

Lattice measurement of B_{B_s} with a chiral light quark action

B. Blossier^a

^aDESY, Platanenallee 6, D-15738 Zeuthen, Germany

The computation on the lattice of the bag parameter B_{B_s} associated to the $B_s - \overline{B}_s$ mixing amplitude in the Standard Model is presented. The estimation has been made by combining the static limit of HQET and the Neuberger light quark action which preserves the chiral symmetry on the lattice. We find $B_{B_s}^{\overline{\text{MS}}\text{stat}}(m_b) = 0.92(3)$.

1. Introduction

$B_s - \overline{B}_s$ mixing is highly important in testing the Standard Model (SM) and constrains strongly its extensions by bounding the unitarity triangle. Since it is a flavor changing neutral process it occurs through loops so that the corresponding mixing amplitude is a sensitive measure of $|V_{ts}|$ and $|V_{tb}|$, as the major SM loop contribution comes from t -quark. The mixing of weak interaction eigenstates B_s and \overline{B}_s induces a mass gap ΔM_s between the mass eigenstates B_{sH} and B_{sL} . Experimentally, D0 has bounded ΔM_s as $17 \text{ ps}^{-1} < \Delta M_s < 21 \text{ ps}^{-1}$ (90 % CL) [1] and the measurement made at CDF gave $\Delta M_s = 17.330^{+0.426}_{-0.221}$ [2]. Theoretically the $B_s - \overline{B}_s$ mixing is described by means of an Operator Product Expansion, *i.e.* the Standard Model Lagrangian \mathcal{L}_{SM} is reduced to an effective Hamiltonian $\mathcal{H}_{eff}^{\Delta B=2}$, up to negligible terms of $\mathcal{O}(1/M_W^2)$:

$$\begin{aligned} \mathcal{H}_{eff}^{\Delta B=2} &= \frac{G_F^2}{16\pi^2} M_W^2 (V_{tb}^* V_{ts})^2 \\ &\times \eta_B S_0(x_t) C(\mu_b) Q_{LL}^{\Delta B=2}(\mu_b), \end{aligned} \quad (1)$$

$$Q_{LL}^{\Delta B=2} = \bar{b} \gamma_{\mu L} s \bar{b} \gamma_{\mu L} s, \quad \mu_b \sim m_b,$$

where $\eta_B = 0.55 \pm 0.01$. $S_0(x_t)$ is a known Inami-Lim function of $x_t = m_t^2/M_W^2$ [3], $C(\mu_b)$ is the Wilson coefficient computed perturbatively to NLO in $\alpha_s(\mu_b)$ in the $\overline{\text{MS}}$ (NDR) scheme, and $Q_{LL}^{\Delta B=2}$ is a four-fermions operator coming from the reduction of the box diagrams in \mathcal{L}_{SM} to a local operator in the effective theory. The mass splitting is

$$\Delta M_{B_s} = 2 |\langle \overline{B}_s | \mathcal{H}_{eff}^{\Delta B=2} | B_s \rangle|. \quad (2)$$

The hadronic matrix element of $Q_{LL}^{\Delta B=2}$, which must be computed non perturbatively, is conventionally parameterized as

$$\langle \overline{B}_s | Q_{LL}^{\Delta B=2}(\mu_b) | B_s \rangle \equiv \frac{8}{3} m_{B_s}^2 f_{B_s}^2 B_{B_s}(\mu_b), \quad (3)$$

where $B_{B_s}(\mu_b)$ is the B_s meson bag parameter and f_{B_s} the decay constant. In the following it will be useful to introduce in addition to the operator $O_1 \equiv Q_{LL}^{\Delta B=2}$ the operators of the so called supersymmetric basis

$$\begin{aligned} O_2 &= \bar{b}^i (1 - \gamma^5) s^i \bar{b}^j (1 - \gamma^5) s^j, \\ O_3 &= \bar{b}^i \gamma_\mu (1 - \gamma^5) s^i \bar{b}^j \gamma_\mu (1 + \gamma^5) s^j, \\ O_4 &= \bar{b}^i (1 - \gamma^5) s^i \bar{b}^j (1 + \gamma^5) s^j, \end{aligned} \quad (4)$$

whose the matrix elements $\langle \overline{B}_s | O_i | B_s \rangle$ are parameterised by

$$\begin{aligned} \langle \overline{B}_s | O_2 | B_s \rangle &= -\frac{5}{3} \left(\frac{m_{B_s}}{m_b(\mu) + m_s(\mu)} \right)^2 f_{B_s}^2 B_2(\mu), \\ \langle \overline{B}_s | O_3 | B_s \rangle &= -\frac{4}{3} \left(\frac{m_{B_s}}{m_b(\mu) + m_s(\mu)} \right)^2 f_{B_s}^2 B_3(\mu), \\ \langle \overline{B}_s | O_4 | B_s \rangle &= 2 \left(\frac{m_{B_s}}{m_b(\mu) + m_s(\mu)} \right)^2 f_{B_s}^2 B_4(\mu). \end{aligned} \quad (5)$$

