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Increasing the effective number of neutrinos with decaying particles

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Abstract

We present models of decaying particles for increasing the effective number of neutrinos N_ν after big bang nucleosynthesis but before the structure formation begins. We point out that our scenario not only solves the discrepancy between the constraints on N_ν from these two epochs, but also provides a possible answer to deeper inconsistency in the estimation of the matter power spectrum amplitude at small scales, represented by σ_8, between the WMAP and some small scale matter power measurements such as the Lyman-α forest and weak lensing. We consider (a) saxion decay into two axions; (b) gravitino decay into axino and axion; (c) Dirac right-handed sneutrino decay into gravitino and right-handed neutrino.

I. INTRODUCTION

Observations of the cosmic microwave background (CMB), galaxy clustering, weak gravitational lensing and Lyman-α forest and so on strongly support the cosmological structures being formed in a universe described by the power-law ΛCDM model. Specifically, recent advancement in those observations enabled us to measure the matter power spectrum from the horizon scale down to about 1 Mpc in a very precise manner.

However, as the data accumulate owing to the recent observations for instance by the WMAP [1, 2, 3, 4] and SDSS [5, 6, 7, 8], possible tensions among different data sets are indicated. This is most easily seen in the term σ_8, the normalization of the matter power spectrum at $8h^{-1}$ Mpc, where h is the Hubble parameter. Namely, the value of σ_8 derived from the WMAP three-year data is slightly lower than that derived from the latest analyses of the Lyman-α forest [9, 10], weak lensing [11, 12] and strong lensing ("giant arc") statistics [13]. The discrepancy surfaced when the WMAP data were updated from the first year data to the three-year data, with significant decrease in the best fit value of σ_8 (decreased from 0.92 ± 0.10 to $0.761^{+0.049}_{-0.048}$) [1, 14].

It is true that all these measurements which favor higher σ_8 than the WMAP3 value are likely to suffer more from systematic errors than the WMAP experiments, but when the ongoing efforts can succeed in decreasing the systematics, they would be more suitable for measuring σ_8 than the CMB experiments. Therefore, we can expect that we will obtain sufficient information to know whether the tension is solved by some systematics not yet accounted for or we have to invoke non-standard cosmology. We, in this paper, assume that the latter case is true and the present discrepancy between the WMAP3 and the observations at smaller scales is real.

Then, what kind of non-standard ingredient do we need ? Actually, this has been already hinted at in the Lyman-α forest analysis of Ref. [10]. In Ref. [10], extensive cosmological parameter estimation was conducted using the latest data set consists of the CMB, galaxy clustering and the Lyman-α forest. They tested a wide range of cosmological models other than the flat ΛCDM model with the adiabatic power-law primordial power spectrum by placing constraints on the tensor mode, the running of the spectral index, massive neutrinos, the effective number of neutrinos, the dark energy equation of state, the curvature of the universe, cosmic strings and isocurvature modes. They have found that the observations prefer these parameters to be consistent with the standard values except for one parameter: the effective number of neutrinos, N_ν. Remarkably, their 2σ limit is $N_\nu = 5.3^{+2.1}_{-1.7}$, not allowing the standard value of $N_\nu = 3.0$ at 2.4σ [1].

This preference of a non-standard value of N_ν by the combined data of the WMAP3 and Lyman-α forest is closely connected to the discrepancy of σ_8 between these data sets. As shown in Ref. [10], the larger value of N_ν enhances the best fit value of the small scale amplitude [2]. This enables the high σ_8 value inferred from the Lyman-α to reconcile with the WMAP3 data, which prefers the low σ_8 when the standard $N_\nu = 3$ is assumed. Although there is no detailed statistical analysis of combined data sets of the WMAP3 and the weak lensing allowing for the possibility of non-standard N_ν, it is reasonable to expect these observations too to be reconciled with the WMAP3 data by $N_\nu > 3$.

On the other hand, as is very well known, the value of N_ν greatly affects big bang nucleosynthesis (BBN), especially the ^{4}He abundance, Y_p. The analysis by Ref. [15], using $Y_p = 0.249 \pm 0.009$ [17, 18], has yielded $N_\nu = 3.1^{+1.4}_{-1.2}$ (95% C.L.), in good agreement with the standard value while still allowing some room for non-standard values. For example, $N_\nu = 4$, which can better fit the combined data of the WMAP3 and Lyman-α forest than $N_\nu = 3$, is acceptable. However, more recent analyses favor $N_\nu = 3$ (see Sec. II for more detailed discussion).

Having seen recent observational constraints on N_ν from the structure formation and nucleosynthesis, we will now consider how the value of N_ν should be in order to satisfy these constraints. Although the simplest choice would be to have $N_\nu \sim 4$ before and after BBN, concerning the central values, it may be preferable to have $N_\nu = 3$ during BBN and increase to $N_\nu = 4 - 5$ well before the structure formation begins. We here note that BBN measures N_ν around the temperature $T = O(\mathrm{MeV})$ while the structure formation data tell us N_ν in a more recent universe, $T \lesssim 100\,\mathrm{eV}$, at which the structure formation of the smallest observable scale (about 1 Mpc) begins [3]. In terms of the cosmological time, they respectively

[1] Ref. [15] recently reexamined this issue using almost the same data set and found $N_\nu = 4.6^{+1.6}_{-1.5}$ at 95% C.L. Although the significance is lower than the one in Ref. [10], tension with the standard value remains. Also, there is an independent analysis by Ref. [16] with a similar data set including Lyman-α which gives $N_\nu = 5 \pm 1$, the 2σ preference for $N_\nu > 3$.

[2] In detail; they report this result in the amplitude at a smaller scale than $8h^{-1}$ Mpc, but similar correlation is expected between N_ν and σ_8.

[3] The present CMB data probe scales larger than $O(10)$ Mpc or equivalently $T \lesssim 10\,\mathrm{eV}$. Meanwhile the

measure N_ν around 1 sec and after 10^8 sec. Thus, the constraints on N_ν from BBN and the structure formation (the CMB, the Lyman-α forest etc.) do not necessarily coincide at face value in general.

In this paper, to realize the latter possibility of increasing N_ν, we investigate models of particles which decay into radiation between BBN and the structure formation. Candidates would have a somewhat long lifetime of 1 to 10^8 sec after which they decay "silently", without destroying the light elements, into very light particles as copious as photons or neutrinos. We show such particles are found in supersymmetric extensions of theories proposed to solve the strong CP problem. Namely, we consider the following possibilities: (a) saxion decay into axions and (b) gravitino decay into axino and axion. We also show a candidate present in models with the right-handed neutrino which are attractive for explaining neutrino masses. In this case, we consider (c) Dirac right-handed sneutrino decay into gravitino and right-handed neutrino. In the next section we will give a review on the present observational status on σ_8 and N_ν and their possible tensions between different experiments. In Sec. III we give details of the models and parameter space where N_ν is successfully increased while meeting cosmological constraints. Then, Sec. IV is devoted to our conclusions and discussion.

