Revised Version

Phys.Rev.D76:093009;2007
DESY 07-097

DESY 07-097
June 2007

Probing New Physics in the Neutrinoless Double Beta Decay Using Electron Angular Correlation

A. Ali [1]

Deutsches Elektronen-Synchrotron, DESY, 22607 Hamburg, Germany

A.V. Borisov[2] , **D.V. Zhuridov**[3]

Faculty of Physics, Moscow State University, 119991 Moscow, Russia

Abstract

The angular correlation of the electrons emitted in the neutrinoless double beta decay ($0\nu2\beta$) is presented using a general Lorentz invariant effective Lagrangian for the leptonic and hadronic charged weak currents. We show that the coefficient K in the angular correlation $d\Gamma/d\cos\theta \propto (1 - K\cos\theta)$ is essentially independent of the nuclear matrix element models and present its numerical values for the five nuclei of interest (^{76}Ge, ^{82}Se, ^{100}Mo, ^{130}Te, and ^{136}Xe), assuming that the $0\nu2\beta$-decays in these nuclei are induced solely by a light Majorana neutrino, ν_M. This coefficient varies between $K = 0.81$ (for the ^{76}Ge nucleus) and $K = 0.88$ (for the ^{82}Se and ^{100}Mo nuclei), calculated taking into account the effects from the nucleon recoil, the S and P-waves for the outgoing electrons and the electron mass. Deviation of K from its values derived here would indicate the presence of New Physics (NP) in addition to a light Majorana neutrino, and we work out the angular coefficients in several $\nu_M +$ NP scenarios for the ^{76}Ge nucleus. As an illustration of the correlations among the $0\nu2\beta$ observables (half-life $T_{1/2}$, the coefficient K, and the effective Majorana neutrino mass $|\langle m\rangle|$) and the parameters of the underlying NP model, we analyze the left-right symmetric models, taking into account current phenomenological bounds on the right-handed W_R-boson mass and the left-right mixing parameter ζ.

[1]e-mail: ahmed.ali@desy.de
[2]e-mail: borisov@phys.msu.ru
[3]e-mail: jouridov@mail.ru

1 Introduction

It is now established beyond any doubt that the observed neutrinos have tiny but non-zero masses and they mix with each other, with both of these features following from the observation of the atmospheric and solar neutrino oscillations and from the long baseline neutrino oscillation experiments [1]. Theoretically, it is largely anticipated that the neutrinos are Majorana particles. Experimental evidence for the neutrinoless double beta decay ($0\nu 2\beta$) would deliver a conclusive confirmation of the Majorana nature of neutrinos, establishing the existence of physics beyond the standard model. This is the overriding interest in carrying out these experiments and in the related phenomenology [2].

We recall that $0\nu 2\beta$-decays are forbidden in the standard model (SM) by lepton number (LN) conservation, which is a consequence of the renormalizability of the SM. However, being the low energy limit of a more general theory, an extended version of the SM could contain nonrenormalizable terms (tiny to be compatible with experiments), in particular, terms that violate LN and allow the $0\nu 2\beta$ decay. Probable mechanisms of LN violation may include exchanges by: Majorana neutrinos ν_Ms [3, 4, 5] (the preferred mechanism after the observation of neutrino oscillations [1]), SUSY particles [6, 7, 8, 9, 10, 11], scalar bilinears (SBs) [12], e.g. doubly charged dileptons (the component ξ^{--} of the $SU(2)_L$ triplet Higgs scalar etc.), leptoquarks (LQs) [13], right-handed W_R bosons [5, 14] etc. From these particles light νs are much lighter than the electron and others are much heavier than the proton. Therefore, there are two possible classes of mechanisms for the $0\nu 2\beta$ decay. With the light νs in the intermediate state the mechanism is called long range and otherwise it is referred to as the short range mechanism. For both these classes, the separation of the lepton physics from the hadron physics takes place [15], which simplifies calculations. According to the Schechter–Valle theorem [16], any mechanism inducing the $0\nu 2\beta$ decay produces an effective Majorana mass for the neutrino, which must therefore contribute to this decay. These various contributions will have to be disentangled to extract information from the $0\nu 2\beta$ decay on the characteristics of the sources of LN violation, in particular, on the neutrino masses and mixing. Measurements of the neutrinoless double beta decay in different nuclei will help in determining the underlying physics mechanism [17, 18].

Our aim in this paper is to examine the possibility to discriminate among the various possible mechanisms contributing to the $0\nu 2\beta$-decays using the information on the angular correlation of the final electrons in the process $N_i(A, Z) \to N_f(A, Z+2) + e^- + e^-$. A preliminary study along these lines was published by us in 2006 [19], with admittedly simplified treatment neglecting the nucleon recoil and the P-wave effects in the outgoing electron wave function. We rectify these shortcomings and provide in this paper a detailed account of the improved treatment. Restricting ourselves to the long-range mechanism, treating the electrons relativistically but with non-relativistic nucleons, we derive the angular correlation between the electrons using the general Lorentz invariant effective Lagrangian involving the leptonic and hadronic charged weak currents. Generally, this angular correlation can be expressed as $d\Gamma/d\cos\theta \sim 1 - K\cos\theta$, where θ is the angle between the electron momenta in the rest frame of the parent nucleus. Expressing $K = \mathcal{B}/\mathcal{A}$, with $-1 < K < 1$, we derive the analytic expressions for $\mathcal{A}$ and $\mathcal{B}$ for the effective Lagrangian characterized by the coefficients $\epsilon_{\alpha i}^{\beta}$ encoding the standard, $(V-A)\otimes(V-A)$, and new physics contributions (see Eq. (1)). Essential steps of these derivations are presented in section 2. The analytic expressions derived here confirm the earlier detailed derivations by Doi et al. [5], and we specify where the treatment presented here transcends the earlier work. Specific cases are relegated to Appendix A (for the decays involving scalar nonstandard terms), Appendix B (for the vector nonstandard terms), and Appendix C (for the tensor nonstandard terms). We hope to return to the discussion of including the short-range mechanism, neglected in this paper, in future work.

Numerical analysis of the electron angular correlation is presented in section 3, and the coefficient K for the various underlying mechanisms in $0\nu 2\beta$-decays are worked out. In particular, numerical values of K for the five nuclei of current experimental interest: ^{76}Ge, ^{82}Se, ^{100}Mo, ^{130}Te, and ^{136}Xe are presented for the light Majorana neutrino ν_M case. Their values range from $K = 0.81$ (for the ^{76}Ge nucleus) and $K = 0.88$ (for the ^{82}Se and ^{100}Mo nuclei). To study the uncertainty in the nuclear matrix elements, we have employed the so-called QRPA model with and without the p-n pairing for the ^{76}Ge nucleus [20], and a more modern QRPA model, fixing the particle-particle pairing strength [21]. While the uncertainty due to the nuclear matrix element model is quite marked for $T_{1/2}$ in some cases, we show that it is rather modest for K, not exceeding 10% for the models discussed here. For the ν_M + NP scenarios, we remark that the nonstandard coefficients $\epsilon^{V-A}_{V\mp A}$, $\epsilon^{T_L}_{T_R}$, and $\epsilon^{T_R}_{T_L}$ do not change the value of the angular coefficient K. The contribution of the scalar nonstandard term from the $\epsilon^{S+P}_{S\mp P}$ coefficients is found to be numerically small. So, what concerns the angular correlation, we have essentially three distinct scenarios: (i) Standard (ν_M), (ii) R-parity violating SUSY ($\nu_M + \epsilon^{T_R}_{T_R}$), and (iii) left-right-symmetric models ($\nu_M + \epsilon^{V\mp A}_{V+A}$). Numerical analysis of the coefficient K in the extended ν_M+NP scenario is carried out for the decay of the ^{76}Ge nucleus using the nuclear matrix element model already specified.

We take a closer look at the underlying physics behind the coefficients $\epsilon^{V\mp A}_{V\mp A}$ in section 4. These coefficients appear in the context of the left-right symmetric models which are theoretically well motivated [22]. Also, the corresponding nuclear matrix elements are available in the literature. Making use of them, we work out the correlations among the angular coefficient K, the half-life $T_{1/2}$ and either the mass of the right-handed W_R boson, m_{W_R}, or the W boson's mixing angle ζ, taking into account the current bounds on the various parameters. Results are presented in Figs. 1 – 4. The differential distribution $d\Gamma/d\cos\theta$ for the $0\nu2\beta$ decay of the ^{76}Ge nucleus is shown in Fig. 5 for some representative values of $|\langle m\rangle|$ for $m_{W_R} = 1,\ 1.5$ TeV and for an infinitely heavy m_{W_R}. It is seen that the effect of the right-handed W_R-boson is more marked in the angular correlation for smaller values of $|\langle m\rangle|$.

2 Angular correlation for the long range mechanism of $0\nu2\beta$ decay

2.1 General effective Lagrangian

For the decay mediated by light ν_Ms, the most general effective Lagrangian is the Lorentz invariant combination of the leptonic j_α and the hadronic J_α currents of definite tensor structure and chirality [23, 24]

$$\mathcal{L} = \frac{G_F V_{ud}}{\sqrt{2}}[(U_{ei} + \epsilon^{V-A}_{V-A,i})j^{\mu i}_{V-A}J^{+}_{V-A,\mu} + {\sum_{\alpha,\beta}}' \epsilon^{\beta}_{\alpha i} j^{i}_{\beta} J^{+}_{\alpha} + \text{H.c.}]\ , \tag{1}$$

where the hadronic and leptonic currents are defined as: $J^{+}_{\alpha} = \bar{u}O_\alpha d$ and $j^{i}_{\beta} = \bar{e}O_\beta \nu_i$; the leptonic currents contain neutrino mass eigenstates and the index i runs over the light eigenstates. Here and thereafter, a summation over the repeated indices is assumed; $\alpha,\beta{=}V\mp A, S\mp P, T_{L,R}$ ($O_{T_\rho} = 2\sigma^{\mu\nu}P_\rho$, $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu,\gamma^\nu]$, $P_\rho = (1\mp\gamma_5)/2$ is the projector, $\rho = L,\ R$); the prime indicates the summation over all the Lorentz invariant contributions, except for $\alpha = \beta = V-A$, U_{ei} is the PMNS mixing matrix [25] and V_{ud} is the CKM matrix element [1]. Note that in Eq. (1) the currents have been scaled relative to the strength of the usual $V-A$ interaction with G_F being the Fermi coupling constant. The coefficients $\epsilon^{\beta}_{\alpha i}$ encode new physics, parametrizing deviations of the Lagrangian from the standard $V-A$ current-current form and mixing of the non-SM neutrinos.

In discussing the extension of the SM for the $0\nu2\beta$ decay, Ref. [5] considered explicitly only nonstandard terms with

$$\epsilon^{V-A}_{V+A,i} = \kappa\frac{g'_V}{g_V}U'_{ei}, \quad \epsilon^{V+A}_{V-A,i} = \eta V'_{ei}, \quad \epsilon^{V+A}_{V+A,i} = \lambda\frac{g'_V}{g_V}V_{ei}\ . \tag{2}$$

Implicitly, also the contributions encoded by the coefficients $\epsilon^{V-A}_{V-A,i}$ are discussed arising from the non-SM contribution to U_{ei} in $SU(2)_L\times SU(2)_R\times U(1)$ models with mirror leptons (see Ref. [5], Eq. (A.2.17)). Here V, U' and V' are the 3×3 blocks of mixing matrices for non-SM neutrinos, e.g., for the usual $SU(2)_L\times SU(2)_R\times U(1)$ model V describes the lepton mixing for neutrinos from right-handed lepton doublets; for $SU(2)_L\times SU(2)_R\times U(1)$ model with mirror leptons [26] U' (V') describes the lepton mixing for mirror left(right)-handed neutrinos [5] etc. The form factors g_V and g'_V are expressed through the mixing angles for left- and right-handed quarks. Thus, $g_V = \cos\theta_C = V_{ud}$ and $g'_V = e^{i\delta}\cos\theta'_C$, with θ_C being the Cabibbo angle, θ'_C is its right-handed mixing analogoue, and the CP violating phase δ arises in these models due to both the mixing of right-handed quarks and the mixing of left- and right-handed gauge bosons (see Ref. [5], Eq. (3.1.11)). The parameters κ, η, and λ characterize the strength of nonstandard effects. Below, we give some illustrative examples relating the coefficients $\epsilon^{V-A}_{V-A,i}$, $\epsilon^{V+A}_{V\pm A,i}$ and the particle masses, couplings and the mixing parameters in the underlying theoretical models.

In the R-parity-violating (RPV) SUSY accompanying the neutrino exchange mechanism [6, 7, 8, 9, 10, 11], SUSY particles (sleptons, squarks) are present in one of the two effective 4-fermion vertices. (The other vertex contains the usual W_L boson.) The nonzero parameters are

$$\epsilon^{V-A}_{V-A,i} = \frac{1}{2}\eta^{n1}_{(q)RR}U_{ni}, \quad \epsilon^{S-P}_{S+P,i} = 2\eta^{n1}_{(l)LL}U_{ni},$$
$$\epsilon^{S+P}_{S+P,i} = -\frac{1}{4}\left(\eta^{n1}_{(q)LR} - 4\eta^{n1}_{(l)LR}\right)U^*_{ni}, \quad \epsilon^{T_R}_{T_R,i} = \frac{1}{8}\eta^{n1}_{(q)LR}U^*_{ni}, \tag{3}$$

where the index n runs over e, μ, τ (1, 2, 3), and the RPV Minimal Supersymmetric Model (MSSM) parameters ηs depend on the couplings of the RPV MSSM superpotential, the masses of the squarks and the sleptons, the mixings among the squarks and among the sleptons. Concentrating on the dominant contributions $\epsilon^{S+P}_{S+P,i}$ and $\epsilon^{T_R}_{T_R,i}$ (as the others are helicity-suppressed), one can express $\eta^{n1}_{(q)LR}$ and $\eta^{n1}_{(l)LR}$ as follows [10]

$$\eta^{n1}_{(q)LR} = \sum_k \frac{\lambda'_{11k}\lambda'_{nk1}}{2\sqrt{2}G_F}\sin 2\theta^d_{(k)}\left(\frac{1}{m^2_{\tilde{d}_1(k)}} - \frac{1}{m^2_{\tilde{d}_2(k)}}\right),$$
$$\eta^{n1}_{(l)LR} = \sum_k \frac{\lambda'_{k11}\lambda_{n1k}}{2\sqrt{2}G_F}\sin 2\theta^e_{(k)}\left(\frac{1}{m^2_{\tilde{e}_1(k)}} - \frac{1}{m^2_{\tilde{e}_2(k)}}\right), \tag{4}$$

where k is the generation index, $\theta^d_{(k)}$ and $\theta^e_{(k)}$ are the squark and slepton mixing angles, respectively, $m_{\tilde{f}_1}$ and $m_{\tilde{f}_2}$ are the sfermion mass eigenvalues, and λ_{ijk} and λ'_{ijk} are the RPV-couplings in the superpotential.

For the mechanism with LQs in one of the effective vertices [13], the nonzero coefficients are

$$\epsilon^{S+P}_{S-P} = -\frac{\sqrt{2}}{4G_F}\frac{\epsilon_V}{M^2_V}, \quad \epsilon^{S+P}_{S+P} = -\frac{\sqrt{2}}{4G_F}\frac{\epsilon_S}{M^2_S},$$
$$\epsilon^{V+A}_{V-A} = -\frac{1}{2G_F}\left(\frac{\alpha^{(L)}_S}{M^2_S} + \frac{\alpha^{(L)}_V}{M^2_V}\right), \quad \epsilon^{V+A}_{V+A} = -\frac{\sqrt{2}}{4G_F}\left(\frac{\alpha^{(R)}_S}{M^2_S} + \frac{\alpha^{(R)}_V}{M^2_V}\right), \tag{5}$$

where

$$\epsilon^\beta_\alpha = U_{ei}\epsilon^\beta_{\alpha i}, \tag{6}$$

the parameters $\epsilon_{S(V)}$, $\alpha^{(L)}_{S(V)}$, and $\alpha^{(R)}_{S(V)}$ depend on the couplings of the renormalizable LQ-quark-lepton interactions consistent with the SM gauge symmetry, the mixing parameters and the common mass scale $M_{S(V)}$ of the scalar (vector) LQs [27].

The nonzero ϵ^β_α for the discussed models are collected in Table 1.

Table 1: Nonzero coefficients ϵ^β_α for various models.

Model	Nonzero ϵs
with W_Rs	ϵ^{V-A}_{V+A}, $\epsilon^{V+A}_{V\mp A}$
RPV SUSY	$\epsilon^{S\mp P}_{S+P}$, ϵ^{V-A}_{V-A}, $\epsilon^{T_R}_{T_R}$
with LQs	$\epsilon^{S+P}_{S\mp P}$, $\epsilon^{V+A}_{V\mp A}$

The upper bounds on some of the ϵ^β_α parameters (6) from the Heidelberg–Moscow experiment were derived in Ref. [28] using the S-wave approximation for the electrons, considering nucleon recoil terms and only one nonzero parameter $\epsilon^\beta_{\alpha i}$ in the Lagrangian (1) at a time.

The coefficients $\epsilon^\beta_{\alpha i}$ entering the Lagrangian (1) can be expressed as

$$\epsilon^\beta_{\alpha i} = \hat{\epsilon}^\beta_\alpha U^{(\alpha,\beta)}_{ei}, \tag{7}$$

where $U^{(\alpha,\beta)}_{ei}$ are mixing parameters for non-SM neutrinos (see, e.g., Eq. (2)). As this Lagrangian describes also ordinary β-decays (without LN violation), the coefficients $\hat{\epsilon}^\beta_\alpha$ are constrained by the existing data on precision measurements in allowed nuclear beta decays, including neutron decay [29]. For example, from these data we obtain the conservative bound

$$\left|\hat{\epsilon}^{V+A}_{V+A}\right| < 7 \times 10^{-2}. \tag{8}$$

From Eqs. (6), (7), (8) and the bound $\left|\epsilon_{V+A}^{V+A}\right| < 7.9 \times 10^{-7}$ (see section 3.2) we can assume that the nonstandard mixing is small:

$$|U_{ei}V_{ei}| \lesssim 10^{-5}, \quad V_{ei} = U_{ei}^{(V+A,V+A)}. \tag{9}$$

2.2 Methods and approximations

We have calculated the leading order in the Fermi constant taking into account the leading contribution of the parameters ϵ_α^β to the decay matrix elements using the approximation of the relativistic electrons and non-relativistic nucleons. The wavefunction of an electron with the asymptotic momentum $\mathbf{p}$ and the spin projection s can be expanded in terms of spherical waves as [5, 30]

$$e_{\mathbf{p}s}(\mathbf{r}) = e_{\mathbf{p}s}^{S_{1/2}}(\mathbf{r}) + e_{\mathbf{p}s}^{P_{1/2}}(\mathbf{r}) + \ldots \tag{10}$$

We take into account the $S_{1/2}$ and the $P_{1/2}$ waves for the outgoing electrons:

$$e_{\mathbf{p}s}^{S_{1/2}}(\mathbf{r}) = \begin{pmatrix} \tilde{g}_{-1}\chi_s \\ \tilde{f}_1 \boldsymbol{\sigma}\cdot\hat{\mathbf{p}}\chi_s \end{pmatrix}, \tag{11}$$

$$e_{\mathbf{p}s}^{P_{1/2}}(\mathbf{r}) = i\begin{pmatrix} \tilde{g}_1 \boldsymbol{\sigma}\cdot\hat{\mathbf{r}}\boldsymbol{\sigma}\cdot\hat{\mathbf{p}}\chi_s \\ -\tilde{f}_{-1}\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}\chi_s \end{pmatrix}, \tag{12}$$

with $\hat{\mathbf{r}} = \mathbf{r}/r$, $\hat{\mathbf{p}} = \mathbf{p}/p$ and the two component spinor χ_s. We use the approximate radial wave functions [5]

$$\begin{pmatrix} \tilde{g}_{-1} \\ \tilde{f}_1 \end{pmatrix} = \tilde{A}_{\mp 1}\left[1 - \frac{1}{6}(\bar{p}r)^2\right], \tag{13}$$

$$(\bar{p}r)^2 = \left(\frac{3}{2}\alpha Z\right)^2 \left(\frac{r}{R}\right)^2 + 3\alpha Z\frac{r}{R}\varepsilon r + (pr)^2, \tag{14}$$

$$\begin{pmatrix} \tilde{g}_1 \\ \tilde{f}_{-1} \end{pmatrix} = \pm\tilde{A}_{\mp 1}\xi_\pm(\varepsilon)\frac{r}{R}, \quad \xi_\pm = \frac{1}{2}\alpha Z + \frac{1}{3}(\varepsilon \pm m_e)R, \tag{15}$$

including the finite de Broglie wave length correction (FBWC) for the $S_{1/2}$ wave. Here R is the nuclear radius, ε is the electron energy and α is the fine structure constant. For the normalization constants $\tilde{A}_{\pm 1}$ we use the approximate Eq. (45) (see below).

The nucleon matrix elements of the color singlet quark currents are [8, 31, 32, 33]

$$\langle P(k')|\bar{u}(1\mp\gamma_5)d|N(k)\rangle = \bar{\psi}(k')\left[F_S^{(3)}(q^2) \mp F_P^{(3)}(q^2)\gamma_5\right]\tau_+\psi(k), \tag{16}$$

$$\langle P(k')|\bar{u}\gamma^\mu(1\mp\gamma_5)d|N(k)\rangle = \bar{\psi}(k')\left[g_V(q^2)\gamma^\mu \mp g_A(q^2)\gamma^\mu\gamma_5 - ig_M(q^2)\frac{\sigma^{\mu\nu}q_\nu}{2m_p} \pm g_P(q^2)\gamma_5 q^\mu\right]\tau_+\psi(k), \tag{17}$$

$$\langle P(k')|\bar{u}\sigma^{\mu\nu}(1\mp\gamma_5)d|N(k)\rangle = \bar{\psi}(k')\left[J^{\mu\nu} \mp \frac{i}{2}\epsilon^{\mu\nu\rho\sigma}J_{\rho\sigma}\right]\tau_+\psi(k), \tag{18}$$

$$J^{\mu\nu} = T_1^{(3)}(q^2)\sigma^{\mu\nu} + \frac{iT_2^{(3)}}{m_p}(\gamma^\mu q^\nu - \gamma^\nu q^\mu) + \frac{T_3^{(3)}}{m_p^2}(\sigma^{\mu\rho}q_\rho q^\nu - \sigma^{\nu\rho}q_\rho q^\mu), \tag{19}$$

where

$$\psi = \begin{pmatrix} P \\ N \end{pmatrix} \tag{20}$$

is a nucleon isodoublet.

The non-relativistic structure of the nucleon currents in the impulse approximation is derived using Refs [32, 34], see Appendices A, B, and C. We have calculated the nucleon recoil terms including the recoil terms due to the pseudoscalar form factor.

Table 2: Expressions for $\mathcal{A}$ in Eqs. (21) and (22) for the stated choice of ϵ_α^β.

