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Froggatt-Nielsen hierarchy and the neutrino mass matrix

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Abstract

We study the neutrino mass matrix derived from the seesaw mechanism in which the neutrino Yukawa couplings and the heavy Majorana neutrino mass matrix are controlled by the Froggatt-Nielsen mechanism. In order to obtain the large neutrino mixings, two Froggatt-Nielsen fields are introduced with a complex vacuum expectation values. As a by-product, CP violation is systematically induced even if the order one couplings of FN fields are real. We show several predictions of this model, such as θ_{13}, the Dirac CP phase, two Majorana CP phases, the effective mass of the neutrinoless double beta decay and the leptogenesis. The prediction of the branching ratio of $\mu \to e\gamma$ is also given in SUSY model.

1 Introduction

The Froggatt-Nielsen (FN) mechanism[1] is one of the attractive mechanisms to explain mass hierarchy of quarks and charged leptons. The idea is that the U(1) global symmetry is taken as a flavor symmetry and the vacuum expectation value (VEV) of a flavor field called FN field gives a proper structure of Yukawa couplings. For quarks and leptons, this mechanism seems to work well by taking an appropriate charge assignment of fields. However it is known that a mass hierarchy of neutrinos is milder than that of charged leptons. In the normal hierarchy case, $m_2/m_3 \simeq \sqrt{\Delta m^2_{\rm sol}/\Delta m^2_{\rm atm}} \sim \mathcal{O}(\lambda)$ in contrast to $m_\mu/m_\tau \sim \mathcal{O}(\lambda^2)$. Here $\lambda \sim 0.2$ is a size of the Cabbibo angle. Then it is a natural question whether the FN mechanism can be applied to the mass hierarchy of neutrinos[2].

A promising approach to get small neutrino masses is the seesaw mechanism[3] where the right-handed heavy Majorana neutrinos are introduced. The dimension five operators are generated after these heavy neutrinos are integrated out. This model has several interesting features. First of all, the light neutrinos are Majorana particles and we expect the neutrinoless double beta decay which provides the information not only of light neutrino mass scale, but also the Majorana nature of neutrinos such as Majorana CP phases[4]. Secondly, the scenario of the leptogenesis[5] is automatically incorporated as a mechanism to generate the baryon asymmetry of universe. In a supersymmetric version of the seesaw

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model, a significant contribution to lepton flavor violation such as $\mu \to e\gamma$ can be generated through the running effect even if the soft scalar masses are taken as flavor universal at the high energy scale[6]. The present bound on $\mu \to e\gamma$, $\mathrm{Br}(\mu \to e\gamma) < 1.2 \times 10^{-11}$ [7] will be improved by two order of magnitudes in MEG experiment[8] and gives the information of neutrino mass matrices.

Applying the FN mechanism to the neutrino Dirac mass matrix and the heavy Majorana neutrino mass matrix would be a natural extension of the model. Since charged leptons and neutrinos are embedded into the same SU(2) doublet in left handed sector, a naive expectation is that the mass hierarchy in the charged lepton mass matrix and the Dirac neutrino mass matrix are similar to each other. In order to get the milder hierarchy for neutrino masses, the hierarchy in the heavy neutrino mass matrix should compensate that of the Dirac mass matrix.

In this paper, we examine how the milder hierarchy for neutrino masses is obtained systematically in the FN scheme. We also discuss the problem which is pointed out by Koide and Takasugi in the analysis of the 2-3 symmetric mass matrices for neutrinos and charged leptons[9]. The 2-3 symmetry[10] is the invariance of the mass Lagrangian under the exchange of ψ_2 and ψ_3, where ψ_i is the i-th generation fermion of $\psi = e_L, e_R, \nu_L, \nu_R$. The problem is as follows: Mass matrices for the neutrino and the charged lepton lead to the maximal 2-3 mixing ($\pi/4$ degree mixing) for both cases, but these mixings cancel each other when they are combined to obtain the neutrino mixing. That is, we obtain the zero 2-3 mixing. This is a serious problem for the 2-3 symmetry, though this is the powerful ansatz to restrict the mass Lagrangian.

We see the general feature of the neutrino mass matrix which arises from the seesaw mechanism with the FN mechanism. We show that the milder hierarchy is naturally obtained by a simple ansatz for a choice of the FN charge of neutrinos. We discuss a case of real Yukawa coupling constants and a case of the mass matrices with the 2-3 symmetry and shows that a cancellation occurs for the 2-3 mixing. This cancellation can be avoided by considering an extended FN mechanism [11], where two FN fields are introduced. Quark mass matrices in this type of model has been discussed in Ref.[11] and it is shown that a CP phase and mixings in the Cabbibo-Kobayashi-Maskawa matrix[12] are obtained and they reproduce the experimental data well. As a by-product, a CP violation is systematically introduced by keeping the Yukawa coupling constants for the matter and FN fields real. We construct a simple model and examine the light neutrino mass hierarchy, the CP phases, and the mixings, which are controlled essentially by VEVs of two FN fields. We also show several predictions of this model, θ_{13}, the Dirac CP phase, two Majorana CP phases, the effective mass of the neutrinoless double beta decay and the leptogenesis. The prediction of the branching ratio of $\mu \to e\gamma$ is also given in SUSY model.

