DESY 18-099

SI-HEP-2018-20

QEFT-2018-12

Discovery potential of stable and near-threshold doubly heavy tetraquarks at the LHC

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(Dated: June 26, 2018)

We study the LHC discovery potential of the double-bottom tetraquarks $bb\bar{u}\bar{d}$, $bb\bar{u}\bar{s}$ and $bb\bar{d}\bar{s}$, the lightest of which having $J^P = 1^+$, called $T^{\{bb\}}_{[\bar{u}\bar{d}]}$, $T^{\{bb\}}_{[\bar{u}\bar{s}]}$ and $T^{\{bb\}}_{[\bar{d}\bar{s}]}$, are expected to be stable against strong decays. Employing the Monte Carlo generators MadGraph5_aMC@NLO and Pythia6, we simulate the process $pp \to \bar{b}b\bar{b}b$ and calculate the bb-diquark jet configurations, specified by the invariant mass interval $m_{bb} < M_{T^{\{bb\}}_{[\bar{q}\bar{q}']}} + \Delta M$. Estimates of ΔM from the measured product $\sigma(pp \to B_c^+ + X)\mathcal{B}(B_c^+ \to J/\psi\pi^+)$ are presented and used to to get the bb-diquark jet cross sections in double-bottom hadrons $\sigma(pp \to H_{\{bb\}} + X)$, where $H_{\{bb\}}$ represent tetraquarks and baryons. This is combined with the LHCb data on the fragmentation $b \to \Lambda_b$ and $b \to B$ to obtain $\sigma(pp \to T^{\{bb\}}_{[\bar{u}\bar{d}]} + X) = (2.4^{+0.9}_{-0.6})$ nb, and about a half of this for the $T^{\{bb\}}_{[\bar{u}\bar{s}]}$ and $T^{\{bb\}}_{[\bar{d}\bar{s}]}$. We also present estimates of the production cross sections for the mixed bottom-charm tetraquarks, $bc\bar{u}\bar{d}$, $bc\bar{u}\bar{s}$ and $bc\bar{d}\bar{s}$, obtaining $\sigma(pp \to T^{[bc]}_{[\bar{u}\bar{d}]} + X) = (48^{+19}_{-12})$ nb, and the related ones having $T^{[bc]}_{[\bar{u}\bar{s}]}$ and $T^{[bc]}_{[\bar{d}\bar{s}]}$. They have excellent discovery potential at the LHC, as their branching ratios in various charge combinations of $BD_{(s)}(\gamma)$ are anticipated to be large.

PACS numbers:

Introduction: The discovery of $X(3872)$, followed by well over a dozen related mesonic states, X, Y, Z, and two baryonic states $P_c(4380)$ and $P_c(4450)$, has opened a second layer of "extraordinary" hadrons in QCD, containing four and five valence quarks and antiquarks [1]. However, their dynamics is not yet deciphered and is under intense study. The competing theoretical models put forward can be roughly classified into two categories: those reflecting the residual QCD long-distance effects, dominated by meson exchanges, and those reflecting genuine short-distance interactions, dominated by gluon exchanges. Their spectroscopy, production and decay characteristics are discussed in a number of reviews [2–6].

Recent theoretical insights, based on heavy quark symmetry (HQS), have brought new perspectives, implying that doubly-heavy tetraquarks (DHTQ) $Q_iQ_j\bar{q}_k\bar{q}_\ell$ must exist in the HQS limit. Here Q_i, Q_j are either b or c quarks, and $\bar{q}_k$, $\bar{q}_\ell$ are light $(\bar{u}, \bar{d}, \bar{s})$ antiquarks. The existence of such tetraquarks was already suggested in the earlier works [7, 8], but this argument has received a great impetus from proofs based on HQS and lattice-QCD [9–15]. In particular, HQS relates the DHTQ masses to those of double-heavy baryons, heavy-light baryons, and heavy-light mesons. As the light degrees of freedom in these hadrons are similar, we anticipate that the heavy quark - heavy diquark symmetry has implications for other non-perturbative aspects as well. In particular, this symmetry can be used as a quantitative guide in the analysis of the current and anticipated data.