ϵ	$\mathcal{A}$
ϵ_{V-A}^{V-A}	$\mathcal{A}_0 + 4C_1\|\mu\|\|\mu_{V-A}^{V-A}\|c_{02} + 4C_1\|\mu_{V-A}^{V-A}\|^2$
ϵ_{V+A}^{V-A}	$\mathcal{A}_0 + 4C_0\|\mu\|\|\mu_{V+A}^{V-A}\|c_{01} + 4C_{1+}\|\mu_{V+A}^{V-A}\|^2$
ϵ_{V-A}^{V+A}	$\mathcal{A}_0 + C_3\|\mu\|\|\epsilon_{V-A}^{V+A}\|c_2 + C_5\|\epsilon_{V-A}^{V+A}\|^2$
ϵ_{V+A}^{V+A}	$\mathcal{A}_0 + C_2\|\mu\|\|\epsilon_{V+A}^{V+A}\|c_1 + C_4\|\epsilon_{V+A}^{V+A}\|^2$
ϵ_{S-P}^{S-P}	$\mathcal{A}_0 + 4C_0^{SP}\|\mu\|\|\mu_{S-P}^{S-P}\|c_{04} + 4C_1^{SP}\|\mu_{S-P}^{S-P}\|^2$
ϵ_{S+P}^{S-P}	$\mathcal{A}_0 + 4C_0^{SP}\|\mu\|\|\mu_{S+P}^{S-P}\|c_{03} + 4C_1^{SP}\|\mu_{S+P}^{S-P}\|^2$
ϵ_{S-P}^{S+P}	$\mathcal{A}_0 + C_2^{SP}\|\mu\|\|\epsilon_{S-P}^{S+P}\|c_4 + C_3^{SP}\|\epsilon_{S-P}^{S+P}\|^2$
ϵ_{S+P}^{S+P}	$\mathcal{A}_0 + C_2^{SP}\|\mu\|\|\epsilon_{S+P}^{S+P}\|c_3 + C_3^{SP}\|\epsilon_{S+P}^{S+P}\|^2$
$\epsilon_{T_L}^{T_L}$	$\mathcal{A}_0 + 4C_0^{T}\|\mu\|\|\mu_{T_L}^{T_L}\|c_{06} + 4C_1^{T}\|\mu_{T_L}^{T_L}\|^2$
$\epsilon_{T_R}^{T_L}$, $\epsilon_{T_L}^{T_R}$	$\mathcal{A}_0$
$\epsilon_{T_R}^{T_R}$	$\mathcal{A}_0 + C_2^{T}\|\mu\|\|\epsilon_{T_R}^{T_R}\|c_5 + C_3^{T}\|\epsilon_{T_R}^{T_R}\|^2$

Table 3: Expressions for $\mathcal{B}$ in Eq. (22) for the stated choice of ϵ_α^β.

ϵ	$\mathcal{B}$
ϵ_{V-A}^{V-A}	$\mathcal{B}_0 + 4D_1\|\mu\|\|\mu_{V-A}^{V-A}\|c_{02} + 4D_1\|\mu_{V-A}^{V-A}\|^2$
ϵ_{V+A}^{V-A}	$\mathcal{B}_0 + 4D_0\|\mu\|\|\mu_{V+A}^{V-A}\|c_{01} + 4D_{1+}\|\mu_{V+A}^{V-A}\|^2$
ϵ_{V-A}^{V+A}	$\mathcal{B}_0 + \|\mu\|\|\epsilon_{V-A}^{V+A}\|(D_3c_2 + D_{3-}s_2) + D_5\|\epsilon_{V-A}^{V+A}\|^2$
ϵ_{V+A}^{V+A}	$\mathcal{B}_0 + \|\mu\|\|\epsilon_{V+A}^{V+A}\|(D_2c_1 + D_{2-}s_1) + D_4\|\epsilon_{V+A}^{V+A}\|^2$
ϵ_{S-P}^{S-P}	$\mathcal{B}_0 + 4D_{0-}^{SP}\|\mu\|\|\mu_{S-P}^{S-P}\|s_{04} + 4D_1^{SP}\|\mu_{S-P}^{S-P}\|^2$
ϵ_{S+P}^{S-P}	$\mathcal{B}_0 + 4D_{0-}^{SP}\|\mu\|\|\mu_{S+P}^{S-P}\|s_{03} + 4D_1^{SP}\|\mu_{S+P}^{S-P}\|^2$
ϵ_{S-P}^{S+P}	$\mathcal{B}_0 + \|\mu\|\|\epsilon_{S-P}^{S+P}\|(D_2^{SP}c_4 + D_{2-}^{SP}s_4) + D_3^{SP}\|\epsilon_{S-P}^{S+P}\|^2$
ϵ_{S+P}^{S+P}	$\mathcal{B}_0 + \|\mu\|\|\epsilon_{S+P}^{S+P}\|(D_2^{SP}c_3 + D_{2-}^{SP}s_3) + D_3^{SP}\|\epsilon_{S+P}^{S+P}\|^2$
$\epsilon_{T_L}^{T_L}$	$\mathcal{B}_0 + 4D_{0-}^{T}\|\mu\|\|\mu_{T_L}^{T_L}\|s_{06} + 4D_1^{T}\|\mu_{T_L}^{T_L}\|^2$
$\epsilon_{T_R}^{T_L}$, $\epsilon_{T_L}^{T_R}$	$\mathcal{B}_0$
$\epsilon_{T_R}^{T_R}$	$\mathcal{B}_0 + D_2^{T}\|\mu\|\|\epsilon_{T_R}^{T_R}\|c_5 + D_3^{T}\|\epsilon_{T_R}^{T_R}\|^2$

2.3 Electron angular correlation

Taking into account the dominant terms introduced in the Appendices A, B, and C in the closure approximation [5] we obtain the differential width in $\cos\theta$ for the $0^+(A,Z) \to 0^+(A,Z+2)e^-e^-$ transitions:

$$\frac{d\Gamma}{d\cos\theta} = \frac{\ln 2}{2}|M_{GT}|^2\mathcal{A}(1 - K\cos\theta), \tag{21}$$

where θ is the angle between the electron momenta in the rest frame of the parent nucleus and the angular correlation coefficient is

$$K = \frac{\mathcal{B}}{\mathcal{A}}\,, \quad -1 < K < 1. \tag{22}$$

The Gamow–Teller nuclear matrix element M_{GT} is defined in Eq. (51) below.

The expressions for $\mathcal{A}$ and $\mathcal{B}$ for different choices of ϵ_α^β, with only one coefficient considered at a time, are shown in Tables 2 and 3.

In these tables

$$c_i = \cos\psi_i, \quad s_i = \sin\psi_i \tag{23}$$

and

$$\mu = \langle m\rangle/m_e, \quad \mu_\alpha^\beta = m_\alpha^\beta/m_e, \tag{24}$$

with the standard effective Majorana mass $\langle m\rangle = \sum_i U_{ei}^2 m_i$ and the nonstandard ones:

$$m_{S\mp P}^{S-P} = \sum_i U_{ei}\epsilon_{S\mp P,i}^{S-P} m_i, \quad m_{V\mp A}^{V-A} = \sum_i U_{ei}\epsilon_{V\mp A,i}^{V-A} m_i, \quad m_{T_{L,R}}^{T_L} = \sum_i U_{ei}\epsilon_{T_{L,R},i}^{T_L} m_i. \tag{25}$$

The quantities $\mathcal{A}$ and $\mathcal{B}$ for all zero ϵ_α^β are

$$\mathcal{A}_0 = C_1|\mu|^2, \quad \mathcal{B}_0 = D_1|\mu|^2 \tag{26}$$

and the relative phases are

$$\begin{aligned}
\psi_{01} &= \arg(\langle\mu\rangle\mu_{V+A}^{V-A*}), \quad \psi_{02} = \arg(\langle\mu\rangle\mu_{V-A}^{V-A*}),\\
\psi_{1} &= \arg(\langle\mu\rangle\epsilon_{V+A}^{V+A*}), \quad \psi_{2} = \arg(\langle\mu\rangle\epsilon_{V-A}^{V+A*}),\\
\psi_{03} &= \arg(\langle\mu\rangle\mu_{S+P}^{S-P*}), \quad \psi_{04} = \arg(\langle\mu\rangle\mu_{S-P}^{S-P*}),\\
\psi_{3} &= \arg(\langle\mu\rangle\epsilon_{S+P}^{S+P*}), \quad \psi_{4} = \arg(\langle\mu\rangle\epsilon_{S-P}^{S+P*}),\\
\psi_{06} &= \arg(\langle\mu\rangle\mu_{T_L}^{T_L*}),\\
\psi_{5} &= \arg(\langle\mu\rangle\epsilon_{T_R}^{T_R*}), \quad \psi_{6} = \arg(\langle\mu\rangle\epsilon_{T_L}^{T_R*}).
\end{aligned} \tag{27}$$

The coefficients C_i and $C_i^{(SP,T)}$ in Table 2 are

$$\begin{aligned}
C_0 &= (\chi_F^2 - 1)A_{01},\\
C_1 &= (\chi_F - 1)^2 A_{01},\\
C_{1+} &= (\chi_F + 1)^2 A_{01},\\
C_2 &= (\chi_F - 1)(\chi_{2-}A_{03} - \chi_{1+}A_{04}),\\
C_3 &= -(\chi_F - 1)(\chi_{2+}A_{03} - \chi_{1-}A_{04} - \chi'_P A_{05} + \chi'_R A_{06}),\\
C_4 &= \chi_{2-}^2 A_{02} - \frac{2}{9}\chi_{1+}\chi_{2-}A_{03} + \frac{1}{9}\chi_{1+}^2 A_{04},\\
C_5 &= \chi_{2+}^2 A_{02} - \frac{2}{9}\chi_{1-}\chi_{2+}A_{03} + \frac{1}{9}\chi_{1-}^2 A_{04} + \chi_P'^2 A_{08} - \chi'_P\chi'_R A_{07} + \chi_R'^2 A_{09};
\end{aligned} \tag{28}$$

$$\begin{aligned}
C_0^{SP} &= -(\chi_F - 1)\chi_F^{SP} A_{00}^{SP},\\
C_1^{SP} &= \chi_F^{SP2} A_{01}^{SP},\\
C_2^{SP} &= (\chi_F - 1)(2\chi_{F0}^{SP} - \chi_{P0}^{SP})A_{02}^{SP},\\
C_3^{SP} &= (2\chi_{F0}^{SP} - \chi_{P0}^{SP})^2 A_{03}^{SP};
\end{aligned} \tag{29}$$

$$\begin{aligned}
C_0^T &= \frac{T_1^{(3)}}{g_A}(\chi_F - 1)A_{00}^T,\\
C_1^T &= \left(\frac{T_1^{(3)}}{g_A}\right)^2 A_{01}^T,\\
C_2^T &= -(\chi_F - 1)\left[(\chi_{RC_\sigma}^{T\prime} + \chi_R^{T\prime} + \chi_{RT_\sigma}^{T\prime} - \chi_{RT}^{T\prime})A_{01} + \left(\frac{1}{3}\chi_{GT}^{T\prime} - 2\chi_T^{T\prime}\right)A_{02}^T\right],\\
C_3^T &= (\chi_{RC_\sigma}^{T\prime} + \chi_R^{T\prime} + \chi_{RT_\sigma}^{T\prime} - \chi_{RT}^{T\prime})^2 A_{09} + \left(\frac{1}{3}\chi_{GT}^{T\prime} - 2\chi_T^{T\prime}\right)^2 A_{03}^T\ .
\end{aligned} \tag{30}$$

The coefficients D_i and $D_i^{(SP,T)}$ entering in Table 3 are:

$$\begin{aligned}
D_0 &= (\chi_F^2 - 1)B_{01},\\
D_1 &= (\chi_F - 1)^2 B_{01}, \quad D_{1+} = (\chi_F + 1)^2 B_{01},
\end{aligned}$$

$$
\begin{aligned}
D_{2-} &= (\chi_F - 1)\chi_{2-}B_{03-}, \quad D_2 = -(\chi_F - 1)\chi_{1+}B_{04}, \\
D_3 &= (\chi_F - 1)(\chi_{2+}B_{03} - \chi'_P B_{05}), \\
D_{3-} &= -(\chi_F - 1)(\chi_{1-}B_{04-} - \chi'_P B_{05-} + \chi'_R B_{06-}), \\
D_4 &= -\chi_{2-}^2 B_{02} + \frac{1}{9}\chi_{1+}^2 B_{04}, \\
D_5 &= \chi_{2+}^2 B_{02} - \frac{1}{9}\chi_{1-}^2 B_{04} - \chi_P'^2 B_{08} + \chi'_P\chi'_R B_{07} - \chi_R'^2 B_{09};
\end{aligned} \tag{31}
$$

$$
\begin{aligned}
D_{0-}^{SP} &= (\chi_F - 1)\chi_F^{SP} B_{00-}^{SP}, \\
D_1^{SP} &= -\chi_F^{SP2} B_{01}^{SP}, \\
D_2^{SP} &= (\chi_F - 1)(2\chi_{F0}^{SP} - \chi_{P0}^{SP})B_{02}^{SP}, \\
D_{2-}^{SP} &= (\chi_F - 1)(2\chi_{F0}^{SP} - \chi_{P0}^{SP})B_{02-}^{SP}, \\
D_3^{SP} &= (2\chi_{F0}^{SP} - \chi_{P0}^{SP})^2 B_{03}^{SP};
\end{aligned} \tag{32}
$$

$$
\begin{aligned}
D_{0-}^T &= -\frac{T_1^{(3)}}{g_A}(\chi_F - 1)B_{00-}^T, \\
D_1^T &= -\left(\frac{T_1^{(3)}}{g_A}\right)^2 B_{01}^T, \\
D_2^T &= -(\chi_F - 1)\left[(\chi_{RC_\sigma}^{T\prime} + \chi_R^{T\prime} + \chi_{RT_\sigma}^{T\prime} - \chi_{RT}^{T\prime})B_{01} + \left(\frac{1}{3}\chi_{GT}^{T\prime} - 2\chi_T^{T\prime}\right) B_{02}^T\right], \\
D_3^T &= (\chi_{RC_\sigma}^{T\prime} + \chi_R^{T\prime} + \chi_{RT_\sigma}^{T\prime} - \chi_{RT}^{T\prime})^2 B_{09} + \left(\frac{1}{3}\chi_{GT}^{T\prime} - 2\chi_T^{T\prime}\right)^2 B_{03}^T,
\end{aligned} \tag{33}
$$

where the integrated phase space factors are

$$
\begin{pmatrix} A_{0k}, & A_{0k}^{(SP,T)} \\ B_{0k}, & B_{0k}^{(SP,T)} \end{pmatrix} = \frac{1}{\ln 2}\frac{a_{0\nu}}{(m_e R)^2}\int \begin{pmatrix} a_{0k}, & a_{0k}^{(SP,T)} \\ b_{0k}, & b_{0k}^{(SP,T)} \end{pmatrix} d\Omega_{0\nu}, \tag{34}
$$

with the phase space element $d\Omega_{0\nu}$ defined as follows:

$$
d\Omega_{0\nu} = m_e^{-5}|\mathbf{p}_1||\mathbf{p}_2|\varepsilon_1\varepsilon_2\delta(\varepsilon_1 + \varepsilon_2 + E_f - E_i)d\varepsilon_1 d\varepsilon_2 d(\hat{\mathbf{p}}_1 \cdot \hat{\mathbf{p}}_2)\ . \tag{35}
$$

The constant $a_{0\nu}$ and the kinematic factors $a_{0k}, a_{0k}^{(S,P,T)}, b_{0k}$ and $b_{0k}^{(S,P,T)}$ entering above are defined as follows:

$$
a_{0\nu} = (G_F g_A)^4 |V_{ud}|^4 m_e^9/(64\pi^5), \tag{36}
$$

$$
\begin{aligned}
&a_{01} = \alpha_+ + \beta_+, \quad a_{02} = \left(\frac{\varepsilon_{21}}{m_e}\right)^2 \beta_+, \quad a_{03} = 2\frac{\varepsilon_{21}}{m_e}\beta_-, \quad a_{04} = \frac{4}{9}\beta_+, \\
&a_{05} = \frac{4}{3}\left(\frac{\zeta\alpha_-}{m_e R} - 2\alpha_+\right), \quad a_{06} = \frac{8}{m_e R}\alpha_-, \quad a_{07} = \frac{1}{3}\left(\frac{4}{m_e R}\right)^2 (\zeta\alpha_+ - 2m_e R\alpha_-), \\
&a_{08} = \left(\frac{2}{3m_e R}\right)^2 [\zeta^2\alpha_+ + 4m_e R(m_e R\alpha_+ - \zeta\alpha_-)], \quad a_{09} = \left(\frac{4}{m_e R}\right)^2 \alpha_+;
\end{aligned} \tag{37}
$$

$$
\begin{aligned}
&a_{00}^{SP} = \alpha_-, \quad a_{01}^{SP} = \alpha_+, \quad a_{02}^{SP} = \frac{\varepsilon_{21}R}{3m_e}[\varepsilon_{21}(\alpha_+ + \beta_+) + 2m_e\beta_-], \\
&a_{03}^{SP} = \left(\frac{\varepsilon_{21}R}{6m_e}\right)^2 [\varepsilon_{21}^2\alpha_+ + (\varepsilon_{21}^2 + 4m_e^2)\beta_+ + 4\varepsilon_{21}m_e\beta_-];
\end{aligned} \tag{38}
$$

$$a_{00}^T = 2\beta_-, \quad a_{01}^T = 16\alpha_+ = 16a_{01}^{SP}, \quad a_{02}^T = \frac{8\zeta\beta_+}{m_eR},$$
$$a_{03}^T = \left(\frac{8\zeta}{m_eR}\right)^2 \beta_+; \tag{39}$$

$$b_{01} = \gamma_+ + \delta_+, \quad b_{02} = \left(\frac{\varepsilon_{21}}{m_e}\right)^2 \delta_+,$$
$$b_{03} = 2\frac{\varepsilon_{21}}{m_e}\delta_+, \quad b_{03-} = 2\frac{\varepsilon_{21}}{m_e}\delta_-,$$
$$b_{04} = \frac{4}{9}\delta_+, \quad b_{04-} = \frac{4}{9}\delta_-,$$
$$b_{05} = \frac{8}{3}\gamma_+, \quad b_{05-} = \frac{4}{3}\frac{\zeta\gamma_-}{m_eR},$$
$$b_{06-} = \frac{8\gamma_-}{m_eR}, \quad b_{07} = \frac{16}{3}\frac{\zeta\gamma_+}{(m_eR)^2},$$
$$b_{08} = \frac{16}{9}\left[\left(\frac{\zeta}{2m_eR}\right)^2 - 1\right]\gamma_+, \quad b_{09} = \left(\frac{4}{m_eR}\right)^2 \gamma_+; \tag{40}$$

$$b_{00-}^{SP} = \gamma_-, \quad b_{01}^{SP} = \gamma_+ = \frac{3}{8}b_{05},$$
$$b_{02}^{SP} = \frac{\varepsilon_{21}^2 R}{3m_e}(\gamma_+ + \delta_+), \quad b_{02-}^{SP} = \frac{2}{3}\varepsilon_{21}R\delta_-,$$
$$b_{03}^{SP} = \left(\frac{\varepsilon_{21}R}{6m_e}\right)^2 \left[\varepsilon_{21}^2(\gamma_+ + \delta_+) - 4m_e^2\delta_+\right]; \tag{41}$$

$$b_{00-}^T = 4\gamma_- = 4b_{00-}^{SP}, \quad b_{01}^T = 16\gamma_+ = 6b_{05},$$
$$b_{02}^T = \frac{8\zeta\delta_+}{m_eR}, \quad b_{03}^T = \left(\frac{8\zeta}{m_eR}\right)^2 \delta_+, \tag{42}$$

where $\varepsilon_{21} = \varepsilon_2 - \varepsilon_1$ is the difference in the electron energy. The characteristic features of the $P_{1/2}$-wave are expressed as

$$\zeta = 3\alpha Z + (\varepsilon_1 + \varepsilon_2)R \tag{43}$$

and the Coulomb corrections appear as the following combinations

$$\alpha_\pm = |\alpha_{-1-1}|^2 \pm |\alpha_{11}|^2, \quad \beta_\pm = |\alpha_{1-1}|^2 \pm |\alpha_{-11}|^2,$$
$$\gamma_+ = 2\mathrm{Re}(\alpha_{11}\alpha_{-1-1}^*), \quad \gamma_- = 2\mathrm{Im}(\alpha_{11}\alpha_{-1-1}^*),$$
$$\delta_+ = 2\mathrm{Re}(\alpha_{-11}\alpha_{1-1}^*), \quad \delta_- = 2\mathrm{Im}(\alpha_{-11}\alpha_{1-1}^*), \tag{44}$$

with $\alpha_{ij} = \tilde{A}_i(\varepsilon_2)\tilde{A}_j(\varepsilon_1)$.

For the normalization constants in the approximation including terms up to $(\alpha Z)^2$ [5]

$$\tilde{A}_{\pm 1} = \sqrt{\frac{\varepsilon \mp m_e}{2\varepsilon}F_0(Z,\varepsilon)},$$
$$F_0 = \frac{4}{\Gamma^2(2\gamma_1 + 1)}(2pR)^{2(\gamma_1 - 1)}|\Gamma(\gamma_1 + iy)|^2 e^{\pi y},$$
$$\gamma_1 = \sqrt{1 - (\alpha Z)^2}, \quad y = \alpha Z\varepsilon/p, \tag{45}$$

we have

$$\begin{pmatrix} \alpha_+ \\ \beta_+ \end{pmatrix} = \frac{1}{2}(\varepsilon_1\varepsilon_2 \pm m_e^2)C_{00}, \quad \begin{pmatrix} \alpha_- \\ \beta_- \end{pmatrix} = \frac{1}{2}(\varepsilon_2 \pm \varepsilon_1)m_e C_{00}, \tag{46}$$

$$\gamma_+ = \delta_+ = \frac{1}{2}|\mathbf{p}_1||\mathbf{p}_2|C_{00}, \quad \gamma_- = \delta_- = 0, \tag{47}$$

where

$$C_{00} = \frac{F_0(Z,\varepsilon_2)F_0(Z,\varepsilon_1)}{\varepsilon_2\varepsilon_1}. \tag{48}$$

Note that using Eq. (47) the expressions for $\mathcal{B}$ from Table 3 are reduced to the form shown in Table 4.

Table 4: Expressions for $\mathcal{B}$ in Eq. (22) for the stated choice of ϵ_α^β for the $\tilde{A}_{\pm 1}$ from Eq. (45).

