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Lepton-flavour violation in the light of leptogenesis and muon $g-2$

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Abstract

A discrepancy between the experimental and theoretical results was reported for the anomalous magnetic moment of the muon. In leptogenesis scenarios which are due to a decay of right-handed neutrinos, this anomaly leads to large violations of lepton flavour within the framework of supersymmetric see-saw models. It is shown that for a hierarchical right-handed neutrino mass spectrum, we generically expect to observe $\mu \to e\gamma$ in the near future experiment.

I. INTRODUCTION

The experiments have measured the neutrino oscillations and established a mass of neutrinos. This cannot be explained in the Standard Model (SM) framework and thus is an evidence of physics beyond SM. From the cosmological side, one of the most challenging mysteries of the universe is the source of the asymmetry of the baryon number density. Since it is quite difficult to explain the asymmetry in SM, we are required to introduce new mechanism to generate it.

Recently, the anomalous magnetic moment $(g-2)$ of the muon suggested a contribution of the new physics. The E821 experiment at Brookhaven measured it at the extremely precise level[1], and the results have been compared with the SM prediction. According to the latest update of the hadronic contribution to the SM value[2], which is based on the e^+e^- collision data, a difference was reported between these two results as (see [3] and references therein for details)

$$a_\mu(\text{exp.}) - a_\mu(\text{SM}) \;=\; 302(88) \times 10^{-11}, \tag{1}$$

which means 3.4σ deviation. One may expect that this discrepancy is explained by hadronic uncertainties in the SM prediction. However, too large corrections are then required to fix (1) in the hadron sector[3]. Instead of pursuing this idea, we consider new contributions from physics beyond SM.

These features are easily achieved by the supersymmetric (SUSY) see-saw scenarios. In the SUSY models, all the SM particles are accompanied by their superpartners, and these new particles affect low-scale phenomena through radiative corrections. In fact, the discrepancy of the muon $g-2$ can be saturated due to a $\tan\beta$ enhancement[4]. On the other hand, the neutrino oscillations are realized by the Type I see-saw mechanism, introducing the right-handed sector[5]. Furthermore, against the mystery of the baryon asymmetry, the see-saw mechanism provides an elegant solution. By producing the right-handed neutrinos at an early stage of the universe, the lepton asymmetry would be generated at the decay of those neutrinos, and then leads to the baryon asymmetry of Universe (BAU) via the sphaleron effect.

As a prediction of the SUSY see-saw scenarios, the lepton flavours are violated at the weak scale. These violations are tightly correlated with the leptogenesis and muon $g-2$.

The measurements of the neutrino oscillations indicate flavour-violations in the neutrino sector. Such violations are naturally transmitted into the charged-lepton sector via the renormalization group evolutions. At the weak scale, the lepton-flavour violating processes are induced by superparticles. Those superparticles contribute as well to the prediction of the muon $g-2$. Thus, they cannot be arbitrary heavy in order to explain the muon $g-2$ anomaly (1). On the other hand, the right-handed neutrino mass scale is required to be large for the leptogenesis to work successfully. Such a large scale enhances the neutrino Yukawa coupling including the flavour-violating effects. Consequently, in the leptogenesis scenarios, it is considered that the lepton-flavour violating processes tend to be sizable in the light of the anomaly of the muon $g-2$. In this letter, we will show that for a hierarchical right-handed neutrino mass spectrum, $\mu \to e\gamma$ is generically expected to be observed in the near future experiment.

II. CORRELATIONS AMONG OBSERVABLES

We first briefly review the muon $g-2$. The SUSY contributions consist of the two diagrams; those mediated by the charginos and neutralinos. Considering the coupling strength, the former usually dominates the SUSY contributions. The result is approximately obtained as [4]

$$\delta a_\mu \simeq \frac{5\alpha_2}{48\pi}\frac{m_\mu^2}{m_{\rm SUSY}^2}\mathrm{sign}(M_2\mu_H)\tan\beta\ , \tag{2}$$

where M_2 is a wino mass, μ_H is a Higgsino mass and $\tan\beta = \langle H_u\rangle/\langle H_d\rangle$ is a ratio of two vacuum expectation values of Higgses. In this expression, we have set all soft masses to be equal for simplicity. In the letter, we choose a sign of $(M_2\mu_H)$ to be positive to explain (1), which is irrelevant for the following two observables.

