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$\mathcal{O}(G_F^2 m_t^4)$ two-loop electroweak correction to Higgs-boson decay to bottom quarks

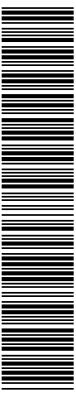
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Abstract

We analytically calculate the dominant two-loop electroweak correction, of $\mathcal{O}(G_F^2 m_t^4)$, to the partial width of the decay of a Higgs boson, with mass $M_H \ll m_t$, into a bottom-quark pair, and describe the most important conceptual and technical details of our calculation. As a by-product of our analysis, we also recover the $\mathcal{O}(\alpha_s G_F m_t^2)$ correction. Relative to the Born result, the $\mathcal{O}(G_F^2 m_t^4)$ correction turns out to be approximately +0.047% and, thus, more than compensates the $\mathcal{O}(\alpha_s G_F m_t^2)$ one, which amounts to approximately -0.022%.

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1 Introduction

The standard model (SM) of elementary particle physics predicts the existence of a last undiscovered particle, the Higgs boson, whose mass M_H is a free parameter of the theory. The direct search for the Higgs boson at the CERN Large Electron-Positron Collider LEP 2 only led to a lower bound of $M_H > 114$ GeV at 95% confidence level [1]. On the other hand, high-precision measurements, especially at LEP and the SLAC Linear Collider SLC, were sensitive to the Higgs-boson mass via electroweak radiative corrections. These indirect measurements yielded the value $M_H = (85^{+39}_{-28})$ GeV and an upper limit of $M_H < 166$ GeV at 95% confidence level [2]. The vacuum-stability and triviality bounds suggest that $130 \lesssim M_H \lesssim 180$ GeV if the SM is valid up to the grand-unification scale (for a review, see Ref. [3]). For these reasons, one hopes to discover the Higgs boson at the CERN Large Hadron Collider (LHC), which will be capable of producing particles with masses up to 1 TeV. The first question after discovering a new scalar particle will be if it actually is the Higgs boson of the SM, or possibly some particle of an extended Higgs sector. Therefore, it is necessary to know the SM predictions for the production and decay rates of the SM Higgs boson with high precision. Its decay into a bottom-quark pair is of special interest, as it is by far the dominant decay channel for $M_H \lesssim 140$ GeV (see, for instance, Ref. [4]).

At this point, we wish to summarise the current status of the calculations of radiative corrections to the $H \rightarrow b\bar{b}$ decay width in the so-called intermediate mass range, defined by $M_W \leq M_H \leq 2M_W$. The correction of order $\mathcal{O}(\alpha_s)$ was first calculated in Ref. [5]. The complete one-loop electroweak correction was found in Ref. [6]. As for the $\mathcal{O}(\alpha_s^2)$ correction, the leading [7] and next-to-leading [8] terms of the expansion in m_b^2/M_H^2 of the diagrams without top quarks are known. The diagrams containing a top quark can be divided into two classes. The diagrams containing gluon self-energy insertions were calculated exactly [9], while for the double-triangle contributions the four leading terms of the expansion in M_H^2/m_t^2 are known [10]. In Ref. [11], the $\mathcal{O}(\alpha_s^3)$ correction without top-quark contributions was calculated in the massless limit. The correction induced by the top quark was subsequently found in Ref. [12] using an appropriate effective field theory. As for the correction of order $\mathcal{O}(\alpha_s G_F m_t^2)$, the universal part, which appears for any Higgs-boson decay to a fermion pair, was calculated in Ref. [13] and the non-universal one, using a low-energy theorem, in Ref. [14]. The latter result was independently found in Ref. [15]. Apart from the Higgs-boson decay into a $t\bar{t}$ pair, only the one into a $b\bar{b}$ pair has such non-universal top-quark-induced contributions, as bottom is the weak-isospin partner of top. The universal and non-universal corrections of order $\mathcal{O}(\alpha_s^2 G_F m_t^2)$ were calculated in Refs. [16] and [17], respectively. Finally, also a result for the universal correction of order $\mathcal{O}(G_F^2 m_t^4)$ was published [18].

In this paper, we calculate the complete correction of order $\mathcal{O}(G_F^2 m_t^4)$, including both the universal and non-universal contributions. To this end, we formally assume that $M_H \ll m_t$. This includes the intermediate mass range of the Higgs boson. Our result for the universal contribution in the on-mass-shell scheme agrees with the one found in Ref. [18], after correcting an obvious mistake in the latter paper. The key results of our

calculation were already presented in a brief communication [19]. Here, the full details are exhibited.

