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Evolution equations for extended dihadron fragmentation functions

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We consider dihadron fragmentation functions, describing the fragmentation of a parton in two unpolarized hadrons, and in particular extended dihadron fragmentation functions, explicitly dependent on the invariant mass, M_h, of the hadron pair. We first rederive the known results on M_h-integrated functions using Jet Calculus techniques, and then we present the evolution equations for extended dihadron fragmentation functions. Our results are relevant for the analysis of experimental measurements of two-particle-inclusive processes at different energies.

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I. INTRODUCTION

The fragmentation of partons into hadrons has been studied in detail in semi-inclusive processes with one hadron detected in the final state, such as e^+e^- annihilation, Semi-Inclusive Deep-Inelastic Scattering (SIDIS) or hadron-hadron collisions. Factorization theorems (see, e.g., [1, 2, 3]) allow to separate perturbatively calculable short-range coefficient functions from long-distance nonperturbative fragmentation functions $D_1^{i\to h}(z)$, describing the "decay" of the hard parton i into an observed hadron h with fractional energy z, provided a hard scale is available. This is the case of the $e^+e^- \to hX$ reaction, where the coefficient function is known at least to $O(\alpha_s)$ [4, 5], with α_s the running strong coupling constant. The same fragmentation functions, $D_1^{i\to h}$, occur in the factorized formula for SIDIS at $O(\alpha_s)$ [4, 5], combined with the specific process-dependent coefficient functions at the same accuracy. For hadronic collisions, factorization is usually assumed, but has not been proven yet.

When considering semi-inclusive processes with two detected hadrons in the final state, e.g., $e^+e^- \to h_1h_2X$, a new class of fragmentation functions, the so-called Dihadron Fragmentation Functions (DiFF), needs to be introduced to guarantee factorization of all collinear singularities [6]. From this perspective, DiFF are analogous to fracture functions in the space-like regime [7, 8]. The DiFF evolution equations have been recently reanalyzed in Ref. [9] and the cross section for $e^+e^- \to h_1h_2X$ has been calculated to $O(\alpha_s)$ in Ref. [10]. At $O(\alpha_s^0)$, the production of two hadrons h_1, h_2 with fractional energies z_1, z_2 and belonging to the same jet, is described by a DiFF, $D_1^{i\to h_1h_2}(z_1, z_2)$, i.e., the fragmentation of a single parton i into the two hadrons. At $O(\alpha_s)$, hadrons produced in the same jet could either come from the fragmentation of a single parton into two hadrons or by the fragmentation of two collinear partons, i and j, into single hadrons. This implies that evolution equations for DiFF contain an inhomogeneous term of the form $D_1^{i\to h_1} \otimes D_1^{j\to h_2}$ [10].

All these studies focused on DiFF as functions of the energy fractions z_1 and z_2, integrated over all the other kinematical variables of the produced hadron pair, including their invariant mass M_h. However, the largest amount of experimental information related to DiFF consists of invariant mass spectra of hadron pairs produced in e^+e^- annihilation [11, 12, 13], Semi-Inclusive Deep-Inelastic Scattering (SIDIS) [14, 15, 16] and proton-proton collisions [17, 18, 19]. In this paper, using the techniques of Jet Calculus [6, 20] we deduce the evolution equations for DiFF with an explicit dependence on M_h. In analogy with Ref. [21], we address them as extended Dihadron Fragmentation Functions (extDiFF).

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DiFF turn out to have important applications in polarization studies, since they can act as spin-analyzers of the fragmenting quark [22]. In particular, the transverse polarization s_T of the fragmenting quark can be related to the azimuthal orientation of the plane containing the two hadron momenta P_1 and P_2, through the mixed product $P_1 \times P_2 \cdot s_T$. The strength of this relation is described by the DiFF $H_1^{\sphericalangle\, i\to h_1 h_2}$ [23]. In SIDIS with transversely polarized targets, this DiFF appears in combination with the transversity distribution function [24, 25, 26], thus providing a way to constrain this elusive partonic distribution (for a review on transversity, see Ref. [27]). The HERMES [28] and COMPASS [29] collaborations have reported preliminary measurements of the induced spin asymmetry (at $\langle Q^2\rangle \approx 2.5$ GeV^2). In the meanwhile, the BELLE collaboration is planning to perform the extraction of the fragmentation function $H_1^{\sphericalangle}$ in e^+e^- annihilation [30], but at the higher scale $\sqrt{s} \approx 10$ GeV [31]. The invariant-mass dependence of this fragmentation function is unknown and nontrivial, as shown, e.g., by model calculations [25, 26, 32]. Therefore, the study of the evolution properties of extDiFF is also timely.