2 The neutrino mass matrix in the hierarchy scheme

We assume that the hierarchy of mass matrices arise from the Froggatt-Nielsen (FN) non-renormalizable interaction as

$$\begin{aligned}
\mathcal{L}_{\mathcal{FN}} &= -\overline{\nu_{Ri}}(Y)_{ij}\ell_j \cdot H_u \left(\frac{\Theta}{\Lambda}\right)^{f_{\nu_{Ri}}+f_{\ell_j}} \\
&\quad -\overline{e_{Ri}}(Y_e)_{ij}\ell_j \cdot H_d \left(\frac{\Theta}{\Lambda}\right)^{f_{E_{Ri}}+f_{\ell_j}} , \\
&\quad -\frac{1}{2}\bar{\nu_{Ri}} m_{ij}(\nu_{Rj})^c \left(\frac{\Theta}{\Lambda}\right)^{f_{\nu_{Ri}}+f_{\nu_{Rj}}} ,
\end{aligned} \tag{1}$$

where $H_{u,d}$ are Higgs doublets, L_j is the left-handed lepton doublets, E_{Rj} and ν_{Rj} are right-handed charged leptons and right-handed neutrinos in the j-th generation, respectively. Λ is a cut-off scale, Y_ν and Y_e are coupling constants, and their U(1) charges are expressed by f. By assigning the U(1) charge of Θ by $f_\Theta = -1$ and $f_{H_u} = f_{H_d} = 0$. The Lagrangian is invariant under $U(1)$ transformation.

When Θ takes a vacuum expectation value

$$\langle \Theta \rangle = \lambda \Lambda , \tag{2}$$

where λ is a small quantity which is of order of Cabbibo angle, ~ 0.2. Then we obtain effective Yukawa couplings matrices and the right-handed neutrino mass matrix as

$$\begin{aligned}
(Y^{\text{eff}})_{ij} &= (Y)_{ij}\lambda^{f_{\nu_{Ri}}+f_{L_j}} . \\
(Y_e^{\text{eff}})_{ij} &= (Y_e)_{ij}\lambda^{f_{e_{Ri}}+f_{L_j}+f_{H_d}} . \\
(m^{\text{eff}})_{ij} &= k_M m_{ij}\lambda^{f_{\nu_{Ri}}+f_{\nu_{Rj}}} .
\end{aligned} \tag{3}$$

Since the power of λ provides the hierarchical structure of mass matrices, coupling constants, $(Y_\nu)_{ij}$ are approximately equal each other. This is true for $(Y_e)_{ij}$ and m_{ij}.

Let us consider neutrino mass matrices. For simplicity, we express FN charges for left-handed lepton doublets and right-handed neutrinos as $(f_{L_1}, f_{L_2}, f_{L_3}) = (f_1, f_2, f_3)$ and $(f_{\nu R1}, f_{\nu R2}, f_{\nu R3}) = (g_1, g_2, g_3)$. Then, the Dirac and Majorana neutrino mass matrices are given by

$$m_D = v_u k_\nu \begin{pmatrix} a_{11}\lambda^{g_1+f_1} & a_{12}\lambda^{g_1+f_2} & a_{13}\lambda^{g_1+f_3} \\ a_{21}\lambda^{g_2+f_1} & a_{22}\lambda^{g_2+f_2} & a_{23}\lambda^{g_2+f_3} \\ a_{31}\lambda^{g_3+f_1} & a_{32}\lambda^{g_3+f_2} & a_{33}\lambda^{g_3+f_3} \end{pmatrix} \tag{4}$$

$$M_R = m \begin{pmatrix} b_{11}\lambda^{2g_1} & b_{12}\lambda^{g_1+g_2} & b_{13}\lambda^{g_1+g_3} \\ b_{12}\lambda^{g_2+g_1} & b_{22}\lambda^{2g_2} & b_{23}\lambda^{g_2+g_3} \\ b_{13}\lambda^{g_3+g_1} & b_{23}\lambda^{g_3+g_2} & b_{33}\lambda^{2g_3} \end{pmatrix} \tag{5}$$

where $k_\nu a_{ij} = (Y)_{ij}$ and $m b_{ij} = m_{ij}$, so that a_{ij} and b_{ij} are normalized to quantities of order 1.

Now, by the seesaw mechanism, the neutrino mass is given by

$$m_\nu = m_D^T M_R^{-1} m_D = \frac{(v_u k_\nu)^2}{m} \begin{pmatrix} D\lambda^{2(f_1-f_3)} & A\lambda^{f_1+f_2-2f_3} & A'\lambda^{f_1-f_3} \\ A\lambda^{f_1+f_2-2f_3)} & B\lambda^{2(f_2-f_3)} & C\lambda^{f_2-f_3} \\ A'\lambda^{f_1-f_3} & C\lambda^{f_2-f_3} & B' \end{pmatrix} , \tag{6}$$

where A, A', B, B', C and D are functions of a_{ij} and B_{ij} and do not contain λ. From Eq.(6), we observe an interesting features: (a) The hierarchical structure of neutrino mass matrix depends only on the FN charge of the left-handed neutrinos, f_i and independent on those of the right-handed neutrinos, g_i (b) If we take $f_2 = f_3 = f_1 + 1$ which is reasonable in view of SU(5) GUT where the FN charges of d_{Ri} quarks are taken as $(2, 1, 1)$ and ν_{Li} and d_{Ri} form a same multiplet.

By taking $f_2 = f_3 = f_1 + 1$, we obtain

$$m_\nu = \frac{(v_u k_\nu)^2 \lambda^{2f_3}}{m} \begin{pmatrix} D\lambda^2 & A\lambda & A'\lambda \\ A\lambda & B & C \\ A'\lambda & C & B' \end{pmatrix} . \tag{7}$$

Needless to say that parameters A, B, B', C, D are expected to be quantities of order one.

This matrix leads an interesting neutrino mass patterns and mixings, the large 2-3 mixing, the reasonably large 1-2 mixing, and the mass squared ratio. In order to see the neutrino mixings, we have to take into account the charged lepton mass matrix, because the neutrino mixing matrix is obtained by multiplying the matrices which diagonalize the charged lepton mass matrix and the neutrino mass matrix.