So far $B_{B_s}(\mu_b)$ has been computed by using lattice QCD [4]-[10]¹. One of the major problems with those computations is in the following: the standard Wilson light quark lattice action breaks explicitly the chiral symmetry, which tremendously complicates the renormalization procedure of $Q_{LL}^{\Delta B=2}$ and its matching to the continuum. To

¹It was also estimated by using QCD sum rules [11] but we will concentrate only on lattice results.

get around that problem we compute $B_{B_s}(\mu_b)$ by using the lattice formulation of QCD in which the chiral symmetry is preserved at finite lattice spacing [12]. On the other hand, it should be stressed that our heavy quark is static, as the currently available lattices do not allow to work directly with the propagating b quark. Thus our results will suffer from $1/m_b$ -corrections.

2. Computation on the lattice

In our numerical simulation we choose to work with the action $S = S_h^{\text{EH}} + S_l^N$, where

$$S_h^{\text{EH}} = a^3 \sum_x \left\{ \bar{h}^+(x) \left[h^+(x) - V_0^{\text{HYP}\dagger}(x - \hat{0}) h^+(x - \hat{0}) \right] - \bar{h}^-(x) \left[V_0^{\text{HYP}}(x) h^-(x + \hat{0}) - h^-(x) \right] \right\}$$

is the static limit of HQET action [13] which has been modified after using the so-called HYP (hypercubic blocking) procedure [14], that is enough to substantially improve the signal/noise ratio [15] [the field $h^+(h^-)$ annihilates the static heavy quark (antiquark)]. $S_l^N = a^3 \sum_x \bar{\psi}(x) D_N^{(m_0)} \psi(x)$ is the overlap light quark action with

$$D_N^{(m_0)} = \left(1 - \frac{1}{2\rho} a m_0 \right) D_N + m_0,$$

$$D_N = \frac{\rho}{a} \left(1 + \frac{X}{\sqrt{X^\dagger X}} \right), \quad X = D_W - \frac{\rho}{a},$$

where D_W is the standard Wilson-Dirac operator. The overlap Dirac operator $D_N^{(m_0)}$ verifies the Ginsparg-Wilson relation $\{\gamma^5, D_N^{(m_0)}\} = \frac{a}{\rho} D_N^{(m_0)} \gamma^5 D_N^{(m_0)}$ and the overlap action is invariant under the chiral light quark transformation [16]

$$\psi \rightarrow \psi + i\epsilon \gamma^5 \left(1 - \frac{a}{\rho} D_N^{(m_0)} \right) \psi, \quad \bar{\psi} \rightarrow \bar{\psi} (1 + i\epsilon \gamma^5),$$

which is essential to prevent mixing of four-fermion operators of different chirality [17]. In other words, in the renormalization procedure, the subtraction of the spurious mixing with $d = 6$ operators will not be needed. We thus compute the two- and three-point functions:

$$\tilde{C}_{AA}^{(2)\pm}(t) = \langle \sum_{\vec{x}} \tilde{A}_0^\pm(\vec{x}, t) \tilde{A}_0^{\pm\dagger}(0) \rangle_v \xrightarrow{t \gg 0} \tilde{Z}_A e^{-\epsilon t}, \quad (6)$$

$$\begin{aligned} \tilde{C}_{VV+AA}^{(3)}(t_i, t) &= \langle \sum_{\vec{x}, \vec{y}} \tilde{A}_0^+(\vec{x}, t_i) \tilde{O}_1(0, 0) \tilde{A}_0^{-\dagger}(\vec{y}, t) \rangle_u \\ &\xrightarrow{t_i \rightarrow 0} \tilde{Z}_A v \langle \overline{B_s} | \tilde{O}_1(\mu) | B_s \rangle_v e^{-\epsilon(t_i - t)}, \quad (7) \\ \tilde{C}_{SS+PP}^{(3)}(t_i, t) &= \langle \sum_{\vec{x}, \vec{y}} \tilde{A}_0^+(\vec{x}, t_i) \tilde{O}_2(0, 0) \tilde{A}_0^{-\dagger}(\vec{y}, t) \rangle_u \\ &\xrightarrow{t_i \rightarrow 0} \tilde{Z}_A v \langle \overline{B_s} | \tilde{O}_2(\mu) | B_s \rangle_v e^{-\epsilon(t_i - t)}, \quad (8) \end{aligned}$$