The paper is organized as follows. In Sec. II, using Jet Calculus we recover the inhomogeneous evolution equations for DiFF derived in Ref. [10]. In Sec. III, following the same lines we deduce the evolution equations for the corresponding extDiFF. Finally, in Sec. IV we draw our conclusions.

II. INTEGRATED DIHADRON FRAGMENTATION FUNCTIONS

The cross section at order $O(\alpha_s)$ for the $e^+e^- \to hX$ process, where a hadron has momentum P_h and energy fraction $z = 2P_h \cdot q/Q^2$ with respect to the center-of-mass (cm) energy $Q^2 \equiv q^2$, can be written formally as

$$\frac{d\sigma^h}{dz} = \sum_i \sigma^i \otimes D_1^{i\to h} \ , \tag{1}$$

where $D_1^{i\to h}$ describes the "decay" at leading twist of the hard parton i into the observed hadron h, the sum on i running over all possible partons species $i = q, \bar{q}, g$. The process-dependent coefficient function σ^i can be calculated and regularized in perturbation theory and it is known at least at $O(\alpha_s)$ [4, 5]. The $D_1^{i\to h}$ at $O(\alpha_s)$, i.e., which absorbs all the collinear singularities, can be accurately parametrized [33, 34], except for the $z \to 0$ portion of phase space.

The generalization of Eq. (1) to the process $e^+e^- \to h_1 h_2 X$ is not straightforward, if one wants to cover the whole phase space accessible to (h_1, h_2). The differential cross section, again at $O(\alpha_s)$, has been recently calculated in Ref. [10] and reads, with obvious notations,

$$\frac{d\sigma^{h_1,h_2}}{dz_1 dz_2} = \sum_{ij} \sigma^{ij} \otimes D_1^{i\to h_1} \otimes D_1^{j\to h_2} + \sum_i \sigma^i \otimes D_1^{i\to h_1 h_2} \ , \tag{2}$$

where the DiFF $D_1^{i\to h_1 h_2}$ contains information on the fragmentation, at leading twist and $O(\alpha_s)$, of the hard parton i directly into the observed hadron pair h_1, h_2. At order $O(\alpha_s^0)$, the first term of Eq. (2) would correspond to the back-to-back emission of a parton and an antiparton, eventually fragmenting in the hadrons h_1 and h_2 belonging to two well separated jets. The second term would apply instead to the case where the hadron pair is produced very close in phase space and it is detected inside the same jet while the other jet is inclusively summed over. However, at order $O(\alpha_s)$ a new kind of collinear singularities arises in the partonic cross section; it corresponds to the configuration where each hadron is obtained from the fragmentation of a single parton, the two partons being almost collinear, i.e., with a very small relative transverse momentum r_T. These $1/r_T^2$ singularities cannot be reabsorbed in each $D_1^{i\to h}$, because they do not correspond to the back-to-back configuration. Hence, they must be reabsorbed in $D_1^{i\to h_1 h_2}$, thus making the two terms in Eq. (2) indistinguishable [10].

As a consequence, after integrating over r_T the DiFF must satisfy the following evolution equation [10]

$$\begin{aligned}\frac{d}{d\ln Q^2} D_1^{i\to h_1 h_2}(z_1, z_2, Q^2) = {} & \frac{\alpha_s(Q^2)}{2\pi} \int_{z_1+z_2}^1 \frac{du}{u^2}\, D_1^{j\to h_1 h_2}\left(\frac{z_1}{u}, \frac{z_2}{u}, Q^2\right) P_{ji}(u) \\ & + \frac{\alpha_s(Q^2)}{2\pi} \int_{z_1}^{1-z_2} \frac{du}{u(1-u)}\, D_1^{j\to h_1}\left(\frac{z_1}{u}, Q^2\right) D_1^{k\to h_2}\left(\frac{z_2}{1-u}, Q^2\right) \hat{P}_{jk}^i(u) \ ,\end{aligned} \tag{3}$$

where here, and in the following, a sum over repeated parton indices is understood. The first term in the right-hand side represents the usual homogeneous evolution for the DiFF $D_1^{i\to h_1 h_2}$ in complete analogy with the case of single-hadron fragmentation: the probability for the parton i to fragment into the hadrons h_1, h_2, is affected by the probability of emitting a parton j with momentum fraction u through the Altarelli-Parisi splitting vertex $P_{ji}(u)$ [35], listed in Eqs. (A.1)–(A.4) of the appendix for convenience. The second term is a new inhomogeneous contribution that corresponds to the probability for the parton i to split in the two partons j and k with momentum fractions u and $(1-u)$, respectively, each one fragmenting in one of the two observed hadrons. The $\hat{P}^i_{jk}(u)$ are the Altarelli-Parisi splitting functions without virtual contributions [20], again listed in Eqs. (A.6)–(A.9) of the appendix for convenience. From this point of view, the situation is similar to the DIS case in the target fragmentation region, since the DiFF can be conceived as the time-like version of the fracture functions in the space-like domain [7, 8].

