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# Nucleon distribution amplitudes from lattice QCD

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We calculate low moments of the leading-twist and next-to-leading twist nucleon distribution amplitudes on the lattice using two flavors of clover fermions. The results are presented in the $\overline{\text{MS}}$ scheme at a scale of $2\,\text{GeV}$ and can be immediately applied in phenomenological studies. We find that the deviation of the leading-twist nucleon distribution amplitude from its asymptotic form is less pronounced than sometimes claimed in the literature.



*Introduction.* — Distribution amplitudes [1, 2, 3, 4, 5, 6] describe the structure of hadrons in terms of valence quark Fock states at small transverse separation and are required in the calculation of (semi)exclusive processes. A simple picture is obtained at very large values of the momentum transfer. For example, the magnetic Sachs form factor of the nucleon $G_M(Q^2)$ can then be expressed as a convolution of the hard scattering kernel $h(x_i, y_i, Q^2)$ and the leading-twist quark distribution amplitude in the nucleon $\varphi(x_i, Q^2)$ [3]:

$$G_M(Q^2) = f_N^2 \int_0^1 [\mathrm{d}x] \int_0^1 [\mathrm{d}y]\, \varphi^\star(y_i, Q^2) h(x_i, y_i, Q^2) \varphi(x_i, Q^2)$$

where $[\mathrm{d}x] = \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3 \delta(1 - \sum_{i=1}^3 x_i)$, and $-Q^2$ is the squared momentum transfer in the hard scattering process. However, in the kinematic region $1\,\text{GeV}^2 < Q^2 < 10\,\text{GeV}^2$, which has attracted a lot of interest recently due to the JLAB data [7, 8] for $G_M$, the situation is more complicated. Here calculations are possible, e.g., within the light-cone sum rule approach [9, 10]. They indicate that higher-twist distribution amplitudes become important while higher Fock states do not play a significant role. In any case, the distribution amplitudes are needed as input.

Being typical nonperturbative quantities, distribution amplitudes are difficult to compute reliably in a model-independent way. Determinations by QCD sum rules have been attempted, but suffer from considerable systematic uncertainties, especially for lower values of $Q^2$. As advocated in the pioneering work [11], lattice QCD can provide valuable additional information.

In this paper we present an improved and extended lattice analysis of the nucleon distribution amplitudes. We find that the asymmetry of the leading-twist amplitude is smaller than in QCD sum rule calculations, in agreement with phenomenological estimates [12, 13], which suggest a less asymmetric form.

*General Framework.* — In the case of the proton, the starting point is the matrix element of a trilocal quark operator,

$$\begin{aligned}
&\langle 0| \left[\exp\left(ig\int_{z_1}^{z_3} A_\mu(\sigma)\mathrm{d}\sigma^\mu\right) u_\alpha(z_1)\right]^a \left[\exp\left(ig\int_{z_2}^{z_3} A_\nu(\tau)\mathrm{d}\tau^\nu\right) u_\beta(z_2)\right]^b d_\gamma^c(z_3)\, |p\rangle \epsilon^{abc} \\
&= \frac{1}{4} f_N \left\{ (p\cdot\gamma C)_{\alpha\beta} (\gamma_5 N)_\gamma V(z_i\cdot p) + (p\cdot\gamma\gamma_5 C)_{\alpha\beta} N_\gamma A(z_i\cdot p) + (i\sigma_{\mu\nu} p^\nu C)_{\alpha\beta} (\gamma^\mu \gamma_5 N)_\gamma T(z_i\cdot p) \right\} + \text{higher twist},
\end{aligned} \tag{1}$$

where path ordering is implied for the exponentials, $a, b, c$ are the color indices, $N$ the proton spinor and $|p\rangle$ denotes a proton state with momentum $p$. We will consider this matrix element for space time separation of the quarks on the light cone $z_i = a_i z$ ($z^2 = 0$) and $\sum_i a_i = 1$.

