

Leptogenesis: Theory & Neutrino Masses

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Abstract

After a brief discussion of baryon and lepton number nonconservation, we review the status of thermal leptogenesis with GUT scale neutrino masses, as well as low scale alternatives with keV neutrinos as dark matter and heavy neutrino masses within the reach of the LHC. Recent progress towards a full quantum mechanical description of leptogenesis is described with resonant leptogenesis as an application. Finally, cosmological $B-L$ breaking after inflation is considered as origin of the hot early universe, generating entropy, baryon asymmetry and dark matter.

Keywords: Leptogenesis, nonequilibrium processes, dark matter

1. Baryon & lepton number nonconservation

¹The basis of leptogenesis [1] are the ‘sphaleron processes’, effective nonperturbative interactions O_{B+L} of all left-handed quarks and leptons in the Standard Model [2] (cf. Fig. 1), which change baryon number (B) and lepton number (L) by a multiple of three, while preserving $B-L$,

$$O_{B+L} = \prod_i (q_{Li} q_{Li} q_{Li} l_{Li}), \quad (1)$$

$$\Delta B = \Delta L = 3N_{CS}. \quad (2)$$

Here N_{CS} , the Chern-Simons number, is an integer characterizing the sphaleron gauge field configuration. At high temperatures, between the critical temperature T_{EW} of the electroweak phase transition and a maximal temperature T_{SPH} ,

$$T_{EW} \sim 100 \text{ GeV} < T < T_{SPH} \sim 10^{12} \text{ GeV}, \quad (3)$$

these processes are believed to be in thermal equilibrium [3]. Although uncontroversial among theorists, it has to be stressed that this important phenomenon has so far not been experimentally tested! It is therefore very

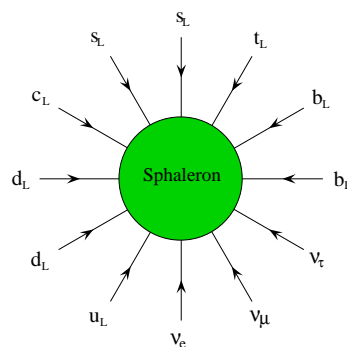


Figure 1: One of the 12-fermion processes which are in thermal equilibrium in the high-temperature phase of the Standard Model.

interesting that the corresponding phenomenon of chirality changing processes in strong interactions might be observable in heavy ion collisions at the LHC [4, 5].

Sphaleron processes relate baryon and lepton number and therefore strongly affect the generation of the cosmological baryon asymmetry. Analyzing the chemical potentials of quarks and leptons in thermal equilibrium, one obtains an important relation between the asymme-

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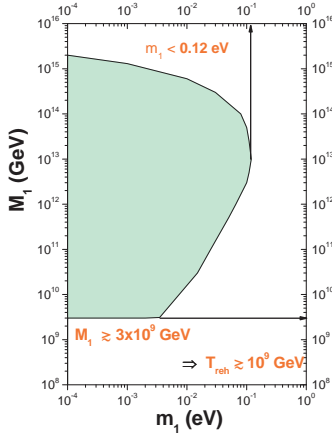


Figure 2: Lower bounds on the smallest heavy neutrino mass M_1 and upper bounds on the smallest light neutrino mass m_1 . From Ref. [12].

tries in B -, L - and B - L -number,

$$\langle B \rangle_T = c_S \langle B - L \rangle_T = \frac{c_S}{c_S - 1} \langle L \rangle_T, \quad (4)$$

where $c_S = \mathcal{O}(1)$. In the Standard Model one has $c_S = 28/79$.

This relation suggests that lepton number violation can explain the cosmological baryon asymmetry. However, lepton number violation can only be weak at late times, since otherwise any baryon asymmetry would be washed out. The interplay of these conflicting conditions leads to important constraints on neutrino properties, and on extensions of the Standard Model in general. Because of the sphaleron processes, lepton number violation can replace baryon number violation in Sakharov's conditions for baryogenesis.