The lightest of the $bb\bar{u}\bar{d}$, $bb\bar{u}\bar{s}$, and $bb\bar{d}\bar{s}$ states are anticipated to be stable against strong decays. Heavier $bb\bar{q}_k\bar{q}_\ell$ states, as well as the double-charm states $cc\bar{q}_k\bar{q}_\ell$, and the mixed bottom-charm tetraquark states $bc\bar{q}_k\bar{q}_\ell$, on the other hand, are estimated to have masses above their respective thresholds. The latter are likely to dissociate into pairs of heavy-light mesons, with large branching ratios, some of which may appear as "double-flavor" narrow resonances [9, 10, 16, 17]. None of these stable or near-threshold DHTQ mesons has so far been seen experimentally. Observing them would establish the existence of tetraquarks, underscoring the role of diquarks, with well-defined color and spin quantum numbers [18–20], as fundamental constituents of hadronic matter.

Our main focus is to develop the expectations about the production of some of the DHTQ mesons., in particular, the double-bottom $J^P = 1^+$ tetraquarks $T^{\{bb\}}_{[\bar{u}\bar{d}]}$, and the related ones $T^{\{bb\}}_{[\bar{u}\bar{s}]}$ and $T^{\{bb\}}_{[\bar{d}\bar{s}]}$. The standard calculational technique, NRQCD and related frameworks [21], however, can not be used at present, as the hadronic matrix elements required for tetraquark production are unknown. The DHTQ decay products are expected to lie in well-collimated double-heavy-diquark jets, which are formed in high energy collisions. These configurations can be calculated in perturbative QCD and, combined with non-perturbative (fragmentation) aspects measured in b-quark jets, enable us to estimate the cross sections of interest.

In a previous paper [22], we have studied the production of double-bottom tetraquarks at a Tera-Z factory in e^+e^- collision, employing the bb-diquark jet configurations in which such tetraquarks are likely to be produced. In this Letter, we study the production of DHTQ states at the LHC, making use of the impressive LHCb data on $pp \to B_c + X$ [23] and b-hadron production fractions in pp collisions [24, 25]. Also, double-bottomonium production has been observed at the LHC, with CMS reporting a cross section $\sigma(pp \to \Upsilon(1S)\Upsilon(1S) + X) = 68 \pm 15$ pb at $\sqrt{s} = 8$ TeV [26]. This is the first step in the searches of double-bottom tetraquarks, such as $pp \to T^{\{bb\}}_{[\bar u \bar d]} + X$, as both final states involve different fragmentation of the same underlying partonic process $pp \to b\bar b b\bar b$. Using the Monte Carlo generators MadGraph5_aMC@NLO [27] and Pythia6 [28], we simulate the process $pp \to \bar b b \bar b b$ and estimate that the production cross section $\sigma(pp \to T^{\{bb\}}_{[\bar u \bar d]} + X)$ can reach a few nb. Replacing a heavy bottom quark by a charm quark, we also simulate the process $pp \to b\bar b c\bar c$ and calculate the production of the mixed bottom-charm tetraquarks $T^{[bc]}_{[\bar u \bar d]}$, $T^{[bc]}_{[\bar u \bar s]}$ and $T^{[bc]}_{[\bar d \bar s]}$, having $J^P = 0^+$, and their $J^P = 1^+$ partners, which are estimated to lie above their corresponding heavy-light mesonic thresholds. We find that the cross sections for these tetraquarks may reach $\mathcal{O}(50)$nb. As LHCb is projected to collect an integrated luminosity of 50 fb^{-1} in Runs 1 -4 [29–31], the prospects of discovering these tetraquarks are excellent.

Production of double-bottom tetraquarks at the LHC: We start by recalling the production and decays of the known doubly-heavy meson $B_c^\pm$ in the process $pp \to b\bar b c\bar c \to B_c^\pm + X$, which serves as the benchmark for our calculations. At $\sqrt{s} = 8$ TeV, the LHCb collaboration has measured the ratio [23][1]

$$R \equiv \frac{\sigma(B_c^+)\mathcal{B}(B_c^+ \to J/\psi\pi^+)}{\sigma(B^+)\mathcal{B}(B^+ \to J/\psi K^+)} = (0.683 \pm 0.018 \pm 0.009)\%, \tag{1}$$

where $0 < p_{\rm T} < 20$ GeV, and $2.0 < y < 4.5$. This value is consistent with the previous LHCb measurement [32]. At $\sqrt{s} = 7$ TeV, the B^+ production cross section is measured as [33]

$$\sigma(B^+) = (43.0 \pm 0.2 \pm 2.5 \pm 1.7)\ \mu\text{b}, \tag{2}$$

with the same kinematic cuts. Using MadGraph [27] and Pythia [28], we find that the 8 TeV cross section is expected to be enhanced by about 19%, compared with the 7 TeV cross section, [2] which is consistent with 20% used in [23]. Using the above results, and the branching ratio [1]

$$\mathcal{B}(B^+ \to J/\psi K^+) = (1.026 \pm 0.031) \times 10^{-3}, \tag{3}$$

we find:

$$\sigma(B_c^+)\mathcal{B}(B_c^+ \to J/\psi\pi^+) = (0.36 \pm 0.03)\ \text{nb}. \tag{4}$$

