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# Next-to-next-to-leading order $\mathcal{O}(\alpha_s^4)$ results for heavy quark pair production in quark–antiquark collisions: The one-loop squared contributions

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We calculate the next-to-next-to-leading order $\mathcal{O}(\alpha_s^4)$ one-loop squared corrections to the production of heavy quark pairs in quark-antiquark annihilations. These are part of the next-to-next-to-leading order $\mathcal{O}(\alpha_s^4)$ radiative QCD corrections to this process. Our results, with the full mass dependence retained, are presented in a closed and very compact form, in the dimensional regularization scheme. We have found very intriguing factorization properties for the finite part of the amplitudes.

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## I. INTRODUCTION

There was recently much activity in the phenomenology of hadronic heavy quark pair production in connection with the Tevatron and the CERN Large Hadron Collider (LHC) which will have its startup this year. There will be much experimental effort dedicated to the discovery of the Higgs boson. There will also be studies of the copious production of top quarks and other heavy particles, which also serve as a background to Higgs boson searches as well as to possible new physics beyond the standard model. Therefore, it is mandatory to reduce the theoretical uncertainty in phenomenological calculations of heavy quark production processes as much as possible.

Several years ago the next-to-next-to-leading order (NNLO) contributions to hadron production were calculated by several groups in massless QCD (see e.g. [1] and references therein). The completion of a similar program for processes that involve massive quarks requires much more dedication and work since the inclusion of an additional mass scale dramatically complicates the whole calculation.

Until very recently there was the belief that the next-to-leading order (NLO) description of heavy charm and bottom production in hadronic collisions considerably underestimates the experimental results. In recent, more refined analyses [2, 3, 4] it was shown that a NLO analysis does in fact properly describe the latest charm and bottom quark production data [5]. The authors of [2] and [3, 4] deal very differently with the problem of large mass logarithms which constitute the central problem in the heavy quark phenomenology. Data on top quark pair production also agrees with the NLO prediction within theoretical and experimental errors (see e.g [6]). In all of these NLO calculations there remains, among others, the problem that the renormalization and factorization scale dependence of the NLO calculations render the theoretical results quite uncertain. This calls for a NNLO calculation of heavy quark production in hadronic collisions which is expected to considerably reduce the scale dependence of the theoretical prediction.

At the lower energies of Tevatron II, top quark pair production is dominated by $q\bar{q}$ annihilation (85 %). The remaining 15% come from gluon fusion. At the higher energy LHC, gluon fusion dominates the production process (90 %) with 10 % left for $q\bar{q}$ annihilation (percentage figures from [6]). This shows that both $q\bar{q}$ annihilation and gluon fusion have to be accounted for in the calculation of top quark pair production.

In general, there are four classes of contributions that need to be calculated for the NNLO corrections to the hadronic production of heavy quark pairs. The first class involves the pure two-loop contribution, which has to be folded with the leading order (LO) Born term. The second class of diagrams consists of the so-called one-loop squared contributions (also called loop–by–loop contributions) arising from the product of one-loop virtual matrix elements. This is the topic of the present paper. Further, there are the one-loop gluon emission contributions that are folded with the one–gluon emission graphs. Finally, there are the squared two-gluon emission contributions that are purely of tree–type.

Bits and pieces of the NNLO calculation are now being assembled. The recent two–loop calculation of the heavy quark vertex form factor [7] can be used as one of the many building blocks in the first class of processes. In this context we would also like to mention the recent work [8] on the NNLO calculation of two-loop virtual amplitudes performed in the domain of high energy asymptotics, where the heavy quark mass is small compared to the other large scales. In this calculation mass power corrections are left out, and only large mass logarithms and finite terms associated with them are retained. The

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authors of the present paper have been involved in a systematic effort to calculate all the contributions from the second class of processes, the one–loop squared contributions. We shall describe the present status of this program in the next paragraph. In the work [9] the full, exact NLO corrections to $t\bar{t}$+jet are presented. When integrating over the full phase space of the jet (or gluon), this calculation can be turned into a NNLO calculation of heavy hadron production of the third class. To our knowledge there does not exist a complete calculation of the fourth class of processes, the squared two-gluon emission contributions.

Let us briefly describe the status of our effort to calculate the one–loop squared contributions for the second class of processes. The highest singularity in the one–loop amplitudes from infrared (IR) and mass singularities (M) is, in general, proportional to $(1/\varepsilon^2)$. This in turn implies that the Laurent series expansion of the one–loop amplitudes has to be taken up to $\mathcal{O}(\varepsilon^2)$ when calculating the one-loop squared contributions. In fact, it is the $\mathcal{O}(\varepsilon^2)$ terms in the Laurent series expansion that really complicate things [10] since the $\mathcal{O}(\varepsilon^2)$ contributions in the one-loop amplitudes involve a multitude of multiple polylogarithms of maximal weight and depth 4 [11]. All scalar master integrals needed in this calculation have been assembled in [10, 11]. Reference [10] gives the results in terms of so-called $L$ functions, which can be written as one-dimensional integral representations involving products of log and dilog functions, while [11] gives the results in terms of multiple polylogarithms. The divergent and finite terms of the one–loop amplitude $q\bar{q} \to Q\bar{Q}$ were given in [12]. The remaining $\mathcal{O}(\varepsilon)$ and $\mathcal{O}(\varepsilon^2)$ amplitudes have been written down in [13]. Squaring the one–loop amplitudes leads to the results of the present paper. In a recent work [14] we have presented closed-form, one-loop squared results for heavy quark production in the fusion of real photons.

In this paper we report on a calculation of the NNLO one–loop squared matrix elements for the process $q\bar{q} \to Q\overline{Q}$. The calculation is carried out in the dimensional regularization scheme [15] with space-time dimension $n = 4 - 2\varepsilon$. In sequels to this paper we shall present results on the square of hadroproduction amplitudes originating from the gluon fusion subprocess $gg \to Q\overline{Q}$ and photoproduction amplitudes $\gamma g \to Q\overline{Q}$.

In our presentation we shall make use of our notation for the coefficient functions of the relevant scalar one-loop master integrals calculated up to $\mathcal{O}(\varepsilon^2)$ in [10]. For the case of gluon-gluon and quark-antiquark collisions, one needs all the scalar integrals derived in [10], e.g. the one scalar one-point function $A$; the five scalar two–point functions $B_1$, $B_2$, $B_3$, $B_4$, and $B_5$; the six scalar three–point functions $C_1, C_2, C_3, C_4, C_5$, and $C_6$; and the three scalar four-point functions $D_1, D_2$, and $D_3$. Taking the *complex* scalar four-point function $D_2$ as an example, we define successive coefficient functions $D_2^{(j)}$ for the Laurent series expansion of $D_2$. One has

$$D_2 = iC_\varepsilon(m^2)\Big\{\frac{1}{\varepsilon^2}D_2^{(-2)} + \frac{1}{\varepsilon}D_2^{(-1)} + D_2^{(0)} + \varepsilon D_2^{(1)} + \varepsilon^2 D_2^{(2)} + \mathcal{O}(\varepsilon^3)\Big\}, \tag{1.1}$$

where $C_\varepsilon(m^2)$ is defined by

$$C_\varepsilon(m^2) \equiv \frac{\Gamma(1+\varepsilon)}{(4\pi)^2}\left(\frac{4\pi\mu^2}{m^2}\right)^\varepsilon. \tag{1.2}$$

We use this notation for both the real and the imaginary parts of $D_2$, i.e. for $\mathrm{Re}D_2$ and $\mathrm{Im}D_2$. Similar expansions hold for the scalar one–point function $A$, the scalar two–point functions $B_i$, the scalar three–point functions $C_i$, and the remaining four-point functions $D_i$. The coefficient functions of the various Laurent series expansions were given in [10] in the form of so–called $L$ functions, and in [11] in terms of multiple polylogarithms of maximal weight and depth 4. It is then a matter of choice which of the two representations are used for the numerical evaluation. The numerical evaluation of the $L$ functions in terms of their one–dimensional integral representations is quite straightforward using conventional integration routines, while there exists a very efficient algorithm to numerically evaluate multiple polylogarithms [16].