II. OBSERVATIONAL TENSIONS IN σ_8 AND N_ν

In this section we give a brief review on the several different observations and analyses of σ_8 and N_ν and their implications. The recent analyses of Lyman-α forest combined with the WMAP three-year data by two independent groups are in Refs. [9] and [10]. The earlier studies by the same groups with the WMAP first year data are in Refs. [20] and [21]. Those analyses used basically the same Lyman-α forest data sets. Their results seem to consistently show that σ_8 derived from the Lyman-α forest data prefers the higher value of the WMAP first year result rather than the three-year value, although two groups conclude that there are no statistically compelling evidence for inconsistency between WMAP3 and Lyman-α data. In Ref. [10], around two sigma discrepancy in the power-law ΛCDM model was reported but they concluded that the difference would be explained by a statistical fluctuation or unknown systematic errors. The analysis of Ref. [9] found weaker significance for the discrepancy and

CMB alone does not practically constrain N_ν (the WMAP three-year alone limit is $N_\nu < 42$ at 95% C.L. [19]).

concluded that the Lyman-α forest data can be in reasonable agreement with WMAP3. However, it is apparent from the figures of likelihood contours in Refs. [9, 10] that the measurements of the small scale fluctuation amplitude by the WMAP3 alone and the Lyman-α forest alone are not fully consistent.

Accumulating data for weak lensing, another efficient probe of σ_8, shows a similar tendency. It was first noted in the WMAP3 paper of Ref. [1] that the ground-based weak lensing measurement by the wide synoptic survey of the Canada-France-Hawaii Telescope [11] favors higher values of $\sigma_8 \approx 0.8 - 1.0$. In Ref. [1], the likelihood contours on the Ω_m-σ_8 plane are drawn for the WMAP3 alone and the weak lensing alone but their overlapping region is small showing some degree of inconsistency. Moreover, a higher σ_8 value is also preferred by a very recently released result of the space-based measurement by the COSMOS survey of the Hubble Space Telescope [12]. The agreement between the largest surveys from ground and space is remarkable and adds to the reliability of the weak lensing result of the high σ_8.

Regarding the fact that the numerous non-standard parameters other than N_ν studied in Ref. [10] cannot solve the discrepancy, we consider that the universe with $N_\nu > 3$ is a strong candidate for explaining both the WMAP3 and the observations which indicate high σ_8.

Now let us look at the constraints on N_ν from BBN. The value of N_ν greatly affects BBN, and in particular, the ^{4}He abundance Y_p is quite sensitive to it. The analysis by Ref. [15], using $Y_p = 0.249 \pm 0.009$ [17, 18], has yielded $N_\nu = 3.1^{+1.4}_{-1.2}$ (95% C.L.), in good agreement with the standard value while still allowing some room for non-standard values. However, there are more recent analyses of Y_p by several groups who give more stringent error bars: $Y_p = 0.250 \pm 0.004$ [22], $Y_p = 0.2427 \pm 0.0028$ [23] and $Y_p = 0.2516 \pm 0.0011$ [24]. We derive the constraints on N_ν from them using the fitting formula in Ref. [25] and the observed deuterium abundance D/H $= (2.82 \pm 0.27) \times 10^{-5}$ [26] on the η-N_ν plane. The 95% C.L. limits are respectively $N_\nu = 3.20^{+0.76}_{-0.68}$, $N_\nu = 3.01^{+0.52}_{-0.48}$ and $N_\nu = 3.32^{+0.23}_{-0.24}$ (their own analysis in Ref. [24] has yielded $N_\nu = 3.28 \pm 0.16$ (2σ), using ^{7}Li data in addition; this is consistent with our calculation). Although it is beyond the scope of this paper to discuss whether their error bars are underestimated or not, we may conclude that three recent analyses of ^{4}He do not favor $N_\nu > 4$.

Thus we are led to consider a cosmological scenario that the effective number of neutrinos N_ν is increased from the standard value to $N_\nu > 4$ during the time between BBN and the structure formation [4]. In the rest of the paper, we will focus on several possible scenarios based on particle physics and discuss each model in detail.

III. MODELS

In this section we provide several models in which the effective number of neutrinos is increased from the standard value, $N_\nu = 3$, after BBN but before the structure formation starts. To this end, we introduce a long-lived particle X with a lifetime τ_X in the range of $\tau_X = O(1\text{–}10^8)$ sec. The lower bound on τ_X comes from the requirement that the additional radiation energy from the decay of X should not change the expansion rate before the neutrino freeze-out. This is because we do not want to change the standard BBN results, especially the ^{4}He abundance. The upper bound corresponds to the cosmic time when the comoving scale of about 1 Mpc enters the horizon.

Before going to the discussion of each model, it will be useful to express the increase of N_ν in terms of the abundance and the lifetime of X. Let us assume that the decay of X produces very weakly interacting relativistic particles collectively denoted by R, which carry a fraction f_R of the energy originally stored in X. The R particles increase the extra effective number of neutrinos by ΔN_ν:

$$\Delta N_\nu \;\simeq\; 3\, f_R \left.\left(\frac{\rho_X}{\rho_\nu}\right)\right|_{T=T_d}, \tag{1}$$

where ρ_X and ρ_ν denote the energy densities of X and the three species of the neutrinos, respectively. We define T_d as the temperature of photons when the decay rate Γ_X becomes equal to $3H$ (H is the Hubble parameter):

$$\Gamma_X \;=\; 3H = \left(\frac{\pi^2 g_*}{10}\right)^{\frac{1}{2}} \frac{T_d^2}{M_P}, \tag{2}$$

where $g_* \simeq 3.36$ counts the relativistic degrees of freedom, and $M_P = 2.4 \times 10^{18}$ GeV is the reduced Planck mass. The lifetime τ_X is related to the decay rate as $\tau_X = 1/\Gamma_X$. To be precise, speaking, the energy density of the R particles also contributes to the right-hand side of Eq. (2). Nevertheless we neglect it here, because it is sub-dominant as long as $\Delta N_\nu \lesssim 1$.

[4] Strictly speaking, the Y_p analysis by Ref. [24] implies $N_\nu > 3$ at 2σ. However, since our models that we will present below are not affected by the value of N_ν at BBN, we assume $N_\nu = 3$ at BBN for simplicity.

The standard value of the neutrino abundance is

$$\left.\frac{\rho_\nu}{s}\right|_{T=T_d} \simeq 0.26\, T_d, \tag{3}$$

where s is the entropy density, and it should be noted here that T_d denotes the temperature of photons, not neutrinos. Substituting Eq. (3) into Eq. (1), we obtain

$$\Delta N_\nu \simeq 1.1\, f_R \left(\frac{T_d}{\mathrm{keV}}\right)^{-1} \left(\frac{\rho_X/s}{10^{-7}\,\mathrm{GeV}}\right), \tag{4}$$

or equivalently,

$$\Delta N_\nu \simeq 1.2\, f_R \left(\frac{\tau_X}{10^6\,\mathrm{sec}}\right)^{\frac{1}{2}} \left(\frac{\rho_X/s}{10^{-7}\,\mathrm{GeV}}\right). \tag{5}$$

It is clear from Eq. (5) that, to increase N_ν by order unity, X must be produced with a sufficiently large abundance and its lifetime should be long enough. In order not to disturb the light element abundances, the decay into the standard-model particles (especially into the hadrons) must be sub-dominant or even forbidden, and the decay products must be "dark", i.e., their interaction with the visible particles should be very weak. In other words, any massive particles that decay into very weakly interacting and relativistic particles can explain the increase of N_ν if and only if they have right abundance and lifetime given by Eq. (5).