To extract the cross section from the above product, we need to know $\mathcal{B}(B_c^+ \to J/\psi\pi^+)$, which is, in general, model-dependent. Noting that there is considerable spread in the predicted value of this quantity in the literature, we use two calculations of the more recent vintage, which we consider more reliable, based on the perturbative QCD approach(PQCD) [34], and on the NLO non-relativistic QCD(NRQCD) [35], which yield:

$$\begin{aligned}\mathcal{B}(B_c^+ \to J/\psi\pi^+) &= (2.6^{+0.6+0.2+0.8}_{-0.4-0.2-0.2}) \times 10^{-3}\ \text{(PQCD)},\\ \mathcal{B}(B_c^+ \to J/\psi\pi^+) &= (2.91^{+0.15+0.40}_{-0.42-0.27}) \times 10^{-3}\ \text{(NRQCD)}.\end{aligned} \tag{5}$$

With this, the production cross section $\sigma(pp \to B_c^+ + X)$ at $\sqrt{s} = 8$ TeV is estimated as:

$$\begin{aligned}\sigma(pp \to B_c^+ X) &= (139^{+34}_{-41})\ \text{nb (PQCD)},\\ \sigma(pp \to B_c^+ X) &= (124^{+28}_{-19})\ \text{nb (NRQCD)}.\end{aligned} \tag{6}$$

The implicit model-dependence can be checked by using the ratios of the semileptonic decays of the $B_c^\pm$ and $B^\pm$, which have a much larger statistics.

Next, we use Madgraph [27] to calculate the cross section for the process $pp \to \bar b b \bar c c$ at $\sqrt{s} = 8$ TeV, which yields

$$\sigma(pp \to \bar b b \bar c c) = (4.79 \pm 0.08) \times 10^3\ \text{nb}. \tag{7}$$

This determines for us the fragmentation fraction:

$$\begin{aligned}f(b\bar c \to B_c^+) &= (2.9^{+0.7}_{-0.8})\%\ \text{(PQCD)},\\ f(b\bar c \to B_c^+) &= (2.6^{+0.5}_{-0.3})\%\ \text{(NRQCD)}.\end{aligned} \tag{8}$$

In the above the B_c^+ mesons survive the cuts $p_{\rm T} < 20$ GeV and $2.0 < y < 4.5$. Noting that for the fragmentation to take place, both the b and the $\bar c$ quarks have to be

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[1] Throughout this Letter, charge conjugation is assumed.

[2] With the cuts $0 < p_{\rm T} < 20$ GeV, and $2.0 < y < 4.5$ and setting $m_b = 4.9$ GeV, we find that the $b\bar b$ cross sections at the 7 and 8 TeV LHC are about 80 μb and 95 μb, respectively. This ratio is less sensitive to the hadronization.

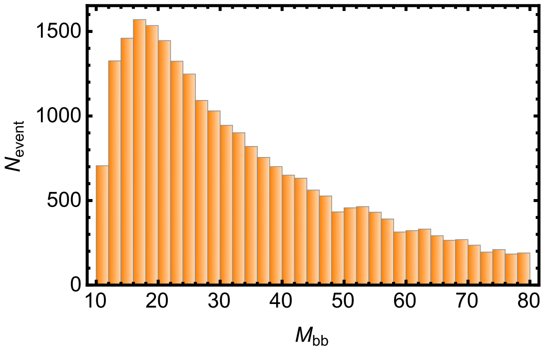

FIG. 1: The bb-quark-pair invariant-mass distribution for the process $pp \to (bb)_{\rm jet} + \bar{b} + \bar{b} + X$ at $\sqrt{s} = 13$ TeV, obtained by generating 10^5 events using MadGraph and Pythia6 at the NLO accuracy.

collinear in a well-collimated jet, defined by an invariant mass interval ΔM. We estimate the value of ΔM so as to reproduce the above fragmentation ratio. This yields:

$$\begin{aligned} \Delta M &= (2.0^{+0.5}_{-0.4})\ \text{GeV (PQCD)}, \\ \Delta M &= (1.9^{+0.3}_{-0.3})\ \text{GeV (NRQCD)}, \end{aligned} \tag{9}$$

which is consistent with $\Delta M = (2.2$ - $4.0)$ GeV, which we obtained from simulating the Z decays in [22], using NRQCD for $\sigma(e^+e^- \to B_c + X)$ [36], but more precise.