Let us summarize the main features of the scalar master integrals. The master integrals $A, B_1, B_3, B_4, C_2, C_3$, and $D_3$ are purely real, whereas $B_2, B_5, C_1, C_4, C_5, C_6, D_1$, and $D_2$ are truly complex. From the form $(AB^* + BA^*) = 2(\mathrm{Re}A\,\mathrm{Re}B + \mathrm{Im}A\,\mathrm{Im}B)$ it is clear that the imaginary parts of the master integrals must be taken into account in the one-loop squared contribution. The master integrals $B_2, B_5, C_1, C_4, C_5$, and $C_6$ are $(t \leftrightarrow u)$ symmetric, where the kinematic variables $t$ and $u$ are defined in Sec. II.

The paper is organized as follows. Section II contains an outline of our general approach and discusses renormalization procedures. Section III presents NLO results for the quark-antiquark annihilation subprocess. In Sec. IV one finds a discussion of the singularity structure of the NNLO squared matrix element for the quark-antiquark annihilation subprocess. In Sec. V we discuss the structure of the finite part of our result. Our results are summarized in Sec. VI. In the Appendix we write down expressions for the building blocks of that part of the finite result that originates from the square of box diagrams.

## II. NOTATION

The heavy flavor hadroproduction proceeds through two partonic subprocesses: gluon fusion and light quark-antiquark annihilation. The first subprocess is the most challenging one in QCD from a technical point of view. It has three production topologies already at the Born level.

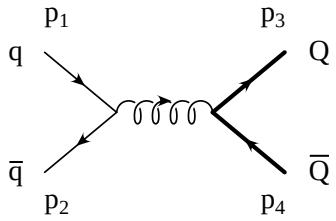


FIG. 1: The lowest order Feynman diagram representing light quark–antiquark annihilation. Normal solid lines represent the light quarks, the curly line represents the gluons and the thick solid lines correspond to the heavy quarks.

Here we consider the second subprocess where there is only one topology at the Born term level (see Fig. 1). Irrespective of the partons involved, the general kinematics is, of course, the same in both processes. In particular, for the quark-antiquark annihilation, Fig. 1, we have

$$q(p_1) + \bar{q}(p_2) \to Q(p_3) + \overline{Q}(p_4), \tag{2.1}$$

The momenta directions correspond to the physical configuration; e.g. $p_1$ and $p_2$ are ingoing whereas $p_3$ and $p_4$ are outgoing. With $m$ being the heavy quark mass, we define

$$\begin{aligned} s \equiv (p_1+p_2)^2, \qquad t \equiv T - m^2 \equiv (p_1-p_3)^2 - m^2, \\ u \equiv U - m^2 \equiv (p_2-p_3)^2 - m^2, \end{aligned} \tag{2.2}$$

so that the energy-momentum conservation reads $s+t+u=0$.

We also introduce the overall factor

$$\mathcal{C} = \left(g_s^4 C_\varepsilon(m^2)\right)^2, \tag{2.3}$$

where $g_s$ is the renormalized strong coupling constant and $C_\varepsilon(m^2)$ is defined in (1.2).

As shown e.g. in [12, 13] the self–energy and vertex diagrams contain ultraviolet (UV) and infrared and collinear (IR/M) poles even after heavy mass renormalization. The UV poles need to be regularized.

Our renormalization procedure is carried out as follows: when dealing with massless quarks we work in the $\overline{\rm MS}$ scheme, while heavy quarks are renormalized in the on–shell scheme, where the heavy quark mass is the pole mass. For completeness we list the set of one-loop renormalization constants that we have used:

$$\begin{aligned} Z_1 &= 1 + \frac{g_s^2}{\varepsilon}\frac{2}{3}\left\{(N_C - n_l)C_\varepsilon(\mu^2) - C_\varepsilon(m^2)\right\}, \\ Z_m &= 1 - g_s^2 C_F C_\varepsilon(m^2)\frac{3-2\varepsilon}{\varepsilon(1-2\varepsilon)}, \\ Z_2 &= Z_m, \\ Z_{1F} &= Z_2 - \frac{g_s^2}{\varepsilon} N_C C_\varepsilon(\mu^2), \\ Z_{1f} &= 1 - \frac{g_s^2}{\varepsilon} N_C C_\varepsilon(\mu^2), \\ Z_3 &= 1 + \frac{g_s^2}{\varepsilon}\left\{(\frac{5}{3}N_C - \frac{2}{3}n_l)C_\varepsilon(\mu^2) - \frac{2}{3}C_\varepsilon(m^2)\right\} \\ &= 1 + \frac{g_s^2}{\varepsilon}\left\{(\beta_0 - 2N_C)C_\varepsilon(\mu^2) - \frac{2}{3}C_\varepsilon(m^2)\right\}, \\ Z_g &= 1 - \frac{g_s^2}{\varepsilon}\left\{\frac{\beta_0}{2}C_\varepsilon(\mu^2) - \frac{1}{3}C_\varepsilon(m^2)\right\}. \end{aligned} \tag{2.4}$$

with $\beta_0 = (11N_C - 2n_l)/3$. $n_l$ is the number of light quarks, $C_F = 4/3$, and $N_C = 3$ is the number of colors. The arbitrary mass scale $\mu$ is the scale at which the renormalization is carried out. The above renormalization constants are as follows: $Z_1$ for the three-gluon vertex, $Z_m$ for the heavy quark mass, $Z_2$ for the heavy quark wave function, $Z_{1F}$ for the $(Q\overline{Q}g)$ vertex, $Z_{1f}$ for the $(q\overline{q}g)$ vertex, $Z_3$ for the gluon wave function, and $Z_g$ for the strong coupling constant $\alpha_s$. Note that $Z_1$ is not actually needed in the present application, but we have presented it for completeness. For the massless quarks there is no mass and wave function renormalization.

The above coefficients (except for $Z_g$) are needed if one renormalizes graph by graph. However, one could choose another route. From the field-theoretical point of view, the renormalized matrix element is obtained from the unrenormalized one by

$$M_{\rm ren} = \prod_n Z_{f_n}^{-1/2} M_{\rm bare}(g_{\rm bare} \to Z_g g_s, m_{\rm bare} \to Z_m m_r), \tag{2.5}$$

where $Z_{f_n}$ are the wave function renormalization constants for all the external on-shell particles under consideration. If one formally expands $M_{\rm bare}$ (e.g. $M_{\rm bare} = M_0 + g_s^2 M_1 + \ldots$) and the renormalization parameters $Z_{f_n}$ as a series of powers in the coupling constant to the requisite order, one arrives at the one-loop order result

$$\begin{aligned} M_{1,\rm ren} = \textstyle\prod_n Z_{f_n,1}^{-1/2} M_0(g_{\rm bare} \to Z_g g_s, m_{\rm bare} \to Z_m m_r) \\ + g_s^2 M_1(g_s, m_r), \end{aligned} \tag{2.6}$$

where now the $Z_{f_n,1}$ correspond to the one–loop renormalization constants for the external particles. In our case one has $Z_{f_1,1} = Z_{f_2,1} = 1$ and $Z_{f_3,1} = Z_{f_4,1} = Z_2$. Thus, one could apply inverse wave function renormalization for external legs and then replace the bare coupling constant $g_{\rm bare} \to Z_g g_s$ (as the mass parameter $m$ does not explicitly enter the leading order Born term matrix element, it is not renormalized at that order). We have verified that, in both ways, we arrive at the same renormalized result.