In the following, we will make use of Jet Calculus [6, 20] and recover the evolution equation (3) within this formalism. We will consider the semi-inclusive production of two hadrons, h_1 and h_2, belonging to the same jet and neglecting the emission of wide-angle hard partons (and related jets). Therefore, we will not perform a fixed-order calculation of the $e^+e^- \to h_1 h_2 X$ cross section. Rather, we will consider a parton i with a large virtuality ϵQ^2 $(0 < \epsilon < 1)$, which fragments in two hadrons h_1 and h_2 inside the same jet. The virtuality can be reconstructed from the invariant mass of the jet by a suitable jet-finding algorithm [36]. The phase-space structure of collinear singularities singled out in fixed-order calculations can be translated in Jet Calculus as a degeneracy in all possible competing mechanisms, which could realize the desidered final state configuration [6, 20]. Thus, the cross section is the sum of all production mechanisms, as in Eq. (2).

We use Q^2 as evolution scale, instead of ϵQ^2. In Leading Logarithmic Approximation (LLA), this substitution induces only subleading corrections and thus is fully justified within this approximation. Moreover, it is convenient to replace this variable with the evolution variable

$$Y = \frac{1}{2\pi\beta_0} \ln\left[\frac{\alpha_s(\mu_R^2)}{\alpha_s(Q^2)}\right] , \tag{4}$$

also named the evolution imaginary time [20]. In Eq. (4), μ_R^2 is the renormalization scale and $\beta_0 = (11N_c - 2N_f)/(12\pi)$ is the one-loop β function with N_c, N_f, the number of colors and flavors, respectively. In LLA, the running of α_s is taken into account at one loop by

$$\alpha_s(Q^2) = \frac{1}{\beta_0 \ln(Q^2/\Lambda^2_{QCD})} , \tag{5}$$

where Λ^2_{QCD} is the infrared scale. Hence, the differential evolution length is just

$$dY = \frac{\alpha_s(Q^2)}{2\pi}\frac{dQ^2}{Q^2} . \tag{6}$$

Let us define the variable y as

$$y = \frac{1}{2\pi\beta_0} \ln\left[\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}\right] , \tag{7}$$

with Q_0^2 and Q^2 two arbitrary scales, and introduce the perturbative parton-to-parton time-like evolution function $E^i_j(x,y)$, which expresses the probability of finding a parton j at the scale Q_0^2 with a momentum fraction x of the parent parton i at the scale Q^2. The function $E^i_j(x,y)$ can be shown to satisfy standard evolution equations [6, 20]

$$\frac{d}{dy}E^i_j(x,y) = \int_x^1 \frac{du}{u} E^k_j\left(\frac{x}{u},y\right) P_{ki}(u) , \tag{8}$$

that can be iteratively solved by using the initial condition

$$E^i_j(x,y)|_{y=0} = \delta_{ji}\delta(1-x) , \tag{9}$$

with δ_{ji} the Kronecker symbol. The $E^i_j(x,y)$ resums leading logarithms of the type $\alpha_s^n \ln^n(Q^2/Q_0^2)$, which show up in the collinear limit of perturbative calculations at the partonic level. As a crosscheck, we can expand Eq. (8) at order $O(\alpha_s)$ with the initial condition (9); neglecting for simplicity the running of α_s in Eq. (7), we get

$$E^i_j(x,y) \equiv E^i_j(x,Q_0^2,Q^2) \approx \delta_{ji}\delta(1-x) + \frac{\alpha_s}{2\pi} P_{ji}(x) \ln\frac{Q^2}{Q_0^2} . \tag{10}$$

Leading logarithmic contributions are therefore automatically accounted for at all orders through the function E. Consider now the fragmentation process $i \to h_1 h_2 X$ where h_1 and h_2 are detected within the same jet and with relative transverse momentum $R_T^2 \lesssim Q^2$. The corresponding cross-section, normalized to the jet-cross section, can be written as

$$\begin{aligned}\frac{1}{\sigma_{jet}}\frac{d\sigma^{i\to h_1h_2}}{dz_1dz_2} &\equiv D_1^{i\to h_1h_2}(z_1,z_2,Y) = D_{1,A}^{i\to h_1h_2}(z_1,z_2,Y) + D_{1,B}^{i\to h_1h_2}(z_1,z_2,Y)\\ &= \int_{z_1+z_2}^1 \frac{dw}{w^2}\, D_1^{j\to h_1h_2}\left(\frac{z_1}{w},\frac{z_2}{w},y_0\right) E_j^i(w,Y-y_0)\\ &+ \int_{y_0}^Y dy \int_{z_1+z_2}^1 \frac{dw}{w^2}\int_{\frac{z_1}{w}}^{1-\frac{z_2}{w}} \frac{du}{u(1-u)} \hat{P}_{lk}^j(u) E_j^i(w,Y-y) D_1^{k\to h_1}\left(\frac{z_1}{wu},y\right) D_1^{l\to h_2}\left(\frac{z_2}{w(1-u)},y\right).\end{aligned} \tag{11}$$