2. Thermal leptogenesis

Leptogenesis is an immediate consequence of the seesaw mechanism, which explains the smallness of light neutrino masses in terms of the largeness of heavy Majorana neutrino masses. The heavy mass eigenstates N and the light mass eigenstates ν are given by

$$N \simeq \nu_R + \nu_R^c : \quad m_N \simeq M, \quad (5)$$

$$\nu \simeq \nu_L + \nu_L^c : \quad m_\nu = -m_D \frac{1}{M} m_D^T, \quad (6)$$

where m_D is the Dirac neutrino mass matrix. For third generation Yukawa couplings $\mathcal{O}(1)$, as in some $\text{SO}(10)$

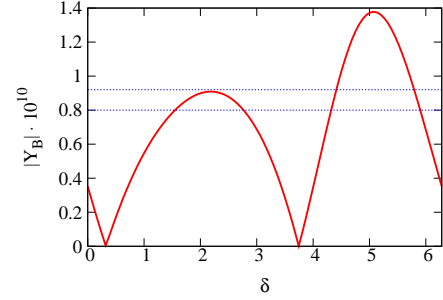


Figure 3: Dependence of the baryon asymmetry $|Y_B|$ on the PMNS phase δ for a particular neutrino mass model with normal hierarchy. From Ref. [17].

GUT models, one obtains the heavy and light neutrino masses,

$$M_3 \sim \Lambda_{\text{GUT}} \sim 10^{15} \text{ GeV}, \quad m_3 \sim \frac{v^2}{M_3} \sim 0.01 \text{ eV}. \quad (7)$$

Remarkably, the light neutrino mass m_3 is compatible with $(\Delta m_{\text{atm}}^2)^{1/2} \equiv m_{\text{atm}} \simeq 0.05 \text{ eV}$, as measured in atmospheric ν -oscillations. This suggests that neutrino physics probes the mass scale of grand unification (GUT), although other interpretations of neutrino masses are possible as well. The heavy Majorana neutrinos have no gauge interactions. Hence, in the early universe, they can easily be out of thermal equilibrium. This makes N_1 , the lightest of them, an ideal candidate for baryogenesis, in accord with Sakharov's condition of departure from thermal equilibrium. In the simplest form of leptogenesis the heavy Majorana neutrinos are produced by thermal processes, which is therefore called 'thermal leptogenesis'. The CP violating N_1 decays into lepton-Higgs pairs lead to a lepton asymmetry $\langle L \rangle_T \neq 0$, which is partially converted to a baryon asymmetry $\langle B \rangle_T \neq 0$ by the sphaleron processes. In early work on leptogenesis, it was anticipated that the light neutrino masses are then required to have masses $m_i < \mathcal{O}(1 \text{ eV})$ [6]. After the discovery of atmospheric neutrino oscillations, more stringent upper bounds on neutrino masses could be derived, and leptogenesis became increasingly popular.

The generated baryon asymmetry is proportional to the CP asymmetry in N_1 -decays. For hierarchical heavy neutrinos it satisfies the upper bound [7, 8]

$$\epsilon_1 = \frac{\Gamma(N_1 \rightarrow l\phi) - \Gamma(N_1 \rightarrow \bar{l}\bar{\phi})}{\Gamma(N_1 \rightarrow l\phi) + \Gamma(N_1 \rightarrow \bar{l}\bar{\phi})} \lesssim 10^{-6} \frac{M_1}{10^{10} \text{ GeV}} \frac{m_{\text{atm}}}{m_1 + m_3} = \epsilon_1^{\text{max}}, \quad (8)$$

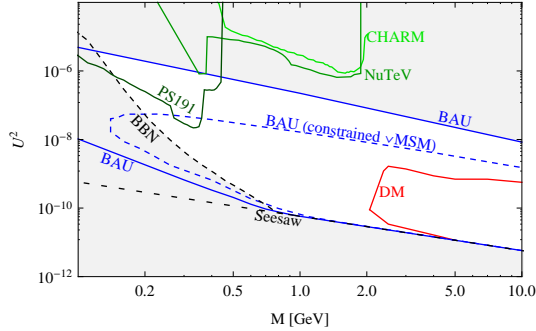


Figure 4: Experimental and cosmological constraints on mixing $U^2 = \text{tr}(\theta^\dagger \theta)$ and $N_{2,3}$ masses $M_{2,3} \approx M$. From Ref. [22].