In momentum space we have

$$V(x_i) \equiv \int V(z_i\cdot p) \prod_{i=1}^3 \exp\left(ix_i(z_i\cdot p)\right) \frac{\mathrm{d}(z_i\cdot p)}{2\pi} \tag{2}$$

with $V(x_i) \equiv V(x_1, x_2, x_3)$ and similarly for $A(x_i)$ and $T(x_i)$. The distribution amplitudes $V(x_i)$, $A(x_i)$ and $T(x_i)$

describe the quark distribution inside the proton as functions of the longitudinal momentum fractions $x_i$. The dependence on the renormalization scale is suppressed for notational simplicity.

We consider the moments of distribution amplitudes, which are defined as

$$V^{lmn} = \int_0^1 [\mathrm{d}x]\; x_1^l x_2^m x_3^n\; V(x_1, x_2, x_3), \tag{3}$$

with analogous definitions for the other distribution amplitudes. Using eqs. (1) and (2) one can relate the moments of the leading-twist nucleon distribution amplitudes to matrix elements of the local operators

$$\begin{aligned}&\mathcal{V}_\tau^{\rho\bar{l}\bar{m}\bar{n}}(0) \equiv \mathcal{V}_\tau^{\rho(\lambda_1\cdots\lambda_l)(\mu_1\cdots\mu_m)(\nu_1\cdots\nu_n)}(0)\\ &= [i^l\mathcal{D}^{\lambda_1}\ldots\mathcal{D}^{\lambda_l}u(0)]^a_\alpha(C\gamma^\rho)_{\alpha\beta}[i^m\mathcal{D}^{\mu_1}\ldots\mathcal{D}^{\mu_m}u(0)]^b_\beta\\ &\times[i^n\mathcal{D}^{\nu_1}\ldots\mathcal{D}^{\nu_n}\gamma_5 d(0)]^c_\tau\;\epsilon^{abc}\end{aligned}$$

by $\langle 0|\mathcal{V}_\tau^{\rho\bar{l}\bar{m}\bar{n}}(0)|p\rangle = -f_N p^{\rho\bar{l}\bar{m}\bar{n}} N_\tau V^{lmn}$ with similar relations [11] for the operators $\mathcal{A}_\tau^{\rho\bar{l}\bar{m}\bar{n}}$ and $\mathcal{T}_\tau^{\rho\bar{l}\bar{m}\bar{n}}$ corresponding to the moments $A^{lmn}$ and $T^{lmn}$, respectively. Here the multi-indices $\bar{l}\bar{m}\bar{n}$ denote the Lorentz structure connected with the covariant derivatives on the r.h.s.

Due to the presence of two *up*-quarks in the proton and isospin symmetry, the three different amplitudes can be expressed in terms of the single amplitude $\phi(x_i)$ with the corresponding moments

$$\phi^{lmn} = \frac{1}{3}(V^{lmn} - A^{lmn} + 2T^{lnm}). \tag{4}$$

The normalization constant $f_N$ is defined by the choice $\phi^{000} = 1$. The moments of the combination $\varphi(x_i) = V(x_i) - A(x_i)$, usually used in sum rule calculations, can easily be obtained as $\varphi^{lmn} = 2\phi^{lmn} - \phi^{nml}$. In the numerical calculation, however, we prefer the combination $\phi^{lmn}$ as the corresponding statistical errors are smaller by a factor of about 3. Note that momentum conservation implies

$$\phi^{lmn} = \phi^{(l+1)mn} + \phi^{l(m+1)n} + \phi^{lm(n+1)}. \tag{5}$$

In particular we have

$$\begin{aligned}1 &= \phi^{100} + \phi^{010} + \phi^{001}\\ &= \phi^{200} + \phi^{020} + \phi^{002} + 2(\phi^{011} + \phi^{101} + \phi^{110}).\end{aligned} \tag{6}$$

In the limit of $Q^2 \to \infty$ one gets $\phi(x_i) = 120x_1x_2x_3$ [4] and the moments $\phi^{lmn}$ are known exactly: $\phi^{100} = \phi^{010} = \phi^{001} = 1/3$, $\phi^{200} = \phi^{020} = \phi^{002} = 1/7$ and $\phi^{011} = \phi^{101} = \phi^{110} = 2/21$. Thus asymmetries of the type $\phi^{100} - \phi^{010}$ are important quantities at low energies as they describe the deviation from the asymptotic case.