ϵ	$\mathcal{B}$
ϵ_{V-A}^{V-A}	$\mathcal{B}_0 + 4D_1\|\mu\|\|\mu_{V-A}^{V-A}\|c_{02} + 4D_1\|\mu_{V-A}^{V-A}\|^2$
ϵ_{V+A}^{V-A}	$\mathcal{B}_0 + 4D_0\|\mu\|\|\mu_{V+A}^{V-A}\|c_{01} + 4D_{1+}\|\mu_{V+A}^{V-A}\|^2$
ϵ_{V-A}^{V+A}	$\mathcal{B}_0 + D_3\|\mu\|\|\epsilon_{V-A}^{V+A}\|c_2 + D_5\|\epsilon_{V-A}^{V+A}\|^2$
ϵ_{V+A}^{V+A}	$\mathcal{B}_0 + D_2\|\mu\|\|\epsilon_{V+A}^{V+A}\|c_1 + D_4\|\epsilon_{V+A}^{V+A}\|^2$
$\epsilon_{S\mp P}^{S-P}$	$\mathcal{B}_0 + 4D_1^{SP}\|\mu_{S\mp P}^{S-P}\|^2$
ϵ_{S-P}^{S+P}	$\mathcal{B}_0 + D_2^{SP}\|\mu\|\|\epsilon_{S-P}^{S+P}\|c_4 + D_3^{SP}\|\epsilon_{S-P}^{S+P}\|^2$
ϵ_{S+P}^{S+P}	$\mathcal{B}_0 + D_2^{SP}\|\mu\|\|\epsilon_{S+P}^{S+P}\|c_3 + D_3^{SP}\|\epsilon_{S+P}^{S+P}\|^2$
$\epsilon_{T_L}^{T_L}$	$\mathcal{B}_0 + 4D_1^T\|\mu_{T_L}^{T_L}\|^2$
$\epsilon_{T_R}^{T_L}$, $\epsilon_{T_L}^{T_R}$	$\mathcal{B}_0$
$\epsilon_{T_R}^{T_R}$	$\mathcal{B}_0 + D_2^T\|\mu\|\|\epsilon_{T_R}^{T_R}\|c_5 + D_3^T\|\epsilon_{T_R}^{T_R}\|^2$

In the definitions of C_i and D_i we use some combinations of nuclear parameters similar to the ones in Ref. [5]. Thus,

$$\chi_{2\pm} = \chi_{GT\omega} \pm \chi_{F\omega} - \frac{1}{9}\chi_{1\mp}; \quad \chi_{1\pm} = (\chi'_{GT} - 6\chi'_T) \pm 3\chi'_F;$$
$$\chi_F = \left(\frac{g_V}{g_A}\right)^2 \frac{M_F}{M_{GT}}; \quad \chi_k = \frac{g_V}{g_A}\frac{M_k}{M_{GT}}, \ k = P,\, R,\, RT;$$
$$\chi_k = \frac{M_k}{M_{GT}}, \ k = T,\, GT,\, RC_\sigma,\, RT_\sigma;$$
$$\chi_F^{SP} = \frac{F_S^{(3)}}{g_V}\chi_F; \quad \chi_{F0}^{SP} = \frac{F_S^{(3)}}{g_V}\left(\frac{g_V}{g_A}\right)^2\frac{M_{F0}}{M_{GT}}; \quad \chi_{P0}^{SP} = \frac{F_S^{(3)}}{g_A}\frac{g_V}{g_A}\frac{M_{P0}}{M_{GT}};$$
$$\chi_k^T = \frac{T_1^{(3)}}{g_A}\chi_k, \ k = R,\, RT,\, RC_\sigma,\, RT_\sigma; \quad \chi_k^T = \frac{T_1^{(3)}}{g_A}\frac{M_k^T}{M_{GT}}, \ k = GT,\, T, \tag{49}$$

where the index F refers to Fermi, GT to Gamow–Teller, T to tensor, P to the P-wave effect and R to the recoil effect. If χ has prime or the index ω than the same has the according matrix element in the numerator. The nuclear matrix elements defined below contain the operator $\tau_+^a = (\tau_1 + i\tau_2)^a/2$ converting the a-th neutron into the a-th proton, and the initial (final) nuclear state are denoted by $|0_i^+\rangle$ ($\langle 0_f^+|$)

$$M_F = \sum_N \langle 0_f^+|| \sum_{a\neq b} h_+(r_{ab}, E_N)\tau_+^a\tau_+^b||0_i^+\rangle, \tag{50}$$

$$M_{GT} = \sum_N \langle 0_f^+|| \sum_{a\neq b} h_+(r_{ab}, E_N)\boldsymbol{\sigma}_a\cdot\boldsymbol{\sigma}_b\tau_+^a\tau_+^b||0_i^+\rangle, \tag{51}$$

$$M_T = \sum_N \langle 0_f^+|| \sum_{a\neq b} h_+(r_{ab}, E_N)\left[(\boldsymbol{\sigma}_a\cdot\hat{\mathbf{r}}_{ab})(\boldsymbol{\sigma}_b\cdot\hat{\mathbf{r}}_{ab}) - \frac{1}{3}\boldsymbol{\sigma}_a\cdot\boldsymbol{\sigma}_b\right]\tau_+^a\tau_+^b||0_i^+\rangle, \tag{52}$$

$$M'_{GT} = \sum_N \langle 0_f^+ || \sum_{a\neq b} h'_+(r_{ab}, E_N) \boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b \tau_+^a \tau_+^b || 0_i^+ \rangle, \tag{53}$$

$$M'_F = \sum_N \langle 0_f^+ || \sum_{a\neq b} h'_+(r_{ab}, E_N) \tau_+^a \tau_+^b || 0_i^+ \rangle, \tag{54}$$

$$M'_T = \sum_N \langle 0_f^+ || \sum_{a\neq b} h'_+(r_{ab}, E_N) \left[(\boldsymbol{\sigma}_a \cdot \hat{\mathbf{r}}_{ab})(\boldsymbol{\sigma}_b \cdot \hat{\mathbf{r}}_{ab}) - \frac{1}{3} \boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b \right] \tau_+^a \tau_+^b || 0_i^+ \rangle, \tag{55}$$

$$M'_P = \sum_N \langle 0_f^+ || \sum_{a\neq b} h'_+(r_{ab}, E_N) \frac{i r_{+ab}}{2 r_{ab}} \{ (\boldsymbol{\sigma}_a - \boldsymbol{\sigma}_b) \cdot [\hat{\mathbf{r}}_{ab} \times \hat{\mathbf{r}}_{+ab}] \} \tau_+^a \tau_+^b || 0_i^+ \rangle, \tag{56}$$

$$M'_R = \sum_N \langle 0_f^+ || \sum_{a\neq b} h'_+(r_{ab}, E_N) \frac{R}{2 r_{ab}} \hat{\mathbf{r}}_{ab} \cdot (\boldsymbol{\sigma}_a \times \mathbf{D}_b + \mathbf{D}_a \times \boldsymbol{\sigma}_b) \tau_+^a \tau_+^b || 0_i^+ \rangle, \tag{57}$$

$$M_{GT\omega} = \sum_N \langle 0_f^+ || \sum_{a\neq b} h_{0\omega}(r_{ab}, E_N) \boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b \tau_+^a \tau_+^b || 0_i^+ \rangle, \tag{58}$$

$$M_{F\omega} = \sum_N \langle 0_f^+ || \sum_{a\neq b} h_{0\omega}(r_{ab}, E_N) \tau_+^a \tau_+^b || 0_i^+ \rangle, \tag{59}$$

$$M_{F0} = \sum_N \langle 0_f^+ || \sum_{a\neq b} h'_0(r_{ab}, E_N) \hat{\mathbf{r}} \cdot \hat{\mathbf{r}}_+ \tau_+^a \tau_+^b || 0_i^+ \rangle, \tag{60}$$

$$M_{P0} = \sum_N \langle 0_f^+ || \sum_{a\neq b} \frac{iR}{2r} h'_0(r_{ab}, E_N) \boldsymbol{\sigma}_+ \cdot [\hat{\mathbf{r}} \times \hat{\mathbf{r}}_+] \tau_+^a \tau_+^b || 0_i^+ \rangle, \tag{61}$$

$$M_{GT}^{T\prime} = \sum_N \langle 0_f^+ || \sum_{a\neq b} h'_+(r_{ab}, E_N) \frac{iR}{r} \boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b \tau_+^a \tau_+^b || 0_i^+ \rangle, \tag{62}$$

$$M_T^{T\prime} = \sum_N \langle 0_f^+ || \sum_{a\neq b} h'_+(r_{ab}, E_N) \frac{iR}{r} \left[(\boldsymbol{\sigma}_a \cdot \hat{\mathbf{r}}_{ab})(\boldsymbol{\sigma}_b \cdot \hat{\mathbf{r}}_{ab}) - \frac{1}{3} \boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b \right] \tau_+^a \tau_+^b || 0_i^+ \rangle, \tag{63}$$

$$M'_{RT} = \sum_N \langle 0_f^+ || \sum_{a\neq b} h'_+(r_{ab}, E_N) \frac{R}{2r} \hat{\mathbf{r}}_{ab} \cdot (\mathbf{T}_a - \mathbf{T}_b) \tau_+^a \tau_+^b || 0_i^+ \rangle, \tag{64}$$

$$M'_{RC_\sigma} = \sum_N \langle 0_f^+ || \sum_{a\neq b} h'_+(r_{ab}, E_N) \frac{iR}{2r} (\hat{\mathbf{r}}_{ab} \cdot \boldsymbol{\sigma}_a C_b - C_a \hat{\mathbf{r}}_{ab} \cdot \boldsymbol{\sigma}_b) \tau_+^a \tau_+^b || 0_i^+ \rangle, \tag{65}$$

$$M'_{RT_\sigma} = \sum_N \langle 0_f^+ || \sum_{a\neq b} h'_+(r_{ab}, E_N) \frac{iR}{2r} \hat{\mathbf{r}}_{ab} \cdot (\boldsymbol{\sigma}_a \times \mathbf{T}_b + \mathbf{T}_a \times \boldsymbol{\sigma}_b) \tau_+^a \tau_+^b || 0_i^+ \rangle \ . \tag{66}$$

In the above expressions, the neutrino potentials $h_i(r_{ab}, \langle E_N \rangle)$ are defined as follows:

$$h_+(r_{ab}, \langle E_N \rangle) = \frac{R}{4\pi^2} \int \frac{d\mathbf{k}}{\omega} \left(\frac{1}{\omega + A_1} + \frac{1}{\omega + A_2} \right) e^{i\mathbf{k}\cdot\mathbf{r}} \simeq R H(r, \bar{A}), \tag{67}$$

$$h_0(r_{ab}, \langle E_N \rangle) = \frac{1}{2\pi^2 \varepsilon_{12}} \int \frac{d\mathbf{k}}{\omega} \left(\frac{1}{\omega + A_1} - \frac{1}{\omega + A_2} \right) e^{i\mathbf{k}\cdot\mathbf{r}}$$

$$\simeq 2H(r, \bar{A}) + r \frac{\partial}{\partial r} H(r, \bar{A}), \tag{68}$$

$$h_{0\omega}(r_{ab}, \langle E_N \rangle) = h_+ - \bar{A} R h_0, \quad h'_+(r_{ab}, \langle E_N \rangle) = h_+ + \bar{A} R h_0, \tag{69}$$

$$h_R(r_{ab}, \langle E_N \rangle) = -\frac{\bar{A}}{m_p} \left[\frac{2}{\pi} \left(\frac{R}{r} \right)^2 - \bar{A} R h_+ \right], \tag{70}$$

with

$$H(r, \bar{A}) = \frac{1}{2\pi^2} \int \frac{d\mathbf{k}}{\omega} \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\omega + \bar{A}}, \tag{71}$$

$$A_j = \varepsilon_j + \langle E_N \rangle - E_i, \ i = 1, 2; \quad \bar{A} = \langle E_N \rangle - (E_i + E_f)/2, \tag{72}$$

where r_{ab} is the distance between the nucleons a and b, and $\langle E_N \rangle$ is the average energy of the intermediate nucleus N.

To derive the expressions for $\mathcal{A}$ and $\mathcal{B}$ shown in Tables 2 and 3 we have used the formulas:

$$\begin{aligned}
&C_1^A\left(\epsilon_{S\mp P}^{S-P}\right) = M_{GT}\frac{m_{S\mp P}^{S-P}}{m_e}\frac{F_S^{(3)}}{g_V}\chi_F^{SP},\\
&C_2^A\left(\epsilon_{S\mp P}^{S-P}\right) = 0, \quad C_2^A\left(\epsilon_{S\mp P}^{S+P}\right)\frac{r}{r_+} = 2M_{GT}\epsilon_{S\mp P}^{S+P}\chi_{F0}^{SP},\\
&C_5^A\left(\epsilon_{S\mp P}^{S-P}\right) = 0, \quad C_5^A\left(\epsilon_{S\mp P}^{S+P}\right)\frac{r}{r_+} = M_{GT}\epsilon_{S\mp P}^{S+P}\chi_{P0}^{SP};
\end{aligned} \tag{73}$$

$$\begin{aligned}
&Z_1^X\left(\epsilon_{V-A}^{V-A}\right) = M_{GT}\left(\mu + 2\mu_{V-A}^{V-A}\right)(\chi_F - 1),\\
&Z_1^X\left(\epsilon_{V+A}^{V-A}\right) = M_{GT}\left[\mu(\chi_F - 1) + 2\mu_{V+A}^{V-A}(\chi_F + 1)\right],\\
&Z_3^X\left(\epsilon_{V\mp A}^{V+A}\right) = \pm M_{GT}\epsilon_{V\mp A}^{V+A}(\chi_{GT\omega} \pm \chi_{F\omega}),\\
&Z_4^X\left(\epsilon_{V\mp A}^{V+A}\right) = \mp\frac{1}{3}M_{GT}\epsilon_{V\mp A}^{V+A}\chi_{1\mp},\\
&Z_6^Y\left(\epsilon_{V-A}^{V+A}\right)\frac{r}{r_+} = M_{GT}\epsilon_{V-A}^{V+A}\chi_P',\\
&Z_{4R}^Y\left(\epsilon_{V-A}^{V+A}\right) = M_{GT}\epsilon_{V-A}^{V+A}\chi_R';
\end{aligned} \tag{74}$$

$$\begin{aligned}
&W_1^U\left(\epsilon_{T_L}^{T_L}\right) = -4M_{GT}\mu_{T_L}^{T_L}\frac{T_1^{(3)}}{g_A},\\
&W_{4R}^V\left(\epsilon_{T_R}^{T_R}\right) = -2M_{GT}\epsilon_{T_R}^{T_R}\frac{T_1^{(3)}}{g_A}(\chi_{RC_\sigma}' + \chi_R' + \chi_{RT_\sigma}^{T\prime} - \chi_{RT}^{T\prime}),\\
&W_2^U\left(\epsilon_{T_R}^{T_R}\right)\frac{r}{r_+} = 2iM_{GT}\epsilon_{T_R}^{T_R}\frac{T_1^{(3)}}{g_A}\chi_{GT}',\\
&W_7^U\left(\epsilon_{T_R}^{T_R}\right)\frac{r}{r_+} = -4iM_{GT}\epsilon_{T_R}^{T_R}\frac{T_1^{(3)}}{g_A}\left(\frac{1}{3}\chi_{GT}' + 2\chi_T'\right).
\end{aligned} \tag{75}$$

For all other arguments ϵ_α^β these nucleon matrix elements have zero values, except for

$$Z_1^X\left(\epsilon_{V\mp A}^{V-A} = 0\right) = M_{GT}\mu(\chi_F - 1). \tag{76}$$

We have calculated the numerical values of the integrated kinematic factors A_{0i}, $A_{0i}^{(SP,T)}$, B_{0i}, and $B_{0i}^{(SP,T)}$ for all the five nuclei of current experimental interest. We shall use them in the results shown below in Table 6 for the angular coefficient K. However, as we will focus in this paper mainly on the $0\nu2\beta$ decay of the ^{76}Ge nucleus, we give the values of these factors for this nucleus in Table 5, where we have used

$$Q = E_i - E_f - 2m_e = 2.039 \text{ MeV}, \tag{77}$$

taken from Ref. [35], and the scaling factor for the neutrino potentials is

$$R = r_0 A^{1/3}, \quad r_0 = 1.1 \text{ fm}. \tag{78}$$

The values of A_{00}^T and B_{03} are of the order of 10^{-44} yr^{-1}. Hence these values are not given in Table 5 and the terms with A_{00}^T and B_{03} can be safely neglected.

Table 5: The integrated kinematic A- and B-factors [in $10^{-15}\mathrm{yr}^{-1}$] for the $0^+ \to 0^+$ transition of the $0\nu2\beta$ decay of ^{76}Ge.

A_{01}	6.69	B_{01}	5.45
A_{02}	1.09×10	B_{02}	8.95
A_{03}	3.76	B_{03}	—
A_{04}	1.30	B_{04}	1.21
A_{05}	2.08×10^{2}	B_{05}	7.27
A_{06}	1.69×10^{3}	—	—
A_{07}	1.05×10^{5}	B_{07}	7.72×10^{4}
A_{08}	6.59×10^{3}	B_{08}	4.97×10^{3}
A_{09}	4.14×10^{5}	B_{09}	3.00×10^{5}
A^{SP}_{00}	2.55	—	—
A^{SP}_{01}	3.77	B^{SP}_{01}	2.73
A^{SP}_{02}	1.18×10^{-1}	B^{SP}_{02}	7.20×10^{-2}
A^{SP}_{03}	1.27×10^{-3}	B^{SP}_{03}	3.71×10^{-4}
A^{T}_{01}	6.03×10	B^{T}_{01}	4.36×10
A^{T}_{02}	1.50×10^{3}	B^{T}_{02}	1.40×10^{3}
A^{T}_{03}	7.67×10^{5}	B^{T}_{03}	7.16×10^{5}

We recall that the analytic expressions associated with the coefficients $\epsilon^{V+A}_{V\mp A}$ given in this section and the values of A_{0i} from Table 5 confirm the results of Ref. [5]. The analytic expressions associated with the coefficients $\epsilon^{V-A}_{V\mp A}$, $\epsilon^{S\mp P}_{S\mp P}$, $\epsilon^{T_{L,R}}_{T_{L,R}}$ and the values of $A^{(SP,T)}_{0i}$, B_{0i}, $B^{(SP,T)}_{0i}$ from Table 5 transcend the earlier work.

3 Analysis of the electron angular correlation

3.1 Qualitative analysis

If the effects of all the interactions beyond the SM extended by the ν_Ms, which we call the "nonstandard" effects, are zero (i.e., all $\epsilon^{\beta}_{\alpha} = 0$), then $K = B_{01}/A_{01}$. Its values are given in Table 6 for various decaying nuclei. We will concentrate on the case of ^{76}Ge nucleus in the following. In this case the correlation (21) is proportional to $1 - 0.81\cos\theta$. (Note that in the limit of $m_e/(E_i - E_f) \to 0$ we have $\alpha_+ + \beta_+ = \gamma_+ + \delta_+$ and $K = 1$.) Tables 2 and 4 show that the presence of the "nonstandard" parameters $\epsilon^{V-A}_{V\mp A}$, $\epsilon^{T_L}_{T_R}$ or $\epsilon^{T_R}_{T_L}$ does not change the value of K and therefore the form of the angular correlation. The presence of any other parameter $\epsilon^{\beta}_{\alpha}$ does change this correlation. From the fact that there are no contributions due to P-wave and recoil effects to the scalar nonstandard terms in the closure approximation (see Appendix A), it follows that the values of A^{SP}_{02}, A^{SP}_{03}, B^{SP}_{02}, and B^{SP}_{03} are small and there are two additional "nonstandard" parameters that do not change significantly the form of the angular correlation, namely, $\epsilon^{S+P}_{S\mp P}$.

Table 6: The values of angular correlation coefficient K for various decaying nuclei for the SM extended by the ν_Ms.

	^{76}Ge	^{82}Se	^{100}Mo	^{130}Te	^{136}Xe
K	0.81	0.88	0.88	0.85	0.84

Using Table 1 and taking into account the fact that $|\mu^{\beta}_{\alpha}|$ are suppressed in comparison with $|\epsilon^{\beta}_{\alpha}|$ by the factor m_i/m_e (the chiral suppression), we find the coefficient K and the set $\{\epsilon\}$ of nonzero $\epsilon^{\beta}_{\alpha}$s that change the $1 - 0.81\cos\theta$ form of the correlation for the SM plus ν_Ms, see Table 7 (the lower two entries). They correspond to the following extensions of the SM: ν_Ms plus RPV SUSY [10], ν_Ms plus right-handed currents (RC) (connected with right-handed W bosons [5] or LQs [13]). Hence, the angular coefficient K can signal the presence of these NP interactions.

Table 7: The angular correlation coefficient K for various SM extensions for decays of ^{76}Ge.

SM extension	$\{\epsilon\}$	K
ν_M	—	0.81
ν_M+RPV SUSY	$\epsilon^{T_R}_{T_R}$	$-1 < K < 1$
ν_M+RC	$\epsilon^{V+A}_{V\mp A}$	$-1 < K < 1$

We remark here that in our earlier analysis [19] we had neglected the P-wave and recoil effects, which is not a good assumption. Our current study shows that these effects give significant contribution to the terms with ϵ^{V+A}_{V-A} and $\epsilon^{T_R}_{T_R}$. Hence, they have to be included in any realistic analysis of the data, as and when it becomes available. Including them, not only the model called ν_M+ RC but also the model ν_M+ RPV can essentially change the angular coefficient K from being 0.81 in the decay of the ^{76}Ge nucleus. Left-right symmetric models belong to the class ν_M+ RC and we have studied these models in detail in section 4, where the correlations among the parameters K, $T_{1/2}$ and either m_{W_R} or ζ are worked out for the case $|\langle m\rangle| \neq 0$, $\cos\psi_i = 0$ considered in section 3.2.

Note that the decay half-life and angular correlation do not give any bounds on the parameters $\epsilon^{T_L}_{T_R}$ and $\epsilon^{T_R}_{T_L}$ because the according expressions for $\mathcal{A}$ and $\mathcal{B}$ do not depend on them.

3.2 Quantitative analysis

Let us now consider some particular cases for the parameter space. We will analyze only the terms with $\epsilon^{V\mp A}_{V\mp A}$ as the corresponding nuclear matrix elements have been workd out in the literature. We use various types of QRPA model for the ^{76}Ge nucleus [20, 21] as a test case.

Using the case of $|\langle m\rangle| = 0$, which gives conservative upper bounds on $|\mu^\beta_\alpha|$ and $|\epsilon^\beta_\alpha|$, the decay half-life is expressed from Eq. (21) as

$$T_{1/2} = \ln 2/\Gamma = \left(|M_{GT}|^2\mathcal{A}\right)^{-1}. \tag{79}$$

From Eq. (79), using Tables 2, 5 and the values of the nuclear matrix elements reported in Refs. [20, 21], we have the following expressions for the half-life [in yr] for various choices of the parameters $|\mu^{V-A}_{V\mp A}|$ and $|\epsilon^{V+A}_{V\mp A}|$, taking only one parameter at a time:

$$T_{1/2} = 1.1(1.3)\times 10^{12}|\mu^{V-A}_{V-A}|^{-2}, \quad T_{1/2} = 3.2(4.0)\times 10^{12}|\mu^{V-A}_{V+A}|^{-2}, \tag{80}$$

$$T_{1/2} = 4.0(21)\times 10^{12}|\mu^{V-A}_{V-A}|^{-2}, \quad T_{1/2} = 4.5(6.8)\times 10^{12}|\mu^{V-A}_{V+A}|^{-2}, \tag{81}$$

$$T_{1/2} = 3.7(27)\times 10^{8}|\epsilon^{V+A}_{V-A}|^{-2}, \quad T_{1/2} = 1.0(9.7)\times 10^{13}|\epsilon^{V+A}_{V+A}|^{-2}. \tag{82}$$

Eq. (80) corresponds to using the pnQRPA model with particle-particle strength parameter g_{pp}=1.02(1.06) [21] and Eqs (81)–(82) correspond to using the QRPA model without (with) the p-n pairing [20] (note that the definitions of the nuclear matrix elements χ'_P and χ_R in Ref. [20] differ from χ'_P and χ'_R in Ref. [5] by the factors 1/2 and $4/(m_e R)$, respectively). Comparing the numerical results in these equations, we note that the dispersion in the half-lifes is less marked for the coefficient $|\mu^{V-A}_{V+A}|$. However, the half-lifes involving the coefficients $|\mu^{V-A}_{V-A}|$ and $|\epsilon^{V\pm A}_{V+A}|$ show a very strong nuclear matrix element dependence. For the QRPA model worked out in [20], it is not clear to us if this is due to a numerical artifact or the treatment of the isoscalar neutron-proton pairing. An important, and related point, is how to fix correctly the particle-particle strength of the nuclear Hamiltonian. Fixing the particle-particle pairing parameter, and varying it as done in [21], leads to rather stable values for the half-life of ^{76}Ge nucleus. Clearly, these issues remain to be further discussed and clarified. A detailed discussion of these nuclear models will take us far afield from the main point of our paper. The theoretical uncertainty in the nuclear matrix elements [2, 36] plays an essential role in the numerical analysis. However, as we show below, the nuclear-model dependence of the angular coefficient K is rather modest.

The fact that the dependence of K on the nuclear matrix elements is much weaker than the uncertainty in $T_{1/2}$ from this source is illustrated in Table 8 for QRPA models [20, 21] for the assumed values of the parameters: $|\mu^{V-A}_{V\mp A}| = |\epsilon^{V+A}_{V+A}| = 5\times 10^{-7}$, $|\epsilon^{V+A}_{V-A}| = 5\times 10^{-9}$. It is clear from Table 8 that measuring K with 10% accuracy (or better) produces useful experimental data that could be sensitive to the new physics. We note that for the parameters $\mu^{V-A}_{V\mp A}$ the angular coefficient does not depend actually on the nuclear matrix elements as it is seen from Tables 2, 3 (for $|\mu| = 0$) and Eqs. (28), (31): $K = (\chi_F \mp 1)^2 B_{01}/\left[(\chi_F \mp 1)^2 A_{01}\right] = B_{01}/A_{01} \simeq 0.81$.

Table 8: $T_{1/2}$ and K for the fixed values of the parameters $|\mu^{V-A}_{V\mp A}|, |\epsilon^{V+A}_{V\mp A}|$ for decay of ^{76}Ge for the case of $|\langle m\rangle| = 0$ in QRPA without (with) p-n pairing [20] [pnQRPA with g_{pp}=1.02(1.06) [21]].

	$\lvert\mu^{V-A}_{V-A}\rvert = 5\times 10^{-7}$	$\lvert\mu^{V-A}_{V+A}\rvert = 5\times 10^{-7}$	$\lvert\epsilon^{V+A}_{V-A}\rvert = 5\times 10^{-9}$	$\lvert\epsilon^{V+A}_{V+A}\rvert = 5\times 10^{-7}$
$T_{1/2}/(10^{25}$ yr)	1.6(8.4)[0.44(0.52)]	1.8(2.7)[1.3(1.6)]	1.5(11)	4.0(39)
K	0.81(0.81)[0.81(0.81)]	0.81(0.81)[0.81(0.81)]	−0.73(−0.73)	−0.79(−0.87)

Using the numerical results given above, the current lower bound $T_{1/2} > 1.6\times 10^{25}$ yr for the ^{76}Ge nucleus [37] yields the upper bounds on the parameters $|\mu^{V-A}_{V\mp A}|$ and $|\epsilon^{V+A}_{V\mp A}|$ shown in Table 9. The bound on $|\epsilon^{V+A}_{V-A}|$ is stronger than the others shown in this table due to the relatively large values of the recoil and P-wave matrix elements in this case. The bounds on $|\epsilon^{V+A}_{V\mp A}|$ given in Table 9 are comparable with the bounds $|\epsilon^{V+A}_{V-A}| < 4\times 10^{-9}$, $|\epsilon^{V+A}_{V+A}| < 6\times 10^{-7}$ given in Ref. [28].