The first term "A" is the convolution of the DiFF at some arbitrary (but still perturbative) factorization scale y_0 with the parton-to-parton evolution function E_j^i . The second term "B", instead, represents the two separate single-hadron fragmentations, integrated over all possible generic intermediate scales y at which the branching $j \to kl$ at partonic level might occur. Integration limits in Eq.(11) are fixed by momentum conservation. Both terms are depicted in Fig. 1. In order to recover Eq.(3), we now take the derivative of Eq. (11) with respect to the variable Y. Using Eq. (8)

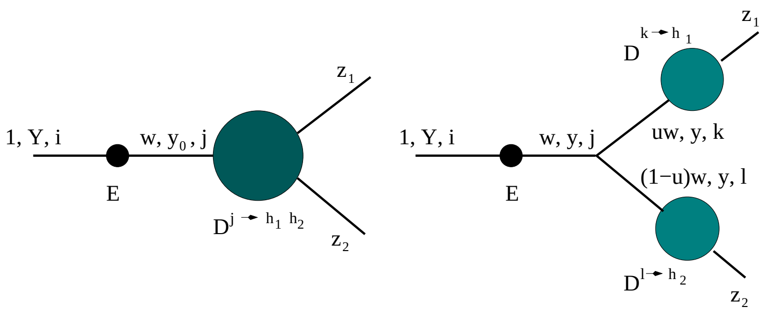

FIG. 1: Double- and single-hadron fragmentations in Eq. (11). The momentum fractions are indicated along with the scale and the parton indices. The black dot represents the parton evolution function E.

and the definition of $D_{1,A}^{i\to h_1h_2}$, we get for the "A" term

$$\begin{aligned}\frac{d}{dY}D_{1,A}^{i\to h_1h_2}(z_1,z_2,Y) &= \int_{z_1+z_2}^1 \frac{dw}{w^2}\, D_1^{j\to h_1h_2}\left(\frac{z_1}{w},\frac{z_2}{w},y_0\right) \frac{d}{dY}[E_j^i(w,Y-y_0)]\\ &= \int_{z_1+z_2}^1 \frac{du}{u^2}\, D_{1,A}^{k\to h_1h_2}\left(\frac{z_1}{u},\frac{z_2}{u},Y\right) P_{ki}(u)\ .\end{aligned} \tag{12}$$

The derivative d/dY of the "B" contribution in Eq. (11) produces two terms, since there is an explicit Y-dependence in the upper integration limit. Using Eq. (9) and the same procedure as before, we get

$$\begin{aligned}\frac{d}{dY}D_{1,B}^{i\to h_1h_2}(z_1,z_2,Y) &= \int_{z_1+z_2}^1 \frac{dw}{w^2}\,\delta_{ji}\delta(1-w)\int_{\frac{z_1}{w}}^{1-\frac{z_2}{w}} \frac{du}{u(1-u)}\,\hat{P}_{lk}^j(u) D_1^{k\to h_1}\left(\frac{z_1}{wu},Y\right) D_1^{l\to h_2}\left(\frac{z_2}{w(1-u)},Y\right)\\ &+ \int_{y_0}^Y dy\int_{z_1+z_2}^1 \frac{dw}{w^2}\int_{\frac{z_1}{w}}^{1-\frac{z_2}{w}} \frac{du}{u(1-u)}\,\hat{P}_{lk}^j(u)\,\frac{d}{dY}E_j^i(w,Y-y)\, D_1^{k\to h_1}\left(\frac{z_1}{wu},y\right) D_1^{l\to h_2}\left(\frac{z_2}{w(1-u)},y\right)\\ &= \int_{z_1}^{1-z_2} \frac{du}{u(1-u)}\,\hat{P}_{lk}^i(u) D_1^{k\to h_1}\left(\frac{z_1}{u},Y\right) D_1^{l\to h_2}\left(\frac{z_2}{1-u},Y\right) + \int_{z_1+z_2}^1 \frac{du}{u^2}\,P_{ki}(u) D_{1,B}^{k\to h_1h_2}\left(\frac{z_1}{u},\frac{z_2}{u},Y\right).\end{aligned} \tag{13}$$