How about the charged lepton mass matrix? Since e_{Li} and ν_{Li} form a doublet of the electroweak symmetry, the FN charges of e_{Li} should be the same as those of ν_{Li}. By assuming the FN charges of e_{Ri} as (k_1, k_2, k_3) $(k_1 > k_2 > k_3)$, we find

$$m_e = v_d k_e \begin{pmatrix} c_{11}\lambda^{k_1+f+1} & c_{12}\lambda^{k_1+f} & c_{13}\lambda^{k_1+f} \\ c_{21}\lambda^{k_2+f+1} & c_{22}\lambda^{k_2+f} & c_{23}\lambda^{k_2+f} \\ c_{31}\lambda^{k_3+f+1} & c_{32}\lambda^{k_3+f} & c_{33}\lambda^{k_3+f} \end{pmatrix} . \tag{8}$$

Then, we find

$$m_e^\dagger m_e = (v_d k_e)^2 \lambda^{2k_3+2f} \begin{pmatrix} D_e\lambda^2 & A_e\lambda & A'_e\lambda \\ A_e^*\lambda & B_e & C_e \\ A'^*_e\lambda & C_e^* & B'_e \end{pmatrix} . \tag{9}$$

Similarly to the neutrino mass matrix case, the same hierarchy is obtained as the neutrino mass matrix due to the fact that the FN charges for e_{Li} are the same as those for ν_{Li}.
(a) The case of real coupling parameters

We consider the case where all coupling constants, Y_{ij}, $(Y_e)_{ij}$ and m_{ij} are real. As we saw, both the neutrino mass matrix and the charged lepton mass matrix lead large 2-3 mixing. The large 2-3 mixing given by the neutrino tends to be cancelled by the large 2-3 mixing given by the charged lepton, so that the resultant atmospheric neutrino mixing, i.e., the mixing between ν_μ and ν_τ mixing is small. This may be a somewhat generic and serious problem.

(b) The case of complex coupling parameters

If we consider complex coupling parameters, there appear too many free parameters including phases, so that the model loses the predictive power. In order to decrease the freedom and make the model predictive, the 2-3 symmetry[9, 10] is frequently used, where the 2-3 symmetry requires the invariance under the exchange of the 2nd and the 3rd generations. If we apply this symmetry for the neutrinos, by assuming $g_3 = g_3$ in addition to $f_2 = f_3$, we find $A = A'$ and $B = B'$. Then,

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} D\lambda^2 & A\lambda & A\lambda \\ A\lambda & B & C \\ A\lambda & C & B \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} D\lambda^2 & \sqrt{2}A\lambda & 0 \\ \sqrt{2}A\lambda & B+C & 0 \\ 0 & 0 & B-C \end{pmatrix} . \quad (10)$$

If we take $|B - C| \gg |B + C| \sim A\lambda$, the mass spectrum is

$$\begin{aligned} m_3 &= B - C \,, \\ m_2 &\simeq B + C + \frac{2A^2}{B+C}\lambda^2 \,, \\ m_1 &\simeq -\frac{2A^2}{B+C}\lambda^2 \end{aligned} \quad (11)$$

and thus

$$\frac{m_2^2 - m_1^2}{m_3^2} \simeq \frac{(B+C)^2 + 4A^2\lambda^2}{(B-C)^2} \quad (12)$$

which is a quantity of order $\lambda^2 \sim 0.04$ and close to $\Delta m^2_{sol}/\Delta m^2_{atom} \sim 0.03$. If we take $(B + C) \sim A\lambda$, which is consistent with the assumed size for the mass squared ratio, then, mass matrix for the 1st and 2nd columns becomes

$$\sim A\lambda \begin{pmatrix} 0 & \sqrt{2} \\ \sqrt{2} & 1 \end{pmatrix} \quad (13)$$

and lead to the solar mixing of $\tan^2\theta_{12} \simeq 1/2$.

For the charged lepton mass matrix, we have two cases. One is the case where $k_2 \neq k_3$. Then, we only require the 2-3 symmetry for e_L. In this case, we find the relations $c_{i2} = c_{i3}$, which lead to $A_e = A'_e$, $B_e = B'_e = C_e = C^*_e$. Another one is to assume the 2-3 symmetry for e_R also by taking $k_2 = k_3$. In this case, we find $A_e = A'_e$, $B_e = B'_e$ and real C_e. For both cases, the matrix is block diagonalized by the rotation with the angle of $\pi/4$ as in the case of Eq.(10), which cancels the $\pi/4$ mixing which came from the diagonalization of the neutrino mass matrix. As a result, the atmospheric neutrino mixing vanishes.

As we saw, the FN type hierarchical model leads either the small atmospheric neutrino mixing, or the loss of the predictive power. In the next section, we give a way to avoid this problem in the two FN fields model, where the relative phase of their vacuum expectation values works to lead the mismatch and also to introduce the CP violation, while coupling parameters are taken to be real.

3 The model of neutrino mixing in the extended Froggatt-Nielsen mechanism

We consider a model which consists of two FN fields, Θ_1, Θ_2 and assume that Y_ν, Y_e and m are real matrices. In this scheme the CP violation is originated solely from the relative phase of vacuum expectation values of two FN fields. As it was shown in Ref. [11], we have to introduce Z_2 symmetry in order for this phase to work as the Dirac CP phase. We take the FN charge and Z_2 parity for them as

$$(f_{\Theta_1}, f_{\Theta_2}, P_{\Theta_1}, P_{\Theta_2}) = (-1, -1, +, -) , \tag{14}$$

where P_{Θ_i} gives the Z_2 parity. For leptons, we take

$$\begin{aligned}
&(f_{\ell_1}, f_{\ell_2}, f_{\ell_3}, P_{\ell_1}, P_{\ell_2}, P_{\ell_3}) = (2, 1, 1, +, +, -), \\
&(f_{e_R1}, f_{e_R2}, f_{e_R3}, P_{e_R1}, P_{e_R2}, P_{e_R3}) = (3, 2, 0, +, +, -), \\
&(f_{\nu_R1}, f_{\nu_R2}, f_{\nu_R3}, P_{\nu_R1}, P_{\nu_R2}, P_{\nu_R3}) = (2, 1, 0, +, +, -).
\end{aligned} \tag{15}$$

