

# Pole Mass, Width, and Propagators of Unstable Fermions

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## Abstract

The concepts of pole mass and width are extended to unstable fermions in the general framework of parity-nonconserving gauge theories, such as the Standard Model. In contrast with the conventional on-shell definitions, these concepts are gauge independent and avoid severe unphysical singularities, properties of great importance since most fundamental fermions in nature are unstable particles. General expressions for the unrenormalized and renormalized dressed propagators of unstable fermions and their field-renormalization constants are presented.

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The conventional definitions of mass and width of unstable bosons are

$$m_{\text{os}}^2 = m_0^2 + \text{Re } A(m_{\text{os}}^2), \quad (1)$$

$$m_{\text{os}}\Gamma_{\text{os}} = -\frac{\text{Im } A(m_{\text{os}}^2)}{1 - \text{Re } A'(m_{\text{os}}^2)}, \quad (2)$$

where  $m_0$  is the bare mass and  $A(s)$  is the self-energy in the scalar case and the transverse self-energy in the vector boson case. The subscript os means that Eqs. (1) and (2) define the on-shell mass and the on-shell width, respectively.

However, it was shown in Ref. [1] that, in the context of gauge theories,  $m_{\text{os}}$  and  $\Gamma_{\text{os}}$  are gauge dependent in next-to-next-to-leading order. It has also been emphasized that Eq. (2) leads to serious unphysical singularities if  $A(s)$  is not analytic in the neighborhood of  $m_{\text{os}}^2$ . This occurs, for example, when  $m_{\text{os}}$  is very close to a physical threshold [2, 3, 4] or, in the resonance region, when the unstable particle is coupled to massless quanta, as in the cases of the  $W$  boson and unstable quarks [5].

In order to solve these severe difficulties, it was proposed in Ref. [1] to base the definitions of mass and width of unstable boson on the complex-valued position of the propagator's pole, namely

$$\bar{s} = m_0^2 + A(\bar{s}). \quad (3)$$

Writing  $\bar{s} = m^2 - im\Gamma$  and taking the real and imaginary parts of Eq. (3), the pole mass  $m$  and the pole width  $\Gamma$  of unstable bosons are defined by the relations

$$m^2 = m_0^2 + \text{Re } A(\bar{s}), \quad (4)$$

$$m\Gamma = -\text{Im } A(\bar{s}). \quad (5)$$

Over the last several years, a number of authors have also advocated the use of  $\bar{s}$  as the basis for the definition of mass and width [6]. If one expands  $A(\bar{s})$  about  $m^2$  and retains only leading terms in  $\Gamma$ , Eqs. (4) and (5) lead back to Eqs. (1) and (2). Thus, Eqs. (1) and (2) may be regarded as the narrow-width approximation of Eqs. (4) and (5). The great advantage of Eqs. (4) and (5) is that  $\bar{s}$  is expected to be gauge independent, since it is the position of a singularity in  $S$ -matrix elements. In fact, formal proofs of the gauge independence of  $\bar{s}$  [as well as of the gauge dependence of Eq. (1)], based on the Nielsen identities, have been presented in the literature [7].

An expression equivalent to Eq. (5) is given by

$$m\Gamma = -Z \text{Im } A(m^2), \quad (6)$$

$$Z = \frac{1}{1 + \text{Im } [A(\bar{s}) - A(m^2)] / (m\Gamma)}. \quad (7)$$

Indeed, inserting Eq. (5) into Eq. (7), Eq. (6) becomes a mathematical identity. Comparing with the conventional expression in Eq. (2), we note two important changes:  $\text{Im } A(m^2)$  is evaluated at the pole mass  $m^2$  rather than the on-shell mass  $m_{\text{os}}^2$ , and the derivative  $-\text{Re } A'(m_{\text{os}}^2)$  in the denominator of Eq. (2) is replaced by a finite difference  $\text{Im } [A(\bar{s}) - A(m^2)] / (m\Gamma)$ . As explained in Ref. [4], this feature solves the threshold singularities mentioned before.