Table 9: Upper bounds on $|\mu^{V-A}_{V\mp A}|$, $|\epsilon^{V+A}_{V\mp A}|$ for decays of ^{76}Ge for the case of $|\langle m\rangle| = 0$ in QRPA.

Nuclear model	$\lvert\mu^{V-A}_{V-A}\rvert$	$\lvert\mu^{V-A}_{V+A}\rvert$	$\lvert\epsilon^{V+A}_{V-A}\rvert$	$\lvert\epsilon^{V+A}_{V+A}\rvert$
pnQRPA with g_{pp}=1.02(1.06) [21]	$2.6(2.9)\times 10^{-7}$	$4.5(5.0)\times 10^{-7}$	—	—
QRPA without (with) p-n pairing [20]	$5.0(11)\times 10^{-7}$	$5.4(6.5)\times 10^{-7}$	$4.8(13)\times 10^{-9}$	$7.9(25)\times 10^{-7}$

To be definite, we use the QRPA model without p-n pairing [20] in the following. The bounds given in Table 9 could be used for deriving the bounds on the parameters of the particular models (see section 2.1). For example, using Eq. (5) we have the following conservative constraints on the couplings of the effective LQ-quark-lepton interactions:

$$|\alpha^{(L)}_I| \le 1.1\times 10^{-9}\left(\frac{M_I}{100\ \text{GeV}}\right)^2, \quad |\alpha^{(R)}_I| \le 2.6\times 10^{-7}\left(\frac{M_I}{100\ \text{GeV}}\right)^2, \quad I = S, V. \tag{83}$$

- Consider a more general case of $|\langle m\rangle| \neq 0$, $\cos\psi_i = 0$, where the index i depends on α, β (as above, we take only one nonzero ϵ^β_α at a time). Using Tables 2 and 4 we have

$$\begin{aligned} \mathcal{A} &= C_1|\mu|^2 + 4C_i|\mu^\beta_\alpha|^2, \\ K\mathcal{A} &= D_1|\mu|^2 + 4D_i|\mu^\beta_\alpha|^2, \end{aligned} \tag{84}$$

  and

$$\begin{aligned} \mathcal{A} &= C_1|\mu|^2 + C_i|\epsilon^\beta_\alpha|^2, \\ K\mathcal{A} &= D_1|\mu|^2 + D_i|\epsilon^\beta_\alpha|^2. \end{aligned} \tag{85}$$

Hence, using Eq. (79) we obtain

$$\begin{aligned} |\mu|^2 &= (\lambda_1 - \lambda_2 K)/T_{1/2}, \\ |\epsilon^\beta_\alpha|^2 &= (-\lambda_3 + \lambda_4 K)/T_{1/2} = 4|\mu^\beta_\alpha|^2, \end{aligned} \tag{86}$$

with the coefficients

$$\lambda_1 = \frac{D_i}{|M_{GT}|^2\Delta_i}, \quad \lambda_2 = \frac{C_i}{|M_{GT}|^2\Delta_i},$$
$$\lambda_3 = \frac{D_1}{|M_{GT}|^2\Delta_i}, \quad \lambda_4 = \frac{C_1}{|M_{GT}|^2\Delta_i}, \tag{87}$$

where $\Delta_i = C_1 D_i - D_1 C_i$.

Using Eqs (86)–(87) we have for $\epsilon_{V+A}^{V+A} \neq 0$

$$|\mu|^2 = (7.9 + 10K) \times 10^{12}/T_{1/2}, \quad |\epsilon_{V+A}^{V+A}|^2 = (5.1 - 6.3K) \times 10^{12}/T_{1/2} \tag{88}$$

and for $\epsilon_{V-A}^{V+A} \neq 0$

$$|\mu|^2 = (7.7 + 10K) \times 10^{12}/T_{1/2}, \quad |\epsilon_{V-A}^{V+A}|^2 = (1.9 - 2.4K) \times 10^{8}/T_{1/2}, \tag{89}$$

with $T_{1/2}$ in years. Fig. 1 shows the correlation among $|\langle m \rangle|$, $T_{1/2}$, K (*left*) and the correlation among $|\epsilon_{V+A}^{V+A}|$, $T_{1/2}$, K (*right*) for the choice of a nonzero ϵ_{V+A}^{V+A}. Fig. 2 shows the same for the parameter ϵ_{V-A}^{V+A}. It is clear from Figs 1 and 2 that the closer is K to 1 for the fixed value of $T_{1/2}$, the weaker is bounded $|\langle m \rangle|$ and stronger is bounded $|\epsilon_{V\mp A}^{V+A}|$. The correlations among $|\epsilon_{V\mp A}^{V+A}|$, $T_{1/2}$, K will be used in the next section in the analysis of left-right symmetric models.

Note that if several ϵ_α^β are nonzero in the considered model than the respective interference terms should be taken into account.

- To extract $|\mu|$, $|\mu_\alpha^\beta|$, $|\epsilon_\alpha^\beta|$, c_i in the general case of $|\langle m \rangle| \neq 0$, $c_i \neq 0$ we need to analyze the data on at least two decaying nuclei. This analysis will be presented for the five nuclei already discussed in a forthcoming paper [38].

4 Electron angular correlation in left-right symmetric models

The experimental bounds on the ϵ_α^β are connected with the masses of new particles, their mixing angles, and other parameters specific to particular extensions of the SM [5, 4, 8, 10, 12, 13]. To illustrate the kind of correlations that the measurements of $T_{1/2}$ and the angular correlation coefficient K in the $0\nu2\beta$ decay would imply, we work out the case of the left-right symmetric models [22]. In the model $SU(2)_L \times SU(2)_R \times U(1)$ the parameters η and λ (see Eq. (2)) are expressed through the masses m_{W_L} and m_{W_R} of the left- and right-handed W bosons and their mixing angle ζ [5]:

$$\eta = -\tan\zeta, \quad \lambda = (m_{W_L}/m_{W_R})^2, \tag{90}$$

under the condition

$$m_{W_L} \ll m_{W_R}. \tag{91}$$

Eqs. (2) and (6) and the relation [5]

$$V_{ei} = V'_{ei} \tag{92}$$

of the $SU(2)_L \times SU(2)_R \times U(1)$ model yield

$$\epsilon_{V+A}^{V+A} = \lambda \frac{g'_V}{g_V} U_{ei} V_{ei}., \quad \epsilon_{V-A}^{V+A} = \eta U_{ei} V_{ei}. \tag{93}$$

To reduce the number of free parameters, we assume the equality of the form factors of the left- and right-handed hadronic currents:

$$g_V = g'_V. \tag{94}$$

The small masses of the observable νs are likely described by the seesaw formula that in the simplest case gives

$$m_i \sim m_D^2/M_R, \quad M_R \gg m_D, \tag{95}$$

with the Dirac mass scale m_D (for the charged leptons and the light quarks $m_D \sim 1$ MeV) and the mass scale M_R of right ν_Ms (in the majority of theories $M_R > 1$ TeV). In the left-right symmetric models these scales arise usually from the two scales of the vacuum expectation values of Higgs multiplets [22]. In the seesaw mechanism, the values of the mixing parameters V_{ei} (for i numbering light mass states) have the same order of magnitude as m_D/M_R. In our discussion we use two rather conservative values (compare with Eq. (9))

$$\epsilon = 10^{-6},\ 5\times 10^{-7} \tag{96}$$

for the mixing parameter

$$\epsilon = |U_{ei}V_{ei}|. \tag{97}$$

We recall that here the summation index i runs only over the light neutrino mass eigenstates (the summation over the total mass spectrum including also heavy states gives strictly zero due to the orthogonality condition [5]).

From Eqs. (90), (93), (94), and (97) we have

$$m_{W_R} = m_{W_L}\left(\epsilon/\left|\epsilon^{V+A}_{V+A}\right|\right)^{1/2}, \quad \zeta = -\arctan\left(\left|\epsilon^{V+A}_{V-A}\right|/\epsilon\right). \tag{98}$$

Using Eq. (91) we note the approximate equality of m_{W_L} and the mass of the observed charged gauge boson W_1 (m_{W_1}=80.4 GeV [1]).

The correlation among m_{W_R} (ζ), K, and $T_{1/2}$ for the case of $|\langle m\rangle| \neq 0$, $\cos\psi_i = 0$ (see section 3.2) is shown in Fig. 3 (4) for the two chosen values of ϵ. The numerical results for these figures have been obtained using Eqs. (88) and (89). It is clear from Fig. 3 (4) that the closer is K to 1 for the fixed value of $T_{1/2}$ the stronger is the lower bound on m_{W_R} (the upper bound on ζ). However this bound is weaker than the one $m_{W_R} > 715$ GeV, obtained from the electroweak fits [1]. There is still a more stringent bound $m_{W_R} > 1.2$ TeV, obtained in Ref. [39] for the $0\nu2\beta$ decay mediated by heavy Majorana neutrinos using arguments based on the vacuum stability [6] and additional theory input. We assume $m_{W_R} \geq 1$ TeV in the next figure.

While experiments in the $0\nu2\beta$ decay would measure the product of the quantities called λ and the neutrino mixing matrix elements $U_{ei}V_{ei}$ in Eq. (93), collider experiments at the Tevatron and the LHC can, in principle, measure λ by determining m_{W_R}. Assuming these logically independent possibilities, we plot the differential width (21) vs. $\cos\theta$ in Fig. 5 for a set of values of $|\langle m\rangle|$ and m_{W_R}, taking ϵ^{V+A}_{V+A} at a time and assuming $\epsilon = 10^{-6}$. In this figure, we consider the values of $|\langle m\rangle|$, starting from $|\langle m\rangle| \leq 0.03$ eV up to $|\langle m\rangle| = 5$ meV, covering two of three scenarios of neutrino mass hierarchies and mixing angles: normal and inverted mass hierarchies (see Ref. [40] for a recent discussion and update). It is seen that the sensitivity of the electron angular correlation to the right-handed W-boson mass m_{W_R} increases with decreasing values of the effective Majorana neutrino mass $|\langle m\rangle|$, as can be seen from Fig. 5 (right), where this correlation is shown for $|\langle m\rangle|$=5 meV, 10 meV.

In conclusion, we have presented a detailed study of the electron angular correlation for the long range mechanism of $0\nu2\beta$ decays in a general theoretical context. This information, together with the ability of observing these decays in several nuclei, would help greatly in identifying the dominant mechanism underlying these decays. At present, no experiment is geared to measuring the angular correlation in $0\nu2\beta$ decays, as the main experimental thrust is on establishing a non-zero signal unambiguously in the first place. We note that the running experiment NEMO3 has already measured the electron angular distribution for the two neutrino double beta decay, and is capable of measuring this correlation in the future for the $0\nu2\beta$ decay as well, assuming that the experimental sensitivity is sufficiently good to establish this decay [41]. The proposed experimental facilities that can measure the electron angular correlation in the $0\nu2\beta$ decays are SuperNEMO [42], MOON [43], and EXO [44]. We have argued in this paper that there is a strong case in building at least one of them.

Acknowledgments

We thank Alexander Barabash and Fedor Šimkovic for helpful discussions, and the referee of our paper for critical comments which helped us in improving and correcting the earlier version of this manuscript. One of us (DVZ) would like to thank DESY for the hospitality in Hamburg where a good part of this work was done.

A $0\nu 2\beta$ decay rate for scalar nonstandard terms

The nucleon currents in the impulse approximation in the nonrelativistic form are used in this paper [32, 34]. Keeping all terms up to order p/m_p in the nonrelativistic expansion we have

$$J^{+}_{S\mp P}(\mathbf{x}) = \sum_a \tau^a_+ \delta(\mathbf{x}-\mathbf{r}_a)\left(F^{(3)}_S \mp F^{(3)}_P B_a\right), \quad B_a = \frac{\boldsymbol{\sigma}_a\cdot\mathbf{q}}{2m_p}, \tag{99}$$

$$J^{\mu +}_{V-A}(\mathbf{x}) = \sum_a \tau^a_+ \delta(\mathbf{x}-\mathbf{r}_a)\left[g^{\mu 0}(g_V I_a - g_A C_a) + g^{\mu m}(g_A \sigma_{am} - g_V D^m_a - g_A P^m_a)\right], \tag{100}$$

$$C_a = \frac{\boldsymbol{\sigma}_a\cdot\mathbf{Q}}{2m_p} - \frac{q^0\boldsymbol{\sigma}_a\cdot\mathbf{q}}{\mathbf{q}^2+m^2_\pi}, \quad D^m_a = \frac{Q^m}{2m_p}I_a - \left(1+\frac{g_M}{g_V}\right)\frac{i[\boldsymbol{\sigma}_a\times\mathbf{q}]^m}{2m_p}, \quad P^m_a = \frac{q^m\boldsymbol{\sigma}_a\cdot\mathbf{q}}{\mathbf{q}^2+m^2_\pi}, \tag{101}$$

where $q^\mu = p^\mu - p'^\mu$ is the 4-momentum transferred from hadrons to leptons, $Q^\mu = p^\mu + p'^\mu$; p^μ and p'^μ are the initial and final 4-momenta of a nucleon; m_p is proton mass and m_π is pion mass.

We neglect the dipole dependence of the form factors $F^{(3)}_S$, $F^{(3)}_P$, g_V, g_A, g_M on the momentum transfer and omit the zero argument of the form factors. Note that $g_V(0)=1$.

Consider the pure SP case assuming $\langle m\rangle = 0$. In terms of the combinations of hadronic currents

$$J^\mu_{\mp L} = \langle F|\tilde{J}^+_\mp|N\rangle\langle N|\hat{J}^{\mu +}_L|I\rangle, \quad J^\mu_{L\mp} = \langle F|\hat{J}^{\mu +}_L|N\rangle\langle N|\tilde{J}^+_\mp|I\rangle, \tag{102}$$

$$\tilde{J}^+_- = \epsilon^{S-P}_{S-P,i}J^+_{S-P} + \epsilon^{S-P}_{S+P,i}J^+_{S+P}, \quad \tilde{J}^+_+ = \epsilon^{S+P}_{S+P,i}J^+_{S+P} + \epsilon^{S+P}_{S-P,i}J^+_{S-P}, \tag{103}$$

$$\hat{J}^{\mu +}_L = U_{ei}J^{\mu +}_{V-A}, \tag{104}$$

and the combinations

$$\ell^{L,R}_\mu = \frac{s^{L,R}_\mu(2\mathbf{y},1\mathbf{x})}{\omega + A_1} - \frac{s^{L,R}_\mu(1\mathbf{y},2\mathbf{x})}{\omega + A_2}, \tag{105}$$

$$\ell^L_{\lambda\mu} = \frac{s^L_{\lambda\mu}(2\mathbf{y},1\mathbf{x})}{\omega + A_1} - \frac{s^L_{\lambda\mu}(1\mathbf{y},2\mathbf{x})}{\omega + A_2} \tag{106}$$

of electron currents

$$s^{L,R}_\mu(2\mathbf{y},1\mathbf{x}) = \bar{e}_2(\mathbf{y})\gamma_\mu(1\mp\gamma_5)e^c_1(\mathbf{x}), \quad s^L_{\lambda\mu}(2\mathbf{y},1\mathbf{x}) = \bar{e}_2(\mathbf{y})\gamma_\lambda(1-\gamma_5)\gamma_\mu e^c_1(\mathbf{x}), \tag{107}$$

$e_i(\mathbf{x}) \equiv e_{\mathbf{p}_i s_i}(\mathbf{x})$, the matrix element is expressed as

$$R^{SP}_{0\nu} = \frac{1}{\sqrt{2!}}\left(\frac{G_F|V_{ud}|}{\sqrt{2}}\right)^2 2\sum_i \int d\mathbf{x}d\mathbf{y}\frac{d\mathbf{k}}{(2\pi)^3}\frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{2\omega}$$
$$\times\sum_N\left[m_i\left(J^\mu_{-L}\ell^R_\mu - J^\mu_{L-}\ell^L_\mu\right) + k^\lambda\left(J^\mu_{+L}\ell^L_{\lambda\mu} - J^\mu_{L+}\ell^L_{\mu\lambda}\right)\right], \tag{108}$$

where $\mathbf{r} = \mathbf{y} - \mathbf{x}$. By using the identities

$$s^{L,R}_\mu(1\mathbf{y},2\mathbf{x}) = s^{R,L}_\mu(2\mathbf{x},1\mathbf{y}), \quad s^L_{\lambda\mu}(1\mathbf{y},2\mathbf{x}) = -s^L_{\mu\lambda}(2\mathbf{x},1\mathbf{y}), \tag{109}$$

the algebraic formula

$$2(am \pm bn) = (a+b)(m\pm n) + (a-b)(m\mp n), \tag{110}$$

the constant

$$C_{0\nu} = \frac{G^2_F|V_{ud}|^2}{8\sqrt{2}\pi}\frac{2m_e}{R} \tag{111}$$

and the neutrino potentials

$$(H_j, H_{\omega j}, H^l_{kj}) = 4\pi \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\omega} \frac{(1, \omega, k^l)}{\omega + A_j} \ , \tag{112}$$

the matrix element (108) is expressed as

$$R^{SP}_{0\nu} = -C_{0\nu} \sum_i \sum_N \left(\frac{m_i}{m_e} M^m_{SP} + M^k_{SP} \right) . \tag{113}$$

Each part of this matrix element is expressed as a sum of nonvanishing (indexed by n) and vanishing (indexed by c) terms, in the closure approximation:

$$M^{m,k}_{SP} = \{M^{m,k}_{SP}\}_n + \{M^{m,k}_{SP}\}_c, \tag{114}$$

$$\begin{aligned} \{M^m_{SP}\}_n &= \frac{R}{2} \int d\mathbf{x} d\mathbf{y} T_N (H_1 + H_2) \\ &\times \left[(A_1 + A_{1R}) F^0_{5+} + (A^i_3 + \tilde{A}^i_{3R}) F^i_{5+} + B_{1R} F^0_- + (B^i_3 + \tilde{B}^i_{3R}) F^i_- \right] , \end{aligned} \tag{115}$$

$$\begin{aligned} \{M^m_{SP}\}_c &= \frac{R}{2} \int d\mathbf{x} d\mathbf{y} T_N (H_1 - H_2) \\ &\times \left[(A_1 + A_{1R}) F^0_{5-} + (A^i_3 + \tilde{A}^i_{3R}) F^i_{5-} + B_{1R} F^0_+ + (B^i_3 + \tilde{B}^i_{3R}) F^i_+ \right] , \end{aligned} \tag{116}$$

$$\begin{aligned} \left\{M^k_{SP}\right\}_n &= \frac{R}{2m_e} \int d\mathbf{x} d\mathbf{y} T_N \, \{ \, (H_{\omega 1} - H_{\omega 2}) \left[-(A^i_4 + \tilde{A}^i_{4R}) E^i_+ + B_{2R} E_- \right] \\ &+ (H^l_{k1} - H^l_{k2}) \left[-(A_2 + A_{2R}) E^l_+ + (A^{lk}_5 + \tilde{A}^{lk}_{5R}) E^k_+ + (B^l_4 + \tilde{B}^l_{4R}) E_- \right] \, \} , \end{aligned} \tag{117}$$

$$\begin{aligned} \left\{M^k_{SP}\right\}_c &= \frac{R}{2m_e} \int d\mathbf{x} d\mathbf{y} T_N \, \{ \, (H_{\omega 1} + H_{\omega 2}) \left[-(A^i_4 + \tilde{A}^i_{4R}) E^i_- + B_{2R} E_+ \right] \\ &+ (H^l_{k1} + H^l_{k2}) \left[-(A_2 + A_{2R}) E^l_- + (A^{lk}_5 + \tilde{A}^{lk}_{5R}) E^k_- + (B^l_4 + \tilde{B}^l_{4R}) E_+ \right] \, \} , \end{aligned} \tag{118}$$

with

$$T_N = g^2_A \langle F| \sum_a \tau^a_+ |N\rangle \langle N| \sum_b \tau^b_+ |I\rangle \delta(\mathbf{x} - \mathbf{r}_a) \delta(\mathbf{y} - \mathbf{r}_b) \ . \tag{119}$$

The electron currents are defined as:

$$\begin{aligned} F_+ &= \tfrac{1}{2} \left[u(\mathbf{yx}) \pm u(\mathbf{xy}) \right] , & F_{5\pm} &= \frac{1}{2} \left[u_5(\mathbf{yx}) \pm u_5(\mathbf{xy}) \right] , \\ F^\mu_+ &= \tfrac{1}{2} \left[u^\mu(\mathbf{yx}) \pm u^\mu(\mathbf{xy}) \right] , & F^\mu_{5\pm} &= \frac{1}{2} \left[u^\mu_5(\mathbf{yx}) \pm u^\mu_5(\mathbf{xy}) \right] , \\ F^{\mu\nu}_+ &= \tfrac{1}{2} \left[u^{\mu\nu}(\mathbf{yx}) \pm u^{\mu\nu}(\mathbf{xy}) \right] , & F^{\mu\nu}_{5\pm} &= \frac{1}{2} \left[u^{\mu\nu}_5(\mathbf{yx}) \pm u^{\mu\nu}_5(\mathbf{xy}) \right] , \\ E_\pm &= F_\pm + F_{5\pm} , & E^i_\pm &= F^{0i}_\pm + F^{0i}_{5\pm} , \end{aligned} \tag{120}$$

with

$$\begin{aligned} u(\mathbf{yx}) &= \bar{e}_2(\mathbf{y}) e^c_1(\mathbf{x}) , & u_5(\mathbf{yx}) &= \bar{e}_2(\mathbf{y}) \gamma_5 e^c_1(\mathbf{x}) , \\ u^\mu(\mathbf{yx}) &= \bar{e}_2(\mathbf{y}) \gamma^\mu e^c_1(\mathbf{x}) , & u^\mu_5(\mathbf{yx}) &= \bar{e}_2(\mathbf{y}) \gamma_5 \gamma^\mu e^c_1(\mathbf{x}) , \\ u^{\mu\nu}(\mathbf{yx}) &= -i\bar{e}_2(\mathbf{y}) \sigma^{\mu\nu} e^c_1(\mathbf{x}) , & u^{\mu\nu}_5(\mathbf{yx}) &= -i\bar{e}_2(\mathbf{y}) \gamma_5 \sigma^{\mu\nu} e^c_1(\mathbf{x}) \ . \end{aligned} \tag{121}$$

The nucleon operator matrix elements are defined as follows:

$$\tilde{A} = A + A^P , \qquad \tilde{B} = B + B^P , \tag{122}$$

$$
\begin{aligned}
A_1 &= 2G_V^0\varepsilon_S, & A_{1R} &= -G_A^0\varepsilon_S C_+ - G_V^0\varepsilon_P B_+, \\
A_2 &= 2G_V^0\varepsilon'_S, & A_{2R} &= -G_A^0\varepsilon'_S C_+ + G_V^0\varepsilon'_P B_+, \\
A_3^i &= G_A^0\varepsilon_S\sigma_+^i, & A_{3R}^i &= -G_A^0\varepsilon_P B_{\sigma+}^i - G_V^0\varepsilon_S D_+^i, \quad A_{3R}^{Pi} = -G_A^0\varepsilon_S P_+^i, \\
A_4^i &= G_A^0\varepsilon'_S\sigma_+^i, & A_{4R}^i &= G_A^0\varepsilon'_P B_{\sigma+}^i - G_V^0\varepsilon'_S D_+^i, \quad A_{4R}^{Pi} = -G_A^0\varepsilon'_S P_+^i, \\
A_5^{lk} &= i\varepsilon_{ilk}A_4^i, & \tilde{A}_{5R}^{lk} &= i\varepsilon_{ilk}\tilde{A}_{4R}^i,
\end{aligned}
\tag{123}
$$

$$
\begin{aligned}
B_{1R} &= -G_A^0\varepsilon_S C_- + G_V^0\varepsilon_P B_-, \\
B_{2R} &= -G_A^0\varepsilon'_S C_- - G_V^0\varepsilon'_P B_-, \\
B_3^i = G_A^0\varepsilon_S\sigma_-^i, \quad B_{3R}^i &= -G_A^0\varepsilon_P B_{\sigma-}^i - G_V^0\varepsilon_S D_-^i, \quad B_{3R}^{Pi} = -G_A^0\varepsilon_S P_-^i, \\
B_4^i = G_A^0\varepsilon'_S\sigma_-^i, \quad B_{4R}^i &= G_A^0\varepsilon'_P B_{\sigma-}^i - G_V^0\varepsilon'_S D_-^i, \quad B_{4R}^{Pi} = -G_A^0\varepsilon'_S P_-^i,
\end{aligned}
\tag{124}
$$

with

$$
B_\pm = B_a I_b \pm I_a B_b \quad B_{\sigma\pm}^i = \sigma_a^i B_b \pm B_a\sigma_b^j, \quad P_\pm^i = P_a^i I_b \pm I_a P_b^i. \tag{125}
$$

Under the exchange of running indices a and b (i.e. $\mathbf{x} \leftrightarrow \mathbf{y}$), nuclear operators A, electron currents E_+ and F_+ and neutrino potentials H_i and $H_{\omega i}$ are even, while B, E_-, F_-, and $\mathbf{H}_{ki}$ are odd.