$$\begin{aligned} \tilde{A}_0^\pm &\equiv \bar{h}^\pm \gamma_0 \gamma^5 s, \quad \tilde{O}_1 = \bar{h}^{(+)} i \gamma_{\mu L} s^i \bar{h}^{(-)} j \gamma_{\mu L} s^j, \\ \tilde{O}_2 &= \bar{h}^{(+)} i P_L s^i \bar{h}^{(-)} j P_L s^j, \quad \sqrt{\tilde{Z}_A} = \langle 0 | \tilde{A}_0^\pm | B_s \rangle_v. \end{aligned}$$

ϵ is the binding energy of the pseudoscalar heavy-light meson. In the computation of $\tilde{C}^{(2)\pm}(t_i, t)$ one current \tilde{A}_0^\pm is local whereas the other is smeared. The role of the smearing is to isolate earlier the ground state [18], as shown in Fig. 1². We see that the same state is isolated when purely local currents are used (with those currents the signal does not exist if V_0^{HYP} is not used in the heavy quark action). The source operators in $\tilde{C}_{VV+AA}^{(3)}(t_i, t)$ and $\tilde{C}_{SS+PP}^{(3)}(t_i, t)$ are the smeared currents \tilde{A}_0^\pm , whereas the four-fermion operators \tilde{O}_1 and \tilde{O}_2 are purely local. In (6), (7) and (8) the subscript "v" and superscript "v" are designed to remind the reader that states and operators are defined in HQET.

Note that in the computation of $\tilde{C}_{VV+AA}^{(3)}(t_i, t)$ and $\tilde{C}_{SS+PP}^{(3)}(t_i, t)$ there are two terms, coming from two different Wick contractions:

$$\sum_i B_{ii}(t) \sum_j B_{jj}(t_i) \quad \text{and} \quad \sum_{i,j} B_{ij}(t) B_{ji}(t_i).$$

i, j are the color indices and

$$B_{ij}(t) = \text{Tr} \left[\sum_{\vec{x}} \gamma_{\mu L} \mathcal{S}_L^{\dagger ik}(0; \vec{x}, t) \gamma_0 \gamma^5 \mathcal{S}_H^{kj}(\vec{x}, t; 0) \right];$$

\mathcal{S}_L and \mathcal{S}_H are the light and heavy propagators respectively and the trace is over spinor indices.

²Even if the time interval from which we extract the binding energy starts at $t = 9$ (green line), the overlap with radial excitations is quite reduced since $t = 6$ when currents are smeared.

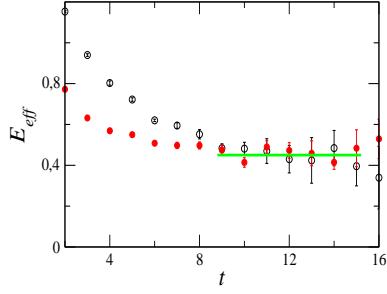


Figure 1. Effective binding energy of the 0^- -state when currents are local (unfilled symbols) or smeared (filled symbols)

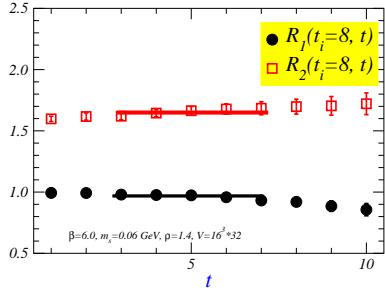


Figure 2. Signals for $R_{1,2}(t_i, t)$ defined in eq (9): lines indicate the time interval on which we fit the signal to a constant to extract $\tilde{B}_1(a)$ and $\tilde{B}_2(a)$ respectively.