which depends on the mass of N_1 and on m_{atm} , the mass splitting in atmospheric neutrino oscillations. The nonequilibrium process of baryogenesis via leptogenesis in the hot early universe is usually described by a set of Boltzmann equations, where also important washout processes have to be taken into account. Solving these equations, one obtains lower bounds on the heavy neutrino mass M_1 , and upper bounds on the light neutrino mass m_1 [9, 10]. In the simplest case of hierarchical heavy neutrinos, and summing over lepton flavours, one obtains a mass window of light neutrino masses, which is favoured by leptogenesis [9]

$$10^{-3} \text{ eV} \lesssim m_i \lesssim 0.1 \text{ eV}. \quad (9)$$

Note, however, that both, lower and upper bound, are modified by lepton flavour effects, which have been extensively studied in recent years (for reviews see, for example, Refs. [11, 12]). In view of Eq. (9), knowledge of the absolute neutrino mass scale is of crucial importance. Hence, a measurement of the neutrino masses m_β in tritium β -decay [13] and $m_{0\nu\beta\beta}$ in neutrinoless double β -decay [14], or the determination of the sum $\sum_i m_i$ from cosmology [15], consistent with Eq. (9), would strongly support the leptogenesis mechanism.

How does leptogenesis depend on the phases of the Dirac and Majorana neutrino mass matrices? Due to the large value of θ_{13} reported at this conference [16], measurement of the PMNS phase δ now appears feasible. This is certainly important, and in some models the generated baryon asymmetry strongly depends on the phase δ (cf. Fig. 3). In general, however, this is model dependent [18], and in the case of hierarchical heavy neutrinos the effect of the PMNS phase δ is unimportant [19, 20].

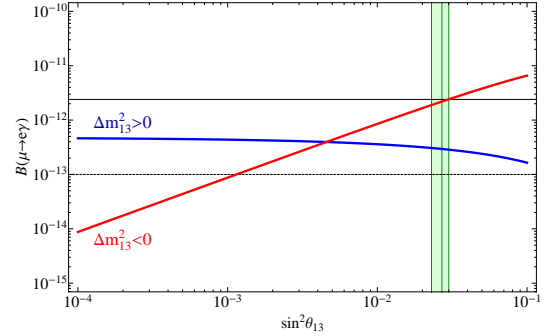


Figure 5: Branching ratio $B(\mu \rightarrow e\gamma)$ as function of $\sin^2 \theta_{13}$ in a scenario of resonant leptogenesis with neutrino masses $M_N = 120 \text{ GeV}$. From Ref. [31].

3. Low Scale Alternatives

3.1. 'Light' Majorana Neutrinos

So far we have considered right-handed neutrinos with GUT scale Majorana masses. In principle, the three right-handed (sterile) neutrinos could have much smaller masses, of order GeV or even keV. It is remarkable that in this case the Standard Model with three sterile neutrinos (νMSSM scenario) can account for neutrino oscillations, baryogenesis and dark matter [21]. The Standard Model Lagrangian is extended by Dirac and Majorana mass terms,

$$\mathcal{L}_{\nu\text{MSSM}} = \mathcal{L}_{\text{SM}} - \bar{L}_L F \nu_R \tilde{\Phi} - \bar{\nu}_R F^\dagger L_L \tilde{\Phi}^\dagger - \frac{1}{2} (\bar{\nu}_R^c M_M \nu_R + \bar{\nu}_R M_M^\dagger \nu_R^c), \quad (10)$$

and the active-sterile mixings are described by the matrix $\theta = m_D M_M^{-1}$ ($U^2 = \text{tr}(\theta^\dagger \theta)$). The scenario has recently been studied in detail quantitatively [22]. The lightest sterile neutrino N_1 provides dark matter, with a mass in the range $1 \text{ keV} < M_1 \lesssim 50 \text{ keV}$, and tiny mixings, $10^{-13} \lesssim \sin^2(2\theta_{\alpha 1}) \lesssim 10^{-7}$, constrained by X-ray observations. Following Ref. [23], baryogenesis is achieved by CP-violating oscillations of N_2 and N_3 . To obtain the right amount of baryon asymmetry and dark matter, resonant enhancement of CP violation is needed, with a high degeneracy of the sterile neutrinos, $|M_2 - M_3|/|M_2 + M_3| \sim 10^{-11}$. The observed dark matter abundance Ω_{DM} requires $N_{2,3}$ masses in the range from 2-10 GeV (cf. Fig. 4).

