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Ghost contributions to charmonium production in polarized high-energy collisions

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Abstract

In a previous paper [Phys. Rev. D **68**, 034017 (2003)], we investigated the inclusive production of prompt J/ψ mesons in polarized hadron-hadron, photon-hadron, and photon-photon collisions in the factorization formalism of nonrelativistic quantum chromodynamics providing compact analytic results for the double longitudinal-spin asymmetry $\mathcal{A}_{LL}$. For convenience, we adopted a simplified expression for the tensor product of the gluon polarization four-vector with its charge conjugate, at the expense of allowing for ghost and anti-ghosts to appear as external particles. While such ghost contributions cancel in the cross section asymmetry $\mathcal{A}_{LL}$ and thus were not listed in our previous paper, they do contribute to the absolute cross sections. For completeness and the reader's convenience, they are provided in this addendum.

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The factorization formalism of nonrelativistic QCD (NRQCD) [1] provides a rigorous theoretical framework for the description of heavy-quarkonium production and decay. This formalism implies a separation of process-dependent short-distance coefficients, to be calculated perturbatively as expansions in the strong-coupling constant α_s, from supposedly process-independent long-distance matrix elements (MEs), to be extracted from experiment, and takes into account the complete structure of the $Q\overline{Q}$ Fock space, which is spanned by the states $n = {}^{2S+1}L_J^{(C)}$ with definite spin S, orbital angular momentum L, total angular momentum J, and color multiplicity $C = 1, 8$. By velocity scaling rules, the MEs are predicted to scale with a definite power of the heavy-quark (Q) velocity $v \ll 1$, so that a small number of these non-perturbative parameters should allow for meaningful predictions in practice.

In Ref. [2], we applied the NRQCD factorization formalism to the inclusive production of prompt J/ψ mesons in polarized hadron-hadron, photon-hadron, and photon-photon collisions and provided compact analytic results for the double longitudinal-spin asymmetry $\mathcal{A}_{LL}$, defined in Eq. (2.1) of Ref. [2]. Specifically, we considered inclusive J/ψ production in polarized pp, γd, and $\gamma\gamma$ collisions, appropriate for the RHIC-Spin experiments at the BNL Relativistic Heavy Ion Collider (RHIC), the SLAC fixed-target experiment E161, and the TeV-Energy Superconducting Linear Accelerator (TESLA) operated in the e^+e^- and $\gamma\gamma$ modes, respectively. We took the J/ψ mesons to be unpolarized.

There is a technical subtlety related to the definition of the polarization four-vector $\varepsilon(p,\xi)$ of an external gluon, with four-momentum p and helicity quantum number $\xi = \pm 1$, which is potentially prone to create confusion. As for the tensor product of $\varepsilon(p,\xi)$ with its charge conjugate, a natural choice, which avoids the introduction of unphysical degrees of gluon polarization, is

$$\varepsilon_\mu(p,\xi)\varepsilon_\nu^*(p,\xi) = \frac{1}{2}\left(-g_{\mu\nu} + \frac{p_\mu\eta_\nu + p_\nu\eta_\mu}{k\cdot\eta} + i\xi\epsilon_{\mu\nu\rho\sigma}\frac{p^\rho\eta^\sigma}{p\cdot\eta}\right), \tag{1}$$

where η is an arbitrary light-like four-vector orthogonal to p, with $\eta^2 = 0 \neq p\cdot\eta$. An obvious disadvantage of Eq. (1) is that it introduces a host of terms involving η in intermediate results. In practical calculations such as the one performed in Ref. [2], it is therefore advantageous to omit the second term on the right-hand side of Eq. (1) and to identify η with the four-momentum p' of another external parton [3], so that

$$\varepsilon_\mu(p,\xi)\varepsilon_\nu^*(p,\xi) = \frac{1}{2}\left(-g_{\mu\nu} + i\xi\epsilon_{\mu\nu\rho\sigma}\frac{p^\rho p'^\sigma}{p\cdot p'}\right), \tag{2}$$

at the expense of endowing the gluon with unphysical degrees of polarization, which must be eliminated by subtracting contributions arising from the presence of its ghost h and anti-ghost $\overline{h}$ as external particles. Since such ghost contributions cancel in the cross section differences appearing in the numerator of $\mathcal{A}_{LL}$, as illustrated below, we did not list them in Ref. [2]. However, they are necessary to recover the well-known expressions for the unpolarized cross sections entering the denominator of $\mathcal{A}_{LL}$, as we did. In this sense, they were included in our numerical analysis.