Summing up Eqs. (12) and (13), we get

$$\begin{aligned}\frac{d}{dY}D_1^{i\to h_1h_2}(z_1,z_2,Y) &= \int_{z_1+z_2}^1 \frac{du}{u^2}\, D_1^{k\to h_1h_2}\left(\frac{z_1}{u},\frac{z_2}{u},Y\right) P_{ki}(u)\\ &+ \int_{z_1}^{1-z_2} \frac{du}{u(1-u)}\, D_1^{k\to h_1}\left(\frac{z_1}{u},Y\right) D_1^{l\to h_2}\left(\frac{z_2}{1-u},Y\right) \hat{P}_{lk}^i(u)\ ,\end{aligned} \tag{14}$$

which is exactly Eq. (3), after changing the scaling variable Y back to the more familiar Q^2. This derivation is useful to adjust the formalism of Jet Calculus from Refs. [6, 20] to the calculation of the $e^+e^- \to h_1 h_2 X$ cross section at $O(\alpha_s)$ from Ref. [10]. However, we want to stress again that even if the two expressions are formally identical, they have been derived from rather different approaches. Eq. (3) from Ref. [10] applies to the full phase space for the production of two hadrons. Instead, Eq. (14) gives the evolution of the parton i in a jet within which we identify the two detected hadrons h_1 and h_2; thus, it is valid only in the portion of phase space defined by the jet. We recall that if Q^2 is the cm energy of the e^+e^- annihilation, the event is characterized by a certain number of jets resulting also from a large-angle hard parton emission; this is not included in the LLA used here. Therefore, the evolution scale in Eq. (14) must be intended as the invariant mass of the considered jet, i.e., ϵQ^2 with $0 < \epsilon < 1$.

III. EXTENDED DIHADRON FRAGMENTATION FUNCTIONS

In the previous section, we considered the inclusive production of two hadrons h_1 and h_2 inside the same jet, summing up all possible values of their invariant mass M_h. If the process starts from the hard scale Q^2 of the fragmenting parton (or, equivalently, ϵQ^2 in the case of the jet), there is no intermediate scale that allows to distinguish the two contributions in Eq. (11): the y_0 in the "A" term is arbitrary, and the scale y for the partonic branching in the "B" term is summed over.

However, most of the experimental information on unpolarized DiFF consists of invariant mass spectra of hadron pairs [11, 12, 13, 14, 15, 16, 17, 18, 19]. In addition, effects related to the partial-wave expansion of DiFF [37] are best explored when the latter explicitly depend on M_h^2. Hence, in this section we will address the evolution equations for DiFF at the fixed scale M_h^2. We will indicate these objects as extended Dihadron Fragmentation Functions (extDiFF), as the time-like analogue of the extended fracture functions that were introduced in Ref. [21] for the space-like SIDIS in the target fragmentation region.

The dependence of the extDiFF upon M_h^2 can be easily mapped into R_T^2, the square of the relative transverse momentum of the hadron pair. In fact, it results [37]

$$R_T^2 \equiv \frac{(P_{1T} - P_{2T})^2}{4} = \frac{z_1 z_2}{z_1 + z_2} \left[\frac{M_h^2}{z_1 + z_2} - \frac{M_1^2}{z_1} - \frac{M_2^2}{z_2} \right] , \tag{15}$$

with M_1 and M_2 the masses of the hadrons h_1 and h_2, respectively. From the first line of Eq. (11), we get the obvious definition

$$\begin{aligned} \frac{1}{\sigma_{jet}} \frac{d\sigma^{i\to h_1 h_2}}{dz_1 dz_2} &\equiv D_1^{i\to h_1 h_2}(z_1, z_2, Y) = D_{1,A}^{i\to h_1 h_2}(z_1, z_2, Y) + D_{1,B}^{i\to h_1 h_2}(z_1, z_2, Y) \\ &\equiv \int dR_T^2 \frac{1}{\sigma_{jet}} \frac{d\sigma^{i\to h_1 h_2}}{dz_1 dz_2 dR_T^2} = \int dR_T^2\, D_{1,A}^{i\to h_1 h_2}(z_1, z_2, R_T^2, Y) + \int dR_T^2\, D_{1,B}^{i\to h_1 h_2}(z_1, z_2, R_T^2, Y) \,. \end{aligned} \tag{16}$$

The inhomogeneous "B" term describes the time-like branching of parton i in two partons k and l with transverse relative momentum r_T, eventually fragmenting in the two hadrons h_1 and h_2 with transverse relative momentum R_T. If the R_T^2 scale is fixed and in the perturbative regime, the scale at the partonic branching is no longer arbitrary as in Eq. (11) [6, 20]. In fact, if u and $(1-u)$ are the fractional momenta of partons k and l, the parton virtualities are related by

$$k_i^2 = \frac{k_k^2}{u} + \frac{k_l^2}{1-u} + \frac{r_T^2}{4u(1-u)} \,. \tag{17}$$