Other important consequences of Eqs. (4)–(7) are the following:

1. It has been shown that the alternative definitions  $\bar{m} = (m^2 + \Gamma^2)^{1/2}$ ,  $\bar{\Gamma} = m\Gamma/\bar{m}$ , constructed from the gauge-independent parameters  $m$  and  $\Gamma$ , can be identified with the measured mass and width of the  $Z^0$  boson [1, 6].
2. Comparison of the pole and on-shell definitions of mass and width leads to the conclusion that the gauge dependences of the latter can be numerically very large, particularly in the case of a heavy Higgs boson [8].
3. It has been shown that Eqs. (6) and (7) can be obtained by imposing a suitable normalization condition on the imaginary part of the renormalized self-energy, and that Eq. (7) can be identified with the field renormalization constant for unstable bosons [9, 10].

The aim of this paper is to extend the concepts of pole mass and width to unstable fermions in the general framework of parity-nonconserving gauge theories, to obtain the expressions analogous to Eqs. (4)–(7), and to derive their unrenormalized and renormalized dressed propagators and field-renormalization constants. Given the fact that, with the exception of the electron, the lightest neutrino, and the proton (or, at the elementary level, the  $u$  quark), all known fundamental fermions in nature are unstable particles, these concepts and expressions are indeed of great significance. In order to simplify the discussion, in the following analysis we will disregard flavor mixing. In the quark sector this means that we are considering a simplified theory in which the Cabibbo-Kobayashi-Maskawa quark mixing matrix is replaced by the unit matrix, while in the leptonic sector absence of flavor mixing naturally occurs if the neutrino masses are neglected.

On covariance grounds, the fermion self-energy is of the form

$$\begin{aligned}\Sigma(p) &= \Sigma_+(p)a_+ + \Sigma_-(p)a_-, \\ \Sigma_{\pm}(p) &= \not{p}B_{\pm}(p^2) + m_0A_{\pm}(p^2),\end{aligned}\tag{8}$$

where  $a_{\pm} = (1 \pm \gamma_5)/2$  are the right/left-handed chiral projectors, and the fermion propagator is

$$iS(p) = \frac{i}{\not{p} - m_0 - \Sigma(p)}.\tag{9}$$

Evaluating the inverse of the denominator in Eq. (9), one finds

$$\begin{aligned}S(p) &= \frac{1}{D(p^2)} \left\{ \{ \not{p}[1 - B_+(p^2)] + m_0[1 + A_-(p^2)] \} a_+ \right. \\ &\quad \left. + \{ \not{p}[1 - B_-(p^2)] + m_0[1 + A_+(p^2)] \} a_- \right\},\end{aligned}\tag{10}$$

$$D(p^2) = [1 - B_+(p^2)][1 - B_-(p^2)] [p^2 - m_0^2 f(p^2)],\tag{11}$$

$$f(p^2) = \frac{[1 + A_+(p^2)][1 + A_-(p^2)]}{[1 - B_+(p^2)][1 - B_-(p^2)]}.$$

The functions  $A_{\pm}(p^2)$  and  $B_{\pm}(p^2)$  are generally complex for  $p^2 > s_{\text{thr}}$ , the threshold of virtual particles contributing to  $\Sigma(p)$ . In the case of unstable fermions,  $s_{\text{thr}} < m^2$ . It is

instructive at this stage to consider the effect of parity ( $\mathcal{P}$ ), charge conjugation ( $\mathcal{C}$ ), and  $\mathcal{CP}$  transformations. One readily finds

$$S(p) \xrightarrow{\mathcal{P}} \gamma^0 S(p') \gamma^0, \quad (12)$$

$$S(p) \xrightarrow{\mathcal{C}} C S^T(-p) C^{-1}, \quad (13)$$

$$S(p) \xrightarrow{\mathcal{CP}} \gamma^0 C S^T(-p') C^{-1} \gamma^0, \quad (14)$$

where  $p' = (p^0, -\vec{p})$ ,  $C = i\gamma^2\gamma^0$ , and  $T$  means *transpose*. If parity is conserved, Eq. (12) leads to  $A_-(p^2) = A_+(p^2)$  and  $B_-(p^2) = B_+(p^2)$ , as expected. If the theory is invariant under charge conjugation, but not parity, Eq. (13) tells us that  $B_-(p^2) = B_+(p^2)$  with no restrictions on  $A_{\pm}(p^2)$ . If the theory is invariant under the  $\mathcal{CP}$  transformation, but not under charge conjugation or parity separately, Eq. (14) leads to  $A_-(p^2) = A_+(p^2)$  with no restrictions on  $B_{\pm}(p^2)$ . As expected, the latter conclusion also follows if the theory is invariant under  $\mathcal{T}$  (time reversal), while no restrictions on  $B_{\pm}(p^2)$  or  $A_{\pm}(p^2)$  are derived by invoking invariance under the  $\mathcal{TCP}$  transformation.