For $pp \to b\bar{b}b\bar{b}$, we have generated 10^5 showered events at $\sqrt{s} = 13$ TeV with MadGraph [27] and Pythia6 [28] at the NLO accuracy. The cross section $\sigma(pp \to b\bar{b}b\bar{b} + X)$, involving the gg and $q\bar{q}$ partons, is evaluated by MadGraph to be (463 ± 4) nb. We also find that the contribution from the Z-induced processes, $(pp \to Z \to b\bar{b}b\bar{b} + X)$ and $(pp \to Zb\bar{b} \to b\bar{b}b\bar{b} + X)$, is down by three orders of magnitude, and hence is not considered any further.

The b-quark pair invariant mass distribution is displayed in Fig. 1. We compare the normalized bb-invariant mass distribution at the LHC ($\sqrt{s} = 13$ TeV) with the corresponding one in e^+e^- collision at the Z pole in Fig. 2, upper panel, while the lower panel shows the ratio of the two. From this figure, we see that the jet-shapes (normalized distributions) are similar in the two cases in the small invariant mass region. Thus, the same jet-resolution criterion can be used in the two processes to estimate the fraction of the bb-invariant mass in which the bb-diquark is likely to fragment into double-bottom hadrons. We use $\Delta M = (2.0^{+0.5}_{-0.4})$ GeV, obtained from the analysis of the data on $\sigma(pp \to B_c^+ + X)$ at $\sqrt{s} = 8$ TeV, discussed earlier, which yields the following fragmentation fraction and the corresponding cross section

$$f(bb \to H_{\{bb\}}) = (3.2^{+1.2}_{-0.8})\%, \tag{10}$$

$$\sigma(pp \to H_{\{bb\}}) = (14.8^{+5.4}_{-3.7})\ \text{nb}. \tag{11}$$

The double-bottom hadrons $H_{\{bb\}}$ include the double-heavy tetraquarks $T^{\{bb\}}_{[\bar{q}\bar{q}']}$ and the double-bottom baryons $\Xi^0_{bb}(bbu)$, $\Xi^-_{bb}(bbd)$, and $\Omega^-_{bb}(bbs)$. The relative fractions of $H_{\{bb\}} \to T^{\{bb\}}_{[\bar{q}\bar{q}']}$ and $H_{\{bb\}} \to \Xi^0_{bb}(bbu)$, $\Xi^-_{bb}(bbd)$, $\Omega^-_{bb}(bbs)$ are not known. In the fragmentation language, they involve the vacuum excitation of a light anti-diquark pair $(\bar{q}\bar{q}')$ in the former, and of a light quark-antiquark pair in the latter. We assume, appealing to the heavy quark - heavy diquark symmetry, that they are similar to the measured ones in a single b-quark jet, for which LHCb has reported the following p_T-dependent ratio [24]:

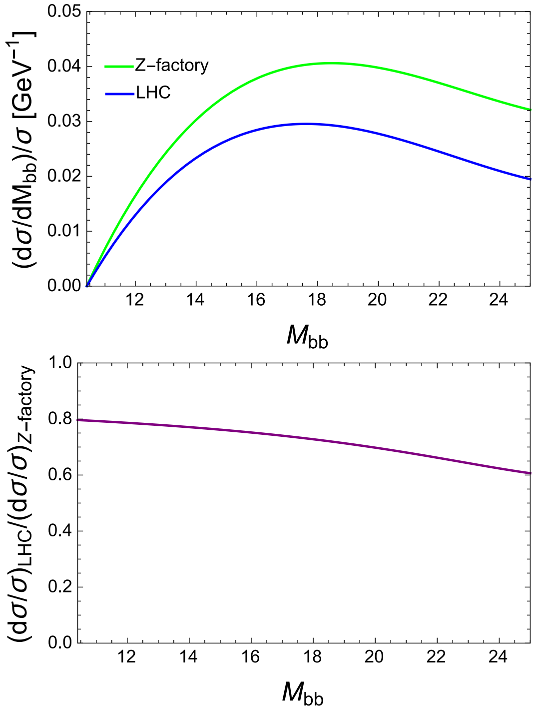

FIG. 2: Normalized differential cross section $\frac{1}{\sigma}[d\sigma(pp \to (bb)_{\rm jet} + \bar{b} + \bar{b} + X)/dM_{bb}]$ at the LHC (13 TeV) versus the corresponding cross section at the Z pole (upper panel) and the ratio of the normalized differential cross sections (lower panel). See text for details.