The constants are defined as:

$$
\begin{aligned}
G_V = \frac{g_V}{g_A}\left[\left(U_{ei} + \epsilon_{V-A,i}^{V-A}\right) + \epsilon_{V+A,i}^{V-A}\right], &\qquad G_A = \left(U_{ei} + \epsilon_{V-A,i}^{V-A}\right) - \epsilon_{V+A,i}^{V-A}, \\
G^0 = G(\epsilon = 0), &\qquad G_V^0 = \frac{g_V}{g_A}U_{ei}, \quad G_A^0 = U_{ei},
\end{aligned}
\tag{126}
$$

$$
\begin{aligned}
\varepsilon_S &= \frac{F_S^{(3)}}{g_A}\left(\epsilon_{S-P,i}^{S-P} + \epsilon_{S+P,i}^{S-P}\right), \quad \varepsilon_P = \frac{F_P^{(3)}}{g_A}\left(\epsilon_{S-P,i}^{S-P} - \epsilon_{S+P,i}^{S-P}\right), \\
\varepsilon'_S &= \frac{F_S^{(3)}}{g_A}\left(\epsilon_{S+P,i}^{S+P} + \epsilon_{S-P,i}^{S+P}\right), \quad \varepsilon'_P = \frac{F_P^{(3)}}{g_A}\left(\epsilon_{S+P,i}^{S+P} - \epsilon_{S-P,i}^{S+P}\right).
\end{aligned}
\tag{127}
$$

Note that in the notations of Ref. [5]:

$$
t = u + u_5, \qquad t^l = u^{0l} + u_5^{0l}. \tag{128}
$$

Since the nucleon recoil term $\mathbf{P}_a$ behaves as an even parity operator while the neutrino momentum $\mathbf{k}$ and the recoil terms B_a, C_a, $\mathbf{D}_a$ as odd ones, each of the A_j, $\mathbf{k}\cdot\mathbf{A}_j$, B_j, $\mathbf{k}\cdot\mathbf{B}_j$ has a definite parity. The operators

$$
\begin{aligned}
A_1,\ A_3^i,\ A_4^i,\ A_{3R}^{Pi},\ A_{4R}^{Pi};\quad B_3^i,\ B_{3R}^{Pi}; \\
\mathbf{r}\cdot\mathbf{B}_{4R},\ r^l A_{2R},\ r^l A_{5R}^{lk}\ ,
\end{aligned}
\tag{129}
$$

have even parity and the operators

$$
\begin{aligned}
A_{1R},\ A_{3R}^i,\ A_{4R}^i;\quad B_{1R},\ B_{2R},\ B_{3R}^i; \\
\mathbf{r}\cdot\mathbf{B}_4,\ \mathbf{r}\cdot\mathbf{B}_{4R}^P,\ r^l A_2,\ r^l A_5^{lk},\ r^l A_{5R}^{Plk}\ ,
\end{aligned}
\tag{130}
$$

have odd parity. The odd-parity operators do not contribute to the $0^+ \to J^+$ transition in the case where both the electrons are in the S-wave state (the $S - S$ case) with no de Broglie wave length correction (no FBWC).

Using the definitions of neutrino potentials

$$
\begin{aligned}
h_+ &= \frac{R}{2}(H_1 + H_2), \quad h_0 = \frac{1}{\varepsilon_{21}}(H_1 - H_2), \quad h_{0\omega} = \frac{R}{\varepsilon_{21}}(H_{\omega 1} - H_{\omega 2}), \\
h'_+\hat{r}^l &= -i\frac{rR}{2}\left(H_{k1}^l + H_{k2}^l\right), \quad h'_0\hat{r}^l = -i\frac{r}{\varepsilon_{21}}\left(H_{k1}^l - H_{k2}^l\right)\ ,
\end{aligned}
\tag{131}
$$

in the $S-S$ case with no FWBC, Eqs. (115), (117) are reduced to

$$\{M_{SP}^m\}_{n,S-S} = \int d\mathbf{x}d\mathbf{y}T_N h_+ \left[A_1 F_{5+}^0 + (A_3^i + A_{3R}^{Pi})F_{5+}^i\right], \tag{132}$$

$$\{M_{SP}^m\}_{c,S-S} = \frac{\varepsilon_{21}R}{2}\int d\mathbf{x}d\mathbf{y}T_N h_0(B_3^i + B_{3R}^{Pi})F_+^i, \tag{133}$$

$$\begin{aligned}\left\{M_{SP}^k\right\}_{n,S-S} &= -\frac{1}{2}\frac{\varepsilon_{21}}{m_e}\int d\mathbf{x}d\mathbf{y}T_N h_{0\omega}(A_4^i + A_{4R}^{Pi})E_+^i \\ &+\frac{1}{2}\frac{\varepsilon_{21}}{m_e}\int d\mathbf{x}d\mathbf{y}T_N \frac{iR}{r}h_0'\hat{r}^l\left(-A_{2R}E_+^l + A_{5R}^{lk}E_+^k\right),\end{aligned} \tag{134}$$

$$\left\{M_{SP}^k\right\}_{c,S-S} = \frac{2}{m_e R}\int d\mathbf{x}d\mathbf{y}T_N \frac{iR}{2r}h_+'\hat{\mathbf{r}}\cdot\mathbf{B}_{4R}E_+, \tag{135}$$

where E, F are taken for $\mathbf{x}$=0, $\mathbf{y}$=0.

For the $0^+\to 0^+$ transition we have

$$\sum_i \frac{m_i}{m_e}\sum_N \{M_{SP}^m\}_{S-S} = g_A^2 C_1^A F_{5+}^0, \tag{136}$$

$$\sum_i\sum_N \left\{M_{SP}^k\right\}_{S-S} = g_A^2\frac{2}{m_e R}\{C_{4R}^B\}_c E_+, \tag{137}$$

with

$$C_1^A = \langle\frac{m_i}{m_e}h_+ A_1\rangle, \quad \{C_{4R}^B\}_c = \langle\frac{iR}{2r}h_+'\hat{\mathbf{r}}\cdot\mathbf{B}_{4R}\rangle, \tag{138}$$

where $\hat{\mathbf{r}}=\mathbf{r}/r$ and $\langle X\rangle = \sum_i\sum_N\langle 0_f^+||X||0_I^+\rangle$, with $h=h(r,E_N)$.

In the $S-P_{1/2}$ case with no FBWC for the $0^+\to 0^+$ transition we have

$$\{M_{SP}^m\}_{n,S-P_{1/2}} = \int d\mathbf{x}d\mathbf{y}T_N h_+\left(A_{3R}^i F_{5+}^i + B_{3R}^i F_-^i\right), \tag{139}$$

$$\{M_{SP}^m\}_{c,S-P_{1/2}} = \frac{\varepsilon_{21}R}{2}\int d\mathbf{x}d\mathbf{y}T_N h_0\left(A_{3R}^i F_{5-}^i + B_{3R}^i F_+^i\right), \tag{140}$$

$$\begin{aligned}\left\{M_{SP}^k\right\}_{n,S-P_{1/2}} &= -\frac{1}{2}\frac{\varepsilon_{21}}{m_e}\int d\mathbf{x}d\mathbf{y}T_N h_{0\omega}A_{4R}^i E_+^i \\ &+\frac{1}{2}\frac{\varepsilon_{21}}{m_e}\int d\mathbf{x}d\mathbf{y}T_N\frac{iR}{r}h_0'\hat{r}^l\left[-A_2E_+^l + (A_5^{lk}+A_{5R}^{Plk})E_+^k\right],\end{aligned} \tag{141}$$

$$\begin{aligned}\left\{M_{SP}^k\right\}_{c,S-P_{1/2}} &= -\frac{1}{m_e R}\int d\mathbf{x}d\mathbf{y}T_N h_{0\omega}A_{4R}^i E_-^i \\ &+\frac{2}{m_e R}\int d\mathbf{x}d\mathbf{y}T_N\frac{iR}{2r}h_+'\hat{r}^l\left[-A_2E_-^l + (A_5^{lk}+A_{5R}^{Plk})E_-^k\right].\end{aligned} \tag{142}$$

The squared modulus of the matrix element (113), summed over the polarizations s_j of the electrons and multiplied by the phase space element (35), yields the differential decay rate for the $0^+\to 0^+$ transition

$$d\Gamma = \sum_{s_1,s_2}|R_{0\nu}^{SP}|^2\frac{m_e^5}{4\pi^3}d\Omega_{0\nu} = \frac{a_{0\nu}}{(m_e R)^2}\left[A_0^{SP} - \hat{\mathbf{p}}_1\cdot\hat{\mathbf{p}}_2 B_0^{SP}\right]d\Omega_{0\nu}, \tag{143}$$

with $a_{0\nu}$ being defined in Eq. (36). Here the coefficients are

$$A_0^{SP} = \sum_{i=1}^4 |M_i|^2, \tag{144}$$

$$B_0^{SP} = \mathrm{Re}(M_1M_2^* + M_1^*M_2 + M_3M_4^* + M_3^*M_4), \tag{145}$$

with

$$M_1 = \alpha^*_{-1-1}\left\{\left[-C_1^A + \frac{2}{m_eR}\{C_{4R}^B\}_c\right] + \left[\left(\frac{m_eR}{3}\left(\frac{\zeta}{m_eR}-2\right)C_{3R}^A + \frac{\varepsilon_{21}R}{3}\{C_{3R}^A\}_c\right)\frac{r}{2R}\right.\right.$$
$$+\frac{\varepsilon_{21}^2R}{6m_e}\left(C_2^A - C_5^A - C_{5R}^A - C_{4R}^A\right)\frac{r}{2R} + \frac{1}{6}\left(\frac{\zeta}{m_eR}-2\right)\left(\{C_2^A\}_c - \{C_5^A\}_c - \{C_{5R}^A\}_c - \{C_{4R}^A\}_c\right)\Big]$$
$$\left.+\left[\frac{(\alpha Z)^2}{2m_eR}\left(\{C_4^A\}_c + \{C_{4R}^A\}_c - 3\{C_{4RF}^B\}_c\right)\right]\right\}, \tag{146}$$
$$M_2 = \alpha^*_{11}\left\{\left[C_1^A + \frac{2}{m_eR}\{C_{4R}^B\}_c\right] + \left[\left(\frac{m_eR}{3}\left(\frac{\zeta}{m_eR}+2\right)C_{3R}^A - \frac{\varepsilon_{21}R}{3}\{C_{3R}^A\}_c\right)\frac{r}{2R}\right.\right.$$
$$+\frac{\varepsilon_{21}^2R}{6m_e}\left(C_2^A - C_5^A - C_{5R}^A - C_{4R}^A\right)\frac{r}{2R} + \frac{1}{6}\left(\frac{\zeta}{m_eR}+2\right)\left(\{C_2^A\}_c - \{C_5^A\}_c - \{C_{5R}^A\}_c - \{C_{4R}^A\}_c\right)\Big]$$
$$\left.+\left[\frac{(\alpha Z)^2}{2m_eR}\left(\{C_4^A\}_c + \{C_{4R}^A\}_c - 3\{C_{4RF}^B\}_c\right)\right]\right\}, \tag{147}$$
$$M_3 = \alpha^*_{1-1}\left\{\left[\frac{2}{m_eR}\{C_{4R}^B\}_c\right] + \left[\frac{\varepsilon_{21}R}{6}\left(\frac{\varepsilon_{21}}{m_e}+2\right)\left(C_2^A - C_5^A - C_{5R}^A - C_{4R}^A\right)\frac{r}{2R}\right.\right.$$
$$+\frac{1}{6}\frac{\zeta}{m_eR}\left(\{C_2^A\}_c - \{C_5^A\}_c - \{C_{5R}^A\}_c - \{C_{4R}^A\}_c\right)\Big]$$
$$\left.+\left[\frac{(\alpha Z)^2}{2m_eR}\left(\{C_4^A\}_c + \{C_{4R}^A\}_c - 3\{C_{4RF}^B\}_c\right)\right]\right\}, \tag{148}$$
$$M_4 = \alpha^*_{-11}\left\{\left[\frac{2}{m_eR}\{C_{4R}^B\}_c\right] + \left[\frac{\varepsilon_{21}R}{6}\left(\frac{\varepsilon_{21}}{m_e}-2\right)\left(C_2^A - C_5^A - C_{5R}^A - C_{4R}^A\right)\frac{r}{2R}\right.\right.$$
$$+\frac{1}{6}\frac{\zeta}{m_eR}\left(\{C_2^A\}_c - \{C_5^A\}_c - \{C_{5R}^A\}_c - \{C_{4R}^A\}_c\right)\Big]$$
$$\left.+\left[\frac{(\alpha Z)^2}{2m_eR}\left(\{C_4^A\}_c + \{C_{4R}^A\}_c - 3\{C_{4RF}^B\}_c\right)\right]\right\}, \tag{149}$$

where $\alpha_{ij} = \tilde{A}_i(\varepsilon_2)\tilde{A}_j(\varepsilon_1)$ and the nucleon matrix elements are

$$C_{3R}^B = \langle\frac{m_i}{m_e}\frac{i}{r}h_+\mathbf{r}\cdot\mathbf{B}_{3R}\rangle, \quad \{C_{3R}^B\}_c = \langle\frac{m_i}{m_e}\frac{i}{2R}h_0\mathbf{r}_+\cdot\mathbf{B}_{3R}\rangle,$$
$$C_{3R}^A = \langle\frac{m_i}{m_e}\frac{i}{2R}h_+\mathbf{r}_+\cdot\mathbf{A}_{3R}\rangle, \quad \{C_{3R}^A\}_c = \langle\frac{m_i}{m_e}\frac{i}{r}h_0\mathbf{r}\cdot\mathbf{A}_{3R}\rangle,$$
$$C_{4R}^A = \langle\frac{i}{2R}h_{0\omega}\mathbf{r}_+\cdot\mathbf{A}_{4R}\rangle, \quad \{C_{4R}^A\}_c = \langle\frac{i}{R}h_{0\omega}\mathbf{r}\cdot\mathbf{A}_{4R}\rangle,$$
$$C_2^A = \langle\frac{1}{2r}h_0'\hat{\mathbf{r}}\cdot\mathbf{r}_+A_2\rangle, \quad \{C_2^A\}_c = \langle h_+'A_2\rangle,$$
$$C_{5(R)}^A = \langle\frac{1}{2R}h_0'\hat{r}^ir_+^jA_{5(R)}^{ij}\rangle, \quad \{C_{5(R)}^A\}_c = \langle\frac{1}{r}h_+'\hat{r}^ir_+^jA_{5(R)}^{ij})\rangle,$$
$$\{C_{4RF}^B\} = \langle\frac{iR}{2r}\frac{r_a^2+r_b^2}{2R^2}h_+'\hat{\mathbf{r}}\cdot\mathbf{B}_{4R}\rangle, \tag{150}$$

with $\mathbf{r}_+ = \mathbf{y}+\mathbf{x} = 2R\hat{\mathbf{r}}_+$.

The terms in the first brackets in Eqs. (146)–(149) come from the $S-S$ case, the terms in the second brackets come from the $S-P_{1/2}$ case and in the third brackets there are the most important terms due to the $P_{1/2}-P_{1/2}$ case and FBWC.

Assuming now $\langle m\rangle \neq 0$ for the dominant terms we have

$$M_1 = \alpha^*_{-1-1}\left\{\left[Z_1^X - C_1^A + \frac{2}{m_eR}\{C_{4R}^B\}_c\right]\right.$$
$$\left.+\left[\frac{\varepsilon_{21}^2R}{6m_e}\left(C_2^A - C_5^A\right)\frac{r}{2R} + \frac{1}{6}\left(\frac{\zeta}{m_eR}-2\right)\left(\{C_2^A\}_c - \{C_5^A\}_c\right)\right]\right\}, \tag{151}$$

$$
\begin{aligned}
M_2 &= \alpha_{11}^* \left\{ \left[Z_1^X + C_1^A + \frac{2}{m_e R}\{C_{4R}^B\}_c \right] \right. \\
&+ \left. \left[\frac{\varepsilon_{21}^2 R}{6 m_e} \left(C_2^A - C_5^A\right) \frac{r}{2R} + \frac{1}{6}\left(\frac{\zeta}{m_e R} + 2\right)\left(\{C_2^A\}_c - \{C_5^A\}_c\right) \right] \right\}, \qquad (152) \\
M_3 &= \alpha_{1-1}^* \left\{ \left[Z_1^X + \frac{2}{m_e R}\{C_{4R}^B\}_c \right] \right. \\
&+ \left. \left[\frac{\varepsilon_{21} R}{6} \left(\frac{\varepsilon_{21}}{m_e} + 2\right) \left(C_2^A - C_5^A\right) \frac{r}{2R} + \frac{1}{6}\frac{\zeta}{m_e R}\left(\{C_2^A\}_c - \{C_5^A\}_c\right) \right] \right\}, \qquad (153) \\
M_4 &= \alpha_{-11}^* \left\{ \left[Z_1^X + \frac{2}{m_e R}\{C_{4R}^B\}_c \right] \right. \\
&+ \left. \left[\frac{\varepsilon_{21} R}{6} \left(\frac{\varepsilon_{21}}{m_e} - 2\right) \left(C_2^A - C_5^A\right) \frac{r}{2R} + \frac{1}{6}\frac{\zeta}{m_e R}\left(\{C_2^A\}_c - \{C_5^A\}_c\right) \right] \right\}. \qquad (154)
\end{aligned}
$$

In the expressions for $M_1, ..., M_4$, the terms with ζ are due to the inclusion of the P-wave in the electron wave function and those with C_{4R}^B are from the inclusion of the nucleon recoil effect. In the closure approximation there are no contributions due to the P-wave and the recoil effects. Note that some of the subdominant terms should be taken into account in case of large cancellation among the dominant terms.

B $0\nu 2\beta$ decay rate for vector nonstandard terms

In this appendix we in general follow the derivation of Ref. [5]. However in addition to Ref. [5] we keep in our calculations the terms associated with the parameters $\epsilon_{V\mp A}^{V-A}$ and the pseudoscalar form factor.

The nucleon currents in the impulse approximation up to order p/m_p in the nonrelativistic expansion are [32, 34]:

$$
J_{V\mp A}^{\mu +}(\mathbf{x}) = \sum_a \tau_+^a \delta(\mathbf{x} - \mathbf{r}_a) \left[g^{\mu 0}(g_V I_a \mp g_A C_a) + g^{\mu m}(\pm g_A \sigma_{am} - g_V D_a^m \mp g_A P_a^m) \right], \qquad (155)
$$

with C_a, D_a^m, P_a^m given in Eq. (101).

In terms of $S_{L\mu\nu}$, $V_{\alpha\mu\nu}$, $J_{\alpha\beta}^{\mu\nu}$ $(\alpha, \beta = L, R)$ [5] the matrix element

$$
R_{0\nu}^{VA} = C_{0\nu} \sum_i \sum_N \frac{R}{2m_e} \int d\mathbf{x} d\mathbf{y}\; 4\pi \frac{d\mathbf{k}}{(2\pi)^3} \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{\omega} \left(m_i J_{LL}^{\mu\nu} S_{L\mu\nu} + J_{LR}^{\mu\nu} V_{L\mu\nu} + J_{RL}^{\mu\nu} V_{R\mu\nu}\right), \qquad (156)
$$

may be expressed as

$$
R_{0\nu}^{VA} = C_{0\nu} \sum_i \sum_N \left(\frac{m_i}{m_e} M_{VA}^m + M_{VA}^k\right), M_{VA}^{m,k} = \{M_{VA}^{m,k}\}_n + \{M_{VA}^{m,k}\}_c. \qquad (157)
$$

The analogues of the Eqs. (C.2.11), (C.2.23), and (C.2.24) from Ref. [5] are as follows:

$$
\begin{aligned}
\{M_{VA}^m\}_n &\equiv \{M_{m_\nu}\}_n = \frac{R}{2} \int d\mathbf{x} d\mathbf{y} T_N (H_1 + H_2) \left[(X_1 + \tilde{X}_{1R}) E_+ + (Y_1^i + \tilde{Y}_{1R}^i) E_-^i \right], \qquad (158) \\
\{M_{VA}^m\}_c &\equiv \{M_{m_\nu}\}_c = \frac{R}{2} \int d\mathbf{x} d\mathbf{y} T_N (H_1 - H_2) \left[(X_1 + \tilde{X}_{1R}) E_- + (Y_1^i + \tilde{Y}_{1R}^i) E_+^i \right], \qquad (159) \\
\{M_{VA}^k\}_n &\equiv \{M_{V+A}(a)\}_n = \frac{R}{m_e} \int d\mathbf{x} d\mathbf{y} T_N \{\, (H_{\omega 1} - H_{\omega 2}) \\
&\times \left[(X_3 + \tilde{X}_{5R}) F_+^0 + Y_{3R} F_{5-}^0 + (X_5^l + \tilde{X}_{4R}^l) F_+^l + (Y_4^l + \tilde{Y}_{6R}^l) F_{5-}^l \right] + (H_{k1}^l + H_{k2}^l) \\
&\times \left[(X_5^l + \tilde{X}_{3R}^l) F_-^0 + (Y_3^l + \tilde{Y}_{5R}^l) F_{5+}^0 + (X_4^{lk} + \tilde{X}_{6R}^{lk}) F_-^k + (Y_6^{lk} + \tilde{Y}_{4R}^{lk}) F_{5+}^k \right] \}, \qquad (160) \\
\{M_{VA}^k\}_c &\equiv \{M_{V+A}(a)\}_c = \frac{R}{m_e} \int d\mathbf{x} d\mathbf{y} T_N \{\, (H_{\omega 1} + H_{\omega 2})
\end{aligned}
$$

$$\times \left[(X_3 + \tilde{X}_{5R})F_-^0 + Y_{3R}F_{5+}^0 + (X_5^l + \tilde{X}_{4R}^l)F_-^l + (Y_4^l + \tilde{Y}_{6R}^l)F_{5+}^l\right] + (H_{k1}^l - H_{k2}^l)$$
$$\times \left[(X_5^l + \tilde{X}_{3R}^l)F_+^0 + (Y_3^l + \tilde{Y}_{5R}^l)F_{5-}^0 + (X_4^{lk} + \tilde{X}_{6R}^{lk})F_+^k + (Y_6^{lk} + \tilde{Y}_{4R}^{lk})F_{5-}^k\right] \}, \tag{161}$$

with $\tilde{X} = X + X^P$, $\tilde{Y} = Y + Y^P$. The operators X and Y are defined in [5], except for the operator $Y_{6R}^l = -Y_{5R}^l$ which is defined to remove the minus sign from the Eqs. (160) and (161); $X_1 = X_{1S}$, $Y_1 = Y_{1S}$.