In the case of the next-to-leading twist distribution amplitudes we restrict ourselves to operators without derivatives, i.e., to the lowest moments. Thus the problem is simplified greatly since the Lorentz decomposition of the relevant matrix element involves only two additional constants $\lambda_1$ and $\lambda_2$ [14]. They describe the coupling to the proton of two independent proton interpolating fields used in QCD sum rules. One of the operators, $\mathcal{L}_\tau$, was suggested in [15] and the other, $\mathcal{M}_\tau$, in [16]:

| | $\beta = 5.29$ | $\beta = 5.40$ |
|---|---|---|
| $f_N \cdot 10^3[\mathrm{GeV}^2]$ | 2.984(60)(157)(65) | 3.144(61)(29)(54) |
| $-\lambda_1 \cdot 10^3[\mathrm{GeV}^2]$ | 39.69(76)(259)(124) | 38.72(70)(43)(106) |
| $\lambda_2 \cdot 10^3[\mathrm{GeV}^2]$ | 78.70(155)(562)(245) | 76.23(139)(84)(207) |
| $\phi^{100}$ | 0.3549(11)(61)(2) | 0.3638(11)(68)(3) |
| $\phi^{010}$ | 0.3100(10)(73)(1) | 0.3023(10)(42)(5) |
| $\phi^{001}$ | 0.3351(9)(11)(2) | 0.3339(9)(26)(2) |
| $\phi^{100} - \phi^{001}$ | 0.0199(23)(46)(4) | 0.0300(23)(93)(1) |
| $\phi^{001} - \phi^{010}$ | 0.0251(16)(84)(3) | 0.0313(17)(12)(7) |
| $\phi^{011}$ | 0.0863(23)(97)(74) | 0.0724(18)(82)(70) |
| $\phi^{101}$ | 0.1135(23)(3)(33) | 0.1136(17)(32)(21) |
| $\phi^{110}$ | 0.0953(21)(58)(31) | 0.0937(16)(3)(38) |
| $\phi^{200}$ | 0.1508(38)(213)(64) | 0.1629(28)(7)(68) |
| $\phi^{020}$ | 0.1207(32)(43)(56) | 0.1289(27)(37)(51) |
| $\phi^{002}$ | 0.1385(36)(47)(64) | 0.1488(32)(77)(73) |
| $\phi^{110} - \phi^{011}$ | 0.0075(33)(69)(44) | 0.0211(27)(78)(32) |
| $\phi^{101} - \phi^{110}$ | 0.0172(29)(82)(57) | 0.0204(21)(134)(50) |
| $\phi^{200} - \phi^{020}$ | 0.0335(43)(26)(78) | 0.0321(33)(69)(55) |
| $\phi^{002} - \phi^{020}$ | 0.0170(36)(8)(56) | 0.0193(24)(32)(42) |

TABLE I: Moments and asymmetries in the $\overline{\mathrm{MS}}$ scheme at 2 GeV for $\phi^{lmn} = (V^{lmn} - A^{lmn} + 2T^{lnm})/3$. The first error is statistical, the second (third) error represents the uncertainty due to the chiral extrapolation (renormalization). The systematic errors should be considered with due caution, see the text for their determination.

$$\begin{aligned}\mathcal{L}_\tau(0) &= \epsilon^{abc}\left[u^{a\,T}(0)C\gamma^\rho u^b(0)\right]\times(\gamma_5\gamma_\rho d^c(0))_\tau,\\ \mathcal{M}_\tau(0) &= \epsilon^{abc}\left[u^{a\,T}(0)C\sigma^{\mu\nu}u^b(0)\right]\times(\gamma_5\sigma_{\mu\nu}d^c(0))_\tau.\end{aligned}$$

Their matrix elements are given by

$$\langle 0|\mathcal{L}_\tau(0)|p\rangle = \lambda_1 m_N N_\tau, \tag{7}$$
$$\langle 0|\mathcal{M}_\tau(0)|p\rangle = \lambda_2 m_N N_\tau. \tag{8}$$

Due to Fierz identities we have

$$(2\lambda_1 + \lambda_2)N = \frac{8}{m_N}\langle 0|\epsilon^{abc}\left(u^{a\,T}Cd^b\right)\gamma_5 u^c|p\rangle, \tag{9}$$

which vanishes in the nonrelativistic limit.

