflationary scales are instead more difficult to realise.

One can study the implications of the condition (2.18) exactly in the same way as was done in refs. [29, 32, 34]. In particular, one can derive a constraint involving only the Kähler potential by minimising the sectional curvature with respect to the variables f^i, taking into account that these variables are normalised to one: $f^i f_i = 1$. This implies the condition that the minimal value of the sectional curvature $R_{\rm min}$ should be less than $2/[3(1+\gamma)]$. Moreover, if $R_{\rm min}$ satisfies that bound, the direction f^i is then constrained to lie within a cone centred around the particular direction that minimises the sectional curvature. This procedure can be performed explicitly for particular classes of models, like for instance those for which the scalar manifold factorises into a product of one-dimensional scalar manifolds or also for coset scalar manifolds. More precisely, for factorisable manifolds it is easy to show that the sectional curvature satisfies a lower bound in terms of the scalar curvatures R_i of the one-dimensional submanifolds which is given by: $R(f^i) \geq (\sum_i R_i^{-1})^{-1}$. For coset manifolds, on the other hand, the Riemann tensor has a very special structure. One can show that in those cases the sectional curvature turns out to be constant and to depend only on some overall curvature scale $R_{\rm all}$, which depends on the particular coset manifold being considered: $R(f^i) = R_{\rm all}$ (see [32] for more details).

It is worth pointing out that the presence of vector multiplets gauging isometries of the chiral multiplet geometry can quantitatively change the right-hand side of the constraint (2.18). This is mainly due to the fact that the D-term contributions to the scalar potential are positive definite. More precisely, for a given value of the potential V, increasing the

ratio between the D-term and the F-term contributions to the potential has the net effect of reducing the left-hand side of (2.18) and therefore making the constraint milder [34]. This could be used to partly compensate the strengthening of the condition induced by increasing γ. A more radical improvement of the situation can be obtained by relying on genuine constant Fayet-Iliopoulos terms [38]. However, this possibility is severely constrained within supergravity, and implies a rather peculiar gauging of the R-symmetry, which does not seem to emerge in any kind of string construction [39].

3 Simple examples

The simplest example one can study is the case of supergravity models involving a single chiral superfield with a canonical Kähler potential:

$$K = \bar{X} X \,. \tag{3.1}$$

For this scalar manifold the Riemann tensor vanishes. From (2.15) we get then that $\hat{\sigma} = 2/3$, and the condition (2.17), or equivalently (2.18), can always be satisfied independently of the value of γ. This implies in particular that there is no obstruction in this case to build a model with any scale of inflation. In models with several fields of this type, that is, with $K = \sum_i \bar{X}^i X^i$, the components of the Riemann tensor will also vanish and therefore the situation is exactly the same.

Another simple case that can be studied is the case of a field with a logarithmic Kähler potential, for which a no-go theorem is discussed in [27, 28]:

$$K = -n \log\left(T + \bar{T}\right), \tag{3.2}$$

which governs the dynamics of moduli fields arising in simple examples of string compactifications. The one-dimensional scalar manifold has in this case a constant sectional curvature which is simply given by $R = 2/n$. From here we get that $\hat{\sigma} = 2/3(1 - 3/n)$. This means that the condition (2.17), or (2.18), can be satisfied only if

$$n \gtrsim 3(1 + \gamma)\,.$$

It is then clear that a model with $\gamma \gg 1$ cannot be built within this setup, as n is typically a number of order 1. For instance the overall Kähler modulus in string models has $n = 3$ and thus, even including subleading corrections to the Kähler potential, one can at best achieve a small γ of the order of the subleading corrections.[4] In models with

[4] In ref. [27] a model of this kind is proposed where a sizable γ is achieved by going to a regime where

several such fields, that is, with logarithmic potentials with coefficients n_i, one finds that the sectional curvature depends on the orientation of the Goldstino direction f^i. However one can proceed exactly as in [29] and minimise the sectional curvature with respect to the variables f^i, taking into account the constraint $f^i f_i = 1$. By doing so it is easy to find that $R(f^i) \geq 2/(\sum_i n_i)$. The condition (2.18) implies therefore that $\sum_i n_i \gtrsim 3(1+\gamma)$. As in the one field case we conclude then that one cannot get an inflationary scale much bigger than the gravitino mass in any model with a small number of moduli with coefficients n_i of order 1.