The additional operators are

$$\begin{aligned}
&X_{1R}^P = G_A^2 P_{\sigma+}^{ii}, \quad X_{3R}^{Pl} = X_{4R}^{Pl} = G_- P_+^l, \quad X_{5R}^P = G_A \varepsilon_A P_{\sigma+}^{ii},\\
&X_{6R}^{Plk} = -G_A \varepsilon_A \left[\delta_{lk} P_{\sigma+}^{ii} - \left(P_{\sigma+}^{lk} + P_{\sigma+}^{kl}\right)\right] + iG_+ \varepsilon_{ilk} P_+^i,\\
&Y_{1R}^{Pi} = G_V G_A P_-^i + G_A^2 i\varepsilon_{ijk} P_{\sigma+}^{jk}, \quad Y_{4R}^{Plk} = -iG_- \varepsilon_{ilk} P_-^i,\\
&Y_{5R}^{Pl} = iG_A \varepsilon_A \varepsilon_{lij} P_{\sigma+}^{ij} - G_+ P_-^l, \quad Y_{6R}^{Pl} = -iG_A \varepsilon_A \varepsilon_{lij} P_{\sigma+}^{ij} - G_+ P_-^l,
\end{aligned} \tag{162}$$

with

$$P_{\sigma+}^{ij} = \sigma_a^i P_b^j + P_a^i \sigma_b^j. \tag{163}$$

Under the exchange of running indices a and b, nuclear operators X, electron currents E_+ and F_+ and neutrino potentials H_i and $H_{\omega i}$ are even, while Y, E_-, F_-, and $\mathbf{H}_{ki}$ are odd.

New constants are defined as:

$$\varepsilon_V = \frac{g_V}{g_A}\left(\epsilon_{V+A,i}^{V+A} + \epsilon_{V-A,i}^{V+A}\right), \quad \varepsilon_A = \epsilon_{V+A,i}^{V+A} - \epsilon_{V-A,i}^{V+A}. \tag{164}$$

The operators

$$\begin{aligned}
&X_1,\ X_{1R}^P;\quad Y_1^i,\ Y_{1R}^{Pi};\\
&X_3,\ X_5^l, X_{5R}^P,\ X_{4R}^{Pl},\ \mathbf{r}\cdot\mathbf{X}_{3R},\ r^l X_{6R}^{lk};\\
&Y_4^l,\ Y_{6R}^{Pl},\ \mathbf{r}\cdot\mathbf{Y}_{5R}, r^l Y_{4R}^{lk}\ ,
\end{aligned} \tag{165}$$

have even parity and the operators

$$\begin{aligned}
X_{1R};\quad Y_{1R}^i; X_{5R},\ X_{4R}^l,\ \mathbf{r}\cdot\mathbf{X}_5,\ \mathbf{r}\cdot\mathbf{X}_{3R}^P,\ r^l X_4^{lk},\ r^l X_{6R}^{Plk};\\
Y_{3R},\ Y_{6R}^l,\ \mathbf{r}\cdot\mathbf{Y}_3,\ \mathbf{r}\cdot\mathbf{Y}_{5R}^P,\ r^l Y_6^{lk},\ r^l Y_{4R}^{Plk}\ ,
\end{aligned} \tag{166}$$

have odd parity.

Using the definitions of the neutrino potentials from Eq. (131) and

$$h_\omega = \frac{R^2}{2}(H_{\omega 1} + H_{\omega 2}) \tag{167}$$

in the $S - S$ case with no FBWC we have

$$\{M_{VA}^m\}_{n,S-S} = \int d\mathbf{x}d\mathbf{y} T_N h_+ (X_1 + X_{1R}^P)E_+, \tag{168}$$

$$\{M_{VA}^m\}_{c,S-S} = \frac{\varepsilon_{21}R}{2}\int d\mathbf{x}d\mathbf{y} T_N h_0 (Y_1^i + Y_{1R}^{Pi})E_+^i, \tag{169}$$

$$\{M_{VA}^k\}_{n,S-S} = \frac{\varepsilon_{21}}{m_e}\int d\mathbf{x}d\mathbf{y} T_N h_{0\omega}\left[(X_3 + X_{5R}^P)F_+^0 + (X_5^l + X_{4R}^{Pl})F_+^l\right]$$
$$+\frac{4}{m_e R}\int d\mathbf{x}d\mathbf{y} T_N \frac{iR}{2r} h_+' \hat{r}^l \left[Y_{5R}^l F_{5+}^0 + Y_{4R}^{lk} F_{5+}^k\right], \tag{170}$$

$$\{M_{VA}^k\}_{c,S-S} = \frac{2}{m_e R}\int d\mathbf{x}d\mathbf{y} T_N h_\omega (Y_4^l + Y_{6R}^{Pl})F_{5+}^l$$
$$+\frac{\varepsilon_{21}}{m_e}\int d\mathbf{x}d\mathbf{y} T_N \frac{iR}{r} h_0' \hat{r}^l \left(X_{3R}^l F_+^0 + X_{6R}^{lk} F_+^k\right), \tag{171}$$

where E and F are taken for $\mathbf{x} = \mathbf{y}$=0.

For the $0^+ \to 0^+$ transition we have

$$\sum_i \frac{m_i}{m_e} \sum_N \{M_{VA}^m\}_{S-S} = g_A^2 (Z_1^X + Z_{1R}^{XP}) E_+, \tag{172}$$

$$\sum_i \sum_N \{M_{VA}^k\}_{S-S} = g_A^2 \left[\frac{\varepsilon_{21}}{m_e} (Z_3^X + Z_{5R}^{XP} + \{Z_{3R}^X\}_c) F_+^0 + \frac{4}{m_e R} Z_{4R}^Y F_{5+}^0 \right], \tag{173}$$

with

$$Z_1^X = \langle \frac{m_i}{m_e} h_+ X_1 \rangle, \qquad Z_{1R}^{XP} = \langle \frac{m_i}{m_e} h_+ X_{1R}^P \rangle, \qquad Z_3^X = \langle h_{0\omega} X_3 \rangle,$$
$$Z_{4R}^Y = \langle \frac{iR}{2r} h_+' \hat{\mathbf{r}} \cdot \mathbf{Y}_{5R} \rangle, \qquad Z_{5R}^{XP} = \langle h_{0\omega} X_{5R}^P \rangle, \qquad \{Z_{3R}^X\}_c = \langle \frac{iR}{r} h_0' \hat{\mathbf{r}} \cdot \mathbf{X}_{3R} \rangle. \tag{174}$$

In the $S - P_{1/2}$ case with no FBWC for the $0^+ \to 0^+$ transition we have

$$\{M_{VA}^m\}_{n,S-P_{1/2}} = \int d\mathbf{x} d\mathbf{y} T_N h_+ Y_{1R}^i E_-^i, \tag{175}$$

$$\{M_{VA}^m\}_{c,S-P_{1/2}} = \frac{\varepsilon_{21} R}{2} \int d\mathbf{x} d\mathbf{y} T_N h_0 Y_{1R}^i E_+^i, \tag{176}$$

$$\{M_{VA}^k\}_{n,S-P_{1/2}} = \frac{\varepsilon_{21}}{m_e} \int d\mathbf{x} d\mathbf{y} T_N h_{0\omega} (X_{4R}^l F_+^l + Y_{6R}^l F_{5-}^l)$$
$$+ \frac{4}{m_e R} \int d\mathbf{x} d\mathbf{y} T_N \frac{iR}{2r} h_+' \hat{r}^l \left[(X_4^{lk} + X_{6R}^{Plk}) F_-^k + (Y_6^{lk} + Y_{4R}^{Plk}) F_{5+}^k \right], \tag{177}$$

$$\{M_{VA}^k\}_{c,S-P_{1/2}} = \frac{2}{m_e R} \int d\mathbf{x} d\mathbf{y} T_N h_\omega (X_{4R}^l F_-^l + Y_{6R}^l F_{5+}^l)$$
$$+ \frac{\varepsilon_{21}}{m_e} \int d\mathbf{x} d\mathbf{y} T_N \frac{iR}{r} h_0' \hat{r}^l \left[(X_4^{lk} + X_{6R}^{Plk}) F_+^k + (Y_6^{lk} + Y_{4R}^{Plk}) F_{5-}^k \right]. \tag{178}$$

The decay rate for the $0^+ \to 0^+$ transition takes the form

$$d\Gamma = \sum_{s_1, s_2} |R_{0\nu}|^2 \frac{m_e^5}{4\pi^3} d\Omega_{0\nu} = \frac{a_{0\nu}}{(m_e R)^2} \left[A_0^{VA} - \hat{\mathbf{p}}_1 \cdot \hat{\mathbf{p}}_2 B_0^{VA} \right] d\Omega_{0\nu}, \tag{179}$$

where the coefficients are

$$A_0^{VA} = \sum_{i=1}^4 |N_i|^2, \tag{180}$$

$$B_0^{VA} = \mathrm{Re}(N_1 N_2^* + N_1^* N_2 + N_3 N_4^* + N_3^* N_4), \tag{181}$$

with

$$N_1 = \alpha_{-1-1}^* \left\{ \left[Z_1^X + Z_{1R}^{XP} - \frac{4}{m_e R} Z_{4R}^Y \right] + \left[\frac{m_e r}{6} \left(\left(\frac{\zeta}{m_e R} - 2 \right) Z_{1R}^Y + \frac{\varepsilon_{21}^2 R}{2 m_e} \{Z_{1R}^Y\}_c \right) + \right.\right.$$
$$\frac{2}{3} \left(\frac{\zeta}{m_e R} - 2 \right) \left(Z_6^Y + Z_{4R}^{YP} + \{Z_{6R}^Y\}_c \right) \frac{r}{2R} + \frac{1}{3} \frac{\varepsilon_{21}^2 R}{m_e} \left(Z_{6R}^Y - \frac{1}{2} (\{Z_6^Y\}_c + \{Z_{4R}^Y\}_c) \right) \Big]$$
$$\left.+ \left[\frac{(\alpha Z)^2}{m_e R} (\{Z_5^X\}_c + 3 Z_{5RF}^Y) \right] \right\}, \tag{182}$$

$$N_2 = \alpha_{11}^* \left\{ \left[Z_1^X + Z_{1R}^{XP} + \frac{4}{m_e R} Z_{4R}^Y \right] + \left[\frac{m_e r}{6} \left(\left(\frac{\zeta}{m_e R} + 2 \right) Z_{1R}^Y + \frac{\varepsilon_{21}^2 R}{2 m_e} \{Z_{1R}^Y\}_c \right) + \right.\right.$$
$$-\frac{2}{3} \left(\frac{\zeta}{m_e R} + 2 \right) \left(Z_6^Y + Z_{4R}^{YP} + \{Z_{6R}^Y\}_c \right) \frac{r}{2R} - \frac{1}{3} \frac{\varepsilon_{21}^2 R}{m_e} \left(Z_{6R}^Y - \frac{1}{2} (\{Z_6^Y\}_c + \{Z_{4R}^Y\}_c) \right) \Big]$$

$$+\left[-\frac{(\alpha Z)^2}{m_e R}(\{Z_5^X\}_c+3Z_{5RF}^Y)\right]\Bigg\}, \quad (183)$$

$$N_3=\alpha^*_{1-1}\Bigg\{\left[Z_1^X+Z_{1R}^{XP}-\frac{\varepsilon_{21}}{m_e}(Z_3^X+Z_{5R}^{XP}+\{Z_{3R}^X\}_c)\right]+\left[\frac{r}{6R}\left(\zeta Z_{1R}^Y+\frac{1}{2}\varepsilon_{21}(\varepsilon_{21}+2m_e)R^2Z_{2R}^Y\right)\right.$$

$$\left.+\frac{1}{3}\frac{\varepsilon_{21}}{m_e}\zeta\left(Z_{4R}^X-\frac{1}{2}(\{Z_4^X\}_c+\{Z_{6R}^{XP}\}_c)\right)\frac{r}{2R}-\frac{1}{3}\left(\frac{\varepsilon_{21}}{m_e}+2\right)(Z_4^X+Z_{6R}^{XP}-2Z_{4R}^X)\right]\Bigg\}, \quad (184)$$

$$N_4=\alpha^*_{-11}\Bigg\{\left[Z_1^X+Z_{1R}^{XP}+\frac{\varepsilon_{21}}{m_e}(Z_3^X+Z_{5R}^{XP}+\{Z_{3R}^X\}_c)\right]+\left[\frac{r}{6R}\left(\zeta Z_{1R}^Y+\frac{1}{2}\varepsilon_{21}(\varepsilon_{21}-2m_e)R^2Z_{2R}^Y\right)\right.$$

$$\left.-\frac{1}{3}\frac{\varepsilon_{21}}{m_e}\zeta\left(Z_{4R}^X-\frac{1}{2}(\{Z_4^X\}_c+\{Z_{6R}^{XP}\}_c)\right)\frac{r}{2R}+\frac{1}{3}\left(\frac{\varepsilon_{21}}{m_e}-2\right)(Z_4^X+Z_{6R}^{XP}-2Z_{4R}^X)\right]\Bigg\}, \quad (185)$$

where the terms in the first brackets in Eqs. (182)–(185) come from the $S-S$ case and the terms in the second ones come from the $S-P_{1/2}$ case. The terms in the third brackets in Eqs. (182)–(183) are the most important terms of those that come from the $P_{1/2}-P_{1/2}$ case and from the $S-S$ case due to FBWC. The nuclear matrix elements are

$$\begin{aligned}
&Z_{1R}^Y=\langle\frac{m_i}{m_e}\frac{i}{2R}h_+\mathbf{r}\cdot\mathbf{Y}_{1R}\rangle,\quad \{Z_{1R}^Y\}_c=\langle\frac{m_i}{m_e}\frac{i}{2R}h_0\mathbf{r}_+\cdot\mathbf{Y}_{1R}\rangle,\\
&Z_6^Y=\langle-\frac{1}{2r}h'_+\hat{r}^ir_+^jY_6^{ij}\rangle,\quad Z_{4R}^{YP}=\langle-\frac{1}{2r}h'_+\hat{r}^ir_+^jY_{4R}^{Pij}\rangle,\quad \{Z_{6R}^Y\}_c=\langle\frac{i}{2R}h_\omega\mathbf{r}\cdot\mathbf{Y}_{6R}\rangle,\\
&Z_{6R}^Y=\langle\frac{i}{2R}h_{0\omega}\mathbf{r}\cdot\mathbf{Y}_{6R}\rangle,\quad \{Z_6^Y\}_c=\langle\frac{1}{r}h'_0\hat{r}^ir^jY_6^{ij}\rangle,\quad \{Z_{4R}^Y\}_c=\langle\frac{1}{r}h'_0\hat{r}^ir^jY_{4R}^{ij}\rangle,\\
&Z_{4R}^X=\langle\frac{i}{2R}h_{0\omega}\mathbf{r}_+\cdot\mathbf{X}_{4R}\rangle,\quad \{Z_4^X\}_c=\langle\frac{1}{r}h'_0\hat{r}^ir_+^jX_4^{ij}\rangle,\quad \{Z_{6R}^{XP}\}_c=\langle\frac{1}{r}h'_0\hat{r}^ir_+^jX_{6R}^{Pij}\rangle,\\
&Z_4^X=\langle\frac{1}{r}h'_+\hat{r}^ir^jX_4^{ij}\rangle,\quad \{Z_5^X\}_c=\langle\frac{ir^2}{2R^2}h_\omega[\hat{\mathbf{r}}_a\times\hat{\mathbf{r}}_b]\cdot\mathbf{X}_5\rangle,\quad Z_{5RF}^Y=\langle\frac{iR}{2r}\frac{r_a^2+r_b^2}{2R^2}h'_+\hat{\mathbf{r}}\cdot\mathbf{Y}_{5R}\rangle. \quad (186)
\end{aligned}$$

The dominant terms give

$$N_1=\alpha^*_{-1-1}\left\{\left[Z_1^X-\frac{4}{m_eR}Z_{4R}^Y\right]+\left[\frac{2}{3}\left(\frac{\zeta}{m_eR}-2\right)Z_6^Y\frac{r}{2R}\right]\right\}, \quad (187)$$

$$N_2=\alpha^*_{11}\left\{\left[Z_1^X+\frac{4}{m_eR}Z_{4R}^Y\right]+\left[-\frac{2}{3}\left(\frac{\zeta}{m_eR}+2\right)Z_6^Y\frac{r}{2R}\right]\right\}, \quad (188)$$

$$N_3=\alpha^*_{1-1}\left\{\left[Z_1^X-\frac{\varepsilon_{21}}{m_e}Z_3^X\right]+\left[-\frac{1}{3}\left(\frac{\varepsilon_{21}}{m_e}+2\right)Z_4^X\right]\right\}, \quad (189)$$

$$N_4=\alpha^*_{-11}\left\{\left[Z_1^X+\frac{\varepsilon_{21}}{m_e}Z_3^X\right]+\left[\frac{1}{3}\left(\frac{\varepsilon_{21}}{m_e}-2\right)Z_4^X\right]\right\}, \quad (190)$$

that agrees with the Eq. (C.3.7) of Ref. [5] taking into account the correspondence with their notations:

$$\begin{aligned}
Z_1^X&=Z_1, & Z_3^X&=Z_3, & Z_6^Y&=Z_6,\\
Z_{4R}^Y&=Z_{4R}, & Z_{4R}^X&=Z_{5R}, & Z_4^X&=Z_5, \quad (191)
\end{aligned}$$

and the fact that Z_2 is absent, as we have calculated only the leading contribution of the parameters ϵ_α^β. Recall that in Ref. [5] the pseudoscalar form factor is not taken into account. However the terms associated with this form factor do not contribute to the dominant terms (187)–(190). Note that in the expressions for N_1 and N_2 given above, the terms with ζ are due to the inclusion of the P-wave in the electron wave function and the ones with Z_{4R}^Y are due to the nucleon recoil effect. We remark that some of the subdominant terms, like those with Z_{4R}^X, $\{Z_4^X\}_c$, $\{Z_{6R}^Y\}_c$, $\{Z_5^X\}_c$ and Z_{5RF}^Y, should be taken into account in the case of large cancellation among the dominant terms. The same is valid for the contribution due to the pseudoscalar form factor $g_AP_a^i$ which yields corrections at about 10 % to the dominant terms.

C $0\nu 2\beta$ decay rate for tensor nonstandard terms

The nucleon currents in the impulse approximation up to order p/m_p in the nonrelativistic expansion are used [32, 34], $J^{\mu+}_{V-A}$ from Eq. (100) and

$$
\begin{aligned}
J^{\mu\nu+}_{T_{L,R}}(\mathbf{x}) &= T_1^{(3)} \sum_a \tau_+^a \delta(\mathbf{x}-\mathbf{r}_a) \left\{ (g^{\mu k} g^{\nu 0} - g^{\mu 0} g^{\nu k}) T_a^k + g^{\mu m} g^{\nu n} \varepsilon_{kmn} \sigma_{ak} \right. \\
&\mp \left. \frac{i}{2} \varepsilon^{\mu\nu\rho\sigma} \left[(g_{\rho k} g_{\sigma 0} - g_{\rho 0} g_{\sigma k}) T_{ak} + g_{\rho r} g_{\sigma s} \varepsilon_{rsk} \sigma_{ak} \right] \right\}, \quad (192) \\
T_a^k &= \left[i \left(T_1^{(3)} - 2T_2^{(3)} \right) q^k I_a + T_1^{(3)} [\boldsymbol{\sigma}_a \times \mathbf{Q}]^k \right] / (2T_1^{(3)} m_p), \quad (193)
\end{aligned}
$$

where, as before, $q^\mu = p^\mu - p'^\mu$ is the 4-momentum transferred from hadrons to leptons, $Q^\mu = p^\mu + p'^\mu$, p^μ and p'^μ are the initial and final 4-momenta of a nucleon. We neglect the dipole dependence of the form factors $T_1^{(3)}$ and $T_2^{(3)}$ on the momentum transfer and omit the zero argument of the form factors.

Consider the pure $T_{L,R}$ case assuming $\langle m \rangle = 0$. In terms of the hadronic currents

$$
\begin{aligned}
& J^{\alpha\mu\nu}_{LT_{L,R}} = \langle F | \hat{J}^{\alpha+}_L | N \rangle \langle N | \tilde{J}^{\mu\nu+}_{T_{L,R}} | I \rangle, \quad J^{\mu\nu\alpha}_{T_{L,R}L} = \langle F | \tilde{J}^{\mu\nu+}_{T_{L,R}} | N \rangle \langle N | \hat{J}^{\alpha+}_L | I \rangle, \quad (194) \\
& \tilde{J}^{\mu\nu+}_{T_L} = \epsilon^{T_L}_{T_L,i} J^{\mu\nu+}_{T_L} + \epsilon^{T_L}_{T_R,i} J^{\mu\nu+}_{T_R}, \quad \tilde{J}^{\mu\nu+}_{T_R} = \epsilon^{T_R}_{T_R,i} J^{\mu\nu+}_{T_R} + \epsilon^{T_R}_{T_L,i} J^{\mu\nu+}_{T_L}, \quad (195) \\
& \hat{J}^{\mu+}_L = U_{ei} J^{\mu+}_{V-A}\ , \quad (196)
\end{aligned}
$$

and the leptonic tensors

$$
\begin{aligned}
\ell^1_{\alpha\mu\nu} &= \frac{t^1_{\alpha\mu\nu}(2\mathbf{y}, 1\mathbf{x})}{\omega + A_1} - \frac{t^1_{\alpha\mu\nu}(1\mathbf{y}, 2\mathbf{x})}{\omega + A_2}, \quad (197) \\
\ell^1_{\alpha\lambda\mu\nu} &= \frac{t^1_{\alpha\lambda\mu\nu}(2\mathbf{y}, 1\mathbf{x})}{\omega + A_1} - \frac{t^1_{\alpha\lambda\mu\nu}(1\mathbf{y}, 2\mathbf{x})}{\omega + A_2}, \quad (198) \\
\ell^2_{\mu\nu\alpha} &= \frac{t^2_{\mu\nu\alpha}(2\mathbf{y}, 1\mathbf{x})}{\omega + A_1} - \frac{t^2_{\mu\nu\alpha}(1\mathbf{y}, 2\mathbf{x})}{\omega + A_2}, \quad (199) \\
\ell^2_{\mu\nu\lambda\alpha} &= \frac{t^2_{\mu\nu\lambda\alpha}(2\mathbf{y}, 1\mathbf{x})}{\omega + A_1} - \frac{t^2_{\mu\nu\lambda\alpha}(1\mathbf{y}, 2\mathbf{x})}{\omega + A_2}\ , \quad (200)
\end{aligned}
$$

with the electron currents defined as

$$
\begin{aligned}
& t^1_{\alpha\mu\nu}(2\mathbf{y}, 1\mathbf{x}) = \bar{e}_2(\mathbf{y}) \gamma_\alpha (1-\gamma_5) \sigma_{\mu\nu} e^c_1(\mathbf{x}), \\
& t^1_{\alpha\lambda\mu\nu}(2\mathbf{y}, 1\mathbf{x}) = \bar{e}_2(\mathbf{y}) \gamma_\alpha (1-\gamma_5) \gamma_\lambda \sigma_{\mu\nu} e^c_1(\mathbf{x}), \\
& t^2_{\mu\nu\alpha}(2\mathbf{y}, 1\mathbf{x}) = \bar{e}_2(\mathbf{y}) \sigma_{\mu\nu} (1-\gamma_5) \gamma_\alpha e^c_1(\mathbf{x}), \\
& t^2_{\mu\nu\lambda\alpha}(2\mathbf{y}, 1\mathbf{x}) = \bar{e}_2(\mathbf{y}) \sigma_{\mu\nu} \gamma_\lambda (1-\gamma_5) \gamma_\alpha e^c_1(\mathbf{x})\ , \quad (201)
\end{aligned}
$$

the matrix element is expressed as

$$
R^T_{0\nu} = \frac{1}{\sqrt{2!}} \left(\frac{G_F |V_{ud}|}{\sqrt{2}} \right)^2 2 \sum_i \int d\mathbf{x} d\mathbf{y} \frac{d\mathbf{k}}{(2\pi)^3} \frac{e^{i\mathbf{k}\cdot\mathbf{r}}}{2\omega}
$$

$$
\times \sum_N \left[m_i \left(J^{\alpha\mu\nu}_{LT_L} \ell^1_{\alpha\mu\nu} + J^{\mu\nu\alpha}_{T_L L} \ell^2_{\mu\nu\alpha} \right) + k^\lambda \left(J^{\alpha\mu\nu}_{LT_R} \ell^1_{\alpha\lambda\mu\nu} + J^{\mu\nu\alpha}_{T_R L} \ell^2_{\mu\nu\lambda\alpha} \right) \right]. \quad (202)
$$