Hence, fixing r_T at the partonic level determines in turn the branching scale k_i^2. At the hadronic level, this is not guaranteed and some assumptions must be made. We will suppose that in the fragmentations $k \to h_1$ and $l \to h_2$ the parton virtualities are negligible, i.e., $k_k^2 \simeq k_l^2 \simeq 0$, meaning that, once the branching $i \to kl$ has occurred, both the perturbative and nonperturbative transverse momenta generated in the fragmentation of the partons k and l are negligible (incidentally, perturbatively generated transverse momenta can be taken into account by using time-like evolution equations depending on transverse momentum [38]). Consequently, the transverse relative momentum r_T between k and l should not be substantially altered in the fragmentation, implying

$$k_i^2 \approx r_T^2 \approx R_T^2 \,. \tag{18}$$

Corrections to the above relation affect our final result only at subleading level. Instead, the above assumption is also consistent with the approximation we are working with. Leading logarithms are known to manifest themselves when the transversa momenta of the emitted partons are strongly ordered along the ladder, e.g., $Q^2 \gg r_{T,1}^2 \gg \ldots \gg r_{T,n}^2 \gg Q_0^2$. Hadron pairs with large relative transverse momentum R_T are thus produced earlier in the imaginary time Y than hadron pairs with small R_T, as appropriate for time-like kinematics.

If R_T^2 is thus in the perturbative domain, in analogy with Eq. (4), we can define the variable y_T as

$$y_T = \frac{1}{2\pi\beta_0} \ln \left[\frac{\alpha_s(\mu_R^2)}{\alpha_s(R_T^2)} \right] , \tag{19}$$

or, in differential form,

$$\frac{d}{dR_T^2} = \frac{\alpha_s(R_T^2)}{2\pi R_T^2} \frac{d}{dy_T} . \tag{20}$$

Since the scale at which the branching occurs is fixed by R_T^2, from Eqs. (11), (16) and (20), we obtain

$$\begin{aligned} & D_{1,B}^{i\to h_1 h_2}(z_1, z_2, R_T^2, Y) \\ & = \frac{\alpha_s(R_T^2)}{2\pi R_T^2} \frac{d}{dy_T} \int_{y_0}^{Y} dy \int_{z_1+z_2}^{1} \frac{dw}{w^2} \int_{\frac{z_1}{w}}^{1-\frac{z_2}{w}} \frac{du}{u(1-u)} \hat{P}_{lk}^{j}(u)\, E_j^i(w, Y-y)\, D_1^{k\to h_1}\left(\frac{z_1}{wu}, y\right) D_1^{l\to h_2}\left(\frac{z_2}{w(1-u)}, y\right) \\ & = \frac{\alpha_s(R_T^2)}{2\pi R_T^2} \int_{z_1+z_2}^{1} \frac{dw}{w^2} \int_{\frac{z_1}{w}}^{1-\frac{z_2}{w}} \frac{du}{u(1-u)} \hat{P}_{lk}^{j}(u)\, E_j^i(w, Y-y_T)\, D_1^{k\to h_1}\left(\frac{z_1}{wu}, y_T\right) D_1^{l\to h_2}\left(\frac{z_2}{w(1-u)}, y_T\right) . \end{aligned} \tag{21}$$

If the scale R_T^2 is fixed in the nonperturbative regime, the above arguments leading to Eq. (20) do not apply. This is the case for the homogeneous "A" term in Eq. (16), which describes the direct fragmentation of parton i in the two hadrons h_1 and h_2: the virtuality k_i^2 of the parent parton cannot be reconstructed from R_T^2 and it is set to the arbitrary factorization scale Q_0^2 (or, in our notations, y_0). From Eqs. (11) and (16), we simply get

$$D_{1,A}^{i\to h_1 h_2}(z_1, z_2, R_T^2, Y) = \int_{z_1+z_2}^{1} \frac{dw}{w^2} D_{1,A}^{j\to h_1 h_2}\left(\frac{z_1}{w}, \frac{z_2}{w}, R_T^2, y_0\right) E_j^i(w, Y-y_0) . \tag{22}$$