and for Higgs, we take $(f_{Hu}, f_{Hd}, P_{Hu}, P_{Hd}) = (0, 0, +, +)$. This choice of the FN charges are consistent to that of quarks in the SU(5) GUT scheme. The interaction Lagrangian is given by,

$$\begin{aligned}
\mathcal{L}_{\mathcal{FN}2} &= -\sum_{n_{\nu1}, n_{\nu2}} \overline{\nu_{Ri}} (Y)_{ij} \ell_j \cdot H_u \left(\frac{\Theta_1}{\Lambda}\right)^{n_{\nu1}} \left(\frac{\Theta_2}{\Lambda}\right)^{n_{\nu2}} \\
&\quad - \sum_{n_{e1}, n_{e2}} \overline{e_{Ri}} (Y_e)_{ij} \ell_j \cdot H_d \left(\frac{\Theta_1}{\Lambda}\right)^{n_{e1}} \left(\frac{\Theta_2}{\Lambda}\right)^{n_{e2}} , \\
&\quad - \sum_{n_{M1}, n_{M2}} \overline{\nu_{Ri}} (m)_{ij} (\nu_{Rj})^C \left(\frac{\Theta_1}{\Lambda}\right)^{n_{M1}} \left(\frac{\Theta_2}{\Lambda}\right)^{n_{M2}} \qquad (16) \\
&. \qquad (17)
\end{aligned}$$

where (n_{X1}, n_{X2}) are taken so as to keep the invariance of the FN U(1) symmetry and Z_2 symmetry. Effective Yukawa couplings and the heavy neutrino masses are given by

$$\mathcal{L}_{\text{eff}} = -\overline{\nu_{Ri}} (y)_{ij} \ell_j \cdot H_u - \overline{e_{Ri}} (y_e)_{ij} \ell_j \cdot H_d - \overline{\nu_{Ri}} (M_R)_{ij} (\nu_{Rj})^C \tag{18}$$

are given by taking

$$\lambda = \frac{\langle \Theta_1 \rangle}{\Lambda} , \; R = \frac{\langle \Theta_2 \rangle}{\langle \Theta_1 \rangle} \equiv |R| e^{i\alpha} , \tag{19}$$

(a) A model of mass matrices

By taking $\langle H_u \rangle = v_u$ and $\langle H_d \rangle = v_d$, mass matrices are obtained. As we stated before, in the spirit of the FN hierarchy model, elements of Y are considered to be approximately equal each other, and for Y_e and m, this should hold. In the following, we consider a simple model where Y, Y_e and m are proportional to the democratic matrix as

$$Y = k_\nu \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} , Y_e = k_e \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} , m = m_M \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} . \quad (20)$$

Then, we find

$$\begin{aligned} m_D &= v_u y = v_u k_\nu \begin{pmatrix} B_4\lambda^4 & B_2\lambda^3 & RB_2\lambda^3 \\ B_2\lambda & B_2\lambda^2 & R\lambda^2 \\ R\lambda^2 & R\lambda & \lambda \end{pmatrix} , \\ M_e &= v_d k_e \begin{pmatrix} B_4\lambda^5 & B_4\lambda^4 & RB_2\lambda^4 \\ B_4\lambda^4 & B_2\lambda^3 & RB_2\lambda^3 \\ R\lambda^2 & R\lambda & \lambda \end{pmatrix} , \\ M_R &= m_M \begin{pmatrix} B_4\lambda^4 & B_2\lambda^3 & R\lambda^2 \\ B_2\lambda^3 & B_2\lambda^2 & R\lambda \\ R\lambda^2 & R\lambda & 1 \end{pmatrix} , \end{aligned} \quad (21)$$

where $B_{2n} = 1 + R^2 + \cdots + R^{2n}$. We are interested in whether this simple model can reproduce the observed data. Firstly, we derive the neutrino mass matrix for the left-handed neutrinos,

$$m_\nu = m_D^T M^{-1} m_D = v_L \begin{pmatrix} B_4\lambda^2 & B_2\lambda & RB_2\lambda \\ B_2\lambda^3 & B_2 & R \\ RB_2\lambda & R & B_2 \end{pmatrix} , \quad (22)$$

where $v_L = (v_u k_\nu)^2 \lambda^2 / m_M$.

(b) Diagonalization

At first, we discuss the neutrino mass diagonalization. By the transformation of the unitary matrix $U^{(1)}$, where

$$U^{(1)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 & 0 & \frac{(1+R)^* B_2^*}{\sqrt{2}(B_2+R)^*}\lambda \\ 0 & 1 & 0 \\ -\frac{(1+R)B_2}{\sqrt{2}(B_2+R)}\lambda & 0 & 1 \end{pmatrix} , \quad (23)$$

m_ν is block diagonalized in a good approximation as

$$(U^{(1)})^T m_\nu U^{(1)} = v_L \begin{pmatrix} (B_4 - \frac{((1+R)B_2)^2}{2(B_2+R)})\lambda^2 & \frac{(1-R)B_2}{\sqrt{2}}\lambda & 0 \\ \frac{(1-R)B_2}{\sqrt{2}}\lambda & B_2 - R & 0 \\ 0 & 0 & B_2 + R \end{pmatrix} , \quad (24)$$

where we assumed that $|B_2 + R| \gg |B_2 - R|$. The matrix in Eq.(21) is diagonalized by the matrix $U^{(3)}$

$$U^{(2)} = \begin{pmatrix} c & -se^{i\rho} & 0 \\ se^{i\rho} & ce^{2i\rho} & 0 \\ 0 & 0 & 1 \end{pmatrix} , \quad (25)$$

where $c = \cos\theta$ and $s = \sin\theta$, and

$$\begin{aligned}
\rho &= \arg\left(\frac{(1-R)B_2}{B_2-R}\right) + \pi \ , \\
\tan 2\theta &= \sqrt{2}\left|\frac{(1-R)B_2}{B_2-R}\right| \lambda \ ,
\end{aligned} \tag{26}$$