Sterile neutrino (N_1) dark matter with mass in the keV range can also be realized in models with left-right symmetric electroweak interactions based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)$. The baryon asymmetry is then generated by N_2 decays, which requires

$N_{2,3}$ masses of order $10^4 - 10^{10}$ GeV [24]. A further ‘low scale alternative’ is leptogenesis at the electroweak scale, with sterile neutrino masses $m_N \sim 100$ GeV and additional scalar fields, which leads to specific predictions testable at the LHC [25]. More models of leptogenesis at the TeV scale can be found in Ref. [11].

3.2. Resonant Leptogenesis

As already discussed in the previous section, the see-saw mechanism does not only work for right-handed neutrino masses at the GUT scale and Yukawa couplings similar to quark and charged lepton Yukawa couplings, it is also applicable for heavy neutrino masses at the TeV scale and very small Yukawa couplings. In this case heavy neutrino self-energy effects have to dominate the CP asymmetry [26, 27], leading to resonant leptogenesis in the case of quasi-degenerate right-handed neutrinos [28, 29]. In a particular neutrino mass model successful leptogenesis is achieved for masses at the electroweak scale, $M_N = 120$ GeV, with a degeneracy $\Delta M_N/M_N \lesssim 10^{-7}$ [31]. It is well known that in supersymmetric models there is a close connection between leptogenesis and lepton flavour changing processes like $\mu \rightarrow e\gamma$ [30]. It is interesting that in the case of resonant leptogenesis, one can have large lepton flavour changing rates also without supersymmetry (cf. Fig. 5), within the reach of the MEG experiment [32].

4. Nonequilibrium theory

Leptogenesis is a nonequilibrium process taking place in an expanding universe with decreasing temperature. It involves quantum interferences in a crucial manner, which implies that the standard treatment by means of Boltzmann equations is theoretically unsatisfactory [35]. In particular, it is currently not possible to quote a theoretical error on the predicted bayon asymmetry. Within quantum field theory, leptogenesis can be treated on the basis of the Schwinger-Keldysh formalism [33, 34], and during the past years significant progress has been made towards a ‘theory of leptogenesis’ [36, 37, 38, 39, 40]. Very important in this context is also the calculation of quantum corrections to decay



Figure 6: Path in the complex time plane for nonequilibrium Green’s functions.

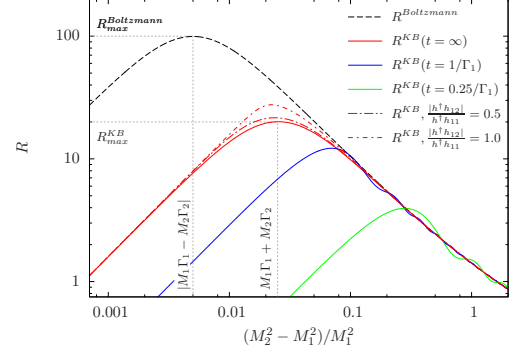


Figure 7: Resonant enhancement R as function of the mass-squared splitting $M_2^2 - M_1^2$ for the Boltzmann result (dashed) and the Kadanoff-Baym result (solid lines), for three fixed times $t = 0.25/\Gamma_1, 1/\Gamma_1, \infty$, and a particular choice of Yukawa couplings. From Ref. [45].

widths and scattering cross sections at high temperature [41, 42, 43]. In the Schwinger-Keldysh formalism one considers Green’s functions Δ for heavy neutrino N_1 , lepton and Higgs on a complex time contour starting at some initial time t_i (cf. Fig. 6). These Green’s functions satisfy Schwinger-Dyson equations with self-energies Π_C ,

$$(\square_1 + m^2)\Delta_C(x_1, x_2) + \int_C d^4x' \Pi_C(x_1, x')\Delta_C(x', x_2) = -i\delta_C(x_1 - x_2). \quad (11)$$