Given the above two substantially different situations, one could then consider a model combining a field with a logarithmic Kähler potential and a field with a canonical Kähler potential (which would act as an uplifting sector):

$$K = -n \log (T + \bar{T}) + \bar{X} X \,. \tag{3.3}$$

In such a case, the scalar manifold spanned by the fields X and T factorises into two one-dimensional manifolds. As before we find that the curvature in the one-dimensional manifold spanned by X vanishes whereas the curvature in the one-dimensional manifold spanned by T is given by $2/n$. This means that the minimal value that the sectional curvature is allowed to take is zero, since the Goldstino direction can be aligned along the direction of zero curvature: $R(f^i) \geq 0$. It is then always possible to satisfy the condition (2.18), independently of the value of n. However, it is clear that in order to achieve a large γ, that is, a scale of inflation bigger than the gravitino scale, the inflationary dynamics must be strongly affected by the uplifting sector. The situation remains qualitatively the same by adding several such building blocks.

Another case that can be easily analysed is that of models with the following Kähler potential:

$$K = -n \log (T + \bar{T} - \bar{X} X) \,. \tag{3.4}$$

In this case the scalar geometry is a maximally symmetric coset space with constant curvature, and one finds $R(f^i) = 2/n$. The situation is then identical to the one obtained with only one field T with a logarithmic Kähler potential and the addition of the X field does not help in satisfying the condition. In particular, it is impossible to realise slow-roll inflation if $n = 3$, unless extra ingredients are added.[5] Again, adding more fields of this

the subleading correction actually induces a significant change in the Kähler curvature. This is achieved thanks to a large numerical coefficient that compensates its parametrical suppression. We believe however that in such a situation there is limited control on the effect of the corrections at higher orders of the low-energy expansion.

[5]For example the model considered in ref. [40] involves an additional uplifting sector. In that case, besides the fields T and X describing the volume and the brane position, one would also have to take into account some extra field Y describing the anti-brane position.

kind in a similar way does not change qualitatively the situation. More involved coset manifolds can be studied as in ref. [32].

4 No-scale models

A general feature of models emerging from string compactifications on a Calabi-Yau is that their moduli sector exhibits, in the large volume limit, the no-scale property:

$$K^i K_i = 3\,. \tag{4.1}$$

As shown in [35], this property constrains the Kähler geometry and, as a consequence of this, the Riemann tensor satisfies certain properties when projected along the particular direction $k^i = K^i/\sqrt{3}$. In particular, one finds that along such a direction the sectional curvature takes precisely the critical value:

$$R(k^i) = \frac{2}{3}\,. \tag{4.2}$$

This means that it is always possible to obtain $\hat{\sigma} = 0$ by choosing the Goldstino direction f^i to be aligned along this special direction k^i. The question is then whether it is possible or not, by departing from the configuration $f^i = k^i$, to get a lower value of the sectional curvature, or equivalently, a larger value of $\hat{\sigma}(f^i)$.

In orbifold models, as well as in smooth compactifications on Calabi-Yau manifolds which are actually K3 fibrations with a large P_1 base, the moduli space is a coset manifold of the type G/H. These spaces are symmetric and the form of the Riemann tensor is further constrained by the presence of isometries. Moreover, they are also homogeneous with a covariantly constant curvature. In these models, the quantity $\hat{\sigma}$ can be easily studied as a function of the direction f^i, and it is possible to prove that the value $\hat{\sigma} = 0$ along the direction k^i corresponds to an absolute maximum [35]. The situation is then identical to that of a single modulus with a logarithmic potential of the form (3.2) with coefficient $n = 3$: Inflation can be realised with the help of subleading corrections to the Kähler potential, but only with a very low scale relative to the gravitino mass ($\gamma \ll 1$). We will come back to this issue in the next section.