and neutrino masses are

$$\begin{aligned}
m_1 &\simeq -v_L[s^2 - \sqrt{2}sc|\frac{(1-R)B_2}{B_2-R}|]|B_2-R|e^{i\alpha_1} = v_L\frac{s^2}{c^2-s^2}|B_2-R|e^{i\alpha_1} \ , \\
m_2 &\simeq v_L[c^2 + \sqrt{2}sc|\frac{(1-R)B_2}{B_2-R}|]|B_2-R|e^{i\alpha_2} = v_L\frac{c^2}{c^2-s^2}|B_2-R|e^{i\alpha_2} \ , \\
m_3 &\simeq v_L|B_2+R|e^{i\alpha_3} \ ,
\end{aligned} \tag{27}$$

where $\cos 2\theta = c^2 - s^2 > 0$ is taken and

$$\begin{aligned}
\alpha_1 &= 2\rho + \arg(B_2-R) + \pi \ , \\
\alpha_2 &= 4\rho + \arg(B_2-R) \ , \\
\alpha_3 &= \arg(B_2+R) \ .
\end{aligned} \tag{28}$$

Now, we define the phase matrix

$$P = \mathrm{diag}(e^{-i\frac{\alpha_1}{2}}, e^{-i\frac{\alpha_2}{2}}, e^{-i\frac{\alpha_3}{2}}) \ , \tag{29}$$

the matrix which diagonalizes the neutrino mass matrix is $U_\nu = U^{(1)}U^{(2)}P$.

Next, we go to the diagonalization of the charged lepton mass matrix. By transforming M_e as $V_e^\dagger M_e U_e$ by the unitary matrix, $U_e = V^{(1)}V^{(2)}$, where

$$\begin{aligned}
V^{(1)} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{1+|R|^2}} & \frac{R^*}{\sqrt{1+|R|^2}} \\ 0 & -\frac{R}{\sqrt{1+|R|^2}} & \frac{1}{\sqrt{1+|R|^2}} \end{pmatrix} \begin{pmatrix} 1 & 0 & \frac{R^*}{\sqrt{1+|R|^2}}\lambda \\ 0 & 1 & 0 \\ -\frac{R}{\sqrt{1+|R|^2}}\lambda & 0 & 1 \end{pmatrix} \ , \\
V^{(2)} &= \begin{pmatrix} 1 & \frac{1+|R|^2R^{*4}}{(\sqrt{1+|R|^2})^{3/2}}\lambda & 0 \\ -\frac{1+|R|^2R^4}{(\sqrt{1+|R|^2})^{3/2}}\lambda & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\end{aligned} \tag{30}$$

M_e is diagonalized in a good approximation and the eigenvalues are

$$\frac{m_\mu}{m_\tau} = \frac{|1-R^4|}{1+|R|^2}\lambda \ , \frac{m_e}{m_\mu} = \frac{|R|^4\sqrt{1+|R|^2}}{|1-R^4|^2}\lambda^2 \ . \tag{31}$$

4 The neutrino mixing matrix and mass spectrum of leptons

At first, we notice that the neutrino mixing which is given by

$$V = U_e^\dagger U_\nu = V^{(2)\dagger} V^{(1)\dagger} U^{(1)} U^{(2)} P \ , \tag{32}$$

and consider how the cancellation of the 2-3 mixing is avoided. The 2-3 mixing is essentially given in the following part

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{1+|R|^2}} & -\frac{R^*}{\sqrt{1+|R|^2}} \\ 0 & \frac{R}{\sqrt{1+|R|^2}} & \frac{1}{\sqrt{1+|R|^2}} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1+R^*}{\sqrt{2(1+|R|^2)}} & \frac{1-R^*}{\sqrt{2(1+|R|^2)}} \\ 0 & -\frac{1-R}{\sqrt{2(1+|R|^2)}} & \frac{1+R}{\sqrt{2(1+|R|^2)}} \end{pmatrix} . \tag{33}$$

Now, the atmospheric mixing is

$$\sin^2 2\theta_{\rm atm} \simeq \frac{|1-R^2|^2}{(1+|R|^2)^2} = 1 - \left(\frac{2\cos\alpha}{|R| + \frac{1}{|R|}} \right)^2 . \tag{34}$$

If there is no phase in R, i.e., $\alpha = 0$, then the 2-3 mixings for the charged lepton and the neutrino cancel each other and leads to the small mixing because we consider $|R| \simeq 1$. Even in the existence of the phase α, it is hard to achieve $\sin^2\theta_{\rm atm} = 1$. Here, we relax this condition and want to reproduce $\sin^2\theta_{\rm atm} \geq 0.9$, then we find

$$|\cos\alpha| \leq \frac{|R| + \frac{1}{|R|}}{2\sqrt{10}} . \tag{35}$$

Since we expect $|R| \simeq 1$, $|\cos\alpha|$ must be around $1/\sqrt{10}$.

Next, we impose the condition which assures the computation given above. The condition is $|B_2 + R| \gg |B_2 - R|$ which is needed to get the hierarchy of neutrino mass $|m_3| \gg |m_2|$. This requires that

$$\frac{((|R| + \frac{1}{|R|})\cos\alpha - 1)^2 + (|R| - \frac{1}{|R|})^2 \sin^2\alpha}{((|R| + \frac{1}{|R|})\cos\alpha + 1)^2 + (|R| - \frac{1}{|R|})^2 \sin^2\alpha} \ll 1 . \tag{36}$$

To fulfill this condition with $\cos\alpha = 1/\sqrt{10}$, we need $(|R| + \frac{1}{|R|})\cos\alpha \sim 1$, so that $|R| \sim 1/|R|$ is required There is another reason. Let us see the 11 element of the neutrino mixing by neglecting $O(\lambda^2)$ term,

$$V_{11} = \left[c - s e^{i\rho} \frac{1}{\sqrt{2}(1+|R|^2)} \left(\frac{1+|R|^2 R^{*4}}{(1-R^*)B_2^*} - (1-R)R^* \right) \lambda \right] e^{-i\frac{\alpha_1}{2}} . \tag{37}$$

Here, we observe that the 2nd term in the parenthesis becomes real in the limit of $|R| = 1$ and works to cancel the first term c when $\cos\alpha = \frac{1}{\sqrt{10}}$. This means that we obtain smaller $|V_{11}|$ element, which in turn leads to a larger solar mixing angle.