Recently, there has been renewed interest in charmonium production by polarized hadron-hadron and photon-hadron collisions [4, 5]. In Ref. [4], the J/ψ and ψ' polarizations were predicted for polarized pp collisions at RHIC-Spin. In Ref. [5], the squares of the helicity amplitudes $\mathcal{M}(a,b,c)$ of the partonic subprocesses $\gamma(a)+g(b)\to Q\overline{Q}[n]+g(c)$ and $g(a)+g(b)\to Q\overline{Q}[n]+g(c)$ were listed for $n={}^1S_0^{(C)},{}^3S_1^{(C)},{}^1P_1^{(C)},{}^3P_J^{(C)}$ with $J=0,1,2$ and $C=1,8$. The longitudinally-polarized differential cross sections evaluated from these helicity amplitudes were found to agree with our results [2] after properly subtracting the ghost contributions mentioned above, which we had provided to the authors of Ref. [5] via private communication. Since these contributions may be useful for applications by other authors as well, we decided to publish them in this addendum to Ref. [2].

In the following, we present the differential cross sections $\mathrm{d}\sigma/\mathrm{d}t$ of the partonic subprocesses

$$\{\gamma,g\}h\to c\overline{c}[n]h. \tag{3}$$

Here and in the following, s, t, and u denote the usual Mandelstam variables. The results for $\{\gamma,g\}\overline{h}\to c\overline{c}[n]\overline{h}$ are identical by charge-conjugation invariance, while those for $h\{\gamma,g\}\to c\overline{c}[n]h$ and $h\overline{h}\to c\overline{c}[n]\{\gamma,g\}$ are related by crossing symmetry, as indicated below. As usual, $\mathrm{d}\sigma/\mathrm{d}t$ is evaluated from the absolute square of the transition matrix element $\mathcal{M}$ through multiplication with factors for flux, phase space, spin, and color, as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t}=\frac{1}{2s}\,\frac{1}{8\pi s}\,\frac{1}{4}\left(\frac{1}{8}\right)^i|\mathcal{M}|^2, \tag{4}$$

where $i=1,2$ is the number of color-octet partons (gluons or ghosts) in the initial state.

The only non-vanishing ghost contributions read

$$\begin{aligned}
|\mathcal{M}|^2(\gamma h\to c\overline{c}[{}^1S_0^{(8)}]h) &= \frac{24e^2g_s^4\langle\mathcal{O}[{}^1S_0^{(8)}]\rangle Q_c^2 su}{Mt(s+u)^2},\\
|\mathcal{M}|^2(\gamma h\to c\overline{c}[{}^3P_0^{(8)}]h) &= \frac{32e^2g_s^4\langle\mathcal{O}[{}^3P_0^{(8)}]\rangle Q_c^2 su}{M^3t(s+u)^4}[(2t+3u)^2+6s(2t+3u)+9s^2],\\
|\mathcal{M}|^2(\gamma h\to c\overline{c}[{}^3P_1^{(8)}]h) &= \frac{32e^2g_s^4\langle\mathcal{O}[{}^3P_1^{(8)}]\rangle Q_c^2}{M^3(s+u)^4}[u^2(t+u)-su^2+s^2(t-u)+s^3],\\
|\mathcal{M}|^2(\gamma h\to c\overline{c}[{}^3P_2^{(8)}]h) &= \frac{32e^2g_s^4\langle\mathcal{O}[{}^3P_2^{(8)}]\rangle Q_c^2}{5M^3t(s+u)^4}[3tu^2(t+u)+su(8t^2+21tu+12u^2)\\
&\quad+3s^2(t^2+7tu+8u^2)+3s^3(t+4u)],\\
|\mathcal{M}|^2(gh\to c\overline{c}[{}^1S_0^{(1)}]h) &= \frac{4g_s^2\langle\mathcal{O}[{}^1S_0^{(1)}]\rangle}{3e^2Q_c^2\langle\mathcal{O}[{}^1S_0^{(8)}]\rangle}|\mathcal{M}|^2(\gamma h\to c\overline{c}[{}^1S_0^{(8)}]h),\\
|\mathcal{M}|^2(gh\to c\overline{c}[{}^3P_J^{(1)}]h) &= \frac{4g_s^2\langle\mathcal{O}[{}^3P_J^{(1)}]\rangle}{3e^2Q_c^2\langle\mathcal{O}[{}^3P_J^{(8)}]\rangle}|\mathcal{M}|^2(\gamma h\to c\overline{c}[{}^3P_J^{(8)}]h),\\
|\mathcal{M}|^2(gh\to c\overline{c}[{}^1S_0^{(8)}]h) &= \frac{5g_s^2}{12e^2Q_c^2}|\mathcal{M}|^2(\gamma h\to c\overline{c}[{}^1S_0^{(8)}]h),
\end{aligned}$$