In the following, we show three cosmologically consistent scenarios in which N_ν increases between BBN and structure formation by order unity. We consider (a) saxion decay into two axions; (b) gravitino decay into axino and axion; (c) Dirac right-handed sneutrino decay into gravitino and right-handed neutrino. We investigate each model in detail below.

A. Saxion decay into axions

One of the most promising solutions to the strong CP problem in quantum chromodynamics (QCD) is the Peccei-Quinn (PQ) mechanism [27], which involves a pseudo-Nambu-Goldstone boson a, the axion, associated with the spontaneous PQ symmetry breaking (for a review, see Ref. [28]). In a supersymmetric theory, the axion forms a supermultiplet, including a fermionic superpartner $\tilde{a}$, the axino, and a scalar partner s, the saxion. In general, the saxion acquires a mass of order $m_{3/2}$ in the presence of the supersymmetry (SUSY) breaking [29, 30]. (Here $m_{3/2}$ is the gravitino mass.) In a class of models, the saxion mainly decays into a pair of the axions (i.e., $f_R \simeq 1$), and these axions contribute to the extra effective number of neutrinos without disturbing the BBN results [5]. However, since the saxion may also decay into two photons, we need to examine whether the photons produced do not spoil the standard cosmology.

We consider a class of models in which the PQ symmetry is spontaneously broken by a single PQ scalar field Φ, whose vacuum expectation value (VEV) sets the scale of the PQ symmetry breaking scale $F_a = \langle \Phi \rangle$. Here we have assumed that the VEV of Φ is real and positive without a loss of generality. The PQ scale F_a is severely constrained from astrophysical and cosmological considerations as $10^{10}\,\mathrm{GeV} \lesssim F_a \lesssim \theta^{-1} 10^{12}\,\mathrm{GeV}$, where θ is an initial misalignment angle of the axion. It is firmly bounded from below by supernova cooling [31, 32], while the upper bound comes from the axion-overclosure limit, which can be relaxed to some extent depending on the cosmological scenarios [33, 34, 35, 36].

Let us express Φ in terms of the saxion s and the axion a as

$$\Phi \;=\; \frac{s}{\sqrt{2}} \exp\left[i \frac{a}{\langle s \rangle} \right]. \tag{6}$$

Expanding the saxion around its VEV as $s = \sqrt{2} F_a + \hat{s}$, we obtain

$$\Phi \;=\; \left(F_a + \frac{\hat{s}}{\sqrt{2}} \right) \exp\left[i \frac{a}{\sqrt{2} F_a} \right], \tag{7}$$

$$\partial_\mu \Phi^\dagger \partial^\mu \Phi \;=\; \frac{1}{2} \partial_\mu \hat{s} \partial^\mu \hat{s} + \frac{1}{2} \partial_\mu a \partial^\mu a + \frac{\hat{s}}{\sqrt{2} F_a} \partial_\mu a \partial^\mu a + \cdots , \tag{8}$$

where the third term induces the saxion decay into axions. The decay rate is given by

$$\Gamma(s \to 2a) \;\simeq\; \frac{1}{64\pi} \frac{m_s^3}{F_a^2}, \tag{9}$$

where m_s is the saxion mass. The lifetime of the saxion then is given as

$$\tau_s \simeq 1.3 \times 10^5\,\mathrm{sec} \left(\frac{m_s}{100\,\mathrm{MeV}} \right)^{-3} \left(\frac{F_a}{10^{12}\,\mathrm{GeV}} \right)^2. \tag{10}$$

Since the axion superfield Φ must not have a SUSY mass, the saxion is a flat direction and acquires only a SUSY breaking mass of the order of the gravitino mass. Therefore, in the early universe, the initial position of the saxion, $s_i \equiv \sqrt{2} |\Phi_i|$, naturally deviates from

[5] Note that it is model dependent whether the saxion dominantly decays into the axions. For instance, in a model where two PQ scalar fields Φ_+ and Φ_-, respectively charged under the PQ symmetry $+1$ and -1, acquire VEVs as $\langle \Phi_+ \rangle \simeq \langle \Phi_- \rangle \sim F_a$, the saxion decay into axions is suppressed.

that in the vacuum $\langle s \rangle = \sqrt{2} F_a$. When the Hubble parameter becomes comparable to the saxion mass m_s, the saxion starts to oscillate around the potential minimum with an initial amplitude, $\delta s_i \simeq |s_i - \sqrt{2} F_a|$. There is *a priori* no way to determine the initial displacement of the saxion, δs_i, but it is expected to be in the range between F_a and M_P.

The saxion abundance depends on the thermal history of the universe, e.g., whether the reheating is completed before or after the saxion starts to oscillate [37]. First let us assume that the saxion starts to oscillate after the reheating. This is the case if the reheating temperature T_R satisfies

$$T_R \gtrsim 2.2 \times 10^8 \, \mathrm{GeV} \left(\frac{m_s}{100 \, \mathrm{MeV}} \right)^{1/2}, \tag{11}$$

where we have used the relativistic degrees of freedom in MSSM, $g_* = 228.75$. The saxion-to-entropy ratio is then given by

$$\begin{aligned} \frac{\rho_s}{s} &= \frac{m_s^2 (\delta s_i)^2/2}{3 H_{\mathrm{osc}}^2 M_P^2} \frac{3 T_{\mathrm{osc}}}{4} \\ &\simeq 4.7 \times 10^{-6} \, \mathrm{GeV} \left(\frac{m_s}{100 \, \mathrm{MeV}} \right)^{1/2} \left(\frac{F_a}{10^{12} \, \mathrm{GeV}} \right)^2 \left(\frac{\delta s_i}{F_a} \right)^2, \end{aligned} \tag{12}$$

where the subscript "osc" denotes that the variables should be evaluated when the saxion starts to oscillate, i.e., $H \simeq m_s$. The saxion decays into axions, increasing the effective number of neutrinos ΔN_ν as

$$\Delta N_\nu \simeq 2.0 \times 10 \left(\frac{m_s}{100 \, \mathrm{MeV}} \right)^{-1} \left(\frac{F_a}{10^{12} \, \mathrm{GeV}} \right)^3 \left(\frac{\delta s_i}{F_a} \right)^2, \tag{13}$$

where we have substituted Eqs. (10) and (12) into Eq. (5), and used $f_R \simeq 1$. However, as we will see below, it is difficult to reconcile the constraint on T_R, Eq. (11), with the gravitino problem.

On the other hand, if the reheating is completed after the oscillation of the saxion commences, the saxion-to-entropy ratio is given by

$$\frac{\rho_s}{s} = \frac{m_s^2 (\delta s_i)^2/2}{3 m_s^2 M_P^2} \frac{3 T_R}{4} \tag{14}$$

$$\simeq 2.2 \times 10^{-8} \, \mathrm{GeV} \left(\frac{T_R}{10^6 \, \mathrm{GeV}} \right) \left(\frac{F_a}{10^{12} \, \mathrm{GeV}} \right)^2 \left(\frac{\delta s_i}{F_a} \right)^2. \tag{15}$$

The increase in the effective number of neutrinos is expressed as

$$\Delta N_\nu \simeq 3.0 \left(\frac{m_s}{10 \, \mathrm{MeV}} \right)^{-3/2} \left(\frac{F_a}{10^{12} \, \mathrm{GeV}} \right)^3 \left(\frac{\delta s_i}{F_a} \right)^2 \left(\frac{T_R}{10^6 \, \mathrm{GeV}} \right). \tag{16}$$

Thus it is possible to increase N_ν by order unity in this scenario.