Our calculations are performed in 't Hooft-Feynman gauge. We adopt the on-mass-shell scheme and regularise the ultraviolet divergences by means of dimensional regularisation, with $D = 4 - 2\epsilon$ space-time dimensions and 't Hooft mass scale μ . We use the anti-commuting definition of γ_5 . As a simplification, we take the Cabibbo-Kobayashi-Maskawa quark mixing matrix to be unity. The Feynman diagrams are generated and drawn using the program **FeynArts** [20] and evaluated using the program **MATAD** [21], which is written in the programming language **FORM** [22].

In order to check our calculations, we also rederive the correction of order $\mathcal{O}(\alpha_s G_F m_t^2)$. Our result agrees with Refs. [13,14,15]. Since this calculation follows the lines of the one leading to the $\mathcal{O}(G_F^2 m_t^4)$ correction, being actually simpler, we refrain from going into details with it.

This paper is organised as follows. In Section 2, we describe in detail the renormalisation procedure underlying our analysis. In Section 3, we present the details of our diagrammatic calculations. In Section 4, we explain how a part of our calculations can be checked through the application of a low-energy theorem. In Section 5, we evaluate the $\mathcal{O}(G_F^2 m_t^4)$ corrections numerically and compare them with the $\mathcal{O}(\alpha_s G_F m_t^2)$ ones. We conclude with a summary in Section 6.

2 Renormalisation procedure

For the reader's convenience, we present in this section the details of the renormalisation procedure which has to be carried out. We derive general expressions for the mass counterterms and wave-function renormalisation constants in the on-shell scheme, valid for any number of loops. Furthermore, we derive the tadpole renormalisation counterterms and describe the treatment of the corrections due to external legs. In our calculations, we do not need to consider electric-charge renormalisation constants, because, to the orders we consider here, there are no such contributions.

Before going into details, we would like to mention that the expressions for the mass and wave-function renormalisation constants to be derived here are only valid for stable particles. Instable particles do have complex self-energy amplitudes, so that their resummed propagators have complex poles. In that case, the renormalisation conditions are more complicated (see, for instance, Ref. [23]). Since all self-energy amplitudes appearing in the calculations of this paper are real, we can restrict ourselves to the case of stable particles.

2.1 Mass and wave-function renormalisation

We write the bare masses in the Lagrangian as sums of the renormalised ones and the mass counterterms. In the on-shell scheme, we fix this splitting by the requirement that the renormalised masses are identical to the poles of the propagators including all radiative

corrections. Furthermore, the wave-function renormalisation constants are obtained as the residues of the propagators at their poles.

2.1.1 Higgs-boson mass and wave-function renormalisation

For the amputated one-particle-irreducible self-energy of the Higgs boson, we write

$$\text{---} \frac{H}{q} \text{---} \left(\text{1-PI} \right) \text{---} \frac{H}{q} \text{---} = i\Sigma_H(q^2). \quad (1)$$

Thus, the dressed propagator, including all radiative corrections, becomes

$$\begin{aligned} S_H^{-1}(q^2) &= \text{---} \dots \text{---} + \text{---} \left(\text{1-PI} \right) \text{---} \dots + \text{---} \left(\text{1-PI} \right) \text{---} \left(\text{1-PI} \right) \text{---} \dots + \dots \\ &= \frac{i}{q^2 - M_{H,0}^2} \sum_{n=0}^{\infty} \left(i\Sigma_H(q^2) \frac{i}{q^2 - M_{H,0}^2} \right)^n \\ &= \frac{i}{q^2 - M_{H,0}^2 + \Sigma_H(q^2)}. \end{aligned} \quad (2)$$

The on-shell renormalisation condition reads

$$S_H(M_H^2) \stackrel{!}{=} 0. \quad (3)$$

Writing the bare mass of the Higgs boson as the sum of the renormalised mass and a counterterm, $M_{H,0}^2 = M_H^2 + \delta M_H^2$, we have

$$\delta M_H^2 = \Sigma_H(M_H^2). \quad (4)$$

Here and in the following, it is understood that, in the expression for a counterterm, all bare quantities have to be replaced by the renormalised ones plus the respective counter-terms. In the case of the Higgs-boson mass counterterm, this means that $\Sigma_H(M_H^2)$ has to be expressed in terms of renormalised quantities. For higher-order expressions, this has to be done iteratively.