$$\begin{aligned} \left[\frac{f_{\Lambda_b}}{f_{B_u} + f_{B_d}}\right](p_{\rm T}) &= (0.404 \pm 0.036) \\ &\times [1 - (0.031 \pm 0.005) p_{\rm T}(\text{GeV})], \end{aligned} \tag{12}$$

where we have added in quadrature the various errors quoted in [24]. To use this input, we need to first calculate the $p_{\rm T}$-distribution of the bb-diquark jet in $pp \to (bb)_{\rm jet} + \bar{b} + \bar{b} + X$. This is shown in FIG 3, where the $(bb)_{\rm jet}$ is defined by the interval $M_{bb}(\Delta M)$ with $\Delta M = 2.0$ GeV.

We convolute this distribution with the one measured by LHCb for $\left[\frac{f_{\Lambda_b}}{f_{B_u}+f_{B_d}}\right](p_{\rm T})$, given above, and estimate the ratio of the $T^{\{bb\}}_{[\bar{u}\bar{d}]}$ production cross section to the

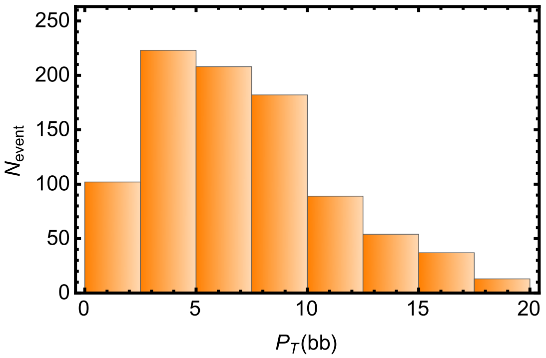

FIG. 3: The bb-quark-pair $P_{\rm T}$ distribution for the process $pp \to (bb)_{\rm jet} + \bar{b} + \bar{b} + X$ at $\sqrt{s} = 13$ TeV, obtained by generating 10^5 events using MadGraph and Pythia6 at the NLO accuracy. The $(bb)_{\rm jet}$ is defined by the interval $M_{bb}(\Delta M)$ with $\Delta M = 2.0$ GeV.

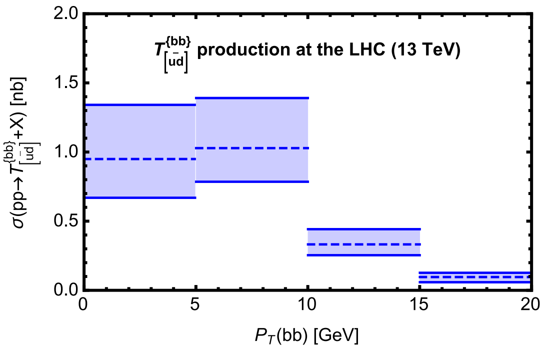

FIG. 4: Projected $p_{\rm T}$-dependence of tetraquark production cross section in $pp \to T^{\{bb\}}_{[\bar{u}\bar{d}]} + X$ at the LHC for $\sqrt{s} = 13$ TeV.

$H_{\{bb\}}$ production cross section:

$$\frac{\sigma(pp \to T^{\{bb\}}_{[\bar{u}\bar{d}]} + X)}{\sigma(pp \to H_{\{bb\}} + X)} = 0.17 \pm 0.01, \tag{13}$$

with both $H_{\{bb\}}$ and $T^{\{bb\}}_{[\bar{u}\bar{d}]}$ having $p_{\rm T} < 20$ GeV. This leads finally to the integrated cross section:

$$\sigma(pp \to T^{\{bb\}}_{[\bar{u}\bar{d}]} + X) = (2.4^{+0.9}_{-0.6}) \text{ nb}. \tag{14}$$

The $p_{\rm T}$-distribution of the differential cross section for $pp \to T^{\{bb\}}_{[\bar{u}\bar{d}]} + X$ is shown in Fig. 4.