In general Calabi-Yau models, the situation is more interesting. Indeed, in those cases the scalar manifold is in general neither symmetric nor homogeneous. The function $\hat{\sigma}$ can then have either a maximum or a saddle point at the special direction k^i, and the space of possible models subdivides into two classes: Models for which it is possible to find a positive value of $\hat{\sigma}$ in a direction different than k^i and models for which it is impossible to get such a positive value.

To illustrate the situation arising in more general Calabi-Yau models, let us consider the Kähler moduli sector of heterotic compactifications. The Kähler potential is here determined by the intersection numbers d_{ijk} of the Calabi-Yau manifold and has the following form

$$K = -\log \mathcal{V}\,, \quad \mathcal{V} = \frac{1}{6} d_{ijk}(T^i + \bar{T}^{\bar{\imath}})(T^j + \bar{T}^{\bar{\jmath}})(T^k + \bar{T}^{\bar{k}}), \tag{4.3}$$

where $\mathcal{V}$ is the classical volume of the Calabi-Yau. This defines a special Kähler geometry, and the Riemann tensor has the special structure $R_{i\bar{\jmath}p\bar{q}} = g_{i\bar{\jmath}} g_{p\bar{q}} + g_{i\bar{q}} g_{p\bar{\jmath}} - e^{2K} d_{ipr} g^{r\bar{s}} d_{\bar{s}\bar{\jmath}\bar{q}}$. The sectional curvature along the Goldstino direction is then given by:

$$R(f^i) = 2 - e^{2K} d_{ipr} g^{r\bar{s}} d_{\bar{s}\bar{\jmath}\bar{q}}\, f^i f^{\bar{\jmath}} f^p f^{\bar{q}}\,. \tag{4.4}$$

This yields:

$$\hat{\sigma}(f^i) = -\frac{4}{3} + e^{2K} d_{ipr} g^{r\bar{s}} d_{\bar{s}\bar{\jmath}\bar{q}}\, f^i f^{\bar{\jmath}} f^p f^{\bar{q}}\,. \tag{4.5}$$

From the explicit form of the Kähler metric derived from (4.3) it follows that $d_{ipr} k^i k^p = 2k_r/\sqrt{3}$. One can then easily verify that along the special direction k^i one indeed has $R(k^i) = 2/3$ and $\hat{\sigma}(k^i) = 0$. It was however shown in ref. [35] that $\hat{\sigma}$ can be made positive or negative along other directions, depending on the intersection numbers d_{ijk}. For instance, in models with only two moduli, the situation simplifies due to the fact that there is only one direction orthogonal to the direction given by k^i. This direction is given by the unit vector n^i defined as:

$$(n^1, n^2) = \frac{(k_2, -k_1)}{\sqrt{\det g}}\,, \quad n^i\, k_i = 0\,. \tag{4.6}$$

One can show [35] that the convexity of the function $\hat{\sigma}(f^i)$ at $f^i = k^i$ is determined by the sign of the discriminant of the cubic polynomial $\mathcal{V}$ defining the volume of the Calabi-Yau, given by:

$$\Delta = -27\Big(d_{111}^2 d_{222}^2 - 3\, d_{112}^2 d_{122}^2 + 4\, d_{111} d_{122}^3 + 4\, d_{112}^3 d_{222} - 6\, d_{111} d_{112} d_{122} d_{222}\Big)\,. \tag{4.7}$$