Summing up Eqs. (22) and (21), and providing each term with an extra step function to separate the two different kinematical regimes, we get the complete expression for the extDiFF at LLA:

$$\begin{aligned} & D_1^{i\to h_1 h_2}(z_1, z_2, R_T^2, Y) = D_{1,A}^{i\to h_1 h_2}(z_1, z_2, R_T^2, Y) + D_{1,B}^{i\to h_1 h_2}(z_1, z_2, R_T^2, Y) \\ & = \int_{z_1+z_2}^{1} \frac{dw}{w^2} D_{1,A}^{j\to h_1 h_2}\left(\frac{z_1}{w}, \frac{z_2}{w}, R_T^2, y_0\right) E_j^i(w, Y-y_0)\, \theta(y_0 - y_T) \\ & + \frac{\alpha_s(R_T^2)}{2\pi R_T^2} \int_{z_1+z_2}^{1} \frac{dw}{w^2} \int_{\frac{z_1}{w}}^{1-\frac{z_2}{w}} \frac{du}{u(1-u)} \hat{P}_{lk}^{j}(u) E_j^i(w, Y-y_T) D_1^{k\to h_1}\left(\frac{z_1}{wu}, y_T\right) D_1^{l\to h_2}\left(\frac{z_2}{w(1-u)}, y_T\right) \theta(y_T - y_0). \end{aligned} \tag{23}$$

Note that, despite the presence of the step functions, the separation between the two regimes is still arbitrary, since it depends on y_0 which is itself arbitrary. The evolution equations for the extDiFF can be obtained, in analogy with Eq. (14), by taking the derivative with respect to Y [or, equivalently, Q^2 via Eq. (6)]. By using Eq. (8) and the definition (23) of the extDiFF themselves, we get

$$\frac{d}{d\ln Q^2} D_1^{i\to h_1 h_2}(z_1, z_2, R_T^2, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_{z_1+z_2}^{1} \frac{du}{u^2} D_1^{j\to h_1 h_2}\left(\frac{z_1}{u}, \frac{z_2}{u}, R_T^2, Q^2\right) P_{ji}(u) . \tag{24}$$

We explicitly checked that by integrating Eq. (24) upon R_T^2 we recover Eq. (3). In conclusion, if the hadron pair is inclusively produced in the same jet at fixed transverse relative momentum R_T (or, equivalently, fixed invariant mass M_h), the evolution equations for the extDiFF are of the standard homogeneous type. The explicit dependence on this new scale breaks the degeneracy of the two production mechanisms described in the previous section. The same arguments apply to the SIDIS target fragmentation region where extended fracture functions, explicitly depending upon the invariant momentum transfer t between the incoming and outgoing hadron, satisfy a homogeneous evolution equation [21]. On the basis of Eq. (24), we argue that the cross section at order $O(\alpha_s)$ for the inclusive production of the two hadrons h_1, h_2, inside the same jet and with invariant mass M_h, can be expressed in the factorized form

$$\frac{d\sigma^{i\to h_1 h_2}}{dz_1 dz_2 dR_T^2} = \sum_i \sigma^i \otimes D_1^{i\to h_1 h_2}(R_T^2, Q^2) , \tag{25}$$

where σ^i are the same coefficient functions found in the single-hadron inclusive cross section of Eq. (1). In our above derivation, we used the techniques of Jet Calculus [6, 20], where the factorization of collinear singularities can be automatically accommodated through the use of the parton-to-parton evolution function E. Exchanges of soft particles are, however, not accounted for.

Eq. (24) can be conveniently diagonalized using a double Mellin transformation. We define

$$D_{n,m}^{i\to h_1 h_2}(R_T^2, Q^2) = \int_0^1 dz_1 \int_0^{1-z_1} dz_2\, z_1^{n-1}\, z_2^{m-1} D_1^{i\to h_1 h_2}(z_1, z_2, R_T^2, Q^2) \;, \tag{26}$$

and the anomalous dimension

$$A_j^i(n+m) = \int_0^1 du\, P_{ji}(u)\, u^{m+n-2} \;. \tag{27}$$

With simple algebra manipulations, it is easy to verify that

$$\frac{d}{d\ln Q^2} D_{n,m}^{i\to h_1 h_2}(R_T^2, Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\, D_{n,m}^{j\to h_1 h_2}(R_T^2, Q^2)\, A_j^i(n+m) \;. \tag{28}$$

The above results can be extended also to polarized extDiFF, in particular to the only one surviving when the hadron pair is collinear, i.e., $H_1^{\sphericalangle\, i\to h_1 h_2}$ [23, 37]. The evolution equations for this function have the same form of the unpolarized case, namely

$$\frac{d}{d\ln Q^2} H_1^{\sphericalangle\, i\to h_1 h_2}(z_1, z_2, R_T^2, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_{z_1+z_2}^1 \frac{du}{u^2}\, H_1^{\sphericalangle\, j\to h_1 h_2}\left(\frac{z_1}{u}, \frac{z_2}{u}, R_T^2, Q^2\right) \delta P_{ji}(u) \;, \tag{29}$$

where the splitting functions δP_{ji} for a transversely polarized fragmenting parton are used [39]. The δP_{ji} are listed in Eqs. (A.10)–(A.13) of the appendix.