Now let us consider the saxion decay into two photons. The decay occurs in the DFSZ axion model [38], as well as in the KSVZ (or hadronic) axion model [39] if the heavy quarks have $U(1)_{\rm em}$ charges. To be concrete, let us consider a hadronic axion model by introducing the coupling of Φ with the heavy quarks Q and $\bar{Q}$ as

$$W \;=\; k\Phi Q\bar{Q}, \tag{17}$$

where k is a coupling constant [6]. We assign the PQ charges as, e.g., $\Phi(+1)$, $Q(-1/2)$, and $\bar{Q}(-1/2)$. Assuming that Q and $\bar{Q}$ furnish $\mathbf{5}$ and $\bar{\mathbf{5}}$ representations of the $SU(5)$ GUT group, Φ couples to the standard-model gauge multiplets as

$$-\mathcal{L}_{\rm int} \;=\; \int d^2\theta \left(\frac{\alpha_i}{8\pi}\right)\frac{\Phi}{F_a} W_\alpha^{(i)} W^{(i)\alpha} + \text{h.c.}, \tag{18}$$

where $\alpha_i = g_i^2/4\pi$ are the gauge coupling constants of the standard model, and $W_\alpha^{(i)}$ are chiral superfields for the gauge multiplets. Thus the saxion decays into two photons with the rate,

$$\Gamma(s\to 2\gamma) \;\simeq\; \frac{\kappa^2\alpha_{\rm em}^2}{512\pi^3}\frac{m_s^3}{F_a^2}, \tag{19}$$

where $\kappa = (3/5)\cos^2\theta_W \simeq 0.5$ (θ_W is the weak mixing angle) and we can see that the branching ratio of two-photon decay is $B_\gamma \simeq 1.7\times 10^{-7}$. The injected photons may destroy the light elements and change the result of BBN for $m_s \gtrsim 40\,\text{MeV}$, while for $m_s \lesssim 40\,\text{MeV}$, the injected photons can never dissociate ^{4}He nuclei [40]. To avoid changing BBN, the following bounds must be satisfied [42, 43]:

$$B_\gamma\left(\frac{\rho_s}{s}\right) \;\lesssim\; \begin{cases} 10^{-14}\,\text{GeV} & \text{for } \; 10^7\,\text{sec} \lesssim \tau_s \lesssim 10^{12}\,\text{sec} \\ 10^{-6} - 10^{-14}\,\text{GeV} & \text{for } \; 10^4\,\text{sec} \lesssim \tau_s \lesssim 10^{7}\,\text{sec} \end{cases}, \tag{20}$$

and the constraints from BBN are very weak for $\tau_s < 10^4\,\text{sec}$. If the saxion mass exceeds about $1\,\text{GeV}$, the saxion decays into gluons with the rate

$$\Gamma(s\to 2g) \;\simeq\; \frac{\alpha_s^2}{64\pi^3}\frac{m_s^3}{F_a^2}, \tag{21}$$

[6] We assume that the PQ symmetry is broken due to the VEV of Φ during inflation. Then the PQ quarks Q and $\bar{Q}$ are not thermalized after inflation and they do not affect the timing when the saxion starts oscillating.

which is much larger than that of Eq. (19). The hadronic branching ratio is $B_h = \alpha_s^2/\pi^2 \simeq 1.4 \times 10^{-3}$. The bound on the saxion abundance in this case is

$$B_h \left(\frac{\rho_s}{s}\right) \lesssim \begin{cases} 10^{-13} - 10^{-14}\,\mathrm{GeV} & \text{for } \; 10^4\,\mathrm{sec} \lesssim \tau_s \lesssim 10^{12}\,\mathrm{sec} \\ 10^{-9} \; - \; 10^{-13}\,\mathrm{GeV} & \text{for } \; 1\,\mathrm{sec} \lesssim \tau_s \lesssim 10^4\,\mathrm{sec} \end{cases} . \tag{22}$$

Thus, if the saxion decays into gluons, the BBN constraints on ρ_s/s become much severer. In particular, for $m_s > 1\,\mathrm{GeV}$ the saxion decay cannot realize $\Delta N_\nu = 1$ due to these constraints. Even in the case $m_s \lesssim 40$ MeV, the energy injection is constrained from the CMB. If the injected photons cannot reach chemical or kinetic equilibrium due to the small rate of interactions with background plasma, it leads to the distortion of the CMB black body spectrum which is constrained from observations [41]. Hence, the constraint comes from the CMB in the region $m_s \lesssim 40\,\mathrm{MeV}$, although this does not give a severe constraint.

Let us here comment on the thermal production of the gravitino. From Eqs. (13) and (16), we can see that the light saxion mass and/or relatively high reheating temperature are required to obtain $\Delta N_\nu \sim 1$ as long as we stick to $\delta s_i \sim F_a$. Since the saxion mass is considered to be of the order of the gravitino mass, the gravitino mass as well must be as light as $O(100)\,\mathrm{MeV}$, and such a light gravitino is realized in gauge-mediated SUSY breaking models [44]. Let us assume that the gravitino is the lightest supersymmetric particle (LSP). If the reheating temperature is too high, the gravitino may overclose the universe [45]. The abundance of the thermally produced gravitino is [46] (see also [43])

$$Y_{3/2} \;\simeq\; 1.9 \times 10^{-16} \left(1 + \frac{m_{\tilde{g}}^2}{3 m_{3/2}^2}\right) \left(\frac{T_R}{10^6\,\mathrm{GeV}}\right), \tag{23}$$

where we have omitted the logarithmic dependence on T_R, and $m_{\tilde{g}}$ is the gluino mass evaluated at $T = T_R$. For $m_{3/2} \ll m_{\tilde{g}}$, the gravitino abundance is given as

$$\Omega_{3/2}^{\mathrm{TP}} h^2 \simeq 7.0 \times 10^{-3} \left(\frac{m_{3/2}}{100\,\mathrm{MeV}}\right)^{-1} \left(\frac{m_{\tilde{g}}}{200\,\mathrm{GeV}}\right)^2 \left(\frac{T_R}{10^6\,\mathrm{GeV}}\right). \tag{24}$$

This should be smaller than the present upper bound on the current dark matter density, $\Omega_{\mathrm{DM}} h^2 \lesssim 0.12$ at 95% C.L. [1]. Therefore, the thermal gravitino production sets the lower bound on the gravitino mass for a fixed reheating temperature. Due to this bound, there is no allowed region in the case that the reheating is completed before the start of saxion oscillations. It should be noted that this constraint cannot be alleviated even in the case of the axino LSP [47]. This is because, although the gravitino eventually decays into the axion and the axino, the decay is too late for such a light gravitino mass. In the next subsection, we consider a much heavier gravitino mass, focusing on the possibility that the axion and the axino produced from the decay of gravitino may explain $\Delta N_\nu \sim 1$.