If $\Delta > 0$, then $\hat{\sigma}(k^i) = 0$ corresponds to the absolute maximum, and it is not possible to meet the condition for slow-roll inflation. If, on the contrary, $\Delta < 0$, the point $\hat{\sigma} = 0$ corresponds to a saddle point and therefore there is a region in the parameter space spanned by the f^i's for which $\hat{\sigma}(f^i)$ can be made positive. Moreover, the value of $\hat{\sigma}$ for $V > 0$ is extremised to a non-vanishing value along some particular direction f^i in between k^i and n^i. Unfortunately, this value is difficult to determine in general, essentially because $\hat{\sigma}$ is defined in terms of the normalised unit vector f^i. Nevertheless, we can still verify

whether it is possible or not to obtain $\hat{\sigma}(f^i)$ larger than the critical value 2/3 required to be able to realise inflation with an arbitrary high scale. A simple way to verify that this is indeed the case is by looking at the particular direction $f^i = z^i$ given by [6]:

$$z^i = \sqrt{\frac{1+a}{9+a}}k^i + \sqrt{\frac{8}{9+a}}n^i\,, \qquad a = -\frac{\Delta}{24}\frac{e^{4K}}{(\det g)^3}\,. \tag{4.8}$$

Notice that $a > 0$ as the factor $e^{4K}/(\det g)^3$ is always positive. Along this particular direction one then obtains[7]:

$$\hat{\sigma}(z^i) = \frac{64\,a}{(a+9)^2}, \tag{4.9}$$

which is positive. Then, assuming that a can be varied over the whole range $[0, +\infty)$ by varying the values of the fields while keeping e^K, $\det g$ and $\mathrm{tr}\, g$ all positive, the largest possible value for $\hat{\sigma}$ is obtained for $a = 9$ and is given by $\hat{\sigma}_{\max} = 16/9$. Since this is larger than 2/3, one should then be able to achieve any arbitrarily large value of γ.

Another interesting situation based on Calabi-Yau manifolds arises in Type II orientifold compactifications. In that case, the scalar geometry that one obtains for a given Calabi-Yau manifold is dual to the one arising for the heterotic model based on the same manifold [41, 42], and one finds opposite signs for the extremal value of $\hat{\sigma}$. In the special case involving only two fields, one can in fact prove that for orientifolds this extremal value is given by $\hat{\sigma} = 64\,a/(a-9)^2$, where a is defined as before but with $\Delta \to -\Delta$, $e^K \to e^{-K}$ and $\det g \to (\det g)^{-1}$, namely $a = (\Delta/24)\,e^{-4K}\,(\det g)^3$. In this case, a viable situation with a positive $\hat{\sigma}$ can therefore be realised only for those Calabi-Yau manifolds for which $\Delta > 0$. One can actually show that in this case $a \in [0, 1]$, and the largest possible value for $\hat{\sigma}$ is obtained for $a = 1$ and is given by $\hat{\sigma}_{\max} = 1$, which is still larger than 2/3.

5 Effect of subleading corrections

We would like now to discuss the role of subleading corrections in the boundary cases when the leading order of the Kähler potential just fulfills the equality in eq. (2.17). As

[6] This direction was found in [35] in the analysis of string compactifications with two moduli. There, it was shown that z^i maximises the quantity $\sigma = (G^k G_k)^2 \hat{\sigma}(f^i)$. One should keep in mind however that in general the function $\hat{\sigma}(f^i)$ is maximised in a direction $f^i \neq z^i$.