It is then convenient to consider two particular correlation functions, the spectral functions Δ^- , which contain information about the system, and the statistical propagators Δ^+ , which depends on the initial state at time t_i ,

$$\Delta^+(x_1, x_2) = \frac{1}{2}\langle\{\Phi(x_1), \Phi(x_2)\}\rangle, \quad (12)$$

$$\Delta^-(x_1, x_2) = i\langle[\Phi(x_1), \Phi(x_2)]\rangle. \quad (13)$$

They satisfy the Kadanoff-Baym equations [44]

$$\square_{1,\mathbf{q}}\Delta_{\mathbf{q}}^-(t_1, t_2) = - \int_{t_2}^{t_1} dt' \Pi_{\mathbf{q}}^-(t_1, t')\Delta_{\mathbf{q}}^-(t', t_2), \quad (14)$$

$$\square_{1,\mathbf{q}}\Delta_{\mathbf{q}}^+(t_1, t_2) = - \int_{t_i}^{t_1} dt' \Pi_{\mathbf{q}}^-(t_1, t')\Delta_{\mathbf{q}}^+(t', t_2) \quad (15)$$

$$+ \int_{t_i}^{t_2} dt' \Pi_{\mathbf{q}}^+(t_1, t')\Delta_{\mathbf{q}}^-(t', t_2), \quad (16)$$

where we have assumed spatial homogeneity, and $\square_{1,\mathbf{q}} = (\partial_{t_1}^2 + m^2 + \mathbf{q}^2)$ is the d’Alembert operator for a particular momentum mode \mathbf{q} . Solving these Kadanoff-Baym equations, one can describe the change of the

system from an initial state of zero baryon number to a final state of non-zero baryon number, i.e. the process of baryogenesis. Recently, this technique has been applied to resonant leptogenesis [45]. The obtained effective enhancement of the CP asymmetry is shown in Fig. 7. The maximal enhancement predicted by Boltzmann equations reads

$$R_{max}^{Boltzmann} = \frac{M_1 M_2}{2|M_1 \Gamma_1 - M_2 \Gamma_2|}. \quad (17)$$

Note that for equal masses and widths of the two heavy neutrinos N_1 and N_2 , R is singular, and therefore unphysical. This singularity is cured by memory effects contained in the Kadanoff-Baym equations, which yield the result

$$R_{max}^{KB} = \frac{M_1 M_2}{2(M_1 \Gamma_1 + M_2 \Gamma_2)}. \quad (18)$$

In summary, the generic effect of a possible resonant enhancement of the CP asymmetry is confirmed by the full quantum mechanical treatment. However, its size is reduced.

5. Cosmological $B-L$ Breaking

Thermal leptogenesis requires a rather large reheating temperature, $T_L \sim 10^{10}$ GeV. In supersymmetric theories this causes a potential problem because of the thermal production of gravitinos, which yields the abundance

$$\Omega_{\tilde{G}} h^2 = C \left(\frac{T_{RH}}{10^{10} \text{ GeV}} \right) \left(\frac{100 \text{ GeV}}{m_{\tilde{G}}} \right) \left(\frac{m_{\tilde{g}}}{1 \text{ TeV}} \right)^2, \quad (19)$$

where $C \sim 0.5$, and T_{RH} is the reheating temperature. For unstable gravitinos, one has to worry about consistency with primordial nucleosynthesis (BBN) whereas stable gravitinos may overclose the universe. As a possible way out, nonthermal production of heavy neutrinos has been suggested [46, 47, 48, 49], which allows to decrease the reheating temperature and therefore the gravitino production. On the other hand, it is remarkable that for typical gravitino and gluino masses in gravity mediated supersymmetry breaking, a reheating temperature $T_{RH} \sim 10^{10}$ GeV yields the right order of magnitude for the dark matter abundance if the gravitino is the LSP. But why should the reheating temperature be as large as the temperature favoured by leptogenesis, i.e. $T_{RH} \sim T_L$?