The axinos, in addition to the gravitinos, are also produced by thermal scattering, and we should check whether the axino is overproduced. Here, since we assume that the axino mass is of the order of the gravitino mass, we do not have to care whether the axino is the LSP or not. The abundance of the thermally produced axinos is calculated as [48, 49],

$$Y_{\tilde{a}}^{\rm TP} \simeq 2.0 \times 10^{-8} \left(\frac{\alpha_s(T_R)^3 \ln[0.098/\alpha_s(T_R)]}{1.1 \times 10^{-4}} \right) \left(\frac{F_a}{10^{12}\,{\rm GeV}} \right)^{-2} \left(\frac{T_R}{10^6\,{\rm GeV}} \right). \tag{25}$$

or equivalently,

$$\Omega_{\tilde{a}}^{\rm TP} h^2 \simeq 5.5 \times 10^{-2} \left(\frac{\alpha_s(T_R)^3 \ln[0.098/\alpha_s(T_R)]}{1.1 \times 10^{-4}} \right) \left(\frac{m_{\tilde{a}}}{10\,{\rm MeV}} \right) \left(\frac{F_a}{10^{12}\,{\rm GeV}} \right)^{-2} \left(\frac{T_R}{10^6\,{\rm GeV}} \right) \tag{26}$$

where $m_{\tilde{a}}$ denotes the mass of the axino. Therefore, if the axino mass is too large and/or the reheating temperature is too high, the thermally produced axino may overclose the universe. Using Eqs. (16) and (26), we derive

$$\Delta N_\nu \simeq 1.2 \left(\frac{\Omega_{\tilde{a}}^{\rm TP} h^2}{0.1} \right)^{-3/2} \left(\frac{m_{\tilde{a}}}{m_s} \right)^{3/2} \left(\frac{T_R}{10^6\,{\rm GeV}} \right)^{5/2} \left(\frac{\delta s_i}{F_a} \right)^2. \tag{27}$$

From this equation, the reheating temperature should be less than 10^6 GeV as long as $\Delta N_\nu \lesssim 1$ and $\delta s_i \gtrsim F_a$ are assumed. Note also that the axino mass in the model (17) is smaller than $m_{3/2}$ unless the PQ scalar has non-minimal coupling with the SUSY breaking sector. So, the upper bound on T_R may be relaxed if $m_{\tilde{a}} \ll m_{3/2} \sim m_s$.

In Fig. 1, we summarize all the constraints discussed above. Here we have taken $\delta s_i = F_a$ and $T_R = 10^6$ GeV as reference values. The thick solid black line labeled (a) shows $\Delta N_\nu = 1$. Note that the region above this line corresponds to $\Delta N_\nu \gtrsim 1$. The dotted blue lines (b) denote the astrophysical and cosmological constraints on the PQ scale, and we have set $\theta = 0.1$. In order to satisfy $1\,{\rm sec} \lesssim \tau_s \lesssim 10^8\,{\rm sec}$, the combination of parameters (m_s, F_a) must lie in the region between two thin solid red lines (c). The constraints from BBN and CMB provide an upper bound on F_a for a fixed m_s, as represented by the dot-dashed green line (d). The thermally produced gravitinos exceed the current observed dark matter abundance if $m_s (\sim m_{3/2})$ is smaller than the value indicated by the vertical long-dashed yellow line (e). For $m_{\tilde{a}} = 0.01\, m_s$, the abundance of the thermally produced axinos exceeds

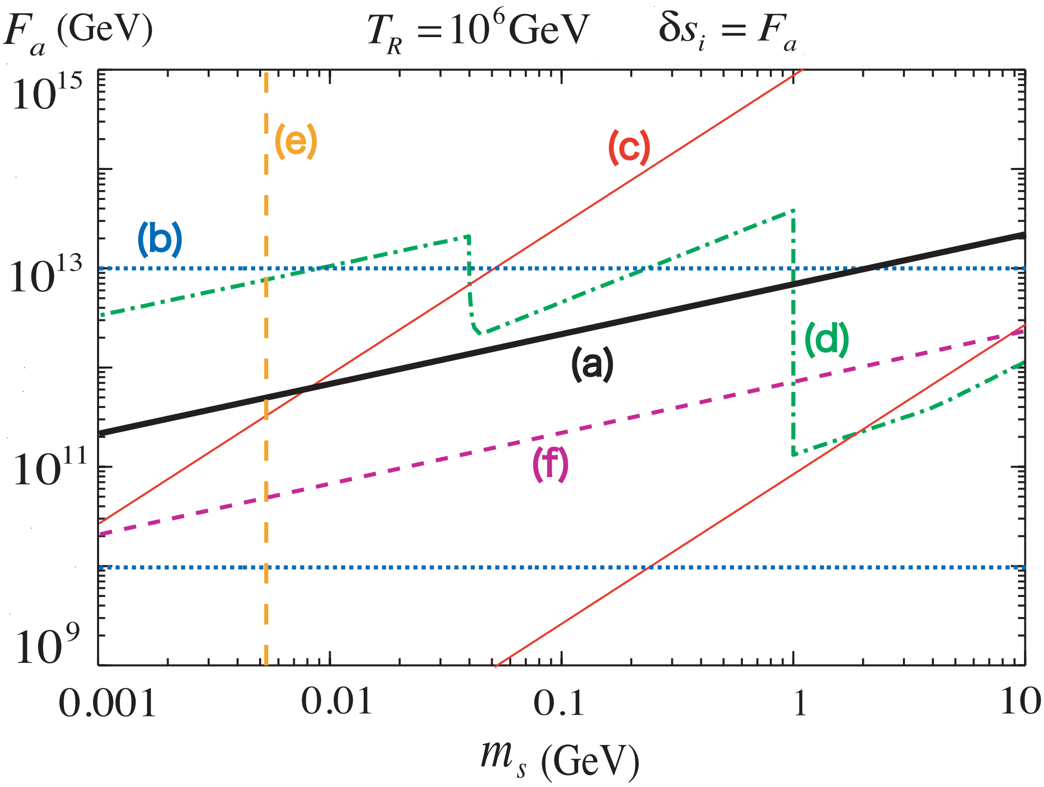

FIG. 1: Constraints on the parameter space m_s and F_a in the saxion decay scenario with $T_R = 10^6$ GeV. We have chosen $\delta s_i = F_a$. The lines labeled (a)–(f) are defined as follows. (a) $\Delta N_\nu = 1$ on this line. (b) Lower and upper bounds on the PQ scale with $\theta \sim 0.1$. (c) Upper line corresponds to $\tau_s \sim 10^8$ sec, and lower line corresponds to $\tau_s \sim 1$ sec. (d) BBN bounds coming from radiative decay for 40 MeV $\lesssim m_s \lesssim 1$ GeV and hadronic decay for $m_s \gtrsim 1$ GeV. For $m_s \lesssim 40$ MeV, the bound comes from the CMB. (e) Lower bound on m_s from gravitino thermal production. (f) Lower bound from axino thermal production for $m_{\tilde{a}} = 0.01\, m_s$. For $m_{\tilde{a}} = m_s$, the constraint coincides with the line (a) accidentally.