[7] This expression can be derived as follows from the results of section 4 of ref. [35]. One starts from the decomposition $\sigma = \omega - 2s^i s_i$, with $\omega = a\,(3\,\det g\,|C|^2)^2$ and $s^i = 0$, taking a general Goldstino direction $G_i = N_i + \alpha K_i$, where N_i is orthogonal to K_i. From the definition of C one easily finds that $3\,\det g\,|C|^2 = N^i N_i$. Moreover, the equation $s^i = 0$ fixes α in terms of N^i and the arbitrary phase of C. One finds in particular that $|\alpha|^2 \geq [(1+a)/24]N^i N_i$, the precise value depending on the phase of C. It then follows that $G^i G_i \geq [(a+9)/8]N^i N_i$. Finally, one computes $\hat{\sigma} = \omega/(G^i G_i)^2$, with $G^i G_i$ taken to assume its minimal value.

we already mentioned in the last section, for no-scale models, the sectional curvature along the direction k^i is $R(k^i) = 2/3$, and therefore $\hat{\sigma} = 0$ along that direction. This means in particular that a general possibility to realise inflation which can arise in all Calabi-Yau string models is to consider subleading corrections to the Kähler potential that break the no-scale property. However this possibility obviously restricts the scale of inflation to be small (compared to the gravitino scale), as the change in $\hat{\sigma}$ is of the order of the subleading correction.

The subleading corrections to the Kähler potential can be of various types, e.g. loop, α' or world-sheet instanton corrections. As a result of these corrections, the no-scale property will be deformed by some small quantity δ, which is parametrically of order $\Delta K/K$:

$$K^i K_i \simeq 3 + \mathcal{O}(\delta)\,. \tag{5.1}$$

In this situation, the extremum of the function $\hat{\sigma}$ along the direction k^i gets in general slightly shifted, and the new value at this extremum becomes of order

$$\hat{\sigma}(k^i) \simeq \mathcal{O}(\delta)\,. \tag{5.2}$$

Comparing this result with the condition (2.17), we see that in this case it would indeed be possible to realise inflation along the direction $f^i \simeq k^i$, provided one can get the right sign for the subleading correction δ. However the parameter γ which sets the scale of inflation is bounded by the parameter $|\delta|$ controlling the relative effect of the subleading corrections in the Kähler potential:

$$\gamma \lesssim \mathcal{O}(|\delta|)\,, \tag{5.3}$$

and therefore one has necessarily $H < m_{3/2}$.

One can consider for instance the effect of α'-corrections to the large volume limit of Calabi-Yau compactifications of the heterotic string [43] and of type IIB orientifolds [44]. These corrections have the effect of shifting the argument of the logarithm in the Kähler potential by some constant parameter ξ [8]:

$$K = -n \log(\mathcal{V} + \xi)\,,$$

where $n = 1, 2$ for heterotic and orientifold models respectively. One can then parametrise the relative effect of these corrections with $\delta \sim \xi/\mathcal{V}$, where $\mathcal{V}$ is the volume of the Calabi-Yau manifold (resp. orientifold). It is easy to check that $\hat{\sigma}$ still has an extremum along the direction k^i, but its value at that point becomes now $\hat{\sigma} \sim \delta \sim \xi/\mathcal{V}$. As a result, the maximal scale of inflation that can be realised within this setup corresponds to $\gamma \sim \xi/\mathcal{V}$, that is $H^2 \sim m_{3/2}^2 \xi/\mathcal{V}$. This is for example the case in the model of ref. [16].

[8] Strictly speaking, in the case of IIB orientifolds the correction is dilaton-dependent. This does not qualitatively modify the effect however.

6 Conclusions

In this paper we have studied the possibility of realising successful slow-roll inflationary scenarios in a general low-energy effective supergravity theory involving only chiral multiplets. We have shown that the condition imposed on the theory for having slow-roll inflation is very similar to the one necessary for obtaining a metastable de Sitter vacuum. In particular, the requirement is that the sectional curvature $R(f^i)$ along the Goldstino direction f^i should be smaller than the critical value $2/[3(1+\gamma)]$, where the parameter $\gamma = V/(3\, m_{3/2}^2)$ depends on the size of the potential relative to the gravitino mass scale. As was shown in [35], the presence of dS vacua with small cosmological constant $\Lambda \ll m_{3/2}^2$, that is, with $\gamma \ll 1$, implies that the sectional curvature is bounded, i.e. $R(f^i) \lesssim 2/3$. For inflation, on the other hand, this condition changes depending on the Hubble scale. In models with $H \gg m_{3/2}$, i.e. $\gamma \gg 1$, such constraint becomes $R(f^i) \lesssim 0$. For models with $H \ll m_{3/2}$ one has instead $\gamma \ll 1$ and the condition takes the form $R(f^i) \lesssim 2/3$ and is therefore similar to the one relevant for metastable dS vacua. This means in particular that models with a scale of inflation higher than the gravitino mass are more difficult to realise than models with a scale of inflation smaller than (or comparable to) the gravitino mass.