*Computation.* — The required matrix elements between the vacuum and the proton state are extracted from two-point correlation functions with the investigated local operators at the sink and a smeared interpolating operator for the proton at the source. In addition one needs the usual proton correlator with both source and sink smeared. We have evaluated these two-point functions on gauge field configurations generated by the QCDSF/DIK collaborations with the standard Wilson gauge action and two flavors of nonperturbatively improved Wilson fermions (clover fermions). The gauge couplings used are $\beta = 5.29$ and $\beta = 5.40$ corresponding to lattice spacings $a \approx 0.075\,\mathrm{fm}$ and $a \approx 0.067\,\mathrm{fm}$ via a Sommer parameter of $r_0 = 0.467\,\mathrm{fm}$ [17, 18]. Our smallest pion masses are $380\,\mathrm{MeV}$ ($\beta = 5.29$) and $420\,\mathrm{MeV}$ ($\beta = 5.40$), while the spatial lattice sizes $L$ are such that $m_\pi L \geq 3.7$.

Due to the discretization of space-time, the mixing pattern of the operators on the lattice is more complicated than in the continuum. It is determined by the transformation behavior of the operators under the (spinorial) symmetry group of our hypercubic lattice. As operators belonging to inequivalent irreducible representations cannot mix, we derive our operators from the irreducibly transforming multiplets of three-quark operators constructed in [19] in order to reduce the amount of mixing to a minimum. These irreducible multiplets constitute also the basis for the renormalization of our operators, which is performed nonperturbatively. To this end we contract our three-quark operators with three quark sources, amputate the external legs from the resulting four-point functions and impose an RI$'$-MOM-like renormalization condition. Finally we use continuum perturbation theory and the renormalization group to convert the results to the $\overline{\mathrm{MS}}$ scheme at a scale of $2\,\mathrm{GeV}$. We estimate the corresponding uncertainty by varying the scale at which our renormalization condition is imposed between $10\,\mathrm{GeV}^2$ and $40\,\mathrm{GeV}^2$. In this procedure, the mixing with "total derivatives" is automatically taken into account.

In the case of the moments considered in this work we can avoid the particularly nasty mixing with lower-dimensional operators completely. Note that the operators $\mathcal{V}_\tau^{\rho\bar{l}\bar{m}\bar{n}}$, $\mathcal{A}_\tau^{\rho\bar{l}\bar{m}\bar{n}}$ and $\mathcal{T}_\tau^{\rho\bar{l}\bar{m}\bar{n}}$ with different multi-indices $\rho\bar{l}\bar{m}\bar{n}$ but the same $lmn$ are related to the same moments $V^{lmn}$, $A^{lmn}$ and $T^{lmn}$, and we make use of this fact not only in order to minimize the mixing problems but also in order to reduce the statistical noise by considering suitable linear combinations.

For the operators without derivatives, i.e., the matrix elements $\lambda_1$, $\lambda_2$ and $f_N$, we have performed a joint fit of all contributing correlators to obtain the values at the simulated quark masses. As these are larger than the physical masses a chiral extrapolation to the physical point is required in the end. To the best of our knowledge there are no results from chiral perturbation theory to guide this extrapolation. Therefore we have adopted a more phenomenological approach aiming at linear (in $m_\pi^2$) fits to our data. It turns out that the ratios $f_N/m_N^2$ and $\lambda_i/m_N$ are particularly well suited for this purpose (see Fig. 1 (upper panel) for an example). Moreover, $f_N/m_N^2$ is dimensionless and hence not affected by any uncertainty in the determination of the lattice spacing. In order to estimate the systematic error due to our linear extrapolation, we also consider a chiral extrapolation including a term quadratic in $m_\pi^2$ and take the difference as the systematic error. The results in the $\overline{\mathrm{MS}}$ scheme at a scale of $2\,\mathrm{GeV}$ are given in Table I. Note that $2\lambda_1 \approx -\lambda_2$, a relation that is expected to hold in the nonrelativistic limit (cf., eq. (9)).