The second observable is $\mathrm{Br}(\mu \to e\gamma)$. The SM contribution is known to be highly suppressed, and the SUSY effects determines the branching ratio. The SUSY diagrams are the same as those of the muon $g-2$ except for the flavour dependence. Let us consider that the soft breaking slepton masses are flavour universal at the cutoff scale, μ_X, which is assumed to be larger than the right-handed neutrino mass scale, μ_R. Even with this flavour-universal condition, in SUSY see-saw models, neutrino Yukawa couplings induce the left-handed slepton mixing through the renormalization running from μ_X down to μ_R. Thus we expect significant SUSY contributions to $\mathrm{Br}(\mu \to e\gamma)$. With the universal slepton mass boundary condition, the branching ratio is represented as[6]

$$\mathrm{Br}(\mu \to e\gamma) \;\sim\; \frac{\alpha^3}{G_F^2} \frac{\left|(Y_N^\dagger Y_N)_{12} \ln\left(\mu_R/\mu_X\right)\right|^2}{m_{\mathrm{SUSY}}^4} \tan^2\beta \,, \tag{3}$$

up to a numerical factor, where Y_N is a neutrino Yukawa coupling matrix in the basis of diagonal charge lepton Yukawa couplings and right-handed neutrino mass matrix. Here, all the soft SUSY breaking parameters including those during the renormalization running were simply set to be equal. Comparing with Eq. (2), a naive relation between δa_μ and $\mathrm{Br}(\mu \to e\gamma)$ is found as $\mathrm{Br}(\mu \to e\gamma) \propto (\delta a_\mu)^2$ for fixed Y_N[7].

It should be mentioned that we showed Eqs. (2) and (3) to clarify the dependence on the model parameters, while for the following numerical analysis, we do not rely on these simplifications. Namely, we will evaluate δa_μ and $\mathrm{Br}(\mu \to e\gamma)$ completely up to the one-loop level after solving a set of renormalization group equations and diagonalizing the mass matrix in the next section.

The production of the right-handed neutrino leads to the generation of the lepton asymmetry. In leptogenesis scenarios such as the thermal and non-thermal ones, the lepton asymmetry is proportional to CP asymmetry in a decay of a lightest right-handed neutrino, N_1, into lepton doublet, L, and Higgs, H_u. The CP asymmetry parameter ϵ_1 is (see [8] for a review)

$$\begin{aligned}\epsilon_1 \;&\equiv\; \frac{\Gamma(N_1 \to L + H_u) - \Gamma(N_1 \to L^c + H_u^c)}{\Gamma(N_1 \to L + H_u) + \Gamma(N_1 \to L^c + H_u^c)} \\ &=\; \frac{1}{8\pi (Y_N Y_N^\dagger)_{11}} \sum_{i\neq 1} \mathrm{Im}\left[(Y_N Y_N^\dagger)_{i1}^2 f(M_i^2/M_1^2)\right] \,,\end{aligned} \tag{4}$$

where M_i's are masses of the right-handed neutrinos and a loop function $f(x)$ is given as

$$f(x) = \sqrt{x} \ln\left(1 + \frac{1}{x}\right) - \frac{2\sqrt{x}}{x-1} \,. \tag{5}$$

Here and in the following analysis, we ignore flavour-dependent effects on the leptogenesis[20] because they are generically negligible. The above expression is valid for a hierarchical right-handed neutrino mass spectrum, *i.e.*, $|M_{2,3} - M_1| \gg \Gamma_{2,3} + \Gamma_1$, where Γ_i is the decay width of the ith right-handed neutrino and estimated at the tree level as

$$\Gamma_i = \frac{(Y_N Y_N^\dagger)_{ii}}{8\pi} M_i \,. \tag{6}$$

When M_2 becomes very close to M_1, the resonant effects contribute to the lepton asymmetry. This topic will be discussed in the final section. In the following analysis, we will focus on the hierarchical (off-resonant) case, $M_1 \ll M_2 \lesssim M_3$.