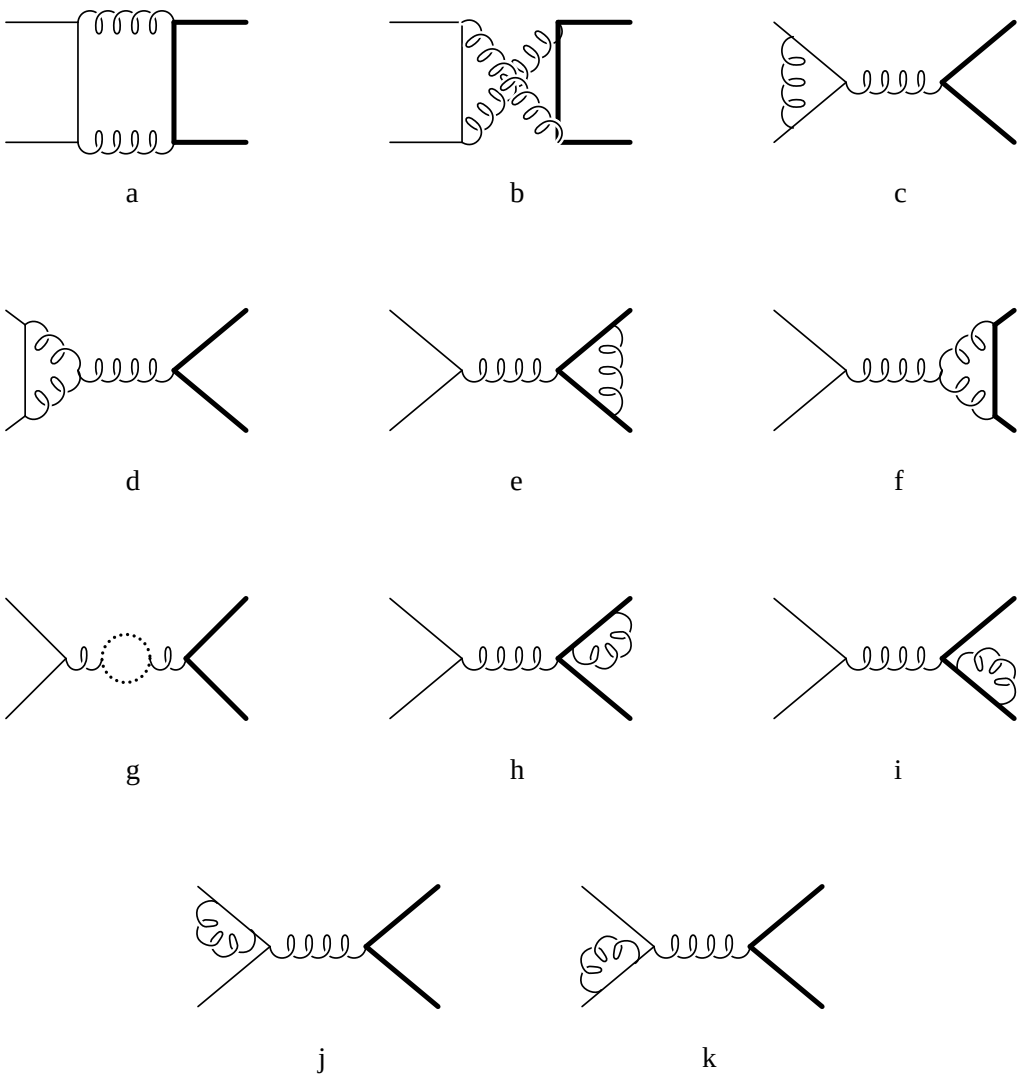


FIG. 2: One-loop Feynman diagrams contributing to the subprocess $q\bar{q} \to Q\overline{Q}$. The loop with the dotted line in (g) represents the gluon, ghost, and light and heavy quarks.

In order to fix our normalization we write down the differential cross section for $q\bar{q} \to Q\overline{Q}$ in terms of the squared amplitudes $|M|^2$. One has

$$d\sigma_{q\bar{q}\to Q\overline{Q}} = \frac{d(\mathrm{PS})_2}{2s}\frac{1}{4N_C^2}|M|^2_{q\bar{q}\to Q\overline{Q}}, \qquad (2.7)$$

where the $n$–dimensional two–body phase space is given by

$$d(\mathrm{PS})_2 = \frac{m^{-2\varepsilon}}{8\pi s}\frac{(4\pi)^\varepsilon}{\Gamma(1-\varepsilon)}\left(\frac{tu - sm^2}{sm^2}\right)^{-\varepsilon}\delta(s+t+u)dtdu \qquad (2.8)$$

and we explicitly show flux $(4p_1p_2)^{-1} = (2s)^{-1}$, initial quark and antiquark spin $(2s_f+1)^{-2} = 1/4$, and color $N_C^{-2}$ averaging factors. Then, at the leading Born term order for $q\bar{q} \to Q\overline{Q}$, we have

$$\frac{1}{g_s^4}|M|^2_{\mathrm{LO}} = 16\left(\frac{t^2+u^2}{s^2} + 2\frac{m^2}{s} - \varepsilon\right) \equiv B. \qquad (2.9)$$

## III. NEXT-TO-LEADING ORDER RESULT

Folding the one–loop matrix elements depicted in Fig. 2 with the LO Born term, Fig. 1, one obtains the virtual part of the NLO result. Although NLO virtual corrections to heavy flavor hadroproduction were calculated before for the $q\bar{q} \to Q\bar{Q}$ case, one cannot find explicit results for this subprocess in the literature. We have therefore recalculated the virtual NLO contribution to $q\bar{q}$–annihilation. In fact, we have calculated the virtual NLO results up to $\mathcal{O}(\varepsilon^2)$. As it turns out, the expressions for the NLO virtual $\varepsilon^1$ and $\varepsilon^2$ contributions considerably simplify the presentation of the corresponding NNLO results, in as much as they appear as important building blocks in the NNLO results.

First, we write down a few abbreviations that we shall use throughout the paper:

$$\beta = \sqrt{1-4m^2/s}, \qquad D = m^2 s - tu, \tag{3.1}$$
$$z_2 = s+2t, \qquad z_{2u} = s+2u, \qquad z_t = 2m^2+t, \qquad z_u = 2m^2+u.$$

Note that $D$ is *not* the space–time dimension. We further define the functions:

$$F_1^{(j)} = \frac{2}{9}(n_l+1) + \frac{28N_C}{9} - \frac{N_C}{\beta^2} - B_2^{(j)}\left(3C_F - \frac{3}{2}N_C + 1 - \frac{\beta^2}{3}\right) \tag{3.2}$$
$$-B_5^{(j)}\left(3C_F - \frac{5N_C}{3} + \frac{2n_l}{3} - \frac{N_C}{2\beta^2}\right) + C_1^{(j)}N_C\frac{m^2}{\beta^2} - \left\{C_4^{(j)}s - C_6^{(j)}(2m^2-s)\right\}(2C_F - N_C),$$

$$F_2^{(j)} = 2\left(s\beta^2(2C_F - N_C) - 12m^2N_C\right) - B_2^{(j)}s\beta^2(2C_F - N_C) + B_5^{(j)}(8m^2+s)N_C + C_1^{(j)}6m^2sN_C, \tag{3.3}$$

