DESY 07-107

New Global Fit to the Total Photon-Proton Cross-Section σ_{L+T} and to the Structure Function F_2

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A fit to world data on the photon-proton cross section σ_{L+T} and the unpolarised structure function F_2 is presented. The 23-parameter ALLM model based on Reggeon and Pomeron exchange is used. Cross section data were reconstructed to avoid inconsistencies with respect to R of the published F_2 data base. Parameter uncertainties and correlations are obtained.

1 Introduction

Deep-inelastic scattering on protons has been studied precisely in the last decades at various energies covering a large kinematic region provided by collider and fixed target experiments, thus providing us with our modern understanding of the proton structure.

The inclusive DIS cross section in the one-photon-exchange approximation is related to the unpolarized structure function $F_2(x, Q^2)$ and the ratio $R(x, Q^2)$ of longitudinal and transverse photo-absoption cross section:

$$\frac{d^2\sigma}{dx\, dQ^2} = \frac{4\pi\alpha_{em}^2}{Q^4}\frac{F_2(x,Q^2)}{x}\left\{1 - y - \frac{Q^2}{4E^2} + \left(1 - \frac{2m^2}{Q^2}\right)\frac{y^2 + Q^2/E^2}{2[1 + R(x,Q^2)]}\right\} . \qquad (1)$$

Here, Q^2 is the square of the photon 4-momentum and $x = Q^2/2M\nu$ with the proton mass M and the photon energy ν in the proton rest frame.

From Eq. (1) it follows that a measurement of the cross section alone is not sufficient to extract both, F_2 and R, and that only a variation of the beam energy E in the proton rest frame for fixed kinematic conditions can give access to both quantities. Alternatively, F_2 can be extracted using parameterizations of world data on R: two common examples are R_{1990} [1] and R_{1998}[2], whose differences reflect the states of world knowledge at the time they were obtained. The sensitivity of the cross section to R increases with y as it can be seen in Eq. (1). The discrepancy in the extracted values of F_2 using the two parameterizations can exceed 4% in the regions of maximum y.

The structure function F_2 is related to the photon-proton cross section σ_{L+T} by the expression:

$$\sigma_{L+T} = \frac{4\pi^2\alpha_{em}}{Q^4}\frac{Q^2 + 4M^2x^2}{1-x}F_2 . \qquad (2)$$

For virtual photons this relation employs the Hand convention for the virtual photon flux. It was used for technical convenience of consistency between real and virtual photon processes.

This paper reports on a new fit of the photon-proton cross section σ_{L+T} which reflects the recent world knowlege on the cross section and is self-consistent with respect to the use of R, since the cross sections were reconstructed in each case using the value of R that had been used to extract the published values of F_2. A result of the fit is a facility to calculate values of F_2 based on a single parameterization of $R = R_{1998}$.

2 The fit

The fit includes 2740 data points: 574 from the SLAC experiments E49a, E49b, E61, E87, E89a, E89b [3]; 292 from NMC [4]; 787 from H1 [5]; 570 from ZEUS [6]; 91 from E665 [7]; 229 points from BCDMS [8]. Real photon data comprise 196 points from Ref. [9] and 1 from ZEUS [10].

The ALLM functional form is a 23-parameter model of σ_{L+T} where F_2 is described by Reggeon and Pomeron exchange, valid for $W^2 > 4\,\mathrm{GeV}^2$, i.e., above the resonance region, and any Q^2 including the real γ process. Here, W^2 is the invariant squared mass of the photon-proton system. For details on the parameterization we refer to the original papers [11, 12]. The new fit was performed by minimizing the χ^2 defined in Eq. (3) where $D_{i,k} \pm \sigma_{i,k}^{stat} \pm \sigma_{i,k}^{syst}$ are the values of σ_{L+T} for data point i within the data set k, δ_k is the normalization uncertainty in data set k quoted by the experiment, ν_k is a parameter for the normalization of each data set in units of the normalization uncertainty, $T(\mathbf{p}, W^2, Q^2)$ is the functional form of the 23-parameter ALLM parameterization.

The χ^2 takes into account uncorrelated point-by-point statistical and systematic uncertainties and overall normalization uncertainties. The normalization parameters ν_k determine the size of the shifts in units of the normalization uncertainties δ_k.