$$
\begin{aligned}
|\mathcal{M}|^2(gh\to c\overline{c}[^3S_1^{(8)}]h) &= \frac{3g_s^6\langle\mathcal{O}[^3S_1^{(8)}]\rangle}{4M^5stu(s+u)^2}[tu^2(t+u)^2(3t-u)+su(-2t^4+2t^3u \\
&\quad +7t^2u^2+4tu^3+u^4)+s^2(3t^4+t^3u-4t^2u^2-3tu^3+u^4) \\
&\quad +s^3(4t^3+7t^2u-2tu^2-u^3)+s^4(t^2+6tu-u^2)], \\
|\mathcal{M}|^2(gh\to c\overline{c}[^1P_1^{(8)}]h) &= \frac{g_s^2\langle\mathcal{O}[^1P_1^{(8)}]\rangle}{e^2Q_c^2\langle\mathcal{O}[^1S_0^{(8)}]\rangle M^2}|\mathcal{M}|^2(\gamma h\to c\overline{c}[^1S_0^{(8)}]h), \\
|\mathcal{M}|^2(gh\to c\overline{c}[^3P_J^{(8)}]h) &= \frac{5g_s^2}{12e^2Q_c^2}|\mathcal{M}|^2(\gamma h\to c\overline{c}[^3P_J^{(8)}]h), \qquad (5)
\end{aligned}
$$

where $e=\sqrt{4\pi\alpha}$, with α being Sommerfeld's fine-structure constant, and $g_s=\sqrt{4\pi\alpha_s}$ are the electromagnetic and strong gauge couplings, Q_c and m_c are the fractional electric charge and mass of the c quark, and $M=2m_c$. By four-momentum conservation, we have $s+t+u=M^2$.

We now explain how the unpolarized and polarized results of Refs. [5, 6] may be recovered from the results of Ref. [2] in combination with Eqs. (4) and (5), considering $\gamma g\to c\overline{c}[^1S_0^{(8)}]g$ as an example. The unpolarized and polarized results of Eqs. (A4) and (A5) in Ref. [6] are obtained from Eq. (4) by inserting

$$
\begin{aligned}
|\mathcal{M}|^2_{\mathrm{unpol}}(\gamma g\to c\overline{c}[^1S_0^{(8)}]g) &= \sum_{\xi_a,\xi_b=\pm1}|\mathcal{M}|^2_{\xi_a,\xi_b}(\gamma g\to c\overline{c}[^1S_0^{(8)}]g)-|\mathcal{M}|^2(\gamma h\to c\overline{c}[^1S_0^{(8)}]h) \\
&\quad -|\mathcal{M}|^2(\gamma\overline{h}\to c\overline{c}[^1S_0^{(8)}]\overline{h}), \\
|\mathcal{M}|^2_{LL}(\gamma g\to c\overline{c}[^1S_0^{(8)}]g) &= \sum_{\xi_a,\xi_b=\pm1}(-1)^{\xi_a\xi_b}|\mathcal{M}|^2_{\xi_a,\xi_b}(\gamma g\to c\overline{c}[^1S_0^{(8)}]g), \qquad (6)
\end{aligned}
$$

respectively, where $|\mathcal{M}|^2_{\xi_a,\xi_b}(\gamma g\to c\overline{c}[^1S_0^{(8)}]g)$ may be gleaned from Eq. (A5) of Ref. [2] and $|\mathcal{M}|^2(\gamma h\to c\overline{c}[^1S_0^{(8)}]h)=|\mathcal{M}|^2(\gamma\overline{h}\to c\overline{c}[^1S_0^{(8)}]\overline{h})$ is given in Eq. (5) above. As mentioned above, all ingredients entering $|\mathcal{M}|^2_{LL}(\gamma g\to c\overline{c}[^1S_0^{(8)}]g)$ are contained in Ref. [2]. By crossing symmetry, we have

$$
\begin{aligned}
|\mathcal{M}|^2(h\gamma\to c\overline{c}[^1S_0^{(8)}]h) &= |\mathcal{M}|^2(\gamma h\to c\overline{c}[^1S_0^{(8)}]h)\Big|_{t\leftrightarrow u}, \\
|\mathcal{M}|^2(h\overline{h}\to c\overline{c}[^1S_0^{(8)}]\gamma) &= |\mathcal{M}|^2(\gamma h\to c\overline{c}[^1S_0^{(8)}]h)\Big|_{s\leftrightarrow t}. \qquad (7)
\end{aligned}
$$

Similar relationships hold for the other partonic subprocesses involving two external gluons considered in Ref. [2].

In conclusion, we complemented the partonic cross sections for the inclusive production of prompt J/ψ mesons in polarized hadron-hadron, photon-hadron, and photon-photon collisions listed in the Appendix of Ref. [2] by providing the ghost contributions, which cancel in the cross section differences entering $\mathcal{A}_{LL}$, but contribute to absolute cross sections, including the unpolarized ones.

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