$$F_3^{(j)} = \frac{56}{3}\left\{2\left[8m^2\left(\frac{1}{t} - \frac{z_2}{s^2\beta^2}\right) - B_1^{(j)}\frac{2}{t}\left(m^2 + \frac{D}{s}\right) - B_5^{(j)}\frac{2z_u}{s\beta^2}\right.\right. \tag{3.4}$$
$$\left. + C_1^{(j)}\left(\frac{4t^2}{s} - z_2\frac{8m^4 - s^2}{s^2\beta^2}\right) - C_3^{(j)}2t\left(1 + 2\frac{T}{s}\right) + (C_4^{(j)} - D_2^{(j)}t)\frac{1}{s}(2D + s^2 + 2t^2)\right]$$
$$+\varepsilon\left[-8m^2\left(\frac{3}{t} - \frac{2z_2}{s^2\beta^2}\right) + B_1^{(j)}2\left(\frac{3z_t}{t} + \frac{2t}{s}\right) - B_5^{(j)}2\left(2 + \frac{z_2}{s\beta^2}\right)\right.$$
$$-C_1^{(j)}\left(8m^2 + 4s + \frac{8m^2t + s^2}{s\beta^2} + 2\frac{m^2s^2 + 2t^3}{D}\right) + C_3^{(j)}2\frac{t}{s}\left(s - 4t - 2st\frac{s-t}{D}\right)$$
$$\left.-(C_4^{(j)} - D_2^{(j)}t)\left(3s + 4t + 2st\frac{s-t}{D}\right)\right]$$
$$\left.+\varepsilon^2\frac{3s^2}{D}\left[C_1^{(j)}z_t + C_3^{(j)}\frac{2t^2}{s} + C_4^{(j)}t - D_2^{(j)}t^2\right]\right\},$$

$$F_4^{(j)} = \frac{16}{3}\left\{2\left[8m^2\left(\frac{1}{u} - \frac{z_{2u}}{s^2\beta^2}\right) - B_{1u}^{(j)}\frac{2}{u}\left(m^2 + \frac{D}{s}\right) - B_5^{(j)}\frac{2z_t}{s\beta^2}\right.\right. \tag{3.5}$$
$$\left. + C_1^{(j)}\left(\frac{4u^2}{s} - z_{2u}\frac{8m^4 - s^2}{s^2\beta^2}\right) - C_{3u}^{(j)}2u\left(1 + 2\frac{U}{s}\right) + (C_4^{(j)} - D_{2u}^{(j)}u)\frac{1}{s}(2D + s^2 + 2u^2)\right]$$
$$+\varepsilon\left[-8m^2\left(\frac{1}{u} + \frac{2z_{2u}}{s^2\beta^2}\right) + B_{1u}^{(j)}2\left(\frac{z_u}{u} - \frac{2u}{s}\right) - B_5^{(j)}2\left(1 - \frac{2z_u}{s\beta^2}\right)\right.$$
$$+C_1^{(j)}\left(4z_u + \frac{8m^2u + s^2}{s\beta^2} + 2m^2s\frac{z_{2u}}{D}\right) + C_{3u}^{(j)}2\frac{u}{s}\left(3s + 4u - 2\frac{stu}{D}\right)$$
$$\left.-(C_4^{(j)} - D_{2u}^{(j)}u)\left(2\frac{m^2s^2}{D} - s - 4u\right)\right]$$
$$\left.-\varepsilon^2\frac{3s^2}{D}\left[C_1^{(j)}z_u + C_{3u}^{(j)}\frac{2u^2}{s} + C_4^{(j)}u - D_{2u}^{(j)}u^2\right]\right\}.$$

The additional subscript "u" in some of the scalar coefficient functions in the expression for $F_4^{(j)}$ (such as $B_{1u}^{(j)}$) is to be understood as an operational definition prescribing a $(t \leftrightarrow u)$ interchange in the argument of that function, i.e. $B_{1u}^{(0)} = B_1^{(0)}\big|_{t\leftrightarrow u}$, etc. Note that $B_5^j$, $C_1^j$, and $C_4^j$ are intrinsically $(t \leftrightarrow u)$ symmetric (see [10]). Taking the $(t \leftrightarrow u)$ symmetry of $B_5^j$, $C_1^j$, and $C_4^j$ into account, one notes a corresponding $(t \leftrightarrow u)$ symmetry for the first and third square brackets in $F_3^{(j)}$ and $F_4^{(j)}$.

Before presenting our result for the NLO matrix element, we would like to comment on its color structure. First note that all the vertex and self-energy (VSE) graphs are proportional to the LO Born term color matrices (see Refs. [12, 13]). Both the parallel ladder box, Fig. 2(a), and the crossed ladder box, Fig. 2(b), have their own color structures. Altogether one obtains the

following three color structures,

$$\begin{aligned}
\mathrm{Tr}(T^a T^b)\ \mathrm{Tr}(T^a T^b) &= \frac{d_A}{4} \Rightarrow 2, \qquad (3.6)\\
\mathrm{Tr}(T^a T^b T^c)\ \mathrm{Tr}(T^b T^a T^c) &= \frac{d_A}{8}\left(N_C - \frac{2}{N_C}\right) \Rightarrow \frac{7}{3},\\
\mathrm{Tr}(T^a T^b T^c)\ \mathrm{Tr}(T^a T^b T^c) &= -\frac{d_A}{4}\frac{1}{N_C} \Rightarrow -\frac{2}{3},
\end{aligned}$$

from folding the Born term with the VSE graphs, the parallel ladder box, Fig. 2(a), and the crossed ladder box, Fig. 2(b), in that order. The common factor $d_A = N_C^2 - 1 = 8$ is the dimension of the adjoint representation of the color group $\mathrm{SU}(N_C)$. We present our NLO result separately for these three color structures.

At NLO the final spin and color summed matrix element can be written down as a sum of three terms:

$$\begin{aligned}
\frac{1}{g_s^2\sqrt{\mathcal{C}}}|M|^2_{\mathrm{Loop}\times\mathrm{Born}} = \mathrm{Re}\Big[&\frac{1}{\varepsilon^2}W^{(-2)}(\varepsilon) + \frac{1}{\varepsilon}W^{(-1)}(\varepsilon)\\
&+W^{(0)}(\varepsilon)\Big], \qquad (3.7)
\end{aligned}$$

where $\mathcal{C}$ has been defined in (2.3). The notation $|M|^2_{\mathrm{Loop}\times\mathrm{Born}}$ means that one is retaining only the $\mathcal{O}(\alpha_s^3)$ part of $|M|^2$.

The first two coefficient functions in (3.7) have a rather simple structure:

$$\begin{aligned}
W^{(-2)}(\varepsilon) &= -2B(2C_F - N_C + 3), \qquad (3.8)\\
W^{(-1)}(\varepsilon) &= -2B\Big(5C_F\Big[C_4^{(-1)}s - C_6^{(-1)}(2m^2 - s)\Big]\\
&\quad\times(2C_F - N_C) - \frac{2}{3}\Big[7\ln(\frac{-t}{m^2}) + 2\ln(\frac{-u}{m^2})\Big]\Big),
\end{aligned}$$

where $B$ is the Born term defined in Eq. (2.9). One should keep in mind that the overall Born term factor $B$ above contains a term multiplied by $\varepsilon$. Therefore, if the expression for $B$, Eq. (2.9), is substituted in $W^{(-2)}$ and $W^{(-1)}$, we will obtain $(\varepsilon)^{-1}$ and finite terms from the first two terms of Eq. (3.7).

The third term in Eq. (3.7) reads

$$W^{(0)}(\varepsilon) = F^{(0)}_{\mathrm{NLO}}, \qquad (3.9)$$

where

$$F^{(j)}_{\mathrm{NLO}} = W_1^{(j)} + W_2^{(j)} + W_3^{(j)}, \qquad (3.10)$$

and where

$$\begin{aligned}
W_1^{(j)} &= 2B\ F_1^{(j)} + 128\frac{m^2 D}{s^4\beta^4}F_2^{(j)},\\
W_2^{(j)} &= -2B\beta_0\ln^{1+j}(\frac{m^2}{\mu^2}), \qquad (3.11)\\
W_3^{(j)} &= F_3^{(j)} + F_4^{(j)}.
\end{aligned}$$

Note that the first term in (3.11) originates entirely from the sum of self-energy and vertex diagrams while the second term is due to renormalization. The terms $F_3^{(j)}$ and $F_4^{(j)}$ in $W_3^{(j)}$ represent the contributions from boxes $a$ and $b$, respectively.