The neutrino Yukawa coupling matrix and the right-handed neutrino mass matrix are connected to the light neutrino mass matrix, m_ν through the see-saw relation,

$$(Y_N)_{ki}\frac{1}{M_k}(Y_N)_{kj}\langle H_u\rangle^2 = (m_\nu)_{ij} = U^*_{ik} m_k U^*_{jk} \ , \tag{7}$$

where m_i is mass eigenvalues of the light neutrinos and U is Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix[11]. From data of neutrino oscillation experiments, parameters in U and the squared mass differences are determined as[12]

$$\begin{aligned}
|\Delta m^2_{\rm atm}| \ &\equiv \ |m_3^2 - m_1^2| \sim 2.5\times 10^{-3}{\rm eV}^2 \ , \quad \sin^2 2\theta_{\rm atm} = 4|U^2_{23}|(1-|U^2_{23}|) \sim 1.0 \ , \\
\Delta m^2_\odot \ &\equiv \ m_2^2 - m_1^2 \sim 8.0\times 10^{-5}{\rm eV}^2 \ , \quad \tan^2\theta_\odot \simeq \frac{|U^2_{12}|}{|U^2_{11}|} \sim 0.4 \ , \\
|U_{13}| \ &\lesssim \ 0.2 \ .
\end{aligned} \tag{8}$$

In order to incorporate the relation Eq. (7) in the analysis, it is useful to use the following parametrization[13],

$$(Y_N)_{ij} = \frac{1}{\langle H_u\rangle}\sqrt{M_i} R_{ik} \sqrt{m_k} U^*_{jk} \ , \tag{9}$$

with R_{ij} satisfying $\sum_k R_{ik}R_{jk} = \delta_{ij}$. R_{ij} has six real parameters.

In a lot of leptogenesis scenarios, the lepton asymmetry is favored to be as large as possible. With the above parametrization, it is easily shown that a size of the CP asymmetry parameter ϵ_1 has an upper bound which is much less than one. With $M_1/M_{2,3} \ll 1$ and $\max(|R_{ij}|) < \mathcal{O}(M_{2,3}/M_1)$ (see [17]), one gets[14]

$$\begin{aligned}
|\epsilon_1| \ &\simeq \ \frac{1}{8\pi(Y_N Y_N^\dagger)_{11}}\left|\sum_{i\neq 1}{\rm Im}\left((Y_N Y_N^\dagger)^2_{i1}\frac{M_1}{M_i}\right)\right| = \frac{M_1}{8\pi\langle H_u\rangle^2}\left|\frac{\sum_i {\rm Im}(m_i^2 R^2_{1i})}{\sum_i m_i |R^2_{1i}|}\right| \\
&\leq \ \frac{M_1}{8\pi\langle H_u\rangle^2}\frac{\Delta m^2_{\rm atm}}{m_1+m_3} \ .
\end{aligned} \tag{10}$$

One can find that $|\epsilon_1|$ is maximized when $R_{12}=0$ and $|{\rm Re}(R_{13})| = |{\rm Im}(R_{13})|$ are satisfied. For a given maximal $\epsilon_1 \equiv \epsilon_1^{\rm max}$, the right-handed neutrino mass is estimated as

$$M_1 \simeq 1.5\times 10^{10}{\rm GeV}\left(\frac{|\epsilon_1^{\rm max}|}{10^{-6}}\right)\left(\frac{\langle H_u\rangle}{174{\rm GeV}}\right)^2\left(\frac{\Delta m^2_{\rm atm}}{2.5\times 10^{-3}{\rm eV}^2}\right)^{-1}\left(\frac{m_1+m_3}{0.05{\rm eV}}\right) \ . \tag{11}$$

We expect that a large asymmetry of the lepton number density leads to a sizable branching ratio of $\mu \to e\gamma$[15, 16]. From Eq. (10), $|\epsilon_1|$ is proportional to M_1, and thus the CP asymmetry, i.e, the lepton asymmetry, is enhanced for heavier M_1. Then, by satisfying the relation $M_1 \ll M_2 \lesssim M_3$, heavier M_1 leads to larger elements of Y_N. As was shown in Eq. (3), large Y_N enhances $\mathrm{Br}(\mu \to e\gamma)$. Thus, for leptogenesis scenarios to successfully work, $\mathrm{Br}(\mu \to e\gamma)$ tends to be large.

One may consider that the SUSY contributions to the $\mu \to e\gamma$ amplitude are suppressed by heavy superparticles. However, the anomaly of the muon $g-2$, Eq. (1), prohibits the particles to be decoupled as long as it is explained by the SUSY contributions. Especially since the branching ratio of $\mu \to e\gamma$, Eq. (3), is tightly correlated with the SUSY contribution to the muon $g-2$, Eq. (2), it is very hard to suppress $\mathrm{Br}(\mu \to e\gamma)$ with keeping both δa_μ and ϵ_1 large.