For the electron currents we have the identities

$$
\begin{aligned}
& t^1_{\alpha\mu\nu}(1\mathbf{y}, 2\mathbf{x}) = -t^2_{\mu\nu\alpha}(2\mathbf{y}, 1\mathbf{x}), \\
& t^1_{\alpha\lambda\mu\nu}(1\mathbf{y}, 2\mathbf{x}) = t^2_{\mu\nu\lambda\alpha}(2\mathbf{y}, 1\mathbf{x}). \quad (203)
\end{aligned}
$$

Using Eqs (110), (111), and (112), the matrix element (202) is expressed as

$$R_{0\nu}^{T} = C_{0\nu}\sum_{i}\sum_{N}\left(\frac{m_i}{m_e}M_T^m + M_T^k\right), \tag{204}$$

$$M_T^{m,k} = \{M_T^{m,k}\}_n + \{M_T^{m,k}\}_c, \tag{205}$$

with nonvanishing (n) and vanishing (c) in the closure approximation parts:

$$\begin{aligned}\{M_T^m\}_n &= R\int d\mathbf{x}d\mathbf{y}T_N(H_1+H_2)\\ &\times\left[(U_1+\tilde{U}_{1R})F_{5+}^0 + (U_3^i+\tilde{U}_{3R}^i)F_{5+}^i + \tilde{V}_{1R}F_-^0 + (V_3^i+\tilde{V}_{3R}^i)F_-^i\right],\end{aligned} \tag{206}$$

$$\begin{aligned}\{M_T^m\}_c &= R\int d\mathbf{x}d\mathbf{y}T_N(H_1-H_2)\\ &\times\left[(U_1+\tilde{U}_{1R})F_{5-}^0 + (U_3^i+\tilde{U}_{3R}^i)F_{5-}^i + \tilde{V}_{1R}F_+^0 + (V_3^i+\tilde{V}_{3R}^i)F_+^i\right],\end{aligned} \tag{207}$$

$$\begin{aligned}\{M_T^k\}_n &= \frac{R}{m_e}\int d\mathbf{x}d\mathbf{y}T_N(H_{\omega1}-H_{\omega2})\left[\tilde{V}_{2R}E_- + (U_4^i+\tilde{U}_{4R}^i)F_+^{0i} + (U_6^{ij}+\tilde{U}_{6R}^{ij})F_+^{ij}\right]\\ &+(H_{k1}^i+H_{k2}^i)\left[(V_4^i+\tilde{V}_{4R}^i)E_+ + (U_2+\tilde{U}_{2R})F_-^{0i} + (U_5^j+\tilde{U}_{5R}^j)F_-^{ij} + (U_7^{ij}+\tilde{U}_{7R}^{ij})F_-^{0j}\right.\\ &\left.+(U_8^{ijk}+\tilde{U}_{8R}^{ijk})F_-^{jk}\right],\end{aligned} \tag{208}$$

$$\begin{aligned}\{M_T^k\}_c &= \frac{R}{m_e}\int d\mathbf{x}d\mathbf{y}T_N(H_{\omega1}+H_{\omega2})\left[\tilde{V}_{2R}E_+ + (U_4^i+\tilde{U}_{4R}^i)F_-^{0i} + (U_6^{ij}+\tilde{U}_{6R}^{ij})F_-^{ij}\right]\\ &+(H_{k1}^i-H_{k2}^i)\left[(V_4^i+\tilde{V}_{4R}^i)E_- + (U_2+\tilde{U}_{2R})F_+^{0i} + (U_5^j+\tilde{U}_{5R}^j)F_+^{ij} + (U_7^{ij}+\tilde{U}_{7R}^{ij})F_+^{0j}\right.\\ &\left.+(U_8^{ijk}+\tilde{U}_{8R}^{ijk})F_+^{jk}\right],\end{aligned} \tag{209}$$

where the nucleon operators are

$$\tilde{U} = U + U^P, \qquad \tilde{V} = V + V^P, \tag{210}$$

$$\begin{aligned}
U_1 &= -2G_A^0(\varepsilon_{T_1}+\varepsilon_{T_2})\boldsymbol{\sigma}_a\boldsymbol{\sigma}_b, \quad U_{1R}^P = G_A^0(\varepsilon_{T_1}+\varepsilon_{T_2})P_{\sigma+}^{ii},\\
U_{1R} &= G_V^0(\varepsilon_{T_1}+\varepsilon_{T_2})D_{\sigma+}^{ii} - iG_A^0(\varepsilon_{T_1}+\varepsilon_{T_2})T_{\sigma+}^{ii},\\
U_2 &= 2iG_A^0(\varepsilon'_{T_1}+\varepsilon'_{T_2})\boldsymbol{\sigma}_a\boldsymbol{\sigma}_b, \quad U_{2R}^P = -iG_A^0(\varepsilon'_{T_1}+\varepsilon'_{T_2})P_{\sigma+}^{ii},\\
U_{2R} &= -iG_V^0(\varepsilon'_{T_1}+\varepsilon'_{T_2})D_{\sigma+}^{ii} + G_A^0(\varepsilon'_{T_1}+\varepsilon'_{T_2})T_{\sigma+}^{ii},\\
U_3^i &= -G_V^0(\varepsilon_{T_1}+\varepsilon_{T_2})\sigma_+^i, \quad U_{3R}^{Pi} = -iG_A^0(\varepsilon_{T_1}+\varepsilon_{T_2})\varepsilon_{ijk}P_{\sigma-}^{jk},\\
U_{3R}^i &= G_A^0(\varepsilon_{T_1}+\varepsilon_{T_2})C_{\sigma+}^i - iG_V^0(\varepsilon_{T_1}+\varepsilon_{T_2})\varepsilon_{ijk}D_{\sigma-}^{jk}\\
&- iG_V^0(\varepsilon_{T_1}+\varepsilon_{T_2})T_+^i - iG_A^0(\varepsilon_{T_1}+\varepsilon_{T_2})\varepsilon_{ijk}T_{\sigma-}^{jk},\\
U_4^i &= -iG_V^0(\varepsilon'_{T_1}+\varepsilon'_{T_2})\sigma_+^i, \quad U_{4R}^{Pi} = -G_A^0(\varepsilon'_{T_1}+\varepsilon'_{T_2})\varepsilon_{ijk}P_{\sigma-}^{jk},\\
U_{4R}^i &= iG_A^0(\varepsilon'_{T_1}+\varepsilon'_{T_2})C_{\sigma+}^i - G_V^0(\varepsilon'_{T_1}+\varepsilon'_{T_2})\varepsilon_{ijk}D_{\sigma-}^{jk}\\
&- G_V^0(\varepsilon'_{T_1}+\varepsilon'_{T_2})T_+^i + iG_A^0(\varepsilon'_{T_1}+\varepsilon'_{T_2})\varepsilon_{ijk}T_{\sigma-}^{jk},\\
U_5^i &= -iG_V^0(\varepsilon'_{T_1}+\varepsilon'_{T_2})\sigma_+^i, \quad U_{5R}^{Pi} = G_A^0\varepsilon'_{T_1}\varepsilon_{ijk}P_{\sigma-}^{jk},\\
U_{5R}^i &= -iG_A^0(\varepsilon'_{T_1}+\varepsilon'_{T_2})C_{\sigma+}^i + G_V^0\varepsilon'_{T_1}\varepsilon_{ijk}D_{\sigma-}^{jk}\\
&- G_V^0(\varepsilon'_{T_1}+\varepsilon'_{T_2})T_+^i + iG_A^0\varepsilon'_{T_2}\varepsilon_{ijk}T_{\sigma-}^{jk},\\
U_6^{ij} &= \frac{1}{2}G_V^0(\varepsilon'_{T_1}+\varepsilon'_{T_2})\varepsilon_{ijk}\sigma_+^k, \quad U_{6R}^{Pij} = iG_A^0(\varepsilon'_{T_1}+\varepsilon'_{T_2})P_{\sigma+}^{ij},
\end{aligned}$$

$$
\begin{aligned}
U^{ij}_{6R} &= -\frac{1}{2}G^0_A(\varepsilon'_{T_1}+\varepsilon'_{T_2})\varepsilon_{ijk}C^k_{\sigma+}-\frac{i}{2}G^0_V(\varepsilon'_{T_1}+\varepsilon'_{T_2})\varepsilon_{ijk}T^k_+\\
&+ iG^0_V(\varepsilon'_{T_1}+\varepsilon'_{T_2})D^{ij}_{\sigma+}-iG^0_A(\varepsilon'_{T_1}+\varepsilon'_{T_2})T^{ij}_{\sigma+},\\
U^{ij}_7 &= +G^0_V(\varepsilon'_{T_1}+\varepsilon'_{T_2})\varepsilon_{ijk}\sigma^k_+-2iG^0_A(\varepsilon'_{T_1}+\varepsilon'_{T_2})(\sigma^i_a\sigma^j_b+\sigma^j_a\sigma^i_b),\\
U^{ij}_{7R} &= -G^0_A(\varepsilon'_{T_1}+\varepsilon'_{T_2})\varepsilon_{ijk}C^k_{\sigma+}-iG^0_V(\varepsilon'_{T_1}+\varepsilon'_{T_2})\varepsilon_{ijk}T^k_+\\
&+ iG^0_V(\varepsilon'_{T_1}+\varepsilon'_{T_2})(\tilde{D}^{ij}_{\sigma+}+\tilde{D}^{ji}_{\sigma+})-G^0_A(\varepsilon'_{T_1}+\varepsilon'_{T_2})(\tilde{T}^{ij}_{\sigma+}+\tilde{T}^{ji}_{\sigma+}),\\
U^{Pij}_{7R} &= +iG^0_A(\varepsilon'_{T_1}+\varepsilon'_{T_2})(\tilde{P}^{ij}_{\sigma+}+\tilde{P}^{ji}_{\sigma+}),\\
U^{ijk}_8 &= +\frac{1}{2}G^0_A(\varepsilon'_{T_1}+\varepsilon'_{T_2})[\varepsilon_{ljk}(\sigma^i_a\sigma^l_b+\sigma^l_a\sigma^i_b)+2\varepsilon_{ilj}(\sigma^l_a\sigma^k_b+\sigma^k_a\sigma^l_b)],\\
U^{ijk}_{8R} &= -\frac{1}{2}G^0_V(\varepsilon'_{T_1}+\varepsilon'_{T_2})\varepsilon_{ljk}\tilde{D}^{li}_{\sigma+}-G^0_V\varepsilon_{ilj}(\varepsilon'_{T_1}\tilde{D}^{lk}_{\sigma+}+\varepsilon'_{T_2}\tilde{D}^{kl}_{\sigma+})\\
&- \frac{i}{2}G^0_A(\varepsilon'_{T_1}+\varepsilon'_{T_2})\varepsilon_{ljk}\tilde{T}^{il}_{\sigma+}-iG^0_A\varepsilon_{ilj}(\varepsilon'_{T_1}\tilde{T}^{lk}_{\sigma+}+\varepsilon'_{T_2}\tilde{T}^{kl}_{\sigma+}),\\
U^{Pijk}_{8R} &= -\frac{1}{2}G^0_A(\varepsilon'_{T_1}+\varepsilon'_{T_2})\varepsilon_{ljk}\tilde{P}^{li}_{\sigma+}-G^0_A\varepsilon_{ilj}(\varepsilon'_{T_1}\tilde{P}^{lk}_{\sigma+}+\varepsilon'_{T_2}\tilde{P}^{kl}_{\sigma+}),
\end{aligned}
\tag{211}
$$

$$
\begin{aligned}
V_{1R} &= -G^0_V(\varepsilon_{T1}+\varepsilon_{T2})D^{ii}_{\sigma-}-iG^0_A(\varepsilon_{T1}+\varepsilon_{T2})T^{ii}_{\sigma-},\quad V^P_{1R}=-G^0_A(\varepsilon_{T1}+\varepsilon_{T_2})P^{ii}_{\sigma-},\\
V_{2R} &= -G^0_V(\varepsilon'_{T1}+\varepsilon'_{T2})D^{ii}_{\sigma-}+iG^0_A(\varepsilon'_{T1}+\varepsilon'_{T2})T^{ii}_{\sigma-},\quad V^P_{2R}=-G^0_A(\varepsilon'_{T1}+\varepsilon'_{T_2})P^{ii}_{\sigma-},\\
V^i_3 &= G^0_V(\varepsilon_{T_1}+\varepsilon_{T_2})\sigma^i_-+2iG^0_A(\varepsilon_{T_1}+\varepsilon_{T_2})[\boldsymbol{\sigma}_a\times\boldsymbol{\sigma}_b]^i,\\
V^i_{3R} &= -G^0_A(\varepsilon_{T1}+\varepsilon_{T2})C^i_{\sigma-}+iG^0_V(\varepsilon_{T_1}+\varepsilon_{T_2})\varepsilon_{ijk}D^{jk}_{\sigma+}\\
&+ iG^0_V(\varepsilon_{T_1}+\varepsilon_{T_2})T^i_--iG^0_A(\varepsilon_{T_1}+\varepsilon_{T_2})\varepsilon_{ijk}T^{jk}_{\sigma+},\\
V^{Pi}_{3R} &= iG^0_A(\varepsilon_{T_1}+\varepsilon_{T_2})\varepsilon_{ijk}P^{jk}_{\sigma+},\\
V^i_4 &= G^0_V(\varepsilon'_{T_1}+\varepsilon'_{T_2})\sigma^i_--2iG^0_A(\varepsilon'_{T_1}+\varepsilon'_{T_2})[\boldsymbol{\sigma}_a\times\boldsymbol{\sigma}_b]^i,\\
V^i_{4R} &= -G^0_A(\varepsilon'_{T1}+\varepsilon'_{T2})C^i_{\sigma-}+iG^0_V(\varepsilon'_{T_1}+\varepsilon'_{T_2})\varepsilon_{ijk}D^{jk}_{\sigma+}\\
&- iG^0_V(\varepsilon'_{T_1}+\varepsilon'_{T_2})T^i_--G^0_A(\varepsilon'_{T_1}+\varepsilon'_{T_2})\varepsilon_{ijk}T^{jk}_{\sigma+},\\
V^{Pi}_{4R} &= iG^0_A(\varepsilon'_{T_1}+\varepsilon'_{T_2})\varepsilon_{ijk}P^{jk}_{\sigma+},
\end{aligned}
\tag{212}
$$

with

$$
T^i_\pm=T^i_aI_b\pm I_aT^i_b,\quad T^{ij}_{\sigma\pm}=\sigma^i_aT^j_b\pm T^i_a\sigma^j_b,\tilde{X}^{ij}_{\sigma\pm}=\sigma^i_aX^j_b\pm X^j_a\sigma^i_b,\quad X=D,\,T,\,P. \tag{213}
$$

Under the exchange of indices a and b, nuclear operators U, electron currents F_+ and neutrino potentials H_i and $H_{\omega i}$ are even, while V, F_-, and $\mathbf{H}_{ki}$ are odd.

The new constants are defined as:

$$
\begin{aligned}
\varepsilon_{T_1}&=\frac{T^{(3)}_1}{g_A}\left(\epsilon^{T_L}_{T_L,i}+\epsilon^{T_L}_{T_R,i}\right),\quad \varepsilon_{T_2}=\frac{T^{(3)}_1}{g_A}\left(\epsilon^{T_L}_{T_L,i}-\epsilon^{T_L}_{T_R,i}\right),\\
\varepsilon'_{T_1}&=\frac{T^{(3)}_1}{g_A}\left(\epsilon^{T_R}_{T_R,i}+\epsilon^{T_R}_{T_L,i}\right),\quad \varepsilon'_{T_2}=\frac{T^{(3)}_1}{g_A}\left(\epsilon^{T_R}_{T_R,i}-\epsilon^{T_R}_{T_L,i}\right).
\end{aligned}
\tag{214}
$$

The even parity operators are

$$
\begin{aligned}
&U_1,\ U^P_{1R},\ k^iU_{2R},\ U^i_3,\ U^{Pi}_{3R},\ U^i_4,\ U^{Pi}_{4R},\ k^iU^j_{5R},\ U^{ij}_6,\ U^{Pij}_{6R},\ k^iU^{ij}_{7R},\ k^iU^{ijk}_{8R};\\
&V^P_{1R},\ V^P_{2R},\ V^i_3,\ V^{Pi}_{3R},\ \mathbf{k}\cdot\mathbf{V}_{4R};
\end{aligned}
\tag{215}
$$

and the odd parity operators are

$$
\begin{aligned}
&U_{1R},\ k^iU_2,\ k^iU^P_{2R},\ U^i_{3R},\ U^i_{4R},\ k^iU^j_5,\ k^iU^{Pj}_{5R},\ U^{ij}_{6R},\ k^iU^{ij}_7,\ k^iU^{Pij}_{7R},\\
&k^iU^{ijk}_8,\ k^iU^{Pijk}_{8R};\quad V_{1R},\ V_{2R},\ V^i_{3R},\ \mathbf{k}\cdot\mathbf{V}_4,\ \mathbf{k}\cdot\mathbf{V}^P_{4R}.
\end{aligned}
\tag{216}
$$

Using the definitions of the neutrino potentials from Eqs. (131) and (167), in the $S-S$ case with no FBWC we have

$$\{M_T^m\}_{n,S-S} = 2\int d\mathbf{x}d\mathbf{y}T_N h_+ \left[(U_1+U_{1R}^P)F_{5+}^0 + (U_3^i+U_{3R}^{Pi})F_{5+}^i\right], \tag{217}$$

$$\{M_T^m\}_{c,S-S} = \varepsilon_{21}R\int d\mathbf{x}d\mathbf{y}T_N h_0 \left[V_{1R}^P F_+^0 + (V_3^i+V_{3R}^{Pi})F_+^i\right], \tag{218}$$

$$\{M_T^k\}_{n,S-S} = \frac{\varepsilon_{21}}{m_e}\int d\mathbf{x}d\mathbf{y}T_N h_{0\omega}\left[(U_4^i+U_{4R}^{Pi})F_+^{0i} + (U_6^{ij}+U_{6R}^{Pij})F_+^{ij}\right]$$
$$+\frac{4}{m_eR}\int d\mathbf{x}d\mathbf{y}T_N\frac{iR}{2r}h'_+\hat{\mathbf{r}}\cdot\mathbf{V}_{4R}E_+, \tag{219}$$
$$\{M_T^k\}_{c,S-S} = \frac{2}{m_eR}\int d\mathbf{x}d\mathbf{y}T_N h_\omega V_{2R}^P E_+$$
$$+\frac{\varepsilon_{21}}{m_e}\int d\mathbf{x}d\mathbf{y}T_N\frac{iR}{r}h'_0\hat{r}^i\left[U_{2R}F_+^{0i} + U_{5R}^jF_+^{ij} + U_{7R}^{ij}F_+^{0j} + U_{8R}^{ijk}F_+^{jk}\right], \tag{220}$$

where E and F are taken for $\mathbf{x}=\mathbf{y}=0$.

For the $0^+\to 0^+$ transition we have

$$\sum_i\frac{m_i}{m_e}\sum_N\{M_T^m\}_{S-S} = g_A^2\left[2(W_1^U+W_{1R}^{UP})F_{5+}^0 + \varepsilon_{21}R\{W_{1R}^{VP}\}_cF_+^0\right], \tag{221}$$

$$\sum_i\sum_N\{M_T^k\}_{S-S} = \frac{2g_A^2}{m_eR}(2W_{4R}^V + \{W_{2R}^{VP}\}_c)E_+, \tag{222}$$

with

$$W_1^U = \langle\frac{m_i}{m_e}h_+U_1\rangle,\quad W_{1R}^{UP} = \langle\frac{m_i}{m_e}h_0U_{1R}^P\rangle,\quad \{W_{1R}^{VP}\}_c = \langle\frac{m_i}{m_e}h_0V_{1R}^P\rangle,$$
$$W_{4R}^V = \langle\frac{iR}{2r}h'_+\hat{\mathbf{r}}\cdot\mathbf{V}_{4R}\rangle,\quad \{W_{2R}^{VP}\}_c = \langle h_\omega V_{2R}^P\rangle. \tag{223}$$

In the $S-P_{1/2}$ case with no FBWC for the $0^+\to 0^+$ transition we have

$$\{M_T^m\}_{n,S-P_{1/2}} = 2\int d\mathbf{x}d\mathbf{y}T_N h_+\left(U_{3R}^iF_{5+}^i + V_{3R}^iF_-^i\right), \tag{224}$$

$$\{M_T^m\}_{c,S-P_{1/2}} = \varepsilon_{21}R\int d\mathbf{x}d\mathbf{y}T_N h_0\left(U_{3R}^iF_{5-}^i + V_{3R}^iF_+^i\right), \tag{225}$$

$$\{M_T^k\}_{n,S-P_{1/2}} = \frac{\varepsilon_{21}}{m_e}\int d\mathbf{x}d\mathbf{y}T_N h_{0\omega}U_{4R}^iF_+^{0i}$$
$$+\frac{4}{m_eR}\int d\mathbf{x}d\mathbf{y}T_N\frac{iR}{2r}h'_+\hat{r}^i\left[(U_2+U_{2R}^P)F_-^{0i} + (U_7^{ij}+U_{7R}^{Pij})F_-^{0j}\right], \tag{226}$$
$$\{M_T^k\}_{c,S-P_{1/2}} = \frac{2}{m_eR}\int d\mathbf{x}d\mathbf{y}T_N h_\omega U_{4R}^iF_-^{0i}$$
$$+\frac{\varepsilon_{21}}{m_e}\int d\mathbf{x}d\mathbf{y}T_N\frac{iR}{r}h'_0\hat{r}^i\left[(U_2+U_{2R}^P)F_+^{0i} + (U_7^{ij}+U_{7R}^{Pij})F_+^{0j}\right]. \tag{227}$$

The decay rate for the $0^+\to 0^+$ transition takes the form

$$d\Gamma = \sum_{s_1,s_2}|R_{0\nu}|^2\frac{m_e^5}{4\pi^3}d\Omega_{0\nu} = \frac{a_{0\nu}}{(m_eR)^2}\left[A_0^T - \hat{\mathbf{p}}_1\cdot\hat{\mathbf{p}}_2B_0^T\right]d\Omega_{0\nu}, \tag{228}$$

where the coefficients are

$$A_0^T = \sum_{i=1}^4|O_i|^2, \tag{229}$$

$$B_0^T = \mathrm{Re}(O_1O_2^* + O_1^*O_2 + O_3O_4^* + O_3^*O_4), \tag{230}$$

with

$$
\begin{aligned}
O_1 &= \alpha^*_{-1-1}\left\{\left[-2(W_1^U + W_{1R}^{UP}) + \frac{2}{m_eR}(W_{4R}^V + \{W_{2R}^{VP}\}_c)\right]\right. \\
&+ \left[\frac{m_er}{3}\left(\frac{\zeta}{m_eR} - 2\right)W_{3R}^U + \frac{\varepsilon_{21}^2 rR}{6}\{W_{3R}^U\}_c\right] + \left.\left[-\frac{3(\alpha Z)^2}{m_eR}\left(W_{4RF}^V + \frac{1}{2}\{W_{2RF}^{VP}\}_c\right)\right]\right\}, \qquad (231)\\
O_2 &= \alpha^*_{11}\left\{\left[2(W_1^U + W_{1R}^{UP}) + \frac{2}{m_eR}(W_{4R}^V + \{W_{2R}^{VP}\}_c)\right] +\right. \\
&+ \left[-\frac{m_er}{3}\left(\frac{\zeta}{m_eR} + 2\right)W_{3R}^U + \frac{\varepsilon_{21}^2 rR}{6}\{W_{3R}^U\}_c\right] + \left.\left[-\frac{3(\alpha Z)^2}{m_eR}\left(W_{4RF}^V + \frac{1}{2}\{W_{2RF}^{VP}\}_c\right)\right]\right\}, \qquad (232)\\
O_3 &= \alpha^*_{1-1}\left\{\left[-\varepsilon_{21}R\{W_{1R}^{VP}\}_c + \frac{2}{m_eR}(W_{4R}^V + \{W_{2R}^{VP}\}_c)\right]\right. \\
&+ \left[\frac{m_er}{3}\left(\frac{\varepsilon_{21}}{m_e} + 2\right)W_{3R}^V + \zeta\frac{\varepsilon_{21}r}{6}\{W_{3R}^V\}_c\right. \\
&+ \frac{\varepsilon_{21}R}{3}\left(\frac{\varepsilon_{21}}{m_e} + 2\right)(W_{4R}^U - \{W_2^U\}_c - \{W_7^U\}_c - \{W_{2R}^{UP}\}_c - \{W_{7R}^{UP}\}_c) \\
&- \left.\frac{4}{3}\frac{\zeta}{m_eR}(W_2^U + W_7^U + W_{2R}^{UP} + W_{7R}^{UP} - \frac{1}{2}\{W_{4R}^U\}_c)\right] + \left.\left[-\frac{3(\alpha Z)^2}{m_eR}\left(W_{4RF}^V + \frac{1}{2}\{W_{2RF}^{VP}\}_c\right)\right]\right\}, \qquad (233)\\
O_4 &= \alpha^*_{-11}\left\{\left[\varepsilon_{21}R\{W_{1R}^{VP}\}_c + \frac{2}{m_eR}(W_{4R}^V + \{W_{2R}^{VP}\}_c)\right]\right. \\
&+ \left[-\frac{m_er}{3}\left(\frac{\varepsilon_{21}}{m_e} - 2\right)W_{3R}^V - \zeta\frac{\varepsilon_{21}r}{6}\{W_{3R}^V\}_c\right. \\
&+ \frac{\varepsilon_{21}R}{3}\left(\frac{\varepsilon_{21}}{m_e} - 2\right)(W_{4R}^U - \{W_2^U\}_c - \{W_7^U\}_c - \{W_{2R}^{UP}\}_c - \{W_{7R}^{UP}\}_c) \\
&- \left.\frac{4}{3}\frac{\zeta}{m_eR}(W_2^U + W_7^U + W_{2R}^{UP} + W_{7R}^{UP} - \frac{1}{2}\{W_{4R}^U\}_c)\right] + \left.\left[-\frac{3(\alpha Z)^2}{m_eR}\left(W_{4RF}^V + \frac{1}{2}\{W_{2RF}^{VP}\}_c\right)\right]\right\}, \qquad (234)
\end{aligned}
$$

where the terms in the first brackets in Eqs. (231)–(234) come from the $S-S$ case and the terms in the second ones come from the $S-P_{1/2}$ case. The terms in the third brackets in Eqs. (231)-(232) are the most important terms of those that come from the $S-S$ case due to FBWC. Note that in the $S-S$ case there is the contribution to Eqs. (231) and (232) from the $(H_{\omega 1}+H_{\omega 2})$ combination in Eq. (209). Therefore the contribution from the $P_{1/2}-P_{1/2}$ case should not be taken into account.