The massless limit of our NLO result Eq. (3.7) without the $\mathcal{O}(\varepsilon)$ and $\mathcal{O}(\varepsilon^2)$ order terms was compared (including also the imaginary part) with corresponding results obtained from the methods developed in Ref. [17] [22]. There was agreement [19]. This serves as a rigorous check on our singularity structure as well as on all the mass logarithms of our original NLO matrix element [12].

## IV. SINGULARITY STRUCTURE OF THE NNLO SQUARED AMPLITUDE

The NNLO final spin and color summed squared matrix element can be written down as a sum of five terms:

$$\begin{aligned}
\frac{1}{\mathcal{C}}|M|^2_{\mathrm{Loop}\times\mathrm{Loop}} = \mathrm{Re}\Big[&\frac{1}{\varepsilon^4}V^{(-4)}(\varepsilon) + \frac{1}{\varepsilon^3}V^{(-3)}(\varepsilon) \qquad (4.1)\\
&+\frac{1}{\varepsilon^2}V^{(-2)}(\varepsilon) + \frac{1}{\varepsilon}V^{(-1)}(\varepsilon) + V^{(0)}(\varepsilon)\Big],
\end{aligned}$$

where $\mathcal{C}$ has been defined in (2.3). Note Eq. (4.1) is *not* a Laurent series expansion in $\varepsilon$ since the coefficient functions $V^{(m)}(\varepsilon)$ are functions of $\varepsilon$ as explicitly annotated in Eq. (4.1). It is nevertheless useful to write the NNLO one-loop squared result in the form of Eq. (4.1) in order to exhibit the explicit $\varepsilon$ structures. All five coefficient functions $V^{(m)}(\varepsilon)$ are bilinear forms in the coefficient functions that define the Laurent series expansion of the scalar master integrals (1.1). Some of these coefficient functions are zero and some of them are just numbers or simple logarithms. In the latter case we will substitute these numbers or logarithms for the coefficient functions $V^{(m)}$ in the five terms above. This will be done for the coefficient functions $A^{(m)}$, $B_1^{(-1)}$, $B_{1u}^{(-1)}$, $B_5^{(-1)}$, $C_3^{(-1)}$, and $C_{3u}^{(-1)}$.

We mention that in the course of our work we took full advantage of the fact that in [12] all the poles in the matrix element for the $q\bar{q} \to Q\overline{Q}$ subprocess are multiplied only by the leading order Born Dirac structure to cast the singular terms of the squared matrix element into an appropriately factorized form.

Before proceeding further, we present three more color structures appearing in the NNLO calculation in addition to the ones presented in Eq. (3.6) :

$$\begin{aligned}
&\mathrm{Tr}(T^a T^b T^{b'} T^{a'})\ \mathrm{Tr}(T^b T^a T^{a'} T^{b'}) = \qquad (4.2)\\
&\qquad\frac{d_A}{16}\left[N_C^2 - 3 + \frac{3}{N_C^2}\right] \Rightarrow \frac{19}{6},\\
&\mathrm{Tr}(T^a T^b T^{b'} T^{a'})\ \mathrm{Tr}(T^a T^b T^{b'} T^{a'}) = \frac{d_A}{16}\left[1 + \frac{3}{N_C^2}\right] \Rightarrow \frac{2}{3},\\
&\mathrm{Tr}(T^a T^b T^{b'} T^{a'})\ \mathrm{Tr}(T^b T^a T^{b'} T^{a'}) =\\
&\qquad -\frac{d_A}{16}\left[1 - \frac{3}{N_C^2}\right] \Rightarrow -\frac{1}{3}.
\end{aligned}$$

The above three color structures arise from folding box $a$ with box $a$, box $b$ with box $b$, as well as the interference of the two boxes, respectively.

Let us first introduce a notation which will help us to present the coefficients of the singular terms in the most concise fashion:

$$\begin{aligned} L_1 &= (2C_F - N_C)\left(C_4^{(-1)} s - C_6^{(-1)}(2m^2 - s)\right), \\ L_2 &= 15C_F - 14\ln(\frac{-t}{m^2}) - 4\ln(\frac{-u}{m^2}), \\ L_3 &= 35C_F - 38\ln(\frac{-t}{m^2}) - 4\ln(\frac{-u}{m^2}), \\ L_4 &= 5C_F - 2\ln(\frac{-t}{m^2}) - 4\ln(\frac{-u}{m^2}). \end{aligned} \tag{4.3}$$

The two most singular terms in (4.1) are proportional to the Born term $B$ defined in (2.9). One has

$$\begin{aligned} V^{(-4)}(\varepsilon) &= (2C_F - N_C + 3)^2 B, \\ V^{(-3)}(\varepsilon) &= 2(2C_F - N_C + 3)B\left[L_1 + \frac{L_2}{3}\right]. \end{aligned} \tag{4.4}$$

We also obtain

$$\begin{aligned} V^{(-2)}(\varepsilon) = \frac{B}{3}\Big[&(3L_1 + L_2)(L_1 + 5C_F)^* \\ &-2\ln(\frac{-t}{m^2})(7L_1 + L_3) \\ &-4\ln(\frac{-u}{m^2})(L_1 + L_4)\Big] \\ &-(2C_F - N_C + 3)F_{\rm NLO}^{(0)}. \end{aligned} \tag{4.5}$$

The last term in Eq. (4.5) is obtained from folding the $\mathcal{O}(\varepsilon^{-2})$ singular term of the matrix element with its finite part, while the rest is obtained from folding the single poles. Note that when one substitutes the Laurent expansions for $B$ and $F_{\rm NLO}^{(0)}$, one gets additional $1/\varepsilon$ poles and finite terms in Eq. (4.5).

The structure of the last term in Eq. (4.1) is a little more complicated. One has

$$\begin{aligned} V^{(-1)}(\varepsilon) = &-L_1^* F_{\rm NLO}^{(0)} - \frac{L_2}{3}(W_1^{(0)} + W_2^{(0)}) \\ &-\frac{L_3}{7}F_3^{(0)} - L_4 F_4^{(0)} \\ &+(2C_F - N_C + 3)\left[-F_{\rm NLO}^{(1)} + V'\right]. \end{aligned} \tag{4.6}$$

The terms multiplied by the $L_m$ functions above are due to folding the single pole terms in the original matrix element with its finite $\mathcal{O}(\varepsilon^0)$ part, while the last term is due to interference $\mathcal{O}(\varepsilon^{-2}) \times \mathcal{O}(\varepsilon)$ terms in the original matrix element. This pole term is due to the Taylor expansion of the original matrix element and cannot be deduced from the knowledge of the LO terms alone. The function $F_{\rm NLO}^{(1)}$ is defined in Eq. (3.10) and is nothing but the finite part of the NLO term with indices of the coefficient functions of the scalar master integrals and the power of the logarithm that multiplies the $\beta_0$ function, shifted upward by 1. For the remaining term $V'$, one obtains