For the higher moments one can proceed in the same way and the constraint (6) is satisfied very well. However, the

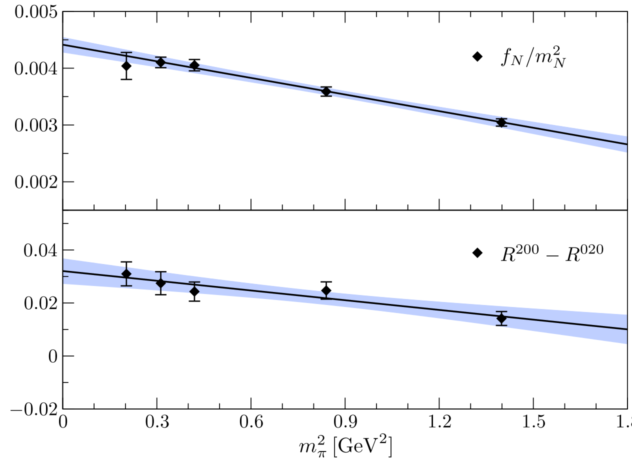


FIG. 1: Linear chiral extrapolation of bare lattice results for $f_N/m_N^2$ (upper panel) and the asymmetry $R^{200} - R^{020}$ (lower panel) with one-$\sigma$ error band.

| | LAT | QCDSR | BLW | BK |
|---|---|---|---|---|
| $V^{001}$ | 0.304 | 0.248 | 0.303 | 0.311 |
| $2A^{010}$ | 0.091 | 0.303 | 0.116 | 0.064 |

TABLE II: Comparison of our results (LAT) to selected sum rule results [20] (QCDSR) and the phenomenological estimates [12] (BLW) and [13] (BK) at the scale $2\,\mathrm{GeV}$.

statistical errors in this approach are too large to allow an accurate determination of the (particularly interesting) asymmetries. We achieved smaller errors by calculating the ratios $R^{lmn} = \phi^{lmn}/S_i$ where $S_1 = \phi^{100} + \phi^{010} + \phi^{001}$ for $l+m+n = 1$, and $S_2 = 2(\phi^{011} + \phi^{101} + \phi^{110}) + \phi^{200} + \phi^{020} + \phi^{002}$ for $l+m+n = 2$. These ratios can be extrapolated linearly to the physical masses. An example is shown in Fig. 1 (lower panel) for the case of the asymmetry $R^{200} - R^{020}$. Requiring that the constraint (6) be satisfied for the renormalized values we can finally extract the moments from the ratios.

*Discussion and Conclusions.* — Since we only have results at two different lattice spacings, we are unable to extrapolate our results to the continuum limit. However, we find that the results obtained at $\beta = 5.29$ and $\beta = 5.40$ are compatible within errors. Hence we take the data from our finer lattice ($\beta = 5.40$) as our final numbers. The values for $\phi^{lmn}$ imply that $\varphi^{100} = 0.394$, $\varphi^{010} = 0.302$ and $\varphi^{001} = 0.304$. These moments can be interpreted as the fraction of momentum carried by the corresponding quarks [5, 6]. Thus we find that the largest fraction of the proton longitudinal momentum is carried by one *up*-quark with spin aligned with the proton spin. However, this asymmetry is not as strong as found in the QCD sum rule calculation. Our results for the first moments are close to phenomenological estimates [12, 13], cf. Table II. On the other hand, our results for $\varphi^{011}$, $\varphi^{101}$ and $\varphi^{110}$ are similar to the sum rule values, while the asymmetries in the moments $\varphi^{200}$, $\varphi^{020}$ and $\varphi^{002}$ are clearly smaller.

Let us now expand the distribution amplitude in terms of

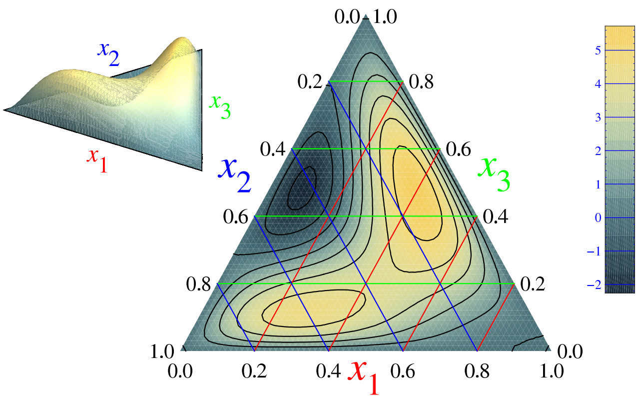


FIG. 2: Barycentric contour plot of the leading-twist distribution amplitude $\varphi(x_1, x_2, x_3)$ at $\mu = \mu_0 = 2\,\mathrm{GeV}$ as obtained from the moments presented in Table I. The lines of constant $x_1$, $x_2$ and $x_3$ are parallel to the sides of the triangle labelled by $x_2$, $x_3$ and $x_1$, respectively.