Introducing the definitions

$$\begin{aligned} \Sigma_{1,2}(p) &= \frac{1}{2}[\Sigma_+(p) \pm \Sigma_-(p)], \\ A_{1,2}(p^2) &= \frac{1}{2}[A_+(p^2) \pm A_-(p^2)], \end{aligned} \quad (15)$$

so that

$$\begin{aligned} \Sigma_{\pm}(p) &= \Sigma_1(p) \pm \Sigma_2(p), \\ A_{\pm}(p^2) &= A_1(p^2) \pm A_2(p^2), \end{aligned} \quad (16)$$

Eq. (10) can be written in the alternative form

$$\begin{aligned} S(p) &= \frac{1}{C(p)[\not{p} - m_0 - \Sigma_1(p)] - \Sigma_2(p)[\Sigma_2(p) - 2m_0A_2(p^2)]} [C(p) - \Sigma_2(p)\gamma_5], \\ C(p) &= \not{p} - \Sigma_1(p) + m_0[1 + 2A_1(p^2)]. \end{aligned} \quad (17)$$

Multiplying numerator and denominator by  $C^{-1}(p)$  on the left, we obtain the compact expression:

$$S(p) = \frac{1}{\not{p} - m_0 - \Sigma_{\text{eff}}(p)} [1 - \Sigma_P(p)\gamma_5], \quad (18)$$

where  $\Sigma_{\text{eff}}(p)$  is an *effective* self-energy defined by

$$\Sigma_{\text{eff}}(p) = \Sigma_1(p) + \frac{\Sigma_2(p)[\Sigma_2(p) - 2m_0A_2(p^2)]}{C(p)}, \quad (19)$$

and

$$\Sigma_P(p) = \frac{\Sigma_2(p)}{C(p)}. \quad (20)$$

The position  $p = M$  of the pole is given by

$$M = m_0 + \Sigma_{\text{eff}}(M). \quad (21)$$

In order to express  $M$  in terms of the original self-energies,  $\Sigma_1(p)$  and  $\Sigma_2(p)$ , and the functions  $A_1(p^2)$  and  $A_2(p^2)$ , we note that  $M$  appears on the l.h.s. of Eq. (21) and in  $C(M)$  in the denominator of the second term in  $\Sigma_{\text{eff}}(M)$  [cf. Eq. (19)]. Therefore,  $M$  satisfies a quadratic equation whose solution is

$$M = \Sigma_1(M) - m_0 A_1(M^2) + \sqrt{m_0^2 [1 + A_1(M^2)]^2 + \Sigma_2(M) [\Sigma_2(M) - 2m_0 A_2(M^2)]}. \quad (22)$$

In Eq. (22) we have chosen the positive square root to ensure that in the parity-conserving case, where  $\Sigma_2(p) = A_2(p^2) = 0$ , Eq. (22) reduces to  $M = m_0 + \Sigma_1(M)$ , which is the correct expression, as  $\Sigma_{\text{eff}}(p) \rightarrow \Sigma_1(p)$  in that limit [cf. Eq. (19)]. It is easy to verify that Eq. (22) is equivalent to the alternative expression

$$M = m_0 \sqrt{f(M^2)}, \quad (23)$$

which is the zero of  $D(p^2)$  in Eq. (11).

Since  $\Sigma_2(p)$  and  $A_2(p^2)$  are parity-nonconserving amplitudes, they are of  $\mathcal{O}(g^2)$ , where  $g$  is a generic weak-interaction gauge coupling. If terms of  $\mathcal{O}(g^8)$  are neglected, Eq. (22) simplifies to

$$M = m_0 + \Sigma_1(M) + \frac{\Sigma_2(M) [\Sigma_2(M) - 2m_0 A_2(M^2)]}{2m_0 [1 + A_1(M^2)]} + \mathcal{O}(g^8). \quad (24)$$

Thus, we see that the parity-nonconserving interactions introduce an explicit correction of  $\mathcal{O}(g^4)$  in  $M$ . Of course, there are also corrections of  $\mathcal{O}(g^2)$  and higher in  $\Sigma_1(M)$ , as well as QCD corrections in the quark cases.