$$\begin{aligned} V' = &-2B\left[\frac{\beta_0}{2}\ln^2(\frac{m^2}{\mu^2}) + 8C_F - \frac{N_C}{\beta^2} - \frac{2n_l + 2 + 28N_C}{27} + B_2^{(0)}\frac{2\beta^2 - 18C_F + 9N_C}{9}\right. \\ &\left. + B_5^{(0)}\frac{2}{9}(5N_C + n_l - 9C_F)\right] - 128\frac{m^2 D}{s^3\beta^4}\left[2(6\beta^2 C_F - N_C) - B_2^{(0)}2\beta^2(2C_F - N_C) - B_5^{(0)}2N_C - C_1^{(0)}sN_C\right] \\ &-\frac{56}{3}\left\{2\left[8m^2\left(\frac{1}{t} - \frac{z_2}{s^2\beta^2}\right) + \left(\frac{2}{s} + \frac{s-t}{D}\right)\left(C_1^{(0)}sz_t + C_3^{(0)}2t^2 + C_4^{(0)}st - D_2^{(0)}st^2\right)\right]\right. \\ &\left. -\varepsilon\left[8m^2\left(\frac{3}{t} - \frac{2z_2}{s^2\beta^2}\right) + \left(\frac{8}{s} + \frac{7s - 4t}{D}\right)\left(C_1^{(0)}sz_t + C_3^{(0)}2t^2 + C_4^{(0)}st - D_2^{(0)}st^2\right)\right]\right\} \\ &-\frac{16}{3}\left\{2\left[8m^2\frac{z_{2u}}{s^2\beta^2} + B_{1u}^{(0)}2\left(\frac{2D}{su} - 1\right) - B_5^{(0)}\frac{2z_{2u}}{s\beta^2} - C_1^{(0)}\left(m^2\left(4 + \frac{sz_{2u}}{D}\right) - \frac{2z_t}{\beta^2}\right)\right.\right. \\ &\left. - \left(\frac{z_{2u}}{s} - \frac{tu}{D}\right)\left(C_{3u}^{(0)}2u + C_4^{(0)}s - D_{2u}^{(0)}su\right)\right] \\ &\left. +\varepsilon\left[-8m^2\left(\frac{1}{u} + \frac{2z_{2u}}{s^2\beta^2}\right) + \left(\frac{8}{s} + \frac{9s + 4u}{D}\right)\left(C_1^{(0)}sz_u + C_{3u}^{(0)}2u^2 + C_4^{(0)}su - D_{2u}^{(0)}su^2\right)\right]\right\}. \end{aligned} \tag{4.7}$$

When one substitutes the Laurent expansions for $F_3^{(0)}$, $F_4^{(0)}$, and $F_{\rm NLO}^{(1)}$, one gets finite and $\mathcal{O}(\varepsilon)$ order terms in Eq. (4.6). However, since we are only interested in the Laurent series expansion up to the finite term, these $\mathcal{O}(\varepsilon)$ contributions should be omitted.

## V. STRUCTURE OF THE FINITE PART

In this section we present the finite part of our result. We calculate the finite part in several pieces, e.g.

$$V^{(0)} = \mathrm{Re}\left[V^{(0)}_{Bf_1} + V^{(0)}_{Bf_2} + V^{(0)}_{f_0 f_0}\right]. \qquad (5.1)$$

The first two terms originate from the interference of the $\mathcal{O}(\varepsilon^{-1}) \times \mathcal{O}(\varepsilon)$ and $\mathcal{O}(\varepsilon^{-2}) \times \mathcal{O}(\varepsilon^2)$ pieces of the initial matrix element. Each of them can be conveniently presented as a sum of five compact expressions:

$$V^{(0)}_{Bf_1} = G_1 + G_2 + G_3 + G_4 + G_5, \qquad (5.2)$$

where

$$\begin{aligned}
G_1 &= -128m^2 D(L_1^* + L_2/3)\Big[F_2^{(1)} \\
&\quad + 12s\beta^2 C_F - 2sN_C - B_2^{(0)} 2s\beta^2(2C_F - N_C) \\
&\quad - B_5^{(0)} 2sN_C - C_1^{(0)} s^2 N_C\Big]/(s^4\beta^4)\,, \\
G_2 &= -2B(L_1^* + L_2/3)\Big[27F_1^{(1)} - 2n_l - 2 \\
&\quad - 28N_C + 216C_F - 27N_C/\beta^2 - B_2^{(0)} 3(18C_F \\
&\quad - 9N_C - 2\beta^2) - B_5^{(0)} 6(9C_F - 5N_C - n_l)\Big]/27\,, \\
G_3 &= \beta_0 B \ln^2\!\left(\frac{m^2}{\mu^2}\right)(L_1 + L_2/3)\,, \qquad (5.3) \\
G_4 &= -16(7L_1^* + L_3)\Big[F_3^{(1)}\, 3/112 \\
&\quad + 8m^2(1/t - z_2/(s^2\beta^2)) + (C_1^{(0)} z_t + C_3^{(0)} 2t^2/s \\
&\quad + C_4^{(0)} t - D_2^{(0)} t^2)(2D + s^2 - st)/D\Big]/3\,, \\
G_5 &= -32(L_1^* + L_4)\Big[F_4^{(1)}\, 3/32 \\
&\quad + 8m^2 z_{2u}/(s^2\beta^2) + B_{1u}^{(0)} 2(2D/(su) - 1) \\
&\quad - B_5^{(0)} 2z_{2u}/(s\beta^2) - C_1^{(0)}(m^2 s z_{2u}/D \\
&\quad - 2(8m^4 + st)/(s\beta^2)) - (C_{3u}^{(0)} 2u/s + C_4^{(0)} \\
&\quad - D_{2u}^{(0)} u)(z_{2u} - stu/D)\Big]/3\,.
\end{aligned}$$

The first three terms above are due to the VSE contributions, and the last two terms are due to the two box diagrams. Similarly, for the second term in Eq. (5.1) we write

$$V^{(0)}_{Bf_2} = H_1 + H_2 + H_3 + H_4 + H_5, \qquad (5.4)$$

with

$$\begin{aligned}
H_1 &= -128(2C_F - N_C + 3)Dm^2\Big[F_2^{(2)} + 4s\beta^2(7C_F + N_C) - 10sN_C - B_2^{(1)} 2s\beta^2(2C_F - N_C) \\
&\quad - B_5^{(1)} 2sN_C - C_1^{(1)} s^2 N_C\Big]/(s^4\beta^4)\,, \\
H_2 &= -(2C_F - N_C + 3)B\Big[F_1^{(2)} 162 + 2(1296C_F + 76N_C - 10n_l - 10 - 243N_C/\beta^2) + B_2^{(0)} 24\beta^2 \\
&\quad - B_2^{(1)} 18(18C_F - 9N_C - 2\beta^2) + B_5^{(0)} 12(N_C + 2n_l) - B_5^{(1)} 36(9C_F - 5N_C - n_l)\Big]/81\,, \\
H_3 &= (2C_F - N_C + 3)B\beta_0 \ln^3\!\left(\frac{m^2}{\mu^2}\right)/3\,, \qquad (5.5) \\
H_4 &= -112(2C_F - N_C + 3)\Big[F_3^{(2)}\, 3/112 + 24m^2(1/t - z_2/(s^2\beta^2)) + (z_t(2C_1^{(0)} + C_1^{(1)}) \\
&\quad + 2t^2(2C_3^{(0)} + C_3^{(1)})/s + t(2C_4^{(0)} + C_4^{(1)}) - t^2(2D_2^{(0)} + D_2^{(1)}))(2D + s^2 - st)/D\Big]/3\,, \\
H_5 &= -32(2C_F - N_C + 3)\Big[F_4^{(2)}\, 3/32 + 8m^2(1/u + z_{2u}/(s^2\beta^2)) + B_{1u}^{(1)} 2(2D/(su) - 1) \\
&\quad - B_5^{(1)} 2z_{2u}/(s\beta^2) - (C_1^{(0)} z_u + C_{3u}^{(0)} 2u^2/s + C_4^{(0)} u - D_{2u}^{(0)} u^2)(4D + 3s^2 + 2su)/D \\
&\quad - C_1^{(1)}(m^2 s z_{2u}/D - 2(8m^4 + st)/(s\beta^2)) - (C_{3u}^{(1)} 2u/s + C_4^{(1)} - D_{2u}^{(1)} u)(z_{2u} D - stu)/D\Big]/3\,.
\end{aligned}$$

Note again that the $\mathcal{O}(\varepsilon)$ and $\mathcal{O}(\varepsilon^2)$ order terms in the above expressions for $V^{(0)}_{Bf_1}$ and $V^{(0)}_{Bf_2}$ can be disregarded. We also mention that the scalar coefficient functions with

superscript "2" above involve multiple polylogarithms.