The production cross sections for the double-bottom baryons (summed over the states) are estimated as

$$\sigma(pp \to (\Xi^0_{bb}, \Xi^-_{bb}, \Omega^-_{bb}) + X) : \sigma(pp \to T^{\{bb\}}_{[\bar{q}\bar{q}']} + X) \approx 2 : 1. \tag{15}$$

Thus, we anticipate about twice as many double-bottom baryons as the double-bottom tetraquarks at the 13 TeV LHC.

The LHCb collaboration is expected to collect about 50 fb^{-1} of data in Runs 1-4 [29–31], which would translate into $\mathcal{O}(10^8)$ $T^{\{bb\}}_{[\bar{u}\bar{d}]}$ events. Taking into account the $s\bar{s}$-suppression, compared to $d\bar{d}$ or $u\bar{u}$, we expect approximately half this number for the other two doubly-bottom tetraquarks $T^{\{bb\}}_{[\bar{u}\bar{s}]}$ and $T^{\{bb\}}_{[\bar{s}\bar{d}]}$. Their lifetimes are expected to be very similar, and estimated as 0.8 ps [24]. Their anticipated discovery modes [22, 37], which have typical branching ratios of $\mathcal{O}(10^{-6})$, suggest that dedicated searches at the LHC will be required to discover them.

Production of $T^{[bc]}_{[\bar{u}\bar{d}]}$ *at the LHC*: As already noted, LHCb has collected an impressive amount of B_c events, with 2.1×10^3 $B_c \to J/\psi\pi$ candidates in 2 fb^{-1} pp collisions at 8 TeV [23]. As the underlying partonic process is the same for the tetraquark $T^{[bc]}_{[\bar{u}\bar{d}]}$ production, but nonperturbative aspects differ, we evaluate the production cross section $\sigma(pp \to T^{[bc]}_{[\bar{u}\bar{d}]} + X)$. For that we generate 10^4 showered $pp \to b\bar{b}c\bar{c}$ events at the pp centre-of-mass energy 8 TeV, using the generators MadGraph [27] and Pythia6 [28] at the NLO accuracy. The cross section $\sigma(pp \to b\bar{b}c\bar{c} + X)$ is evaluated by MadGraph to be $(4.79 \pm 0.08) \times 10^3$ nb, which on using $\Delta M = (2.0^{+0.5}_{-0.4})$ GeV yields the following fragmentation fraction and the corresponding cross section:

$$f(bc \to H_{\{bc\}}) = (5.7^{+2.4}_{-1.4})\%, \tag{16}$$

$$\sigma(pp \to H_{\{bc\}} + X) = (273^{+113}_{-\ 66}) \text{ nb}. \tag{17}$$

Combined with Eq. (12) for the fragmentation fraction, we get

$$\sigma(pp \to T^{[bc]}_{[\bar{u}\bar{d}]} + X) = (48^{+19}_{-12}) \text{ nb}, \tag{18}$$

with $p_{\rm T}(T^{[bc]}_{[\bar{u}\bar{d}]}) < 20$ GeV. Assuming a detection efficiency of 10^{-6}, we anticipate $O(10^3)$ $T^{[bc]}_{[\bar{u}\bar{d}]}$ candidate events in the currently available LHCb data set, and approximately a half of this number for the related tetraquarks $T^{[bc]}_{[\bar{u}\bar{s}]}$ and $T^{[bc]}_{[\bar{d}\bar{s}]}$. There is considerable uncertainty in these estimates as the mixed bottom-charm tetraquarks, as opposed to the stable bb-tetraquarks, have $J^P = 0^+$, and $J^P = 1^+$, and their relative production rates in the fragmentation of a cb-diquark is an additional unknown parameter. They apply to the sum of both the J^P states. The mass of $T^{[bc]}_{[\bar{u}\bar{d}]}$ is estimated in Ref. [10] to be 7229 MeV, some 83 MeV above the BD threshold, and one expects a narrow resonance in this channel. The masses of the other two tetraquarks with an s-quark, are pitched at 7406 MeV, some 170 MeV above the B_sD threshold [10], considerably broadening the resonances.