Parameterizing

$$M = m - i \frac{\Gamma}{2}, \quad (25)$$

and taking the real and imaginary parts of Eq. (21), we obtain

$$m = m_0 + \text{Re} \Sigma_{\text{eff}}(M), \quad (26)$$

$$\frac{\Gamma}{2} = -\text{Im} \Sigma_{\text{eff}}(M), \quad (27)$$

which are the counterparts of Eqs. (4) and (5) and define the pole mass  $m$  and the pole width  $\Gamma$  of the unstable fermion. In analogy with Eqs. (6) and (7), Eq. (27) can be rewritten as

$$\frac{\Gamma}{2} = -Z \text{Im} \Sigma_{\text{eff}}(m), \quad (28)$$

$$Z = \frac{1}{1 + \text{Im} [\Sigma_{\text{eff}}(M) - \Sigma_{\text{eff}}(m)] / (\Gamma/2)}. \quad (29)$$

Indeed, inserting Eq. (27) into Eq. (29), Eq. (28) becomes a mathematical identity.

Returning to Eq. (18) and using  $a_+ + a_- = 1$  and  $a_+ - a_- = \gamma_5$ , we write the propagator in the form

$$\begin{aligned} iS(p) &= i[S_+(p)a_+ + S_-(p)a_-], \\ S_{\pm}(p) &= \frac{1 \mp \Sigma_P(p)}{\not{p} - m_0 - \Sigma_{\text{eff}}(p)}. \end{aligned} \quad (30)$$

Dividing numerator and denominator by  $1 \mp \Sigma_P(p)$ , we note that  $S_{\pm}(p)$  can be expressed as

$$S_{\pm}(p) = \frac{1}{\not{p} - m_0 - \Sigma_{\pm}^{\text{eff}}(p)}, \quad (31)$$

$$\Sigma_{\pm}^{\text{eff}}(p) = \Sigma_{\text{eff}}(p) \mp \frac{[\not{p} - m_0 - \Sigma_{\text{eff}}(p)]\Sigma_P(p)}{1 \mp \Sigma_P(p)}. \quad (32)$$

Thus, in the denominator of Eq. (31) the effective self-energy  $\Sigma_{\text{eff}}(p)$  has been replaced by  $\Sigma_{\pm}^{\text{eff}}(p)$ . We note, however, that the two self-energies coincide at  $\not{p} = M$ , on account of Eq. (21). Namely, we have

$$\Sigma_{\pm}^{\text{eff}}(M) = \Sigma_{\text{eff}}(M). \quad (33)$$

In order to construct the renormalized propagator,

$$iS^{(r)}(p) = i \left[ S_+^{(r)}(p)a_+ + S_-^{(r)}(p)a_- \right], \quad (34)$$

it is convenient to use the representation of the unrenormalized amplitude  $S(p)$  given in Eq. (10). Recalling that  $S(p)$  is the Fourier transform of  $\langle 0|T[\psi(x)\bar{\psi}(0)]|0\rangle$  and splitting  $\psi(x) = \psi_+(x) + \psi_-(x)$  into right- and left-handed components  $\psi_{\pm}(x) = a_{\pm}\psi(x)$ , one finds that the contributions of  $\langle 0|T[\psi_-(x)\bar{\psi}_-(0)]|0\rangle$ ,  $\langle 0|T[\psi_+(x)\bar{\psi}_-(0)]|0\rangle$ ,  $\langle 0|T[\psi_+(x)\bar{\psi}_+(0)]|0\rangle$ , and  $\langle 0|T[\psi_-(x)\bar{\psi}_+(0)]|0\rangle$  are given by the 1st, 2nd, 3rd, and 4th terms of Eq. (10), respectively. Shifting  $\psi_{\pm}(x)$  as

$$\psi_{\pm}(x) = \sqrt{Z_{\pm}}\psi'_{\pm}(x), \quad (35)$$

where  $\psi'_{\pm}(x)$  are the renormalized fields, it follows that the renormalized expressions are obtained by dividing the 1st, 2nd, 3rd, and 4th terms of Eq. (10) by  $|Z_-|$ ,  $\sqrt{Z_+Z_-^*}$ ,  $|Z_+|$ , and  $\sqrt{Z_-Z_+^*}$ , respectively. Thus, we obtain

$$S_{\pm}^{(r)}(p) = \frac{\not{p}[1 - B_{\pm}(p^2)] + \sqrt{Z_{\mp}/Z_{\pm}}m_0[1 + A_{\mp}(p^2)]}{|Z_{\mp}|D(p^2)}. \quad (36)$$