The nuclear matrix elements are

$$
\begin{aligned}
W_{3R}^U &= \langle\frac{m_i}{m_e}\frac{i}{2R}h_+\mathbf{r}_+\cdot\mathbf{U}_{3R}\rangle, \quad \{W_{3R}^U\}_c = \langle\frac{m_i}{m_e}\frac{i}{r}h_0\mathbf{r}\cdot\mathbf{U}_{3R}\rangle,\\
W_{3R}^V &= \langle\frac{m_i}{m_e}\frac{i}{r}h_+\mathbf{r}\cdot\mathbf{V}_{3R}\rangle, \quad \{W_{3R}^V\}_c = \langle\frac{m_i}{m_e}\frac{i}{2R}h_0\mathbf{r}_+\cdot\mathbf{V}_{3R}\rangle,\\
W_{4R}^U &= \langle\frac{i}{2R}h_{0\omega}\mathbf{r}_+\cdot\mathbf{U}_{4R}\rangle,\\
\{W_2^U\}_c &= \langle\frac{R}{r}h_0'\hat{\mathbf{r}}\cdot\hat{\mathbf{r}}_+U_2\rangle, \quad \{W_{2R}^{UP}\}_c = \langle\frac{R}{r}h_0'\hat{\mathbf{r}}\cdot\hat{\mathbf{r}}_+U_{2R}^P\rangle,\\
\{W_7^U\}_c &= \langle\frac{R}{r}h_0'\hat{r}^i\hat{r}_+^jU_7^{ij}\rangle, \quad \{W_{7R}^{UP}\}_c = \langle\frac{R}{r}h_0'\hat{r}^i\hat{r}_+^jU_{7R}^{Pij}\rangle. \qquad (235)
\end{aligned}
$$

Assuming now $\langle m\rangle \neq 0$ for the dominant terms we have

$$
O_1 = \alpha^*_{-1-1}\left\{\left[Z_1^X - 2W_1^U + \frac{2}{m_eR}(W_{4R}^V + \{W_{2R}^{VP}\}_c)\right] + \right\}, \qquad (236)
$$

$$
O_2 = \alpha^*_{11}\left\{\left[Z_1^X + 2W_1^U + \frac{2}{m_eR}(W_{4R}^V + \{W_{2R}^{VP}\}_c)\right] + \right\}, \qquad (237)
$$

$$O_3 = \alpha^*_{1-1} \left\{ \left[Z_1^X + \frac{2}{m_e R}(W^V_{4R} + \{W^{VP}_{2R}\}_c) \right] \right.$$
$$\left. + \left[\frac{\varepsilon_{21} R}{3} \left(\frac{\varepsilon_{21}}{m_e} + 2 \right) (W^U_{4R} - \{W^U_2\}_c - \{W^U_7\}_c) - \frac{4}{3} \frac{\zeta}{m_e R}(W^U_2 + W^U_7) \right] + \right\}, \tag{238}$$
$$O_4 = \alpha^*_{-11} \left\{ \left[Z_1^X + \frac{2}{m_e R}(W^V_{4R} + \{W^{VP}_{2R}\}_c) \right] \right.$$
$$\left. + \left[\frac{\varepsilon_{21} R}{3} \left(\frac{\varepsilon_{21}}{m_e} - 2 \right) (W^U_{4R} - \{W^U_2\}_c - \{W^U_7\}_c) - \frac{4}{3} \frac{\zeta}{m_e R}(W^U_2 + W^U_7) \right] + \right\}. \tag{239}$$

Again, in the above expressions, the terms with ζ are due to the inclusion of the P-wave in the electron wave function and the ones with W^{VP}_{2R} and W^X_{4R} $(X = U, V)$ are due to the nucleon recoil effect.

References

[1] Particle Data Group: W.-M. Yao et al., *J. Phys.* **G33**, 1 (2006).

[2] S.R. Elliot and P. Vogel, *Ann. Rev. Nucl. Part. Sci.* **52**, 115 (2002); P. Vogel, arXiv:hep-ph/0611243.

[3] Ya.B. Zeldovich and M.Yu. Khlopov, *JETP Lett.* **34**, 141 (1981); *Sov. Phys. Usp.* **24**, 755 (1981).

[4] M.G. Shchepkin, *Sov. Phys. Usp.* **27**, 555 (1984).

[5] M. Doi, T. Kotani, and E. Takasugi, *Prog. Theor. Phys. Suppl.* **83**, 1 (1985).

[6] R.N. Mohapatra, *Phys. Rev.* **D34**, 3457 (1986).

[7] J.D. Vergados, *Phys. Lett.* **B184**, 55 (1987).

[8] M. Hirsch, H.V. Klapdor-Kleingrothaus, and S.G. Kovalenko, *Phys. Rev. Lett.* **75**, 17 (1995); *Phys. Lett.* **B352**, 1 (1995); *Phys. Lett.* **B403**, 291 (1997); *Nucl. Phys. Proc. Suppl.* **A52**, 257 (1997); *Phys. Rev.* **D57**, 1947 (1998).

[9] K.S. Babu and R.N. Mohapatra, *Phys. Rev. Lett.* **75**, 2276 (1995).

[10] M. Hirsch, H.V. Klapdor-Kleingrothaus, and S.G. Kovalenko, *Phys. Lett.* **B372**, 181 (1996); **B381**, 488 (Erratum) (1996); H. Päs, M. Hirsch, and H.V. Klapdor-Kleingrothaus, *Phys.Lett.* **B459**, 450 (1999).

[11] A. Faessler, S.G. Kovalenko, F. Simkovic, and J. Schwieger, *Phys. Rev. Lett.* **78**, 183 (1997).

[12] H.V. Klapdor-Kleingrothaus and U. Sarkar, *Phys. Lett.* **B554**, 45 (2003).

[13] M. Hirsch, H.V. Klapdor-Kleingrothaus, and S.G. Kovalenko, *Phys. Rev.* **D54**, 4207 (1996).

[14] M. Hirsch, H.V. Klapdor-Kleingrothaus, and O. Panella, *Phys. Lett.* **B374**, 7 (1996).

[15] J.D. Vergados, *Nucl. Phys. B (Proc. Suppl.)* **143**, 211 (2005) [arXiv:hep-ph/0409319].

[16] J. Schechter and J.W. Valle, *Phys. Rev.* **D25**, 2951 (1982); E. Takasugi, *Phys. Lett.* **B149**, 372 (1984); J.F. Nieves, *Phys. Lett.* **B147**, 375 (1984).

[17] F. Deppisch and H. Päs, *Phys. Rev. Lett.* **98**, 232501 (2007) [arXiv:hep-ph/0612165].

[18] V.M. Gehman and S.R. Elliott, J. Phys. G **34**, 667 (2007) [arXiv:hep-ph/0701099].

[19] A. Ali, A.V. Borisov, and D.V. Zhuridov, arXiv:hep-ph/0606072.

[20] G. Pantis, F. Šimkovic, J.D. Vergados, and A. Faessler, *Phys. Rev.* **C53**, 695 (1996).

[21] M. Kortelainen and J. Suhonen, Phys. Rev. C **75**, 051303 (2007) [arXiv:0705.0469 [nucl-th]].

[22] J.C. Pati and A. Salam, *Phys. Rev.* **D10**, 275 (1974); R.N. Mohapatra and J.C. Pati, *Phys. Rev.* **D11**, 566, 2558 (1975); G. Senjanovic and R.N. Mohapatra, *Phys. Rev.* **D12**, 1502 (1975); R.N. Mohapatra and G. Senjanovic, *Phys. Rev.* **D23**, 165 (1981).

[23] H. Päs, M. Hirsch, H.V. Klapdor-Kleingrothaus, and S.G. Kovalenko, *Phys. Lett.* **B453**, 194 (1999).

[24] G. Gamov and E. Teller, *Phys. Rev.* **49**, 895 (1936); S.F. Novaes, in *"Particle and Fields"*, Proc. 10th J.A. Swieca Summer School, São Paulo, Brazil, 31 Jan – 12 Feb 1999 (World Scientific, Singapore, 2000) [arXiv:hep-ph/0001283].

[25] B. Pontecorvo, *Sov. Phys. JETP* **6**, 429 (1958); Z. Maki, M. Nakagawa, and S. Sakata, *Prog. Theor. Phys.* **28**, 870 (1962).

[26] J.C. Pati and A. Salam, *Phys. Lett.* **B58**, 333 (1975).

[27] W. Buchmüller, R. Rückl, and D. Wyler, *Phys. Lett.* **B191**, 442 (1987).

[28] H.V. Klapdor-Kleingrothaus and H. Päs, in COSMO 99: 3rd International Conference on Particle Physics and the Early Universe, Trieste, Italy, 27 Sep – 3 Oct 1999; in H.V. Klapdor-Kleingrothaus (Ed.), *"Sixty Years of Double Beta Decay"* (World Scientific, Singapore, 2001), p. 755 [arXiv:hep-ph/0002109].

[29] N. Severijns, M. Beck, and O. Naviliat-Cuncic, *Rev. Mod. Phys.* **78**, 991 (2006) [arXiv:nucl-ex/0605029].

[30] M.E. Rose, *Relativistic Electron Theory* (Wiley, New York, 1961).

[31] S.L. Adler et al., *Phys. Rev.* **D11**, 3309 (1975).

[32] T. Tomoda, *Rep. Prog. Phys.* **54**, 53 (1991).

[33] J.D. Vergados, *Phys. Rep.* **361**, 1 (2002).

[34] T. Ericson and W. Weise, *Pions and Nuclei* (Clarendon Press, Oxford, 1988).

[35] K. Zuber, Preprint IPPP-05-56, DCPT-05-114 (2005). Presented at IPPP Workshop on Matrix Elements for Neutrinoless Double Beta Decay, Durham, England, 23–24 May 2005 [arXiv:nucl-ex/0511009].

[36] V.A. Rodin, A. Faessler, F. Šimkovic, and P. Vogel, *Czech. J. Phys.* **56**, 495 (2006); arXiv:0706.4304 [nucl-th].

[37] C.E. Aalseth et al., *Phys. Rev.* **D70**, 078302 (2004).

[38] A. Ali, A.V. Borisov, and D.V. Zhuridov (work in progress).

[39] H.V. Klapdor-Kleingrothaus and U. Sarkar, *Proc. Indian Natn. Sci. Acad.* **70A**, 251 (2004).

[40] S.M. Bilenky, Annales de la Fondation Louis de Broglie **31**, no. 2–3, 139 (2006) [arXiv:hep-ph/0605172].

[41] A.S. Barabash and the NEMO Collaboration, *Nucl. Phys. B (Proc. Suppl.)* **138**, 207 (2005); R. Arnold et al., *Phys. Rev. Lett.* **95**, 182302 (2005).

[42] F. Piquemal, *Phys. Atom. Nucl.* **69**, 2096 (2006).

[43] M. Nomachi et al., *Nucl. Phys. B (Proc. Suppl.)* **138**, 221 (2005).

[44] D. Akimov et al., *Nucl. Phys. B (Proc. Suppl.)* **138**, 224 (2005).

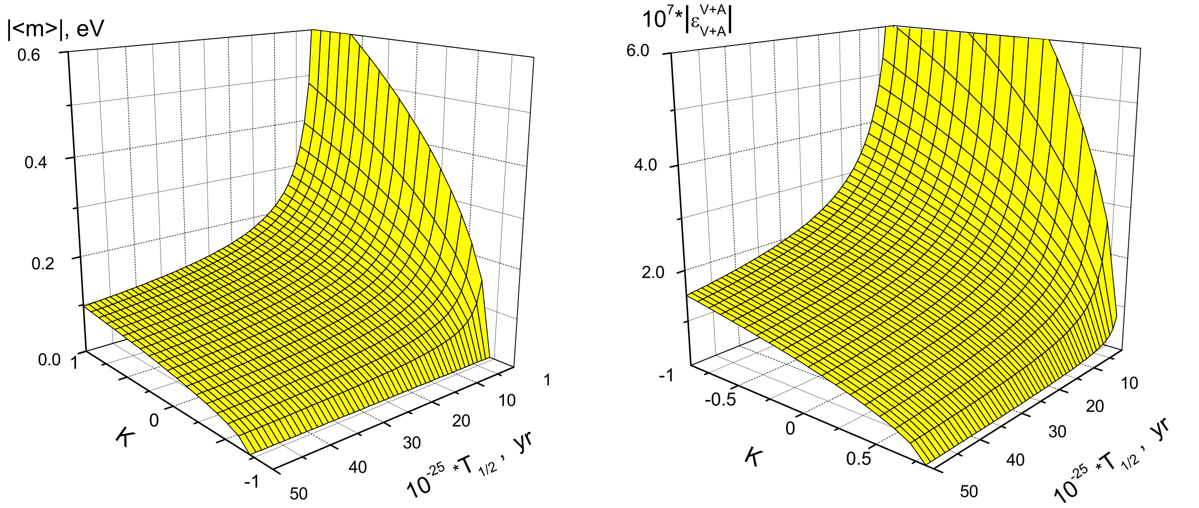

Figure 1: Correlation between the neutrino effective mass $|\langle m\rangle|$ (*left*) [$|\epsilon^{V+A}_{V+A}|$ (*right*], the angular correlation coefficient K, and the half-life $T_{1/2}$ for the $0\nu2\beta$ decay of ^{76}Ge for the case $\cos\psi_1 = 0$.

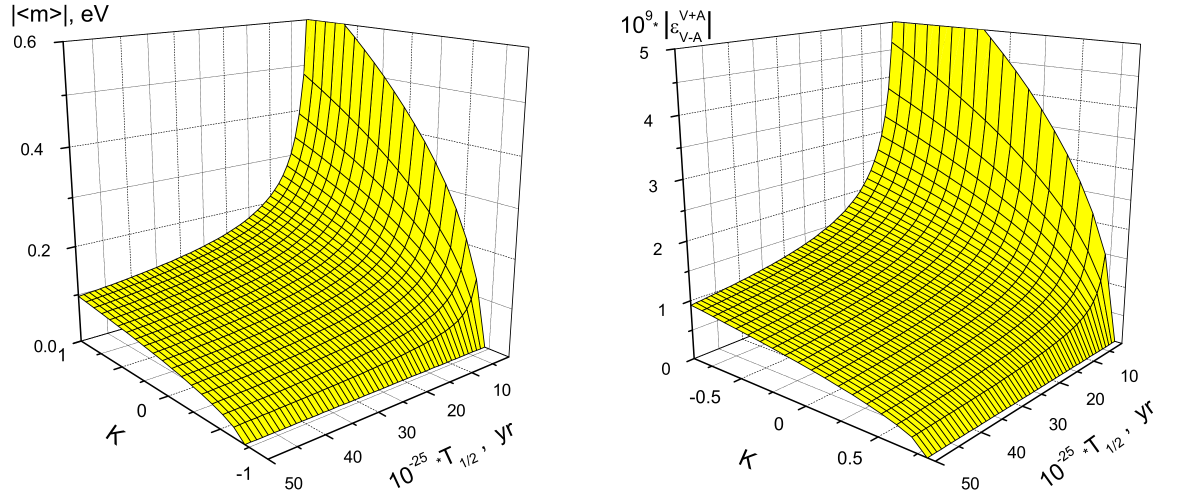

Figure 2: Correlation between the neutrino effective mass $|\langle m\rangle|$ (*left*) [$|\epsilon^{V+A}_{V-A}|$ (*right*)], the angular correlation coefficient K, and the half-life $T_{1/2}$ for the $0\nu2\beta$ decay of ^{76}Ge for the case $\cos\psi_1 = 0$.

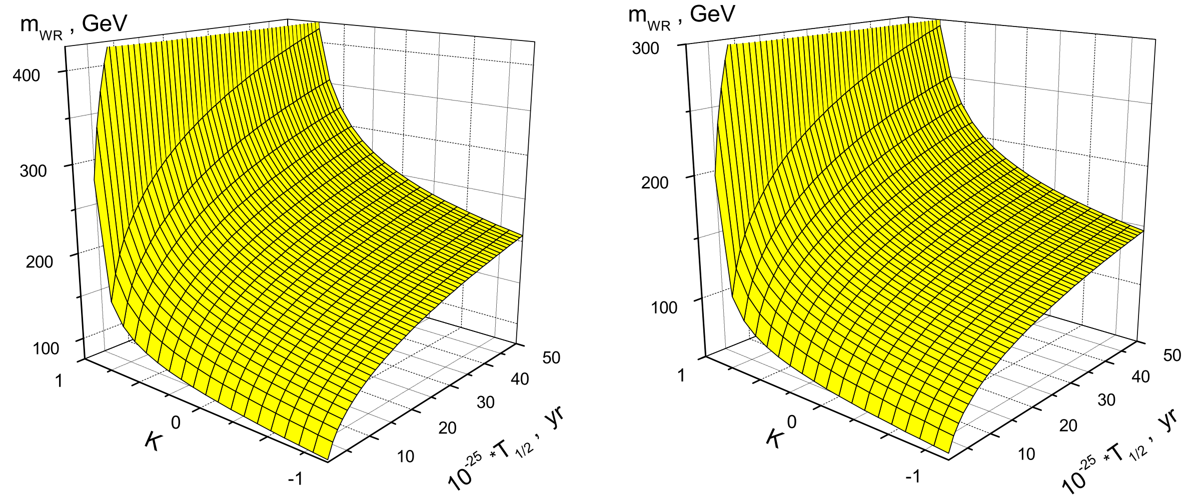

Figure 3: Correlation between the right-handed W-boson mass m_{W_R}, the angular correlation coefficient K, and the half-life $T_{1/2}$ for the $0\nu 2\beta$ decay of ^{76}Ge for the case $\cos\psi_1 = 0$ and $\epsilon = 10^{-6}$ (*left*) and for $\epsilon = 5 \times 10^{-7}$ (*right*).

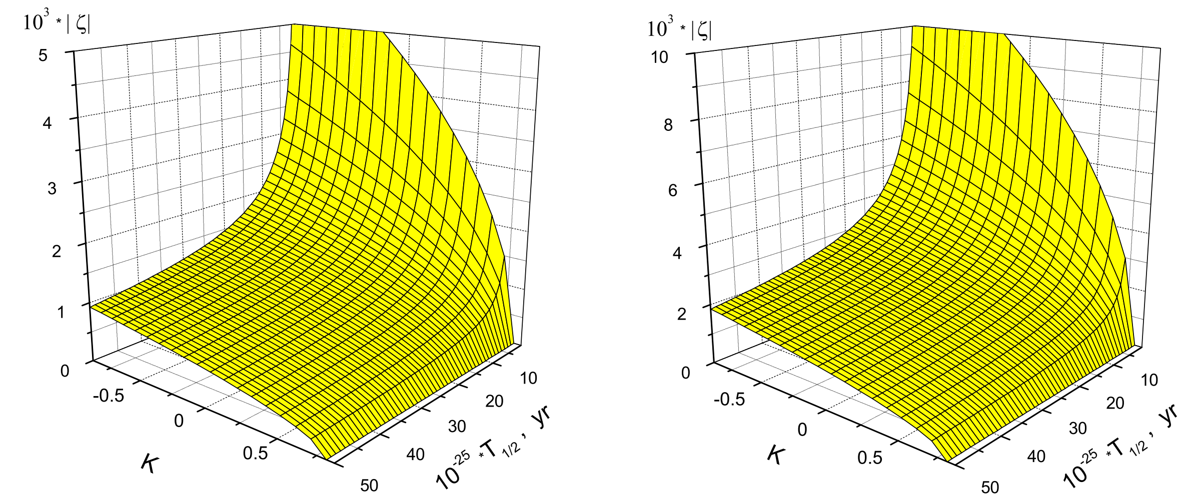

Figure 4: Correlation between the mixing parameter ζ, the angular correlation coefficient K, and the half-life $T_{1/2}$ for the $0\nu 2\beta$ decay of ^{76}Ge for the case $\cos\psi_1 = 0$ and $\epsilon = 10^{-6}$ (*left*) and for $\epsilon = 5 \times 10^{-7}$ (*right*).

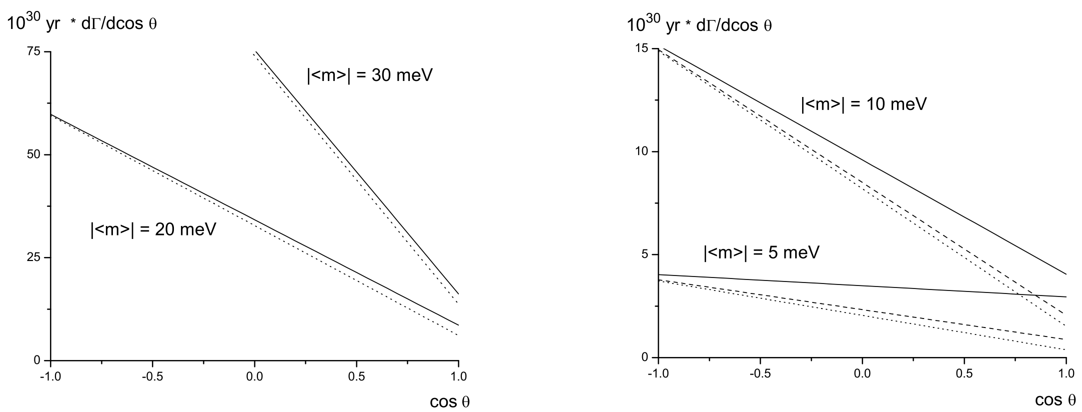

Figure 5: *Left*: Differential width in $\cos\theta$ for the $0\nu2\beta$ decay of ^{76}Ge for a fixed value of $\epsilon = 10^{-6}$ and $|\langle m\rangle| = 20, 30$ meV. The straight and dotted lines correspond to $m_{W_R} = 1$ TeV, ∞, respectively (the latter is the conventional case of the light Majorana neutrino exchange mechanism). *Right*: The same as the left figure but for smaller values of $|\langle m\rangle| = 5, 10$ meV. In addition, the dashed lines correspond to $m_{W_R} = 1.5$ TeV.