Before going into the detailed analysis, we comment about the number of parameters. There are three parameters, λ, $|R|$ and α aside from the overall normalization of m_D, M_R and m_ℓ. Therefore, if we fix the above three parameters, three neutrino mixings, one Dirac CP phase, two Majorana CP phases, the ratios of masses of the left-handed neutrinos and the right-handed neutrinos, and those of charged leptons. If we go to the leptogenesis and the LFV, we need to fix the overall factors, which we see later.

In the following analysis, we take the values of three parameters as

$$\lambda = \frac{1}{4}\ , \quad \cos\alpha = \frac{1}{\sqrt{10}}\ , \sin\alpha = -\frac{3}{\sqrt{10}}\ , \quad |R| = 1\ , \tag{38}$$

as we explained before. Then, we find

$$V = P' \begin{pmatrix} 0.863 & 0.585 & (-0.012 + 0193i) \\ (-0.531 - 0.054i) & (0.678 + 0.069i) & 0.585 \\ (0.259 + 0.026i) & (-0.528 - 0.054i) & 0.811 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-0.50\pi i} & 0 \\ 0 & 0 & e^{0.53\pi i} \end{pmatrix} , \tag{39}$$

where P' is a physically meaningless diagonal phase matrix. We see the Dirac phase $\delta = -0.52\pi$ in the standard phase convention[13] and two Majorana phases as $\beta = -0.50\pi$ and $\gamma = 0.53\pi$.

We obtain the ratios of masses as

$$\begin{aligned} \frac{m_1}{m_3} &= 0.027\ , & \frac{m_2}{m_3} &= 0.27 \\ \frac{M_1}{M_3} &= 0.0035\ , & \frac{M_2}{M_3} &= 0.060 \\ \frac{m_e}{m_\tau} &= 0.00062\ , & \frac{m_\mu}{m_\tau} &= 0.037\ . \end{aligned} \tag{40}$$

The absolute values of elements of the mixing matrix are

$$\begin{pmatrix} 0.863 & 0.585 & 0.194 \\ 0.533 & 0.681 & 0.585 \\ 0.260 & 0.531 & 0.811 \end{pmatrix} \tag{41}$$

which gives

$$\tan^2\theta_{\rm sol} = 0.455\ , \quad \sin^2 2\theta_{\rm atm} = 0.9\ , \tag{42}$$

which are in a good agreement with the data, and

$$\frac{\Delta m_{\rm sol}}{\Delta m_{\rm atm}} \simeq \frac{m_2^2 - m_1^2}{m_3^2} = 6.3 \times 10^{-2}\ , \tag{43}$$

which are in a reasonable agreement with the experimental data, in view of this simple model.

By taking $m_\tau = 1.75$GeV and $m_{\nu\tau} = m_{\nu 3} \simeq \sqrt{\Delta m^2_{\rm atm}} \simeq 0.05$eV, we find $m_e = 1.1$MeV and $m_\mu = 64$MeV, and $m_{\nu e} = m_{\nu 1} = \mathcal{O}(10^-3)$eV and $m_{\nu\mu} = m_{\nu 2} = 0.012$eV. In order to obtain $m_{\nu_3} = 0.05$eV, $v_L = 3.0 \times 10^{-11}$GeV is required. Though the predicted masses in our very simple model don't have excellent agreements with the experimental data, *i.e.* the predicted value of m_e is about twice of the experimental value and that of m_μ is about half, we may say that our model is still successful. Such a discrepancy can be improved by relaxing our assumption that Y, Y_e and m are proportional to the democratic matrix.

We comment that if $\sin\alpha = \frac{3}{\sqrt{10}}$ is taken, then the mixing matrix V changes to V^* in comparison with the case of $\sin\alpha = -\frac{3}{\sqrt{10}}$.

5 Predictions

Since we fixed all parameters except for m_M which determines the absolute magnitude of the right-handed neutrino masses, we can compute predictions for various observables.

(a) The neutrinoless double beta decay

As a feasible experiment which has a potential to give an information of Majorana CP violation phases at low energy scale, we consider the neutrinoless double beta decay [4], $(A, Z) \to (A, Z+2) + e^- + e^-$. If the neutrinoless double beta decay is observed, the measurement of the neutrinoless double beta decay half-life combined with information on the absolute values of neutrino masses might give a constraint on the neutrino mass parameters or determine them. Predictions on the neutrino less double beta decay can be controlled by a effective mass, $\langle m_\nu \rangle = |\sum V_{ej}^2 m_j|$[4]. In our model, the effective mass is predicted as

$$\frac{\langle m_\nu \rangle}{m_3} = 0.03 \; . \tag{44}$$

Therefore, with $m_3 = \sqrt{\Delta m_{atm}^2} \sim 0.05$eV, we find that the effective mass is as small as 0.0015eV, so that it seems hard to be observed.

(b) Baryon number asymmetry

We consider the thermal leptogenesis scenario[5] in which the baryon number asymmetry of the universe is generated by the conversion of the Lepton number asymmetry produced by CP violating decay of heavy right-handed neutrinos. Recently the effect of flavors is studied [14, 15], and this effect is shown to be significant in several cases, *e.g.*, a case that primordial $B-L$ asymmetries are considered[16] or one that total CP asymmetry parameter is strongly suppressed by the cancellation between the flavor dependent CP asymmetries as $\epsilon_1^e + \epsilon_1^\mu + \epsilon_1^\tau \sim 0$[17]. However, in most cases, the contribution from flavor effect is within 10%[15]. Here we consider the zero primordial asymmetry case and the total CP asymmetry parameter is not small. Then one flavor approximation can be used.