In this context it is interesting to note that for typical neutrino mass parameters in leptogenesis, $\tilde{m}_1 \sim$

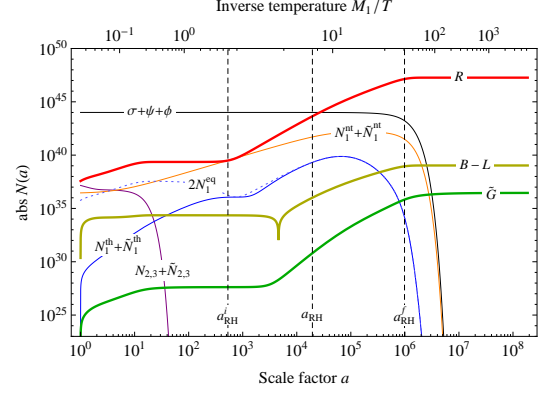


Figure 8: Comoving number densities for particles of the symmetry breaking sector (Higgs σ + higgsinos ψ + inflatons ϕ), (non)thermally produced (s)neutrinos of the first generation ($N_1^{\text{th}} + \tilde{N}_1^{\text{th}}$, $N_1^{\text{nt}} + \tilde{N}_1^{\text{nt}}$), (s)neutrinos of the second and third generation ($N_{2,3} + \tilde{N}_{2,3}$), MSSM radiation (R), lepton asymmetry ($B-L$) and gravitinos (\tilde{G}) as functions of the cosmic scale factor a . From Ref. [51].

0.01 eV, $M_1 \sim 10^{10}$ GeV, the heavy neutrino decay width takes the value

$$\Gamma_{N_1}^0 = \frac{\tilde{m}_1}{8\pi} \left(\frac{M_1}{v_{EW}} \right)^2 \sim 10^3 \text{ GeV}. \quad (20)$$

If the early universe in its evolution would reach a state where the energy density is dominated by nonrelativistic heavy neutrinos, their decays to lepton-Higgs pairs would lead to a relativistic plasma with temperature

$$T_{RH} \sim 0.2 \cdot \sqrt{\Gamma_{N_1}^0} M_P \sim 10^{10} \text{ GeV}, \quad (21)$$

which is indeed the temperature wanted for gravitino dark matter! Is this an intriguing hint or just a misleading coincidence?

It is remarkable that an intermediate heavy neutrino dominance indeed occurs in the course of the cosmological evolution if the initial inflationary phase is driven by the false vacuum energy of unbroken $B-L$ symmetry [50, 51]. Consider the supersymmetric standard model with right-handed neutrinos, described by the superpotential (in $SU(5)$ notation: $\mathbf{10} = (q, u^c, e^c)$, $\mathbf{5} = (d^c, l)$),

$$W_M = h_{ij}^u \mathbf{10}_i \mathbf{10}_j H_u + h_{ij}^d \mathbf{5}_i^* \mathbf{10}_j H_d + h_{ij}^y \mathbf{5}_i^* n_j^c H_u + h_i^n n_i^c S_1, \quad (22)$$

supplemented by a term which enforces $B-L$ breaking,

$$W_{B-L} = \frac{\sqrt{\lambda}}{2} \Phi (v_{B-L}^2 - 2S_1 S_2). \quad (23)$$

The Higgs fields $H_{u,d}$ and $S_{1,2}$ break electroweak symmetry and $B-L$ symmetry, respectively. It is very in-

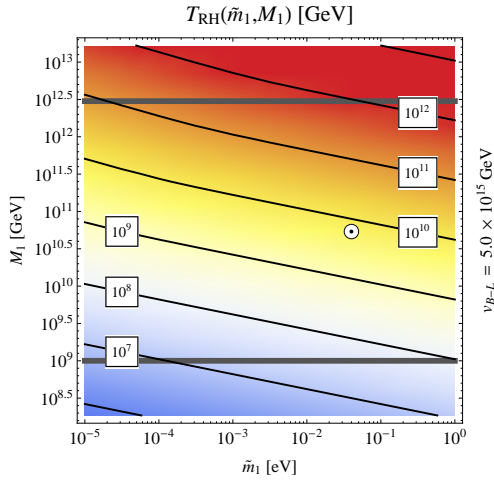


Figure 9: Contour plot of the reheating temperature T_{RH} as function of the effective light neutrino mass \tilde{m}_1 and the heavy neutrino mass M_1 . The thick gray horizontal lines represent lower and upper bound on M_1 , respectively, which arise from requiring consistency with hybrid inflation and production of cosmic strings during the $B-L$ phase transition. The small white circle marks the position of the parameter point used for Fig. 8. From Ref. [51].