We should mention specific cases in which $\mathrm{Br}(\mu \to e\gamma)$ is suppressed. First of all, the flavour structure of the Yukawa coupling potentially causes a cancellation in the decay amplitude. Actually, the amplitude is proportional to $(Y_N^\dagger Y_N)_{12} = (Y_N)^*_{11}(Y_N)_{12} + (Y_N)^*_{21}(Y_N)_{22} + (Y_N)^*_{31}(Y_N)_{32}$. Thus, an accidental cancellation may happen among these complex numbers with δa_μ and ϵ_1 fixed. We might also obtain cancellations between the chargino and neutralino contributions, or by taking into account the initial flavour-changing components of the soft SUSY breaking parameters. These cases will be commented in the final section. In any case, the cancellations are considered to be accidental in general and thus regarded as a fine-tuning. As the second case, let us consider texture structures of the right-handed Yukawa coupling. Assigning texture zeros properly for the Yukawa coupling, $\mathrm{Br}(\mu \to e\gamma)$ can be suppressed[16]. Then, instead of $\mu \to e\gamma$, it is likely to observe other lepton flavour violating processes, e.g., $\tau \to \mu(e)\gamma$. Note that, since the light-neutrino mass spectrum is predicted to be specific in this framework, it is expected to identify such a case in future. In the following study, we will discuss a lower bound of $\mathrm{Br}(\mu \to e\gamma)$ in generic conditions. Namely, we will assume no fine-tunings in the neutrino sector, and not include the specific texture setup.

III. ANALYSIS

Lower bounds of the predicted $\mathrm{Br}(\mu \to e\gamma)$ are displayed in Fig. 1 as a function of ϵ_1 for fixed δa_μ. Here, we scanned the SUSY parameters [a] and the neutrino Yukawa couplings satisfying the experimental data (9). In the analysis, we set μ_X to be the grand-unification (GUT) scale and imposed the following assumptions; neglecting resonant effects of the leptogenesis and generic structure for the lepton sector. In particular, as for the latter assumption, we assumed no fine-tunings in the right-handed neutrino Yukawa coupling and sought the parameter points which minimize $|(Y_N)^*_{11}(Y_N)_{12}| + |(Y_N)^*_{21}(Y_N)_{22}| + |(Y_N)^*_{31}(Y_N)_{32}|$ for fixed ϵ_1. This means that $|(Y_N^\dagger Y_N)_{12}|$ is minimized without an accidental cancellation among three different terms in the summation on the lines.

It can be said that the lower bounds in Fig. 1 are conservative under the above assumptions. We can check that on the lines, $m_1 = 0$ and $U_{13} = 0$ are satisfied, and $|\epsilon_1|$ is maximized by satisfying $R_{12} = 0$ and $\arg(R_{13}) = \pi/4$. When we increase the lightest neutrino mass, according to Eq. (10) the maximal value of ϵ_1 is suppressed by $\Delta m^2_{\mathrm{atm}}/(m_1 + m_3)$. Namely, with ϵ_1 fixed, M_1 increases for larger m_1, and thus, the lower bound of $\mathrm{Br}(\mu \to e\gamma)$ goes up. In addition, $\mathrm{Br}(\mu \to e\gamma)$ is minimized when both $m_1 = 0$ and $U_{13} = 0$ are satisfied. This is because $(Y_N)_{11}$ becomes naturally small. From (9), if either m_1 or U_{13} is finite, the branching ratio of $\mu \to e\gamma$ receives an additional contribution from $(Y_N)^*_{11}(Y_N)_{12}$. Also, the CP violation phases in the PMNS matrix are taken to be zero, otherwise $\mathrm{Br}(\mu \to e\gamma)$ becomes larger. Although we assumed that the slepton mass matrix is universal at the cutoff scale, introducing the off-diagonal components is just additive to the branching ratio. In conclusion, the lower bounds in Fig. 1 are conservative.