Evaluating the inverse of Eq. (34),  $S^{(r)}(p)$  can also be written as

$$\begin{aligned} S^{(r)}(p) &= \frac{1}{I_+^{(r)}(p)a_+ + I_-^{(r)}(p)a_-}, \\ I_{\pm}^{(r)}(p) &= |Z_{\pm}| \left\{ \not{p}[1 - B_{\pm}(p^2)] - \sqrt{\frac{Z_{\mp}^*}{Z_{\pm}^*}}m_0[1 + A_{\pm}(p^2)] \right\}. \end{aligned} \quad (37)$$

In the case of stable fermions, it is customary to define the field-renormalization constants so that the pole residue in  $S^{(r)}(p)$  equals unity. In the case of unstable fermions, this is generally not possible, since the analysis involves two complex functions  $S_{\pm}^{(r)}(p)$  that would require four constants, while the adjustable constants at our disposal,  $Z_-/Z_+$ ,  $|Z_+|$ , and  $|Z_-|$ , allow for only three independent real parameters. A particularly simple example of this restriction is provided by unstable fermions in parity-conserving theories. In that case, there is a single self-energy  $\Sigma(p)$ , the residue of the pole is  $1/[1 - \Sigma'(M)]$ , a complex amplitude, while the field-renormalization constant  $Z$  is real. Returning to the parity-nonconserving theories, and taking these observations into account, we normalize  $S_{\pm}^{(r)}(p)$  by requiring that the absolute values of their pole residues equal unity. A simple and symmetric determination of  $Z_{\pm}$  that satisfies these constraints is

$$\begin{aligned} Z_{\pm} &= \frac{1 + R_{\pm}}{2F[1 - B_{\pm}(M^2)]}, \\ R_{\pm} &= \frac{1 + A_{\pm}(M^2)}{1 + A_{\mp}(M^2)}, \\ F &= 1 - M^2 \frac{f'(M^2)}{f(M^2)}. \end{aligned} \quad (38)$$

Writing  $Z_{\pm} = |Z_{\pm}|e^{i\theta_{\pm}}$ , the residues of  $S_{\pm}^{(r)}(p)$  are  $e^{i\theta_{\mp}}$ , respectively, in agreement with our requirements. This implies that in the resonance region the renormalized propagator behaves as  $i(e^{i\theta_-} a_+ + e^{i\theta_+} a_-)/(\not{p} - M)$ , which, in leading order, reduces to the Breit-Wigner form  $i/(\not{p} - m + i\Gamma/2)$ .

Inserting Eqs. (23) and (38) in Eq. (36),  $S_{\pm}^{(r)}(p)$  can be expressed completely in terms of the functions  $A_{\pm}(p^2)$  and  $B_{\pm}(p^2)$ , as

$$\begin{aligned} S_{\pm}^{(r)}(p) &= \frac{2Fe^{i\theta_{\mp}}}{1 + R_{\mp}} \frac{1 - B_{\mp}(M^2)}{1 - B_{\mp}(p^2)} \\ &\times \frac{\not{p} + M[1 + A_{\mp}(p^2)][1 - B_{\pm}(M^2)]/\{[1 + A_{\pm}(M^2)][1 - B_{\pm}(p^2)]\}}{p^2 - M^2 f(p^2)/f(M^2)}. \end{aligned} \quad (39)$$

In the case of CP conservation, we have  $A_+(p^2) = A_-(p^2)$  and  $R_{\pm} = 1$ , so that Eqs. (38) and (39) simplify considerably.