the current observed dark matter abundance below the long-dashed purple line (f). For $m_{\tilde{a}} = m_s$, the constraint from the thermally produced axinos coincides with the line (a) accidentally, so it is not explicitly drawn. The region below (a) is excluded if $m_{\tilde{a}} \simeq m_s$. We have found regions for $\Delta N_\nu \sim 1$ consistent with all the constraints. For $\delta s_i = F_a$, they are 1 MeV $\lesssim m_s \lesssim 1$ GeV, $F_a \sim 10^{12}$ GeV and 10^5 GeV $\lesssim T_R \lesssim 10^6$ GeV. Since the PQ scale F_a is close to the upper bound coming from the axion-oveclosure limit, the axion can also play

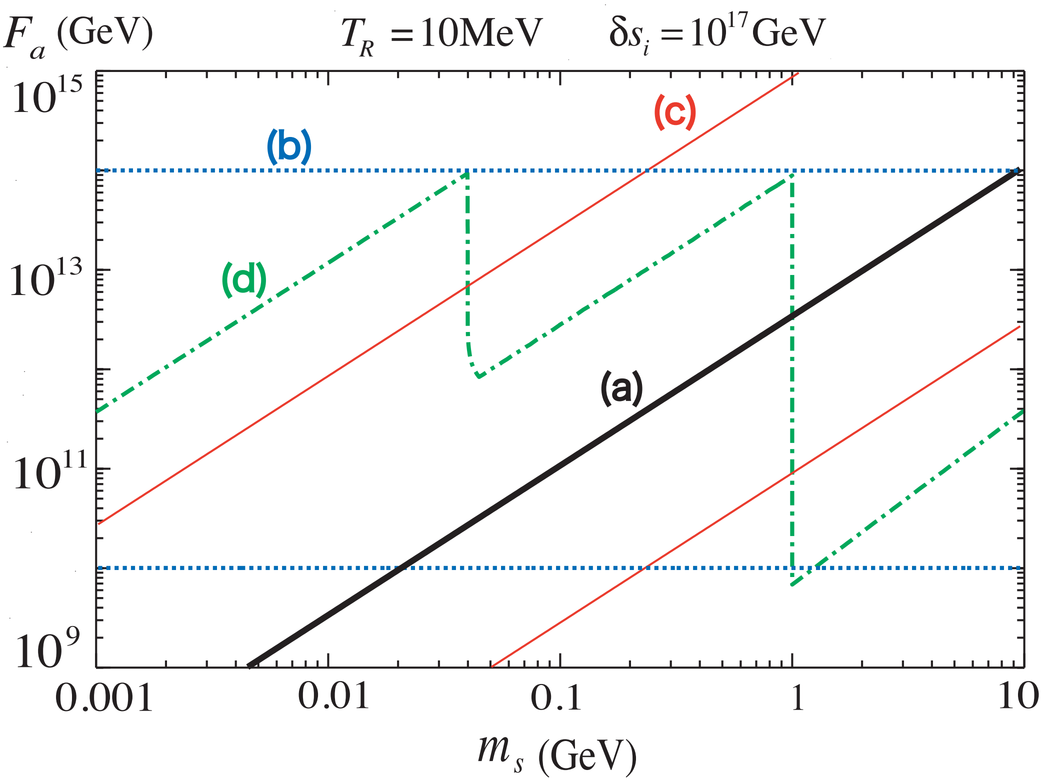

FIG. 2: Same as Fig. 1, except for $T_R = 10\,\mathrm{MeV}$ and $\delta s_i = 10^{17}\,\mathrm{GeV}$.

a role of dark matter of the universe. Moreover, dark matter may be also explained by the thermally produced axinos (see Eq. (27)).

On the other hand, δs_i may be as large as the Planck scale. We also show the result for $\delta s_i = 10^{17}\,\mathrm{GeV}$ and $T_R = 10\,\mathrm{MeV}$ in Fig. 2, while keeping the other parameters the same as in Fig. 1. Note that for such a low reheating temperature, the axion is diluted and the upper bound on the PQ scale F_a is relaxed [36]. Since δs_i is independent of F_a in this case, the energy density of the saxion is also independent of F_a. Hence, the BBN and CMB constraints is given in terms of τ_s and the dependence on F_a is only through τ_s, as is seen from Fig. 2. For such a low reheating temperature, the axino does not give any meaningful constraint. For $\delta s_i \sim 10^{16}$–$10^{18}\,\mathrm{GeV}$, there are allowed parameter regions, $1\,\mathrm{MeV} \lesssim m_s \lesssim 1\,\mathrm{GeV}$, $10^{10}\,\mathrm{GeV} \lesssim F_a \lesssim 10^{14}\,\mathrm{GeV}$, and a few $\mathrm{MeV} \lesssim T_R \lesssim 100\,\mathrm{MeV}$ [7].

[7] For the precise value of the lower bound on T_R, see Refs. [19, 50, 51, 52].

B. Gravitino decay into axino and axion

Next we consider the gravitino decay into the axion and the axino at late times [47]. The axino mass is model dependent, and it can be (much) smaller than the gravitino mass [30]. Here, from a phenomenological point of view, we treat the axino mass as a free parameter, but it should be kept in mind that one may need to contrive a model that realizes a specific value of the axino mass, especially if it is much smaller than the gravitino mass. If the gravitino is the next-to-lightest supersymmetric particle (NLSP) and the axino is the LSP, the gravitino decays into the axion and the axino. Both the axion and the axino produced by the gravitino decay contribute to the extra effective number of neutrinos, so $f_R = 1$.

The lifetime of the gravitino is

$$\begin{aligned} \tau(\tilde{G} \to a + \tilde{a}) &\simeq \left(\frac{1}{192\pi} \frac{m_{3/2}^3}{M_P^2} \right)^{-1} \\ &\simeq 8.7 \times 10^7 \,\mathrm{sec} \left(\frac{m_{3/2}}{300\,\mathrm{GeV}} \right)^{-3}. \end{aligned} \tag{28}$$

Therefore $m_{3/2}$ must be larger than about $300\,\mathrm{GeV}$ in order to satisfy the requirement $\tau \lesssim 10^8\,\mathrm{sec}$. The needed gravitino abundance is given by Eq. (5) as

$$\frac{\rho_{3/2}}{s} \simeq 8.8 \times 10^{-9}\,\mathrm{GeV}\ \Delta N_\nu \left(\frac{m_{3/2}}{300\,\mathrm{GeV}} \right)^{3/2}, \tag{29}$$

which is given in terms of Y as

$$Y_{3/2} \simeq 2.9 \times 10^{-11}\ \Delta N_\nu \left(\frac{m_{3/2}}{300\,\mathrm{GeV}} \right)^{1/2}. \tag{30}$$

The gravitino may be produced both thermally and non-thermally. First we assume that gravitinos are dominantly produced by particle scatterings in thermal plasma. From Eq. (23), in order to obtain the gravitino abundance Eq. (30), the reheating temperature T_R must be as high as $O(10^{10})\,\mathrm{GeV}$ with $m_{\tilde{g}} \sim O(1)\,\mathrm{TeV}$. For such a high reheating temperature, however, axinos are also efficiently produced by thermal scatterings. Their thermal abundance is given by Eqs. (25) or (26). Thus, for the axino abundance not to exceed the current observed dark matter abundance, the axino mass must be smaller than $O(1)\,\mathrm{keV}$.