It is stressed that the lower bound of $\mathrm{Br}(\mu \to e\gamma)$ depends on M_2/M_1 but is independent of M_3. It is because on the lines of the lower bound, R_{ij} still has enough degrees of freedom to give the same minimal value of $\mathrm{Br}(\mu \to e\gamma)$ for different values of M_3/M_2. In other words, the minimum of $\mathrm{Br}(\mu \to e\gamma)$ has a flat direction in the parameter space. It is obtained that as long as M_2/M_1 is larger than 10, the lower bound is propotional to M_2/M_1.While for smaller M_2/M_1, the lower bound of $\mathrm{Br}(\mu \to e\gamma)$ is not a linear function of M_2/M_1, because contributions from the next leading order in loop function $f(M_i^2/M_1^2)$ which is a term of

[a] In the analysis, we set the soft mass of the selectron to be degenerate with that of the smuon. When the selecton is much heavier than the smuon, $\mathrm{Br}(\mu \to e\gamma)$ can be suppressed for fixed δa_μ.

order M_1^3/M_2^3 are non-negligeble. On the other hand, the lower bound is almost independent of $\tan\beta$. This is simply because $\mathrm{Br}(\mu \to e\gamma)$ is proportional to $(\delta a_\mu)^2$. Thus, the branching ratio remains the same for fixed δa_μ.

From Fig. 1, in order to obtain $\delta a_\mu > 2.1 \times 10^{-9}$ (1σ), the leptogenesis scenarios with $\epsilon_1 > 10^{-5}$ provide the generic lower bound of $\mathrm{Br}(\mu \to e\gamma) > 10^{-13}$ for the hierarchy $M_2/M_1 > 10$ with $M_1 \gtrsim 1.5 \times 10^{11}$GeV. This is the sensitivity at which we expect to observe $\mu \to e\gamma$ in near future such as in the MEG experiment[18]. For smaller $\epsilon_1 = 10^{-6}$ with $M_2/M_1 = 10$, there is the parameter region where we will not detect $\mu \to e\gamma$, while for larger M_2/M_1 such as $= 100$, the lower bound exceeds the experimental sensitivity with $\delta a_\mu > 2.1 \times 10^{-9}$. On the other hand, for a smaller hierarchy case, $M_2/M_1 = 3$, the lower bound of $\mathrm{Br}(\mu \to e\gamma)$ reaches $O(10^{-13})$ when ϵ_1 is larger than 10^{-4}, which corresponds to $M_1 \gtrsim 10^{12}$GeV. As a result, since a lot of leptogenesis models practically require large CP asymmetry, we expect to observe the lepton-flavour violating muon decay in near future in the light of the muon $g-2$ anomaly when M_2/M_1 is hierarchical.

IV. CONCLUSIONS AND DISCUSSION

The reported discrepancy of the muon $g-2$ has impacts on low-energy phenomena. In this letter, we focused on the lepton flavour violation. We showed that in the light of this anomaly, successful leptogenesis scenarios prefere a sizable branching ratio of the lepton-flavour violating muon decay. Since the muon $g-2$ anomaly favors superparticles to stay in rather low-energy regime, e.g., $\lesssim 1$TeV, the SUSY contributions to the muon $g-2$ can be checked directly by probing the those particles in the forthcoming Large Hadron Collider. When the SUSY contributions will be confirmed in the experiment, neglecting resonant leptogenesis and setting $\mu_X > \mu_R$, we found that leptogenesis scenarios generically predict $\mathrm{Br}(\mu \to e\gamma)$ to become larger than $O(10^{-13})$ in wide parameter regions. Thus, we expect to detect $\mu \to e\gamma$ in the near future experiment.

In the analysis, we have focused on the CP asymmetry parameter, ϵ_1, to discuss leptogenesis. The lepton asymmetry depends on the thermal history of the universe as well, particularly on the production channels of the right-handed neutrinos. As a natural channel, they are produced in the thermal bath effectively when the reheating temperature exceeds the right-handed neutrino scale. In this thermal leptogenesis scenarios, the lepton asymme-