Since the right-handed neutrino mass eigenvalues are hierarchical, the CP asymmetry parameter and washout mass parameter in our model with $\lambda = 1/4$, $|R| = 1$ and $\cos\alpha = 1/\sqrt{10}$, $\sin\alpha = -3/\sqrt{10}$ for the standard model (SM) case and minimal supersymmetric standard model (MSSM) case are

$$\epsilon_1 \equiv \frac{\Gamma(\nu_{R1} \to l H_u) - \Gamma(\nu_{R1} \to l^c H_u^*)}{\Gamma(\nu_{R1} \to l H_u) + \Gamma(\nu_{R1} \to l^c H_u^*)}$$
$$\sim \begin{cases} -\frac{3}{16\pi (YY^\dagger)_{11}} \sum_{j\neq 1} \frac{M_1}{M_j} \mathrm{Im}\left((YY^\dagger)_{j1}^2\right) \simeq -1.3\times 10^{-6} \left(\frac{m_M}{10^{13}\mathrm{GeV}}\right) \ , & \text{SM case} \ , \\ -\frac{1}{8\pi (YY^\dagger)_{11}} \sum_{j\neq 1} \frac{M_1}{M_j} \mathrm{Im}\left((YY^\dagger)_{j1}^2\right) \simeq -8.3\times 10^{-7} \left(\frac{m_M}{10^{13}\mathrm{GeV}}\right) \ , & \text{MSSM case} \ , \end{cases} \tag{45}$$

and

$$\tilde{m}_1 = \frac{(YY^\dagger)_{11}}{M_1} v_u^2 \simeq 0.032\mathrm{eV} \ . \tag{46}$$

Using the approximate efficiency function[5]

$$\eta(m) = \left(\frac{8.25\times 10^{-3}\mathrm{eV}}{m} + \left(\frac{m}{2\times 10^{-4}\mathrm{eV}}\right)\right)^{-1} \ , \tag{47}$$

the predicted BAU in thermal leptogenesis scenario is given as

$$\frac{n_B}{s} = -\frac{10}{31 g_*}\epsilon_1 \eta(\tilde{m}_1) = \begin{cases} 1.0\times 10^{-11} \left(\frac{m_M}{10^{13}\mathrm{GeV}}\right) \ , & \text{SM case} \ , \\ 3.2\times 10^{-12} \left(\frac{m_M}{10^{13}\mathrm{GeV}}\right) \ , & \text{MSSM case} \ , \end{cases} \tag{48}$$

with $g_* = 108.5$ in SM case and $g_* = 232.5$ in MSSM case. Note that m_M is the only one parameter which are left free in our simplest model. In order to reproduce $\eta_B/s = 8.7\times 10^{-11}$, we should set $m_M = 8.4\times 10^{13}$GeV for SM case and $m_M = 2.7\times 10^{14}$GeV for MSSM case which give $(M_1, M_2, M_3) = (3.1\times 10^{11}, 5.3\times 10^{12}, 8.9\times 10^{13})$GeV in SM case and $(M_1, M_2, M_3) = (4.6\times 10^{11}, 7.9\times 10^{12}, 1.3\times 10^{14})$GeV in MSSM case.

(c) The lepton flavor violation processes

It is interesting to consider $\mu \to e\gamma$ in the MSSM case. Even if the flavor universal boundary condition is taken at high energy scale such as GUT scale where all the sfermion mass matrices are proportional to $m_0^2\mathbf{1}$ with $\mathbf{1}$ being unit matrix and all the trilinear coupling matrix is proportional to the Yukawa coupling matrix with a dimensionfull proportionality coefficient A_0, the off-diagonal elements of neutrino Yukawa coupling matrix induce the flavor mixings in slepton sector and this affects the prediction on lepton flavor violating processes, such as $\mu \to e\gamma$.

In general, the SUSY contribution to $\mu \to e\gamma$ strongly depends on the right-handed neutrino mass scale in addition to the SUSY parameters. The lower bound on the right-handed neutrino mass scale from successful leptogenesis [4] has an implication for the prediction of lepton flavor violation processes[18]. In our simplest model, all the parameters are fixed. Especially the successful leptogenesis gives $m_M \simeq 2.7\times 10^{14}$GeV.

[4]We don't consider the gravitino problem though it is very serious problem[19] in supersymmetric models. This topic is outside the scope of present work.

The normalization factor for the right-handed neutrino mass matrix $m_M = 2.7 \times 10^{14}$GeV with $v_L = 3.0 \times 10^{-11}$GeV determines the normalization of neutrino Yukawa matrix as $k_\nu = \sqrt{v_L m_M/(v_u \lambda)^2} \simeq 2.1$. With these normalization factors, one get

$$Y^\dagger L Y = \begin{pmatrix} -0.0045 & -0.0095 - 0.012i & -0.012 + 0.0040i \\ -0.0095 + 0.012i & -0.063 & -0.013 + 0.038i \\ -0.012 - 0.0040i & -0.013 - 0.038i & -2.5 \end{pmatrix} \tag{49}$$

where $L = \mathrm{diag}(\ln \frac{M_1}{M_X}, \ln \frac{M_2}{M_X}, \ln \frac{M_3}{M_X})$ with $M_X = 2.0 \times 10^{16}$GeV and we take the base where M_R and M_e are diagonalized. As easily seen from form of mass matrices, m_D, M_R, and M_e, large mixing angles in the neutrino sector come from seesaw enhancement[20], *i.e.* the off-diagonal elements of m_D are not large even in the basis where M_e and M_R are diagonalized. Therefore the off-diagonal elements of $Y^\dagger L Y$ is suppressed to be much smaller than one.