More concretely, we have shown that the condition for successful inflation can be generically satisfied in any no-scale model by taking into account the effect of subleading corrections, although in those cases the scale of inflation has to be suppressed with respect to $m_{3/2}$. On the other hand, models with a scale of inflation that is comparable or even larger than the gravitino mass can instead be realised only in certain Calabi-Yau compactifications, those ones allowing for a value of $\hat{\sigma} \sim 2/3$. We have also shown through some simple examples that the conditions necessary for slow-roll inflation can also be achieved by adding to the moduli sector of the theory an uplifting sector. In those cases the size of the parameter γ, which gives the ratio between the scale of inflation and the gravitino mass during inflation, depends on the influence that the uplifting sector has on the inflationary dynamics. For example in models with a Kähler potential of the type (3.2) with $n = 3$ it is clear that in order to have $\gamma \gg 1$ the uplifting sector should dominate the inflationary dynamics. If this is not the case, the uplifting sector only mildly changes the condition (2.17) and one has a model with $H \lesssim m_{3/2}$. This is actually the typical situation in inflationary scenarios based on the KKLT setup, as was pointed out in [45].

Recall however that the gravitino mass during inflation is not necessarily the same as the gravitino mass at the vacuum. In order to construct models with $H \gg m_{3/2}$, one possibility is then to perform an additional tuning to make the gravitino mass during inflation much bigger than the gravitino mass at the vacuum [45, 46]. We have shown in

this paper that another possibility to realise $H \gg m_{3/2}$ without performing an additional tuning is to consider Calabi-Yau compactifications allowing for a sizable value of $\hat{\sigma}$, or equivalently, for a small value of the sectional curvature.

It is interesting to note that from (2.18), and by taking into account the definition of γ in (1.1), one can compute the following bound on the value of the inflationary Hubble parameter:

$$H^2 \lesssim R_{\rm min}^{-1}\Big(\frac{2}{3} - R_{\rm min}\Big)\, m_{3/2}^2\,, \tag{6.1}$$

where $R_{\rm min}$ denotes the minimal value that the sectional curvature of the moduli space is allowed to take. In the vacuum of the theory the same kind of bound can be computed for the mass m of the lightest scalar. Actually following the same reasoning as the one used to derive (2.13) and imposing that at the vacuum $V = \nabla_i V = 0$, one easily deduces that:

$$m^2 \lesssim\ f^i f^{\bar{\jmath}}\, \nabla_i \nabla_{\bar{\jmath}} V = 3\Big(\frac{2}{3} - R_{\rm min}\Big) m_{3/2}^2\,. \tag{6.2}$$

As we already mentioned, the two gravitino scales in (6.1) and (6.2) may differ, but in the absence of additional tuning of the parameters in the theory, both scales are naturally expected to be of the same order of magnitude.

One can compute now the ratio of the bounds (6.1) and (6.2). This yields the following simple relation:

$$\frac{H^{\rm max}}{m^{\rm max}} \sim R_{\rm min}^{-1/2}\,. \tag{6.3}$$

This is perhaps the most objective measure of the tension against making the scale of inflation much larger than the scale of supersymmetry breaking, and shows that the only way to relax such tension in a robust way (that is, without extra fine-tuning) is to choose for inflation a direction in field space where the Kähler curvature is very small.

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