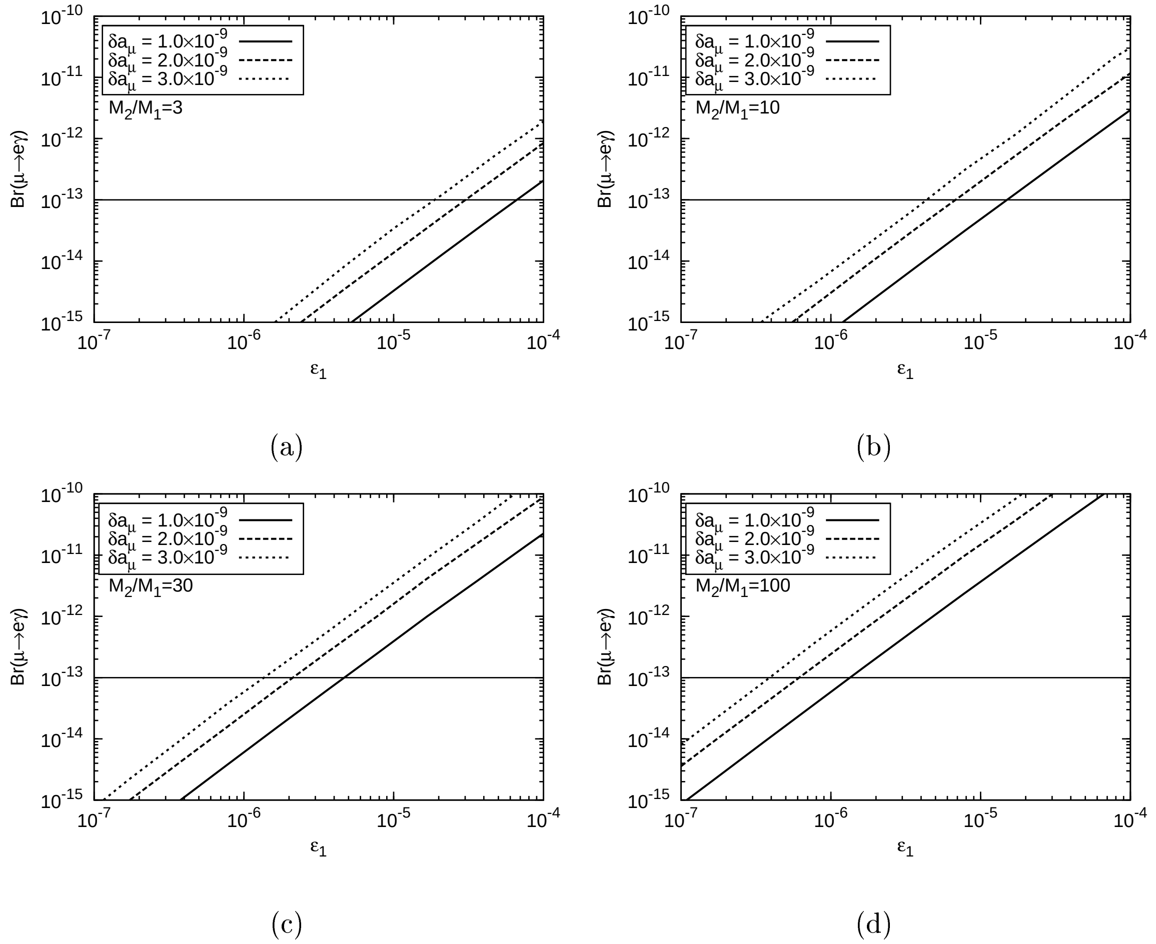

FIG. 1: Generic lower bound on the prediction of $\mathrm{Br}(\mu \to e\gamma)$ for fixed δa_μ and ϵ_1. Here, μ_X is set to be the GUT scale. In the analysis, we assumed no fine-tunings and did not take specific texture structures so that we avoid accidental suppression of $\mathrm{Br}(\mu \to e\gamma)$. We also neglect the resonant effects for the leptogenesis, which become significant when M_2 is very close to M_1. In the graphs, the ratio M_2/M_1 is taken as (a) $M_2/M_1 = 3$, (b) $= 10$, (c) $= 30$, and $= 100$ with M_1 varied. The horizontal line at $\mathrm{Br}(\mu \to e\gamma) = 10^{-13}$ represents the sensitivity of the near future experiment.