With such a light mass, however, its free streaming may erase the cosmological structure and conflict with the observation. The maximal abundance consistent with the observational data including Lyman-α forest can be inferred from the upper bound on the HDM component, or the neutrino masses. According to Ref. [10], the 95% C.L. limit obtained from the data set including the Lyman-α forest is $\sum m_\nu < 0.17\,\mathrm{eV}$ which can be converted to $\Omega_\nu h^2 < 1.8 \times 10^{-3}$. Therefore, it is reasonable to expect that the contribution to the energy density of the universe from such light axino must be less than 1% of the dark matter, in order to be consistent with the observed Lyman-α forest. This further constrains the axino mass down to be smaller than $O(10)\,\mathrm{eV}$. Note that, for the axino mass lighter than $O(10)\,\mathrm{eV}$, the axino abundance produced from the gravitino decay is negligibly small.

So far we have assumed that the gravitino with the abundance Eq. (30) is thermally produced. This requires a quite high reheating temperature, which limits the axino mass to being much smaller than the gravitino mass. Since the axino mass is generically of the order of the gravitino mass, such a hierarchy may pose a difficulty to build a viable axion model that realizes the axino mass. If the gravitino is non-thermally produced from, e.g., inflaton decay [53, 54, 55, 56] (or modulus decay [57, 58]), the tension can be greatly relaxed. The gravitino abundance is then dependent on the inflaton mass m_ϕ and VEV $\langle\phi\rangle$. For instance, in a high scale inflation model, the inflaton decays into the SUSY breaking sector, producing the gravitino as [56] [8]

$$Y_{3/2} \simeq O(10^{-11}) \left(\frac{T_R}{10^3\,\mathrm{GeV}}\right)^{-1} \left(\frac{\langle\phi\rangle}{10^{15}\,\mathrm{GeV}}\right)^2 \left(\frac{m_\phi}{10^{12}\,\mathrm{GeV}}\right)^2, \tag{31}$$

where the precise abundance depends on the details of the SUSY breaking sector. The values adopted for m_ϕ and $\langle\phi\rangle$ in Eq. (31) can be realized in e.g. a hybrid inflation model [59]. For such a low reheating temperature, the thermal production of the axino does not set any severe bound on the axino mass. In particular, note that Eq. (25) is not applicable for the reheating temperature smaller than the weak scale. Instead, the axino produced from the gravitino decay puts an upper bound as $m_{\tilde a} \lesssim O(100)\,\mathrm{MeV}$. This can be derived as follows. The axino abundance from the gravitino decay is

$$\Omega_{\tilde a} h^2 \simeq 8 \times 10^{-4} \Delta N_\nu \left(\frac{m_{\tilde a}}{100\,\mathrm{MeV}}\right) \left(\frac{m_{3/2}}{300\,\mathrm{GeV}}\right)^{1/2}. \tag{32}$$

Requiring the axino abundance to be smaller than 1% of the dark matter, we obtain an upper bound on the axino mass as $m_{\tilde a} \lesssim O(100)\,\mathrm{MeV}$. Although the axino mass cannot be as large as the gravitino mass, the required hierarchy of the two is rather mild, compared

[8] Note that $T_R \sim 10^3\,\mathrm{GeV}$ is naturally realized from the spontaneous decay of the inflaton through the top Yukawa coupling [55] for $m_\phi = 10^{12}\,\mathrm{GeV}$ and $\langle\phi\rangle = 10^{15}\,\mathrm{GeV}$.

to the previous case. In a similar fashion, we can show that the LSPs produced by this non-thermal process cannot be the dominant component of the dark matter in the models described below.

In the present model, both the axion and axino are produced from the gravitino decay as relativistic particles. In contrast to the axion, the axino becomes non-relativistic at some time depending on its mass. But it is typically well after the matter-radiation equality epoch, and the axino abundance amounts to only a small fraction of the energy density of the universe. Thus both the axion and the axino contribute to the effective number of the neutrinos, that is, $f_R = 1$. The same argument is applied to the following model as well.

So far we have neglected the saxion abundance. Since the saxion mass is roughly equal to the gravitino mass, the saxion decays much earlier than BBN begins (see Eq. (10)). In addition, if δs_i is of $O(F_a)$, since the saxion does not dominate the universe, our arguments above remain unchanged.

The final comment is that one cannot exchange the roles of the gravitino and the axino in the above scenario. Similar arguments show that the gravitino must be much lighter than the axino. This is because, as long as we require $\Delta N_\nu \simeq 1$, the gravitinos produced from the axino are so abundant that the small scale fluctuations ($\gtrsim$ a few Mpc) would be smoothed out unless the gravitino mass is small enough. However, since the axion multiplet cannot have a SUSY mass, the axino mass cannot be much larger than the gravitino mass and the scenario does not seem to work.

C. Dirac right-handed sneutrino decay into gravitino and right-handed neutrino

The neutrino oscillation experiments have revealed that the neutrinos have finite but small masses. To explain the tiny neutrino masses one introduces right-handed neutrinos into the standard model. The right-handed neutrinos may be allowed to have large Majorana masses as large as GUT scale, because they are singlets with respect to the standard-model gauge group. However, the Majorana mass term can be forbidden by some symmetry such as the lepton-number symmetry. Thus, if this is the case, the neutrino mass is given by the Dirac mass term, and the mass of the right-handed neutrino is very light. The smallness of the neutrino mass is explained by the small Yukawa coupling of $\sim m_\nu/\langle H_u \rangle \lesssim O(10^{-13})$, where m_ν is a neutrino mass and $\langle H_u \rangle$ denotes the VEV of the up-type Higgs. On the other hand, the right-handed sneutrino acquires a mass from SUSY breaking effects. Since the Yukawa coupling is rather small, the lifetime of right-handed sneutrinos is very long, and their decay into the right-handed neutrino and the gravitino can increase N_ν.

First, the lifetime of right-handed sneutrinos is given as

$$\begin{aligned}\tau_{\tilde{\nu}_R} &\simeq \left(\frac{1}{48\pi}\frac{m_{\tilde{\nu}_R}^5}{m_{3/2}^2 M_P^2}\right)^{-1} \\ &\simeq 1.4\times 10^8\,\mathrm{sec}\left(\frac{m_{3/2}}{500\,\mathrm{keV}}\right)^2\left(\frac{m_{\tilde{\nu}_R}}{1\,\mathrm{GeV}}\right)^{-5},\end{aligned} \tag{33}$$

where the right-handed sneutrino mass $m_{\tilde{\nu}_R}$ should be less than $\sim 1\,\mathrm{GeV}$ as discussed later. From Eq. (5), the abundance of the right-handed sneutrinos should be

$$\frac{\rho_{\tilde{\nu}_R}}{s} \simeq 6.8\times 10^{-9}\,\mathrm{GeV}\ \Delta N_\nu \left(\frac{m_{3/2}}{500\,\mathrm{keV}}\right)^{-1}\left(\frac{m_{\tilde{\nu}_R}}{1\,\mathrm{GeV}}\right)^{5/2}. \tag{34}$$