Finally, the cross section $\sigma(pp \to b\bar{b}c\bar{c} + X)$ at 13 TeV is evaluated by MadGraph to be $(8.76 \pm 0.19) \times 10^3$ nb. Repeating the steps indicated for the 8 TeV case, we estimate the cross section at 13 TeV to increase by approximately a factor 1.9, yielding

$$\sigma(pp \to T^{[bc]}_{[\bar{u}\bar{d}]}) = (88^{+33}_{-22})\ \text{nb}. \tag{19}$$

with $p_{\mathrm{T}}(T^{[bc]}_{[\bar{u}\bar{d}]}) < 20$ GeV.

With the LHCb integrated luminosity of 50 fb^{-1}, to be reached in Runs 1-4, well over 10^9 $T^{[bc]}_{[\bar{u}\bar{d}]}$ events will be produced.

We would like to thank Estia Eichten, Tim Gershon, Marek Karliner, Luciano Maiani, Alexander Parkhomenko, Antonello Polosa, Gerrit Schierholz, Sheldon Stone, Cen Zhang and Zhi-Jie Zhao for helpful discussions. This work is supported in part by the National Natural Science Foundation of China under Grant Nos. 11575110, 11655002, 11735010, the Natural Science Foundation of Shanghai under Grant No. 15DZ2272100, and the DFG Forschergruppe FOR 1873 "Quark Flavour Physics and Effective Field Theories".

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[1] M. Tanabashi *et al.* (Particle Data Group) Phys. Rev. D **98**, 030001 (2018).

[2] A. Ali, J. S. Lange and S. Stone, Prog. Part. Nucl. Phys. **97**, 123 (2017) doi:10.1016/j.ppnp.2017.08.003 [arXiv:1706.00610 [hep-ph]].

[3] A. Esposito, A. Pilloni and A. D. Polosa, Phys. Rept. **668**, 1 (2016) doi:10.1016/j.physrep.2016.11.002 [arXiv:1611.07920 [hep-ph]].

[4] H. X. Chen, W. Chen, X. Liu and S. L. Zhu, Phys. Rept. **639**, 1 (2016) doi:10.1016/j.physrep.2016.05.004 [arXiv:1601.02092 [hep-ph]].

[5] F. K. Guo, C. Hanhart, U. G. Meiner, Q. Wang, Q. Zhao and B. S. Zou, Rev. Mod. Phys. **90**, no. 1, 015004 (2018) doi:10.1103/RevModPhys.90.015004 [arXiv:1705.00141 [hep-ph]].

[6] S. L. Olsen, T. Skwarnicki and D. Zieminska, Rev. Mod. Phys. **90**, no. 1, 015003 (2018) doi:10.1103/RevModPhys.90.015003 [arXiv:1708.04012 [hep-ph]].

[7] J. P. Ader, J. M. Richard and P. Taxil, Phys. Rev. D **25**, 2370 (1982). doi:10.1103/PhysRevD.25.2370

[8] A. V. Manohar and M. B. Wise, Nucl. Phys. B **399**, 17 (1993) doi:10.1016/0550-3213(93)90614-U [hep-ph/9212236].

[9] M. Karliner and J. L. Rosner, Phys. Rev. Lett. **119**, no. 20, 202001 (2017) doi:10.1103/PhysRevLett.119.202001 [arXiv:1707.07666 [hep-ph]].

[10] E. J. Eichten and C. Quigg, Phys. Rev. Lett. **119**, no. 20, 202002 (2017) doi:10.1103/PhysRevLett.119.202002 [arXiv:1707.09575 [hep-ph]].

[11] A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, Phys. Rev. Lett. **118**, no. 14, 142001 (2017) doi:10.1103/PhysRevLett.118.142001 [arXiv:1607.05214 [hep-lat]].

[12] P. Bicudo, M. Cardoso, A. Peters, M. Pflaumer and M. Wagner, Phys. Rev. D **96**, no. 5, 054510 (2017) doi:10.1103/PhysRevD.96.054510 [arXiv:1704.02383 [hep-lat]].

[13] P. Junnarkar, M. Padmanath and N. Mathur, EPJ Web Conf. **175**, 05014 (2018) doi:10.1051/epjconf/201817505014 [arXiv:1712.08400 [hep-lat]].

[14] T. Mehen, Phys. Rev. D **96**, no. 9, 094028 (2017) doi:10.1103/PhysRevD.96.094028 [arXiv:1708.05020 [hep-ph]].

[15] A. Czarnecki, B. Leng and M. B. Voloshin, Phys. Lett. B **778**, 233 (2018) doi:10.1016/j.physletb.2018.01.034 [arXiv:1708.04594 [hep-ph]].