After having computed the correlation functions (6), (7) and (8) we build the following two ratios $R_1(t_i, t)$ and $R_2(t_i, t)$:

$$\begin{aligned}
 R_1(t_i, t) &= \frac{\tilde{C}_{VV+AA}^{(3)}(t_i, t)}{\frac{8}{3}\tilde{Z}_A^2\tilde{C}_{AA}^{(2)+}(t_i)\tilde{C}_{AA}^{(2)-}(t)} \\
 &\xrightarrow[t_i-t \gg 0]{} \frac{v\langle\bar{B}_s|\tilde{O}_1|B_s\rangle_v}{\frac{8}{3}|0|\tilde{A}_0^-|B_s\rangle_v|^2} \equiv \tilde{B}_1(a), \\
 R_2(t_i, t) &= \frac{\tilde{C}_{SS+PP}^{(3)}(t_i, t)}{-\frac{5}{3}\tilde{Z}_A^2\tilde{C}_{AA}^{(2)+}(t_i)\tilde{C}_{AA}^{(2)-}(t)} \\
 &\xrightarrow[t_i-t \gg 0]{} \frac{v\langle\bar{B}_s|\tilde{O}_2|B_s\rangle_v}{-\frac{5}{3}|0|\tilde{A}_0^-|B_s\rangle_v|^2} \equiv \tilde{B}_2(a). \quad (9)
 \end{aligned}$$

Those ratios are calculated either with a fixed time $t \in [-6, -8, -10, -12, -14, -16]$ and t_i free, or by fixing $t_i \in [6, 8, 10, 12, 14, 16]$ while letting t free. We take the average of the two options.

In Fig. 2 we show the quality of the signals for $R_{1,2}(t_i, t)$, with $t_i = 6$ fixed. The signal for $\tilde{B}_1(a)$ is quite stable as a function of t_i , whereas the signal for $\tilde{B}_2(a)$ rapidly deteriorates for larger t_i , and is completely lost for $t_i > 10$.

3. Extraction of physical B_{B_s}

Three steps are required to extract $B_{B_s} \equiv B_1$ from the lattice:

- (1) $\tilde{B}_{1,2}(a)$ are matched onto the continuum $\overline{\text{MS}}(\text{NDR})$ scheme at NLO in perturbation theory at the renormalization scale $\mu = 1/a$ [17],
- (2) $\tilde{B}_{1,2}$ are evolved from $\mu = 1/a$ to $\mu = m_b$ by using the HQET anomalous dimension matrix, known to 2-loop accuracy in perturbation theory [8,19],
- (3) $\tilde{B}_{1,2}(\mu = m_b)$ are then matched onto their QCD counterpart, $B_{1,2}(m_b)$, in the $\overline{\text{MS}}(\text{NDR})$ scheme at NLO [19].

The scales chosen to do the matchings are such that neither $\ln(a\mu)$ in the step (1) nor $\ln(\mu/m_b)$ in the step (3) correct strongly the matching constants. The advantage of using a chiral light quark action for the step (1) lies in the fact that four-fermion operators can mix only with a dimension 6 four-fermion operator of the same chirality. In other words we have not more than 4 independent renormalization constants in the renormalization matrix, because \tilde{O}_1 and \tilde{O}_2 can mix neither with $\tilde{O}_3 \equiv \bar{h}^+\gamma_\mu L s \bar{h}^-\gamma_\mu R s$, nor with $\tilde{O}_4 \equiv \bar{h}^+(1-\gamma^5)s \bar{h}^-(1+\gamma^5)s$. Actually, thanks to the heavy quark symmetry, those constants are not all independent and we have [17]

$$\begin{pmatrix} \tilde{B}_1^{\overline{\text{MS}}}(\mu) \\ \tilde{B}_2^{\overline{\text{MS}}}(\mu) \end{pmatrix} = \begin{pmatrix} Z_{11}(a\mu) & 0 \\ \frac{Z_{22}(a\mu) - Z_{11}(a\mu)}{4} & Z_{22}(a\mu) \end{pmatrix} \begin{pmatrix} \tilde{B}_1(a) \\ \tilde{B}_2(a) \end{pmatrix}.$$

4. Results and discussion

Our results are based on two simulations, with the parameters given in Tab. 1.

We find $B_{B_s}^{\overline{\text{MS}} \text{ stat}}(m_b) = 0.922(12)(25)$, where the first error is statistical, the second is systematic and contains the error from the estimation of $\alpha_s(1/a)$ and the finite a effects. From Fig. 3 it can be seen that our value is larger than the previous static result [5]. This difference is likely

β	N_{conf}	action	ρ	am_0^s	κ_l
6.0	100	overlap Wilson	1.4	0.06	0.1435
5.85	40	overlap	1.6	0.09	

Table 1

Parameters of our simulations: am_0^s and ρ have been chosen following [20,21]; the volume is $16^3 \times 32$.