try is roughly estimated as $Y_L \sim 10^{-2}\kappa\epsilon_1$, where κ is the efficient factor which is determined by the initial abundance of the right-handed neutrino and the wash-out effects (see [8]). In the absence of the initial right-handed neutrinos, κ can be as large as $O(10^{-1})$, and thus $\epsilon_1 \gtrsim 10^{-6}$ is needed to explain the present measurements of BAU. This maximal κ is achieved when a washout mass parameter, $\tilde{m}_1 = (Y_N)_{1i}(M_i)^{-1}(Y_N^*)_{1i}\langle H_u\rangle^2$, is tuned to be about 0.001eV. We checked that this condition can be realized on the lines of the lower bound of $\mathrm{Br}(\mu \to e\gamma)$. On the other hand, since the efficiency factor decreases very quickly when $\tilde{m}_1$ differs from the maximal efficiency value, the efficiency factor easily takes smaller value. In practice, we need $\epsilon_1 \gtrsim 2 \times 10^{-6}$[8], and then the lower bound of $\mathrm{Br}(\mu \to e\gamma)$ is obtained to be larger than $O(10^{-13})$ for $M_2/M_1 > 30$ with $\delta a_\mu > 2.1 \times 10^{-9}$ [b]. In this parameter region, we expect to measure the flavour-changing decay of the muon in near future. In another scenario, the right-handed neutrino may be produced non-thermally. In this case, the resultant lepton asymmetry strongly depends on physics in high-energy scale. Even in this case, the lepton asymmetry is proportional to the CP asymmetry parameter, ϵ_1, and we obtain the lower bound of $\mathrm{Br}(\mu \to e\gamma)$.

If the lapton-flavour violations will not be observed in the future experiments, the absence of signals does not always exclude the leptogenesis scenarios as a source of BAU. In this letter, we imposed several conditions to obtain the lower bound. First of all, we assumed no accidental cancellations in the decay amplitude of the lepton-flavour violation. However, it is possible to suppress the process in some specific cases. Actually, $\mathrm{Br}(\mu \to e\gamma)$ can be lower than the bounds in Fig. 1 when the right-handed neutrinos have a special flavour structure which suppresses $|(Y_N^\dagger Y_N)_{12}|$. Another cancellation may happen between the chargino and neutralino contributions, at some spots in the parameter space. Also, taking into account initial flavour-changing components of the soft SUSY breaking parameters at the mediation scale, cancellations may also happen because their contribution can destructively interfere with those from the right-handed neutrino Yukawa coupling. The second possibility is obtained by concerning the assumption of $\mu_X > \mu_R$. The charged-lepton sector receives the flavour-changing corrections through the renormalization evolutions only when the soft SUSY breaking effects are mediated before the right-handed neutrinos decouple. We have assumed that the mediation takes place at the GUT scale, while the gauge-mediated SUSY breaking scenarios generally have lower μ_X. The scale dependence is just logarithmic and very weak when the messenger scale is far from the right-handed neutrino mass scale. However, if the messenger scale approaches very close to the right-handed neutrino scale, the lepton-flavour violating processes become suppressed[19]. In those cases, the lepton-flavour violations will not be measured in near future even in leptogenesis scenarios with the muon $g-2$ anomaly.

[b] Since lager ϵ_1 corresponds to heavier M_1, a higher reheating temperature is required, and thus the cosmological problem of the thermal gravitino production tends to be severer.

The lepton asymmetry may be enhanced compared with the evaluation in this letter. Actually, the right-handed neutrinos can have a (quite) degenerate mass spectrum. In the case of the hierarchical spectrum, the lepton asymmetry is obtained only from the decay of the lightest right-handed neutrino, while when the heavier right-handed neutrino mass becomes close to the lightest one, *i.e.*, $M_2 \to M_1$, CP asymmetric decay of N_2 simultaneously contributes to the lepton asymmetry[9, 10]. Actually, Eq. (4) diverges for $M_2 \to M_1$, and taking into account ϵ_2 in addition to ϵ_1, the CP asymmetry is enhanced and becomes maximized when $|M_2 - M_1| \sim \Gamma_{2,3} + \Gamma_1$[9]. As an another possibility, the flavour effect might affect the leptogenesis. This effect can modify the estimation of the CP asymmetry of the right-handed neutrino decay. For instance, even if the total CP asymmetry, ϵ_1, is canceled, sufficient CP asymmetry is potentially produced by flavour effects[21]. Anyway, all of these possibilities restrict the right-handed neutrino structure. Thus, we may observe distinct signatures, e.g., $\tau \to \mu\gamma$ and $\tau \to e\gamma$, in future experiments in the light of the muon $g-2$ anomaly. We will discuss these contents in future works.

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