Parameter	ALLM97	this fit	uncertainty
$m_0^2(\mathrm{GeV}^2)$	0.31985	0.454	0.137
$m_{\mathcal{P}}^2(\mathrm{GeV}^2)$	49.457	30.7	13.4
$m_{\mathcal{R}}^2(\mathrm{GeV}^2)$	0.15052	0.118	0.224
$Q_0^2(\mathrm{GeV}^2)$	0.52544	1.13	1.47
$\Lambda_0^2(\mathrm{GeV}^2)$	0.06527	0.06527	-
$a_{\mathcal{P}1}$	-0.0808	-0.105	0.024
$a_{\mathcal{P}2}$	0.44812	-0.496	0.154
$a_{\mathcal{P}3}$	1.1709	1.31	1.04
$b_{\mathcal{P}4}$	0.36292	-1.43	2.31
$b_{\mathcal{P}5}$	1.8917	4.50	2.46
$b_{\mathcal{P}6}$	1.8439	0.554	0.531
$c_{\mathcal{P}7}$	0.28067	0.339	0.093
$c_{\mathcal{P}8}$	0.22291	0.128	0.104
$c_{\mathcal{P}9}$	2.1979	1.17	1.14
$a_{\mathcal{R}1}$	0.584	0.373	0.150
$a_{\mathcal{R}2}$	0.37888	0.994	0.443
$a_{\mathcal{R}3}$	2.6063	0.781	0.524
$b_{\mathcal{R}4}$	0.01147	2.70	1.84
$b_{\mathcal{R}5}$	3.7582	1.83	2.39
$b_{\mathcal{R}6}$	0.49338	1.26	1.33
$c_{\mathcal{R}7}$	0.80107	0.837	0.500
$c_{\mathcal{R}8}$	0.97307	2.34	2.34
$c_{\mathcal{R}9}$	3.4942	1.79	0.93

Table 1: Parameters of the functional form used in the ALLM parameterization [11]. Results of the ALLM97 fit [12] without uncertainties in comparison to the results discussed in this paper with uncertainties. These uncertainties correspond only to the diagonal elements of the full covariance matrix which must be used to calculate uncertainties in F_2 or cross sections. The parameter Λ_0^2 has no uncertainty as it was fixed in the fit.

$$\begin{aligned}\chi^2(\mathbf{p},\boldsymbol{\nu}) &= \sum_{i,k} \frac{[D_{i,k}(W^2,Q^2)\cdot(1+\delta_k\nu_k) - T(\mathbf{p},W^2,Q^2)]^2}{({\sigma_{i,k}^{stat}}^2 + {\sigma_{i,k}^{syst}}^2)\cdot(1+\delta_k\nu_k)^2} + \sum_k \nu_k^2 \\ &\approx \sum_{i,k} \frac{[D_{i,k}(W^2,Q^2) - T(\mathbf{p},W^2,Q^2)\cdot(1-\delta_k\nu_k)]^2}{{\sigma_{i,k}^{stat}}^2 + {\sigma_{i,k}^{syst}}^2} + \sum_k \nu_k^2\ , \qquad (3)\end{aligned}$$

In order to keep the number of free parameters as small as possible, the normalization parameters are determined analytically in each minimization step using the relation

$$\nu_k = \frac{\sum_i \delta_k T_{i,k}(T_{i,k} - D_{i,k})/\sigma_{i,k}^2}{\sum_i T_{i,k}^2 \delta_k^2/\sigma_{i,k}^2 + 1}, \quad (4)$$

obtained by requiring $\partial\chi^2/\partial\nu_k = 0$ in the context of the approximation for χ^2 in the second line of Eq. (3); here $\sigma_{i,k}^2 = \sigma_{i,k}^{stat\,2} + \sigma_{i,k}^{syst\,2}$. This separate extraction is possible since the normalization parameters are not correlated and depend only on the involved data points and the functional parameters. The resulting fit has a reduced χ^2 equal to 0.94; the contributions from each data set, together with the normalization parameters can be found in Ref. [13]. Table 1 shows the final parameters from this fit with the corresponding uncertainties and, for comparison, the parameters from the ALLM97 fit. Figure **??** shows the new fit in comparison with world data and with the ALLM97 fit. A full comparison between the two fits is not possible as in the ALLM97 fit parameter uncertainties were not provided. Presumely, these uncertainties are larger than the those of the new fit, since the size of the current data set is nearly twice as large. The uncertainties in the cross sections calculated from the fit as represented by the error bands in the figure are much smaller than individual error bars on the original data points because of the smoothness constraint inherent in the fitted model. The fit evaluated at any kinematic point is effectively an average of a number of data points.

In conclusion, a new fit of world data on σ_{L+T} and F_2 is presented. Such a fit is consistent in the choice of the R parameterization R_{1998}. Also, for the first time, parameter and fit uncertainties are calculated. A subroutine that allows the calculation of σ_{L+T} and F_2 with their fit uncertainties is available upon request from the authors.

3 Acknowledgments

The authors would like to thank E.C. Aschenauer, A. Miller and W.-D. Nowak for valuable discussions.

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