Such large abundance of the sneutrino is unlikely to be produced by thermal scatterings or decays of other superparticles due to the smallness of $m_{\tilde{\nu}_R}$ and the Yukawa coupling [60]. But, the sufficient energy density of right-handed sneutrino can be non-thermally produced in the form of coherent oscillations. The right-handed sneutrino can develop a large field value during inflation, and after the inflation ends it begins to oscillate coherently, which can induce a large abundance of the right-handed sneutrino [61, 62]. Its energy density-to-entropy ratio $\rho_{\tilde{\nu}_R}/s$ is fixed at the end of the reheating process,

$$\begin{aligned}\frac{\rho_{\tilde{\nu}_R}}{s} &= \frac{m_{\tilde{\nu}_R}^2|\tilde{\nu}_{Ri}|^2 T_R}{4H_{\mathrm{osc}}^2 M_P^2} \\ &\simeq 4.3\times 10^{-8}\,\mathrm{GeV}\left(\frac{|\tilde{\nu}_{Ri}|}{10^{14}\,\mathrm{GeV}}\right)^2\left(\frac{T_R}{100\,\mathrm{GeV}}\right),\end{aligned} \tag{35}$$

where $\tilde{\nu}_{Ri}$ denotes the initial amplitude of the right-handed sneutrino. Here we used the Hubble parameter H_{osc} at the start of the oscillations is equal to $m_{\tilde{\nu}_R}$.

As discussed above, the abundance of gravitinos produced by $\tilde{\nu}_R$ decay should be subdominant component of the dark matter. The abundance is given by

$$\Omega_{3/2}h^2 \simeq 9.3\times 10^{-4}\left(\frac{m_{\tilde{\nu}_R}}{1\,\mathrm{GeV}}\right)^{3/2}. \tag{36}$$

It is interesting that the abundance is independent of $m_{3/2}$. In order not to significantly affect the observed small scale structure ($\gtrsim$ a few Mpc), $m_{\tilde{\nu}_R}$ must be smaller than $\sim 1\,\mathrm{GeV}$. With the constraint $\tau \lesssim 10^8\,\mathrm{sec}$, $m_{3/2} \lesssim 500\,\mathrm{keV}$ is also required. Note that such a hierarchical mass relation may be realized in gauge-mediated SUSY breaking models with some extended gauge interaction which involves right-handed neutrinos and is broken at an intermediate scale.

Finally, we comment on the gravitino decay into $\tilde{\nu}_R$ and ν_R. This case leads to the same result as in the previous section, after exchanging $(\tilde{\nu}_R, \nu_R)$ with $(\tilde{a}, a)$. Therefore, for $\Delta N_\nu \sim 1$, $m_{3/2} \gtrsim 300\,\mathrm{GeV}$ and $m_{\tilde{\nu}_R} \lesssim 1\,\mathrm{MeV}$ are required. However, this hierarchical mass relation, $m_{3/2} \gg m_{\tilde{\nu}_R}$ is unlikely in SUSY models. Hence, this case is not expected to explain increasing ΔN_ν.

IV. CONCLUSIONS AND DISCUSSION

In this paper, we present models of decaying particles for increasing the effective number of neutrinos N_ν after BBN but before the structure formation begins. In the model (a) where the saxion decays into two axions, broad regions are allowed. For instance, T_R can take a value from $10\,\mathrm{MeV}$ up to $10^6\,\mathrm{GeV}$, depending on the initial displacement of the saxion and $m_{\tilde{a}}/m_s$ (see Figs. 1 and 2 for details). In particular, the saxion mass needs to lie in the range between $1\,\mathrm{MeV}$ and $1\,\mathrm{GeV}$, which suggests the light gravitino. In the model (b) where the gravitino decays into the axino and the axion, we require $m_{3/2} \gtrsim 300\,\mathrm{GeV}$ together with $m_{\tilde{a}} \lesssim 10\,\mathrm{eV}$ or $m_{\tilde{a}} \lesssim 100\,\mathrm{MeV}$ depending on the gravitino production processes. The former (latter) bound is the case with the thermal (non-thermal) production. In particular, one needs a hierarchy between the gravitino mass and the axino mass. In the model (c) where the Dirac right-handed sneutrino decays into the gravitino and the right-handed neutrino, $m_{\tilde{\nu}_R} \lesssim 1\,\mathrm{GeV}$ and $m_{3/2} \lesssim 500\,\mathrm{keV}$ are required. This case works only for the non-thermal origin of the right-handed sneutrino in the form of scalar condensates.

Such a scenario is motivated because non-standard values of $N_\nu > 3$ are preferred by the combined data of the CMB, galaxy clustering and the Lyman-α forest [10, 15, 16] whereas most of the recent analyses of primordial ^{4}He abundance favor standard $N_\nu = 3$ [17, 22, 23]. As is discussed in Ref. [10], the preference for $N_\nu > 3$ of the Lyman-α combined data stems from the inconsistency in the estimation of the matter power spectrum amplitude at small scales, represented by σ_8, between the WMAP and the Lyman-α forest: the latter yields somewhat higher value of σ_8. We note that such higher σ_8 values are also derived by other probes of the small scale matter power spectrum by the weak lensing [11, 12] and strong lensing [13]. Thus, we would like to stress that the models proposed here can not only solve the discrepancy between BBN and the structure formation (the CMB, the Lyman-α forest and so on) but also give a possible answer to the inconsistency between the WMAP and small scale matter power measurements such as the Lyman-α forest and weak lensing.

It should be emphasized that N_ν is increased by the "free-streaming" relativistic particles like massless neutrinos in our models. Our scenario of increasing N_ν may recall the readers to the scenario of "interacting" neutrinos discussed e.g. in Refs. [63, 64] whose prediction includes the increase in N_ν after BBN by the recoupling. Even though N_ν changes by the same amount, there is a stark contrast between the free-streaming particles and interacting ones as regards the effects on the structure formation. The consequence is that, although N_ν can be increased in the interacting neutrino scenario, it cannot solve the problem. This is explicitly verified in Ref. [16]. Their Fig. 5 (a) shows that the free-streaming particles can better fit the Lyman-α data by increasing N_ν from 3 but such is not the case for interacting particles as shown in Fig. 5 (b).

Finally, since the discrepancy which we have addressed in this paper is about 2σ level, further data and studies are necessary in order to see whether inconsistency exists in the standard cosmological model or in the interpretation of one or more observations. Recently, Ref. [19] have obtained the constraint on N_ν from the WMAP [1, 2, 3, 4] and the SDSS luminous red galaxy power spectrum [6] to be $0.9 < N_\nu < 8.2$ (95% C.L.). This is not in conflict with the one derived using the Lyman-α and the earlier galaxy power spectrum as mentioned in the Introduction, $N_\nu = 4.6^{+1.6}_{-1.5}$ [15], but it does not have sufficient sensitivity to test the need for $N_\nu > 3$. Since we cannot expect the galaxy power spectrum data to increase significantly in near future, improvement in the reliability of the Lyman-α forest and weak lensing will be needed to solve the issue. We believe the ongoing works in the communities to understand sources of systematic errors will accomplish this task and, together with the future CMB experiments (the PLANCK sensitivity for N_ν is forecasted to be ~ 0.2, see e.g. [65]), this will tell us whether the scenario of increasing N_ν is realized in Nature.

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