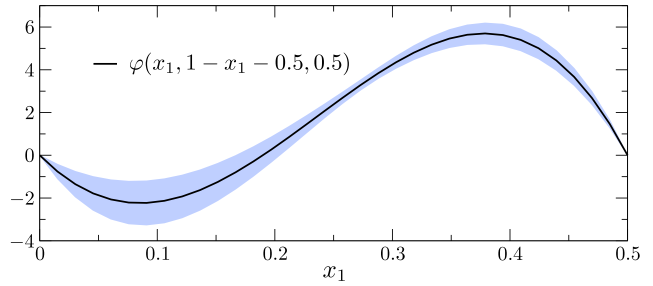


FIG. 3: The model distribution amplitude $\varphi(x_1, x_2, x_3)$ for $x_3 = 0.5$ as a function of $x_1$ with statistical errors.

polynomials $P_n$ to order $N = 2$ chosen such that the mixing matrix is diagonal [21, 22]:

$$\varphi(x_i, \mu) = 120 x_1 x_2 x_3 \sum_{n=0}^{N} c_n(\mu_0) P_n(x_i) \left( \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\omega_n} .$$

Calculating the coefficients $c_n(\mu_0)$ from an independent subset of the moments $\phi^{lmn}(\mu_0 = 2\,\mathrm{GeV})$, we obtain a model function for the distribution amplitude presented in Fig. 2. While the (totally symmetric) asymptotic amplitude $120\, x_1 x_2 x_3$ has a maximum for $x_1 = x_2 = x_3 = 1/3$, inclusion of the first moments (i.e., choosing $N = 1$) moves this maximum to $x_1 \approx 0.46$, $x_2 \approx 0.27$, $x_3 \approx 0.27$ giving the first quark substantially more momentum than the others. The second moments then turn this single maximum into the two local maxima in Fig. 2. The approximate symmetry in $x_2$ and $x_3$ is due to the approximate symmetry $\varphi^{lmn} \approx \varphi^{lnm}$ of our results. It is also seen in QCD sum rule calculations as well as in several models such as BLW and BK and may indicate the formation of a diquark system. To illustrate the statistical uncertainty we show in Fig. 3 the profile of $\varphi$ at $x_3 = 0.5$ and the corresponding error band. Note that higher-order polynomials have been disregarded in this model and thus Figs. 2 and 3 should be interpreted with due caution.

In order to establish a link to observable quantities we are calculating nucleon form factors using light cone sum rules. As our moments are close to the phenomenological estimates shown in Table II we can expect reasonable agreement with the experimental data. It is, however, to be stressed that these calculations still involve some model dependence, while the moments presented in this work were obtained from first principles.

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[1] V. L. Chernyak and A. R. Zhitnitsky, "Asymptotic Behavior of Hadron Form-Factors in Quark Model. (In Russian)," *JETP Lett.* **25** (1977) 510.

[2] A. V. Efremov and A. V. Radyushkin, "Factorization and Asymptotical Behavior of Pion Form- Factor in QCD," *Phys. Lett.* **B94** (1980) 245–250.

[3] G. P. Lepage and S. J. Brodsky, "Exclusive Processes in Quantum Chromodynamics: The Form- Factors of Baryons at Large Momentum Transfer," *Phys. Rev. Lett.* **43** (1979) 545–549.

[4] G. P. Lepage and S. J. Brodsky, "Exclusive Processes in Perturbative Quantum Chromodynamics," *Phys. Rev.* **D22** (1980) 2157.

[5] V. L. Chernyak and I. R. Zhitnitsky, "Nucleon Wave Function and Nucleon Form-Factors in QCD," *Nucl. Phys.* **B246** (1984) 52–74.