interesting that the last term, W_{B-L} , can successfully describe inflation with Φ as inflaton field [52, 53]. Inflation ends in tachyonic preheating where $B-L$ is spontaneously broken. The false vacuum energy density is then rapidly converted into a nonrelativistic gas of $B-L$ Higgs bosons and a small admixture of a relativistic plasma of Standard Model particles produced during preheating. Heavy neutrinos N_1 are thermally produced from the plasma and nonthermally in decays of the $B-L$ Higgs bosons (σ). This generates entropy, baryon asymmetry and gravitino abundance.

The result of a quantitative analysis is shown in Fig. 8 for typical parameters: $M_1 = 5.4 \times 10^{10}$ GeV, $\tilde{m}_1 = 4.0 \times 10^{-2}$ eV, $m_{\tilde{G}} = 100$ GeV and $m_{\tilde{g}} = 1$ TeV. The final baryon asymmetry and dark matter abundance are $\eta_B \lesssim 4 \times 10^{-9}$, $\Omega_{\tilde{G}} h^2 \simeq 0.11$. A systematic parameter scan yields a lower bound on the gravitino mass, $10 \text{ GeV} \lesssim m_{\tilde{G}}$, and a range of heavy neutrino masses, $2 \times 10^{10} \text{ GeV} \lesssim M_1 \lesssim 2 \times 10^{11} \text{ GeV}$. Note that the described mechanism for the generation of dark matter also works for very heavy gravitinos, whose decays before BBN produce nonthermal higgsino or wino dark matter [54].

As Fig. 8 shows, most radiation, $B-L$ asymmetry and gravitino abundance are generated during a reheating period where the cosmic scale factor increases from $a_{RH}^i \sim 10^3$ to $a_{RH}^f \sim 10^6$, and the equation of state changes from matter dominance to radiation dom-

inance. On the other hand, the ‘reheating temperature’ T_{RH} is roughly constant, since there is a balance between temperature decrease due to expansion and temperature increase due to $B-L$ Higgs boson decays. Note that contrary to conventional reheating mechanisms, the reheating temperature T_{RH} is now determined by neutrino parameters! For neutrino masses consistent with leptogenesis and dark matter, the reheating temperature varies between 10^7 GeV and 10^{12} GeV (cf. Fig. 9).

6. Summary and Outlook

- Standard thermal leptogenesis is an elegant explanation of the cosmological matter-antimatter asymmetry. It is consistent with GUT scale heavy neutrino masses, and it (successfully) predicts bounds on light neutrino masses and, in a model dependent way, also restricts CP phases.
- There exist various viable low-scale alternatives, with keV sterile neutrinos as dark matter or right-handed neutrinos with $\mathcal{O}(100 \text{ GeV})$ masses. One then assumes highly degenerate masses to obtain a resonance enhancement of CP violation in heavy neutrino decays, which allows successful leptogenesis. Contrary to GUT scale leptogenesis, these models can be directly tested in astrophysical observations and at the LHC.
- Recently, considerable progress has been made in treating the nonequilibrium process of leptogenesis within quantum field theory, based on the Schwinger-Keldysh formalism, with a first application to resonant leptogenesis. Furthermore, important quantum corrections to decay rates and scattering processes have been evaluated at finite temperature. One can expect that these efforts will eventually lead to a quantitative ‘theory of leptogenesis’.
- The possible connection between leptogenesis, dark matter and inflation is a fascinating possibility. It is intriguing that in a supersymmetric extension of the Standard Model with right-handed neutrinos, a false vacuum with unbroken $B-L$ symmetry indeed provides an initial state whose decay, via an intermediate state of heavy neutrino dominance, can explain the observed entropy, matter-antimatter asymmetry and dark matter abundance.

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