In summary, in the approximation of neglecting flavor mixing, we have derived general and closed expressions for the pole mass and width of unstable fermions in parity-nonconserving gauge theories [Eqs. (26) and (27)], their unrenormalized and renormalized propagators [Eqs. (9), (10), (18) and Eqs. (36), (37), (39)], and their field-renormalization constants [Eq. (38)]. We also note that the discussion after Eq. (38) provides a theoretical framework to employ  $i/(\not{p} - m + i\Gamma/2)$  as leading-order propagator, particularly in the resonance region. In turn, it was emphasized in Ref. [5] that the systematic use of this propagator in the evaluation of the gluonic and photonic contributions to the fermion self-energy avoids the emergence in the resonance region of catastrophic on-shell singularities proportional to  $[m_{\text{os}}\Gamma_{\text{os}}/(p^2 - m_{\text{os}}^2)]^n$  ( $n = 2, 3, \dots$ ). Furthermore, in the quark case,

the same diagrams lead to an unbounded gauge dependence of  $\mathcal{O}(\alpha_s(m)\Gamma)$  in the on-shell mass  $m_{\text{os}}$ , which is neatly avoided by employing the pole mass  $m$ .

The significance of the concepts of pole mass and width we have discussed is that they are gauge independent, and thus satisfy a fundamental tenet of gauge theories to be identified with physical observables.

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## References

- [1] A. Sirlin, Phys. Rev. Lett. **67**, 2127 (1991); Phys. Lett. B **267**, 240 (1991).
- [2] J. Fleischer and F. Jegerlehner, Phys. Rev. D **23**, 2001 (1981); D. Y. Bardin, B. M. Vilensky, and P. K. Khristova, Sov. J. Nucl. Phys. **53**, 152 (1991) [Yad. Fiz. **53**, 240 (1991)]; *ibid.* **54**, 833 (1991) [Yad. Fiz. **54**, 1366 (1991)]; B. A. Kniehl, Nucl. Phys. **B357**, 439 (1991); **B376**, 3 (1992); Z. Phys. C **55**, 605 (1992); Phys. Rept. **240**, 211 (1994).
- [3] T. Bhattacharya and S. Willenbrock, Phys. Rev. D **47**, 4022 (1993).
- [4] B. A. Kniehl, C. P. Palisoc, and A. Sirlin, Nucl. Phys. **B591**, 296 (2000) [arXiv:hep-ph/0007002]; Phys. Rev. D **66**, 057902 (2002) [arXiv:hep-ph/0205304].
- [5] M. Passera and A. Sirlin, Phys. Rev. D **58**, 113010 (1998) [arXiv:hep-ph/9804309]; Acta Phys. Polon. B **29**, 2901 (1998) [arXiv:hep-ph/9807218]; A. Sirlin, in *Proceedings of the Fourth International Symposium on Radiative Corrections: Application of Quantum Field Theory to Phenomenology*, edited by J. Sola, (World Scientific, Singapore, 1999), p. 546.
- [6] S. Willenbrock and G. Valencia, Phys. Lett. B **259**, 373 (1991); R. G. Stuart, *ibid.* B **262**, 113 (1991); **272**, 353 (1991); Phys. Rev. Lett. **70**, 3193 (1993); H. Veltman, Z. Phys. C **62**, 35 (1994); M. Passera and A. Sirlin, Phys. Rev. Lett. **77** (1996) 4146 [arXiv:hep-ph/9607253]; A. Sirlin, in *Proceedings of the Ringberg Workshop on The Higgs Puzzle — What Can We Learn from LEP2, LHC, NLC and FMC?*, edited by B. A. Kniehl, (World Scientific, Singapore, 1997), p. 39.
- [7] P. Gambino and P. A. Grassi, Phys. Rev. D **62**, 076002 (2000) [arXiv:hep-ph/9907254]; P. A. Grassi, B. A. Kniehl and A. Sirlin, Phys. Rev. Lett. **86**, 389 (2001) [arXiv:hep-th/0005149]; Phys. Rev. D **65**, 085001 (2002) [arXiv:hep-ph/0109228].
- [8] B. A. Kniehl and A. Sirlin, Phys. Rev. Lett. **81**, 1373 (1998) [arXiv:hep-ph/9805390]; Phys. Lett. B **440**, 136 (1998) [arXiv:hep-ph/9807545].



- [9] B. A. Kniehl and A. Sirlin, Phys. Lett. B **530**, 129 (2002) [arXiv:hep-ph/0110296].
- [10] M. L. Nekrasov, Phys. Lett. B **531**, 225 (2002) [arXiv:hep-ph/0102283].