[6] V. L. Chernyak, A. A. Ogloblin, and I. R. Zhitnitsky, "The wave functions of the octet baryons," *Z. Phys.* **C42** (1989) 569.

[7] **Jefferson Lab Hall A** Collaboration, O. Gayou *et al.*, "Measurement of $G_{Ep}/G_{Mp}$ in $\vec{e}p \to e\vec{p}$ to $Q^2 = 5.6\,\mathrm{GeV}^2$," *Phys. Rev. Lett.* **88** (2002) 092301, arXiv:nucl-ex/0111010.

[8] V. Punjabi *et al.*, "Proton elastic form factor ratios to $Q^2 = 3.5\,\mathrm{GeV}^2$ by polarization transfer," *Phys. Rev.* **C71** (2005) 055202, arXiv:nucl-ex/0501018.

[9] V. M. Braun, A. Lenz, N. Mahnke, and E. Stein, "Light-cone sum rules for the nucleon form factors," *Phys. Rev.* **D65** (2002) 074011, arXiv:hep-ph/0112085.

[10] A. Lenz, M. Wittmann, and E. Stein, "Improved light-cone sum rules for the electromagnetic form factors of the nucleon," *Phys. Lett.* **B581** (2004) 199–206, arXiv:hep-ph/0311082.

[11] G. Martinelli and C. T. Sachrajda, "The quark distribution amplitude of the proton: a lattice computation of the lowest two moments," *Phys. Lett.* **B217** (1989) 319.

[12] V. M. Braun, A. Lenz, and M. Wittmann, "Nucleon form

factors in QCD," *Phys. Rev.* **D73** (2006) 094019, `arXiv:hep-ph/0604050`.

[13] J. Bolz and P. Kroll, "Modelling the nucleon wave function from soft and hard processes," *Z. Phys.* **A356** (1996) 327, `arXiv:hep-ph/9603289`.

[14] V. Braun, R. J. Fries, N. Mahnke, and E. Stein, "Higher twist distribution amplitudes of the nucleon in QCD," *Nucl. Phys.* **B589** (2000) 381–409, `arXiv:hep-ph/0007279`.

[15] B. L. Ioffe, "Calculation of Baryon Masses in Quantum Chromodynamics," *Nucl. Phys.* **B188** (1981) 317–341.

[16] Y. Chung, H. G. Dosch, M. Kremer, and D. Schall, "Baryon Sum Rules and Chiral Symmetry Breaking," *Nucl. Phys.* **B197** (1982) 55.

[17] A. Ali Khan *et al.*, "Axial coupling constant of the nucleon for two flavours of dynamical quarks in finite and infinite volume," *Phys. Rev.* **D74** (2006) 094508, `arXiv:hep-lat/0603028`.

[18] C. Aubin *et al.*, "Light hadrons with improved staggered quarks: Approaching the continuum limit," *Phys. Rev.* **D70** (2004) 094505, `arXiv:hep-lat/0402030`.

[19] T. Kaltenbrunner, M. Göckeler, and A. Schäfer, "Irreducible Multiplets of Three-Quark Operators on the Lattice: Controlling Mixing under Renormalization," *Eur. Phys. J.* **C55** (2008) 387–401, `arXiv:0801.3932 [hep-lat]`.

[20] I. D. King and C. T. Sachrajda, "Nucleon Wave Functions and QCD Sum Rules," *Nucl. Phys.* **B279** (1987) 785.

[21] V. M. Braun, S. E. Derkachov, G. P. Korchemsky, and A. N. Manashov, "Baryon distribution amplitudes in QCD," *Nucl. Phys.* **B553** (1999) 355–426, `arXiv:hep-ph/9902375`.

[22] N. G. Stefanis, "The physics of exclusive reactions in QCD: Theory and phenomenology," *Eur. Phys. J. direct* **C7** (1999) 1, `arXiv:hep-ph/9911375`.

[23] **SciDAC** Collaboration, R. G. Edwards and B. Joó, "The Chroma software system for lattice QCD," *Nucl. Phys. Proc. Suppl.* **140** (2005) 832, `arXiv:hep-lat/0409003`.

[24] P. A. Boyle, 2005. `http://www.ph.ed.ac.uk/~paboyle/bagel/Bagel.html`