The branching ratio of $\mu \to e\gamma$ within mass insertion approximation is calculated as

$$\mathrm{Br}(\mu \to e\gamma) \simeq \frac{\alpha^3}{G_F^2} \frac{|6m_0^2 + 2A_0^2|^2}{(4\pi)^4 m_S^8} |(Y^\dagger L Y)_{12}|^2 \tan^2\beta \; . \tag{50}$$

If we take the grand unified gaugino mass, $m_{1/2}$ at M_X, m_S^8 is approximately $m_S^8 = 0.5 m_0^2 (m_0^2 + 0.6 m_{1/2}^2)^2$[21]. The $\mu \to e\gamma$ constraint on our model is displayed in Fig. 1. As shown in the figure, MEG experiment which is expected to reach $\mathrm{Br}(\mu \to e\gamma) < 10^{-13}$ can test very wide region of SUSY parameter space in our model. A 3.4σ deviation from the SM was reported in muon $g-2$[22]. If this discrepancy comes from the SUSY contribution, rather light SUSY spectrum, *i.e.* $m_0, m_{1/2} < 500$GeV, is favored, so that our simple model promises a measurable size of $\mathrm{Br}(\mu \to e\gamma)$ in the light of $g-2$.

6 Summary and Discussions

We examined whether neutrino mass matrix derived from the seesaw mechanism is compatible with the hierarchical mass matrices based on the Froggatt-Nielsen (FN) mechanism. We showed the followings:

1. The milder hierarchy of neutrino masses is obtained for the FN charge of the left-handed lepton doublet ℓ_{Li} taken as $f_1 = f_2 + 1 = f_3 + 1$. Since the FN charges for d_{Ri} are taken as (2,1,1), this charge assignment of ℓ_{Li} is compatible with SU(5) GUT models with $f_1 = 2$.

2. We showed the problem which arises from the 2-3 symmetry is evaded by introducing two FN fields with opposite Z_2 parity. The relative phase of vacuum expectation values of two FN fields acts an important role for this.

3. We constructed a model where coupling constants are real. In this framework, the relative phase becomes the sole origin of complex phases in mass matrices. In particular, we examined a simple case where coupling matrices are proportional to the

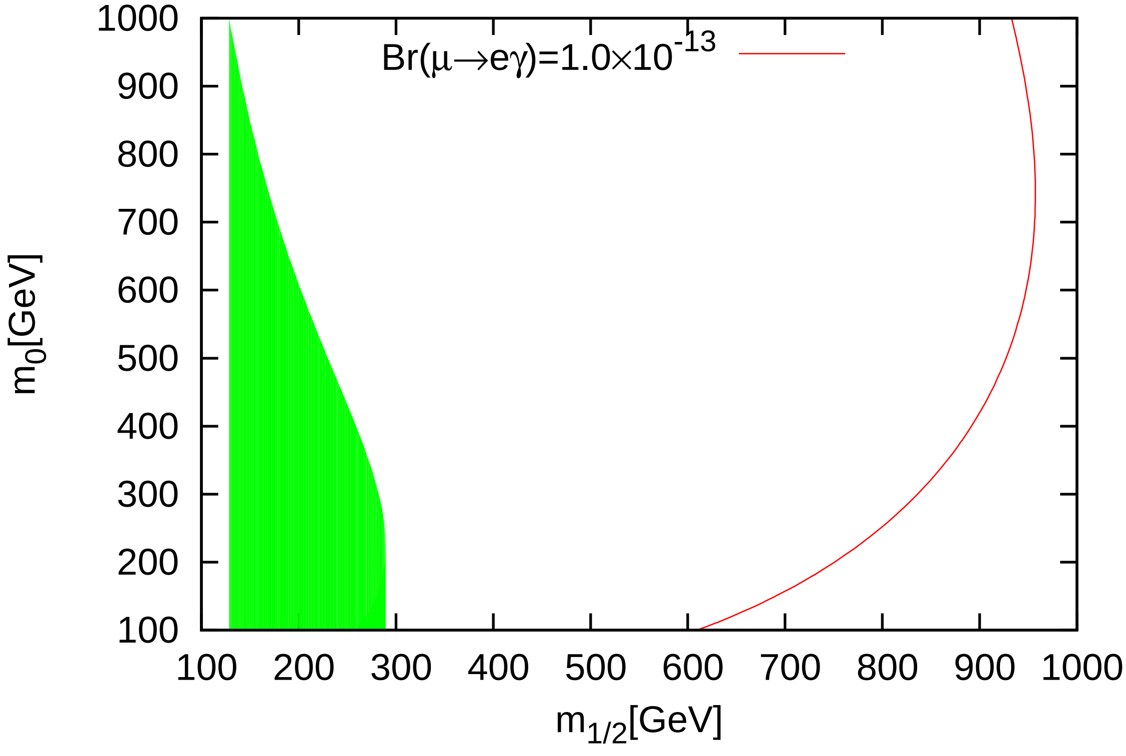

Figure 1: The contour of predicted value of $\mathrm{Br}(\mu \to e\gamma) = 1.0 \times 10^{-13}$ is shown on the $m_{1/2}$ and m_0 plane. The shaded region is already excluded, *i.e.* $\mathrm{Br}(\mu \to e\gamma) > 1.2 \times 10^{-11}$. For the other SUSY parameters, $A_0 = 0$ and $\tan\beta = 10$ are taken.

democratic matrix and obtained various predictions. The model predicts the normal hierarchy case and reproduces neutrino mixings, the ratio of neutrino squared masses well. The θ_{13}, the Dirac CP phase and the two Majorana CP phases are predicted. The predicted effective mass of the double beta decay is small and the successful leptogenesis scenario is obtained. Also, the $\mu \to e + \gamma$ is discussed in the SUSY model.

The intention of this paper is to show that the matching between the FN mechanism and the neutrino mass matrix derived from the seesaw mechanism is good, although the neutrino mass matrix must be quite different from that for the charged lepton mass matrix. Another interest is that this kind of model gives a possibility of the common origin of the CP violation for the quark system and the neutrino system, when we consider these mass matrices simultaneously.

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