[16] A. Esposito, M. Papinutto, A. Pilloni, A. D. Polosa and N. Tantalo, Phys. Rev. D **88**, no. 5, 054029 (2013) doi:10.1103/PhysRevD.88.054029 [arXiv:1307.2873 [hep-ph]].

[17] S. Q. Luo, K. Chen, X. Liu, Y. R. Liu and S. L. Zhu, Eur. Phys. J. C **77**, no. 10, 709 (2017) doi:10.1140/epjc/s10052-017-5297-4 [arXiv:1707.01180 [hep-ph]].

[18] R. L. Jaffe, Phys. Rev. D **15**, 267 (1977). doi:10.1103/PhysRevD.15.267

[19] R. L. Jaffe and F. Wilczek, Phys. Rev. Lett. **91**, 232003 (2003) doi:10.1103/PhysRevLett.91.232003 [hep-ph/0307341].

[20] L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. D **71**, 014028 (2005) doi:10.1103/PhysRevD.71.014028 [hep-ph/0412098].

[21] For a comprehensive review, see N. Brambilla *et al.*, Eur. Phys. J. C **71**, 1534 (2011) doi:10.1140/epjc/s10052-010-1534-9 [arXiv:1010.5827 [hep-ph]].

[22] A. Ali, A. Y. Parkhomenko, Q. Qin and W. Wang, Phys. Lett. B **782**, 412 (2018) doi:10.1016/j.physletb.2018.05.055 [arXiv:1805.02535 [hep-ph]].

[23] R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **114**, 132001 (2015) doi:10.1103/PhysRevLett.114.132001 [arXiv:1411.2943 [hep-ex]].

[24] R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. D **85**, 032008 (2012) doi:10.1103/PhysRevD.85.032008 [arXiv:1111.2357 [hep-ex]].

[25] R. Aaij *et al.* [LHCb Collaboration], JHEP **1408**, 143 (2014) doi:10.1007/JHEP08(2014)143 [arXiv:1405.6842 [hep-ex]].

[26] V. Khachatryan *et al.* [CMS Collaboration], JHEP **1705**, 013 (2017) doi:10.1007/JHEP05(2017)013 [arXiv:1610.07095 [hep-ex]].

[27] J. Alwall *et al.*, JHEP **1407**, 079 (2014) doi:10.1007/JHEP07(2014)079 [arXiv:1405.0301 [hep-ph]].

[28] T. Sjostrand, S. Mrenna and P. Z. Skands, JHEP **0605**, 026 (2006) doi:10.1088/1126-6708/2006/05/026 [hep-ph/0603175].

[29] R. Aaij *et al.* [LHCb Collaboration], Eur. Phys. J. C **73**, no. 4, 2373 (2013) doi:10.1140/epjc/s10052-013-2373-2 [arXiv:1208.3355 [hep-ex]].

[30] I. Bediaga *et al.* [LHCb Collaboration], CERN-LHCC-2012-007, LHCb-TDR-12.

[31] See, for example, A. Carbone, in the proceedings of the *9th. Int. Workshop on Charm Physics*, Novosibirsk, Rusia.

[32] R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **109**, 232001 (2012) doi:10.1103/PhysRevLett.109.232001 [arXiv:1209.5634 [hep-ex]].

[33] R. Aaij *et al.* [LHCb Collaboration], JHEP **1712**, 026 (2017) doi:10.1007/JHEP12(2017)026 [arXiv:1710.04921 [hep-ex]].

[34] Z. Rui, H. Li, G. x. Wang and Y. Xiao, Eur. Phys. J. C **76**, no. 10, 564 (2016) doi:10.1140/epjc/s10052-016-4424-y [arXiv:1602.08918 [hep-ph]].

[35] C. F. Qiao, P. Sun, D. Yang and R. L. Zhu, Phys. Rev. D **89**, no. 3, 034008 (2014) doi:10.1103/PhysRevD.89.034008 [arXiv:1209.5859 [hep-ph]].

[36] Z. Yang, X. G. Wu, G. Chen, Q. L. Liao and J. W. Zhang, Phys. Rev. D **85**, 094015 (2012) doi:10.1103/PhysRevD.85.094015 [arXiv:1112.5169 [hep-ph]].

[37] Y. Xing and R. Zhu, arXiv:1806.01659 [hep-ph].