We emphasize that the factorized forms of all the expressions given in this paper hold only when one retains the full $\varepsilon$ dependence in the Born and NLO terms.

The last term in Eq. (5.1) comes from the square of the $O(\varepsilon^0)$ term of the matrix element. It can also be written as a sum of five terms:

$$V^{(0)}_{f_0 f_0} = M_{VSE} + M_{BVSE} + M_{aa} + M_{ba} + M_{bb}. \quad (5.6)$$

The first term is the square of the finite parts of vertex and self-energy graphs; the second one is an interference of the vertex and self-energy graphs with the two box diagrams. These two terms can be presented in a very compact form:

$$\begin{aligned} M_{\rm VSE} &= F_1^{(0)} \left(W_1^{(0)} + W_2^{(0)} - B\,F_1^{(0)}\right)^* \\ &\quad - \left|F_2^{(0)}\right|^2 32m^2 D/(s^5\beta^6) \qquad (5.7) \\ &\quad - \beta_0 \ln(\frac{m^2}{\mu^2}) \left(W_2^{(0)}/2 + W_1^{(0)} - 2B\,F_1^{(0)}\right); \\ M_{BVSE} &= 7P + 2P|_{t\leftrightarrow u} + \left(F_1^{(0)} - \beta_0 \ln(\frac{m^2}{\mu^2})\right) \\ &\quad \times (F_3^{(0)} + F_4^{(0)})^*, \qquad (5.8) \end{aligned}$$

with

$$\begin{aligned} P = 64m^2 F_2^{(0)*} \Big[&2D/t - B_1^{(0)} D/t + C_1^{(0)} T z_2 - C_3^{(0)} 2tT \\ &+ (C_4^{(0)} - D_2^{(0)} t)(D + t^2)\Big]/(3s^3\beta^4). \qquad (5.9) \end{aligned}$$

When writing out $P|_{t\leftrightarrow u}$ one has to do the $t \leftrightarrow u$ operation in all the terms in the function $P$, i.e. for $z, t, F_2^{(0)}, B_1^{(0)}, C_3^{(0)}, T$, and $D_2^{(0)}$ ($C_1^{(0)}$ and $C_4^{(0)}$ are $t \leftrightarrow u$ symmetric).

Finally, we are left with the last three terms in Eq. (5.6), which are the longest terms in our NNLO result. However, to our surprise, we were able to discover nice factorization properties of the square of the two box diagrams. This part of the cross section can be put together with the help of several building blocks; e.g. each of the last three terms in Eq. (5.6) can be written as a sum of bilinear products. Each of the factors in the bilinear products are linear combinations of scalar integral coefficient functions multiplied by some combination of kinematic variables. To be more specific, we write

$$\begin{aligned} M_{aa} &= \frac{76}{3}\Big[s^{-1}Q_1 Q_8^* + 4m^2 Q_2 Q_3^* + Q_4 Q_{10}^* \\ &\quad + m^2 Q_5 Q_{11}^* - 2s^{-1} Q_6 Q_{12}^* + Q_7 Q_{13}^*\Big], \\ M_{bb} &= \frac{4}{19} M_{aa}|_{t\leftrightarrow u}, \qquad (5.10) \\ M_{ba} &= \frac{16}{3}\Big[s^{-1}Q_8 Q_{14}^* + 4m^2 Q_9 Q_{15}^* + Q_{10} Q_{16}^* \\ &\quad + 2m^2 Q_{11} Q_{16}^* + 2s^{-1} Q_{12} Q_{17}^* + Q_{13} Q_{18}^*\Big]. \end{aligned}$$

Explicit expressions for the 18 linear forms $Q_i$ are given in the Appendix. The bilinear forms above arise from folding certain pairs of Dirac structures in our original matrix element. The expression for $M_{ba}$ represents the result of the interference of the finite parts of box $a$ and box $b$.

It is quite obvious that the factorized forms for the finite part of the NNLO result for the $q\bar{q} \to Q\overline{Q}$ subprocess should also hold for the corresponding massless amplitudes. We have not seen this being displayed in the literature.

In the finite contribution, Eq. (5.1), one can see the interplay of the product of powers of $\varepsilon$ resulting from the Laurent series expansion of the scalar integrals [cf. Equation (1.1)] on the one hand, and powers of $\varepsilon$ resulting from doing the spin algebra in dimensional regularization on the other hand. For example, for the finite part one has a contribution from $C_6^{(-1)} B_1^{(0)*}$ as well as a contribution from $C_6^{(-1)} B_1^{(1)*}$. Terms of the type $C_6^{(-1)} B_1^{(0)*}$, where the superscripts corresponding to $\varepsilon$ powers do not compensate, would be absent in the regularization schemes where traces are effectively taken in four dimensions, i.e. in the so-called four-dimensional schemes or in dimensional reduction.

We want to emphasize that all our factorized results given in this paper take up about 10 Kb of hard disk space. This has to be compared with the length of the original computer output. The original computer output for the one-loop squared cross section of the $q\bar{q} \to Q\overline{Q}$ subprocess turned out to be very long and took up approximately 4 MB of hard disk space. Therefore, the reduction is of the order of 400 in the present case.

As a final remark we want to emphasize that we have done two independent calculations using REDUCE [20] and FORM [21] when squaring the one–loop amplitudes. After casting the results into the above compact form, we have checked the final result against the original untreated versions using again the REDUCE Computer Algebra System.

## VI. CONCLUSIONS

We have presented analytical $\mathcal{O}(\alpha_s^4)$ NNLO results for the one-loop squared contributions to heavy quark pair production in quark–antiquark annihilation. These are the first exact results for the hadroproduction of heavy quarks at NNLO where the heavy quark mass dependence is fully retained. Our results form part of the NNLO description of heavy quark pair production relevant for the NNLO analysis of ongoing experiments at the Tevatron and planned experiments at the LHC.

Our analytical results are presented in factorized forms. For the divergent parts, the squared matrix elements are given in terms of the Laurent series expansion of the corresponding LO and NLO contributions expanded up to $\mathcal{O}(\varepsilon)$ and $\mathcal{O}(\varepsilon^2)$, respectively. In this way we could transfer parts of the finite part of the squared

amplitudes to the coefficient functions of the pole terms. After this we found that the remaining parts of the finite contribution could be further factorized, partly in terms of the corresponding LO and NLO pieces, and, for the box graphs, in terms of factorizing forms as described in Sec. V. Writing our analytical results in factorized forms led to a reduction of the length of the original output by a factor of 400. To the best of our knowledge these nice factorization properties of the squared amplitude were not known before. It would be interesting to find out the underlying reason for this factorization.

The present paper deals with unpolarized quarks in the initial and final states. Since our results for the matrix elements in [13] contain the full spin information of the process, an extension to the polarized case with polarization in the initial state and/or in the final state including spin correlations is not difficult.

The present calculation constitutes a first step in obtaining the full NNLO corrections to the heavy quark production processes in QCD. A further next step is to provide one–loop squared results for gluon-initiated heavy quark pair production. Work on the gluon–initiated channel is in progress.

Analytical results in electronic format for all the terms in Eq. (4.1), including the $(t \leftrightarrow u)$ symmetric terms explicitly written out, as well as combined full results, are readily available [23].