These evolution equations can be conveniently used for phenomenological analyses, since they can connect experimental data taken at different energies.

IV. CONCLUSIONS

We have shown that in leading logarithm approximation the so-called extended Dihadron Fragmentation Functions (extDiFF), describing the inclusive production of two hadrons inside the same jet at fixed invariant mass M_h, satisfy evolution equations of the same homogeneous type as in the single-hadron fragmentation case. We stress that the explicit dependence on the scale M_h^2 is required to break the degeneracy at $O(\alpha_s)$ between the fragmentation from a single parton or after the branching in two collinear partons. While the first contribution pertains to the nonperturbative regime, in the latter the transverse relative momentum R_T of the two hadrons can be traced back to the transverse relative momentum of the two collinear partons after the branching, and, ultimately, to the hard scale of the originating parton. The analysis of the corresponding contribution to extDiFF shows that the dependence on this perturbative scale can be predicted.

In our derivation, we used the techniques of Jet Calculus [6, 20]. Factorization of collinear singularities can be automatically accommodated through the use of the parton-to-parton evolution function E. On the basis of the simple result for the evolution equations of extDiFF, we argue that the cross section at order $O(\alpha_s)$ for the inclusive production of the two hadrons h_1, h_2, inside the same jet and with invariant mass M_h, can be expressed in a factorized form involving the same coefficient functions as in the single-hadron inclusive cross section. A complete proof of this statement would require however the inclusion of soft particle exchanges, which are not accounted for in Jet Calculus approach.

Evolution equations can be extended also to polarized extDiFF, in particular to $H_1^{\sphericalangle\, i\to h_1 h_2}$ [23]. Such fragmentation function can be extracted from e^+e^- annihilation [30], evolved to the scale of semi-inclusive deep inelastic scattering measurements and allow the extraction of the transversity distribution function [24, 25, 26].

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APPENDIX: SPLITTING FUNCTIONS

The unpolarized leading-order splitting functions $P(u)$ read [35]

$$
\begin{aligned}
P_{qq}(u) &= C_F\left[2\left(\frac{1}{1-u}\right)_+ + \frac{3}{2}\,\delta(1-u) - 1 - u\right], && \text{(A.1)}\\
P_{qg}(u) &= T_R[1-2u+2u^2], && \text{(A.2)}\\
P_{gq}(u) &= C_F\left[\frac{2}{u} - 2 + u\right], && \text{(A.3)}\\
P_{gg}(u) &= 2C_A\left[\left(\frac{1}{1-u}\right)_+ + \frac{1}{u} + u(1-u) - 2\right] + \frac{11C_A - 2N_f}{6}\,\delta(1-u), && \text{(A.4)}
\end{aligned}
$$

where N_f is the number of flavors, $C_A = 3$, $C_F = 4/3$, $T_R = 1/2$; the " + " prescription is defined as usual by

$$
\int_0^1 dz\, f(z)[g(z)]_+ = \int_0^1 dz\, g(z)\,[f(z) - f(1)]\,. \qquad \text{(A.5)}
$$

The unpolarized leading-order splitting functions $\hat{P}$ are readily obtained from the previous ones by dropping virtual contributions at the endpoint. In our notations, they read:

$$
\begin{aligned}
\hat{P}^q_{gq}(u) &= C_F\left[\frac{2}{1-u} - 1 - u\right], && \text{(A.6)}\\
\hat{P}^g_{q\bar{q}}(u) &= T_R[1-2u+2u^2], && \text{(A.7)}\\
\hat{P}^q_{qg}(u) &= C_F\left[\frac{2}{u} - 2 + u\right], && \text{(A.8)}\\
\hat{P}^g_{gg}(u) &= 2C_A\left[\frac{1}{1-u} + \frac{1}{u} + u(1-u) - 2\right]. && \text{(A.9)}
\end{aligned}
$$

The transversely polarized leading-order splitting functions δP read [39, 40]

$$
\begin{aligned}
\delta P_{qq}(u) &= C_F\left[2\left(\frac{1}{1-u}\right)_+ + \frac{3}{2}\delta(1-u) - 2\right], && \text{(A.10)}\\
\delta P_{qg}(u) &= 0, && \text{(A.11)}\\
\delta P_{gq}(u) &= 0, && \text{(A.12)}\\
\delta P_{gg}(u) &= 2C_A\left[\left(\frac{1}{1-u}\right)_+ - 1\right] + \frac{11C_A - 2N_f}{6}\,\delta(1-u)\,. && \text{(A.13)}
\end{aligned}
$$

Due to angular momentum conservation, there is no mixing between quarks and gluons. The last splitting function can be used to evolve the DiFF for linearly polarized gluons, $\delta\hat{G}^{\triangleleft}$ [41].

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