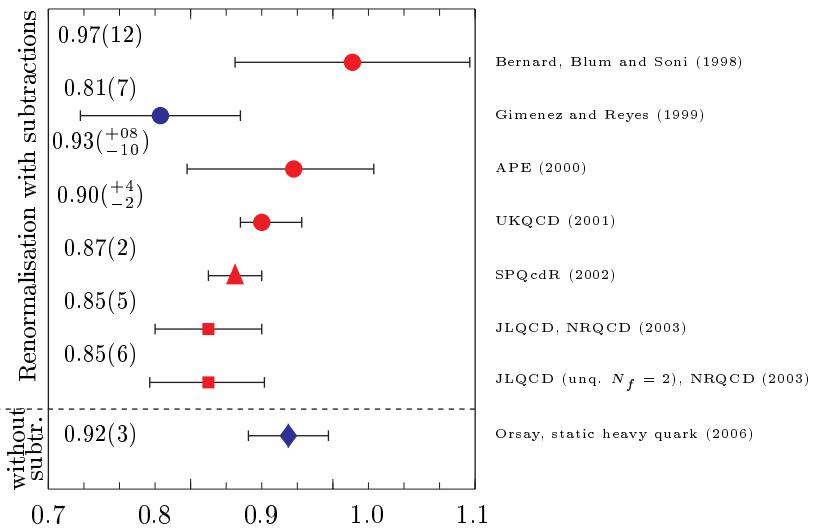


Figure 3. Various lattice values of $B_{B_s}^{\overline{\text{MS}}}(m_b)$ [4]-[10]; blue symbols correspond to a computation made with a static heavy quark

due to the use of Neuberger light quark action (no subtractions), due to the use of the HYP procedure, or the combination of both. To answer to this question we made a computation with the Wilson light quark action. In that case we have subtractions in the renormalisation procedure:

$$\langle \tilde{O}_1 \rangle^{\text{con}} = Z_{11} \langle \tilde{O}_1 \rangle^{\text{lat}} \left(1 + Z_{13} \frac{\langle \tilde{O}_3 \rangle^{\text{lat}}}{\langle \tilde{O}_1 \rangle^{\text{lat}}} + Z_{14} \frac{\langle \tilde{O}_4 \rangle^{\text{lat}}}{\langle \tilde{O}_1 \rangle^{\text{lat}}} \right).$$

A correction on Z_{13} and Z_{14} , coming from a different definition of the HQET action, induces a substantial systematic error on B_{B_s} as illustrated on the Tab. 2. From Fig. 3 we also notice that our value is also somewhat larger than the results obtained with the propagating heavy quark. We

$\frac{\langle \tilde{O}_3 \rangle^{\text{lat}}}{\langle \tilde{O}_1 \rangle^{\text{lat}}}$	-1.011(1)
$\frac{\langle \tilde{O}_4 \rangle^{\text{lat}}}{\langle \tilde{O}_1 \rangle^{\text{lat}}}$	1.013(2)

HYP	Z_{13}	Z_{14}	$B_{B_s}^{\text{stat}}(m_b)$
no	-0.459	-0.919	0.763(5)
yes	-0.235	-0.470	0.873(5)

Table 2

Ratios $\langle \tilde{O}_{3,4} \rangle^{\text{lat}} / \langle \tilde{O}_1 \rangle^{\text{lat}}$ extracted from the simulation with a Wilson light quark and the improved HQET action (left table); comparison of Z_{13} , Z_{14} and $B_{B_s}^{\text{stat}}(m_b)$ obtained with the Wilson action in function of HQET action improvement (right table).

can take account of the $1/m_b$ effects by interpolating linearly through M_{B_s} :

$$B_{B_s}^{\overline{\text{MS}}, M_{B_s}}(m_b) = B_{B_s}^{\overline{\text{MS}}, \text{stat}}(m_b) \left(1 + \frac{C}{M_{B_s}} \right), \quad (10)$$