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## APPENDIX

Here we present the expressions for the terms $Q_m$ appearing in Equation (5.10).

$$
\begin{aligned}
Q_1 &= \Big[8m^2(s/t - z_2/(s\beta^2)) - B_1^{(0)}2(m^2 s + D)/t - B_5^{(0)}2z_u/\beta^2 + C_1^{(0)}(2D + s^2 + 2tz_t + 2m^2 z_2/\beta^2) \\
&\quad - C_3^{(0)}2t(s + 2T) + C_4^{(0)}(2D + s^2 + 2t^2) - D_2^{(0)}t(2D + s^2 + 2t^2)\Big]/D \,, \\
Q_2 &= 2/t - B_1^{(0)}/t + C_1^{(0)}Tz_2/D - C_3^{(0)}2tT/D + C_4^{(0)}(1 + t^2/D) - D_2^{(0)}t(1 + t^2/D) \,, \\
Q_3 &= 4(2m^2 z_t - D)/(s\beta^2 t) + B_1^{(0)}2T/t + B_5^{(0)}2z_t/(s\beta^2) + C_1^{(0)}(z_t/\beta^2 + (m^2 st - t^3)/D) \\
&\quad + C_3^{(0)}2t^3/D + C_4^{(0)}st^2/D - D_2^{(0)}st^3/D \,, \\
Q_4 &= \Big[8m^2 z_u/(s\beta^2) + B_1^{(0)}2m^2 + B_5^{(0)}2m^2 z_2/(s\beta^2) + C_1^{(0)}(2tT + m^2 z_2/\beta^2) \\
&\quad - C_3^{(0)}2m^2 t + C_4^{(0)}(m^2 s + 2t^2) - D_2^{(0)}t(m^2 s + 2t^2)\Big]/D \,, \\
Q_5 &= \Big[16m^2 z_u/(s\beta^2) + B_1^{(0)}4m^2 + B_5^{(0)}4m^2 z_2/(s\beta^2) + C_1^{(0)}2(2tT + m^2 z_2/\beta^2) \\
&\quad - C_3^{(0)}4m^2 t + C_4^{(0)}2(m^2 s + 2t^2) - D_2^{(0)}2t(m^2 s + 2t^2)\Big]/D \,, \\
Q_6 &= 16m^2/(s\beta^2) + B_1^{(0)}2 - B_5^{(0)}2/\beta^2 - C_1^{(0)}(4t(D + m^2 t) + s^2 T/\beta^2 + 4m^2 t^2/\beta^2)/D \\
&\quad + C_3^{(0)}2tTz_2/D - C_4^{(0)}z_2(D + t^2)/D + D_2^{(0)}tz_2(D + t^2)/D \,, \\
Q_7 &= \Big[8m^2(s/t - 4 - 5z_t/(s\beta^2)) - B_1^{(0)}2(2D/t - 3m^2 + u) + B_5^{(0)}2(m^2 s + 6m^2 t - su)/(s\beta^2) \\
&\quad + C_1^{(0)}(2m^2 s + 10tT + (m^2 + s)z_2/\beta^2) - C_3^{(0)}2t(5m^2 + z_2) + C_4^{(0)}(s(5m^2 + z_2) + 10t^2) \\
&\quad - D_2^{(0)}t(s(5m^2 + z_2) + 10t^2)\Big]/D \,,
\end{aligned}
\tag{A.1}
$$

$$
\begin{aligned}
Q_8 &= 8m^2(D/t - tz_2/(s\beta^2)) - B_1^{(0)}2T(2D/t - s) + B_5^{(0)}2(D + tz_t/\beta^2) \\
&\quad - C_1^{(0)}s(m^2s - t^2 - tz_t - tz_t/\beta^2 - t^2(m^2s - t^2)/D) - C_3^{(0)}2stT(1 + st/D) \\
&\quad + C_4^{(0)}s(m^2s + 2t^2 + st^3/D) - D_2^{(0)}st(m^2s + 2t^2 + st^3/D) \ , \\
Q_9 &= -4(T/t + z_t/(s\beta^2)) + B_1^{(0)}2T/t + B_5^{(0)}2z_t/(s\beta^2) + C_1^{(0)}(z_t/\beta^2 + t(m^2s - t^2)/D) + C_3^{(0)}2t^3/D \\
&\quad + C_4^{(0)}st^2/D - D_2^{(0)}st^3/D \ , \\
Q_{10} &= \Big[8m^2D - B_1^{(0)}2Dz_t + B_5^{(0)}2tD - C_1^{(0)}st(m^2s + 4m^2t + t^2) + C_3^{(0)}2t^2(m^2s - t^2) \\
&\quad + C_4^{(0)}st(m^2s - t^2) - D_2^{(0)}st^2(m^2s - t^2)\Big]/D \ , \\
Q_{11} &= \Big[8Dz_u/(s\beta^2) + B_1^{(0)}2D + B_5^{(0)}2z_2D/(s\beta^2) - C_1^{(0)}s(m^2s - t^2 - z_2D/(s\beta^2)) - C_3^{(0)}2st^2 \\
&\quad - C_4^{(0)}s^2t + D_2^{(0)}s^2t^2\Big]/D \ , \\
Q_{12} &= 8m^2z_t/(s\beta^2) + B_1^{(0)}2T - B_5^{(0)}2(D - tz_t)/(s\beta^2) + C_1^{(0)}sz_t((2m^2 - s)/(s\beta^2) + t^2/D) \\
&\quad - C_3^{(0)}2st^2T/D - C_4^{(0)}s^2tT/D + D_2^{(0)}s^2t^2T/D \ , \\
Q_{13} &= C_1^{(0)}sz_t + C_3^{(0)}2t^2 + C_4^{(0)}st - D_2^{(0)}st^2 \ , \\
Q_{14} &= \Big[8m^2(s/u - z_{2u}/(s\beta^2)) - B_{1u}^{(0)}2(m^2s + D)/u - B_5^{(0)}2z_t/\beta^2 + C_1^{(0)}(2D + s^2 + 2uz_u + 2m^2z_{2u}/\beta^2) \\
&\quad - C_{3u}^{(0)}2u(s + 2U) + C_4^{(0)}(2D + s^2 + 2u^2) - D_{2u}^{(0)}u(2D + s^2 + 2u^2)\Big]/D \ , \\
Q_{15} &= 2/u - B_{1u}^{(0)}/u + C_1^{(0)}Uz_{2u}/D - C_{3u}^{(0)}2uU/D + C_4^{(0)}(1 + u^2/D) - D_{2u}^{(0)}u(1 + u^2/D) \ , \\
Q_{16} &= \Big[8m^2(s/u - z_u/(s\beta^2)) - B_{1u}^{(0)}2(D - m^2t)/u - B_5^{(0)}2(t - m^2z_{2u}/(s\beta^2)) \\
&\quad + C_1^{(0)}(s^2 - tz_u + D/\beta^2 - 2m^2uz_{2u}/(s\beta^2)) - C_{3u}^{(0)}2u(m^2 + z_{2u}) \\
&\quad + C_4^{(0)}(D + s^2 - tu) - D_{2u}^{(0)}u(D + s^2 - tu)\Big]/D \ , \\
Q_{17} &= 16m^2/(s\beta^2) + B_{1u}^{(0)}2 - B_5^{(0)}2/\beta^2 - C_1^{(0)}(4u(D + m^2u) + s^2U/\beta^2 + 4m^2u^2/\beta^2)/D + C_{3u}^{(0)}2uUz_{2u}/D \\
&\quad - C_4^{(0)}z_{2u}(D + u^2)/D + D_{2u}^{(0)}uz_{2u}(D + u^2)/D \ , \\
Q_{18} &= \Big[8m^2(4s/u - 1 - 5z_u/(s\beta^2)) - B_{1u}^{(0)}2(5D - 3m^2t + tu)/u - B_5^{(0)}2(4t - 5m^2z_{2u}/(s\beta^2)) \\
&\quad + C_1^{(0)}(4s^2 + 2(4s + 5u)U + 5m^2z_{2u}/\beta^2) - C_{3u}^{(0)}2u(5m^2 + 4z_{2u}) + C_4^{(0)}(5m^2s + 4sz_{2u} + 10u^2) \\
&\quad - D_{2u}^{(0)}u(5m^2s + 4sz_{2u} + 10u^2)\Big]/D \ .
\end{aligned}
$$


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[23] All the relevant results are available in REDUCE and FORM format. The results can be retrieved from the preprint server `http://arXiv.org` by downloading the source of this article or can be obtained directly from the authors.