where $C = -0.24(6)$ GeV [6]. With $M_{B_s} = 5.37$ GeV, $B_{B_s}^{\overline{\text{MS}}, M_{B_s}} = 0.955(11) \times B_{B_s}^{\overline{\text{MS}}, \text{stat}}(m_b)$, we obtain after the simulation at $\beta = 6.0$ $B_{B_s}^{\overline{\text{MS}}, M_{B_s}} = 0.881(15)$. JLQCD collaboration showed that the errors due to quenching are likely to be small [9,10]. That issue has to be addressed by unquenching the $B_s - \overline{B}_s$ mixing amplitude in the static limit and by avoiding the subtraction procedure as well. The first step would be to make a simulation with 2 degenerate Wilson sea light quarks and an overlap valence strange quark. Eventually with $f_{B_s}^{\text{und}} = 230$ MeV [22], $V_{ts} = 04076$ [23], $V_{tb} = 0.99912$ [24], we obtain $\Delta M_{B_s}^{\text{SM}} = 20.7 \text{ ps}^{-1}$. However this value has to be taken very carefully because the uncertainty on $f_{B_s}^{\text{und}}$ is 30%.

5. Conclusion

For the first time the bag parameter associated to the $B_s - \overline{B}_s$ mixing amplitude in the Standard Model has been computed on the lattice by combining the static limit of HQET and a light quark action which preserves the chiral symmetry on the lattice. Thus systematic error induced by subtractions in the renormalisation procedure are absent, since there is no mixing among dimension 6 four-fermion operators of different chirality.

$1/m_b$ corrections and quenching effects have still to be studied carefully.

REFERENCES

1. A. Abazov *et al* [D0 Collaboration], hep-ex/0603029
2. A. Abulencia *et al* [CDF Collaboration], hep-ex/0606027
3. T. Inami and C.S. Lim, *Prog. Theor. Phys.* **65**, 297 (1981).
4. C.W. Bernard, T. Blum and A. Soni, *Phys. Rev. D* **58**, 014501 (1998) [hep-lat/9801039].
5. V. Gimenez and J. Reyes, *Nucl. Phys. B* **545**, 576 (1999) [hep-lat/9806023].
6. D. Bećirević *et al*, *Nucl. Phys. B* **618**, 241 (2001) [hep-lat/0002025].
7. L. Lellouch and C. J. Lin, *Phys. Rev. D* **64**, 094501 (2001) [hep-ph/0011086].
8. D. Bećirević *et al*, JHEP **0204**, 25 (2002) and *Nucl. Phys. Proc. Suppl.* **106**, 385 (2002) [hep-lat/0110091].
9. S. Aoki *et al*, *Phys. Rev. D* **67**, 014506 (2003) [hep-lat/0208038].
10. S. Aoki *et al*, *Phys. Rev. Lett.* **91**, 212001 (2003) [hep-ph/0307039].
11. K. Hagiwara, S. Narison and D. Nomura, *Phys. Lett. B* **540**, 233 (2002)
12. H. Neuberger, *Phys. Lett. B* **417**, 141 (1998) [hep-lat/9707022].
13. E. Eichten and B. Hill, *Phys. Lett. B* **240**, 193 (1990).
14. A. Hasenfratz and F. Knechtli, *Phys. Rev. D* **64**, 034504 (2001) [hep-lat/0103029].
15. M. Della Morte *et al*, *Phys. Lett. B* **581**, 93 (2004) [hep-lat/0307021].
16. M. Lüscher, *Phys. Lett. B* **428**, 342 (1998) [hep-lat/9802011].
17. D. Bećirević and J. Reyes, *Nucl. Phys. Proc. Suppl.* **129**, 435 (2004) [hep-lat/0309131].
18. P. Boyle [UKQCD Collaboration], *J. Comput. Phys.* **179**, 349 (2002) [hep-lat/9903033].
19. D. J. Broadhurst and A. G. Grozin, *Phys. Rev. D* **52**, 4082 (1995) [hep-ph/9410240]; V. Gimenez, *Nucl. Phys. B* **582**, 375 (1992); X. Ji and M. J. Musolf, *Phys. Lett. B* **409**, 257 (1991).
20. L. Giusti, C. Hoelbling and C. Rebbi, *Phys. Rev. D* **64**, 114508 (2001); Erratum-ibid *Phys. Rev. D* **65**, 079903 (2002) [hep-lat/0108007].
21. P. Hernandez, K. Jansen, L. Lellouch and H. Wittig, *Nucl. Phys. Proc. Suppl.* **106**, 766 (2002) [hep-lat/0110199].
22. S. Hashimoto, *Int. J. Mod. Phys. A* **20**, 5133 (2005)
23. M. Bona *et al* [UTfit group], hep-ph/0606167
24. J. Charles [CKMfitter group], hep-ph/0606046