Expanding Eq. (2) about $q^2 = M_H^2$ and taking the limit $q^2 \rightarrow M_H^2$,

$$\begin{aligned} S_H^{-1}(q^2) &= \frac{i}{q^2 - M_H^2} \frac{1}{1 + \Sigma'_H(M_H^2) + \mathcal{O}(q^2 - M_H^2)} \\ &\xrightarrow{q^2 \rightarrow M_H^2} \frac{iZ_H}{q^2 - M_H^2}, \end{aligned} \quad (5)$$

we read off the Higgs-boson wave-function renormalisation constant as

$$Z_H = \frac{1}{1 + \Sigma'_H(M_H^2)}. \quad (6)$$

Writing $Z_H = 1 + \delta Z_H$ and performing a loop expansion of Eq. (6), we have

$$\delta Z_H^{(1)} = -\Sigma_H^{(1)\prime}(M_H^2), \quad (7)$$

$$\delta Z_H^{(2)} = -\Sigma_H^{(2)\prime}(M_H^2) + \left(\Sigma_H^{(1)\prime}(M_H^2)\right)^2. \quad (8)$$

Here and in the following, numbers placed in parentheses as superscripts specify the loop order of the perturbative expression.

2.1.2 Fermion mass and wave-function renormalisation

The amputated one-particle-irreducible self-energy of fermion f has the form

$$\frac{f}{q} \rightarrow [1\text{-PI}] \rightarrow \frac{f}{q} = i\Sigma_f(q) = iq\omega_-\Sigma_{f,L}(q^2) + iq\omega_+\Sigma_{f,R}(q^2) + im_{f,0}\Sigma_{f,S}(q^2), \quad (9)$$

where $m_{f,0}$ is the bare mass of fermion f and $\omega_{\pm} = (1 \pm \gamma_5)/2$ are the projectors onto the helicity eigenstates.

The fermion field f is composed of left- and right-handed components, l and r , respectively, as

$$f = l + r, \quad l = \omega_- f, \quad r = \omega_+ f. \quad (10)$$

In the electroweak theory, l and r interact differently, which has to be accounted for in the renormalisation procedure. In terms of these components, the purely fermionic part of the SM Lagrangian reads:

$$\mathcal{L} = \bar{f}(i\cancel{\partial} - m_{f,0})f = i\bar{l}\cancel{\partial}l + i\bar{r}\cancel{\partial}r - m_{f,0}\bar{r}l - m_{f,0}\bar{l}r. \quad (11)$$

We see that l and r are massless fermion fields with propagators

$$\frac{l}{q} = \frac{r}{q} = \frac{i}{\cancel{q}}. \quad (12)$$

In addition, we have the following $r-l$ transition vertices:

$$\frac{l}{q} \bullet \frac{r}{q} = \frac{r}{q} \bullet \frac{l}{q} = -im_{f,0}. \quad (13)$$

From Eq. (9), we read off the amputated one-particle-irreducible self-energies pertaining to the four different helicity combinations as

$$\begin{aligned} \frac{l}{q} \rightarrow [1\text{-PI}] \rightarrow \frac{l}{q} &= iq\Sigma_{f,L}(q^2), \\ \frac{r}{q} \rightarrow [1\text{-PI}] \rightarrow \frac{r}{q} &= iq\Sigma_{f,R}(q^2), \\ \frac{l}{q} \rightarrow [1\text{-PI}] \rightarrow \frac{r}{q} &= \frac{r}{q} \rightarrow [1\text{-PI}] \rightarrow \frac{l}{q} = im_{f,0}\Sigma_{f,S}(q^2). \end{aligned} \quad (14)$$

Note that above expressions do not yet include the tree-level contributions from Eqs. (12) and (13). Equations (12)–(14) are the ingredients out of which we construct the propagators of the left- and right-handed fields including all radiative corrections. This is done in close analogy to the case of γ - Z -mixing (see, e.g., Ref. [24]). To this end, we introduce the propagator-type symbols

$$\begin{aligned} \overline{\frac{l}{\not{q}}} \longrightarrow &:= \overline{l} \longrightarrow + \overline{l} \longrightarrow \text{1-PI} \longrightarrow + \overline{l} \longrightarrow \text{1-PI} \longrightarrow \text{1-PI} \longrightarrow + \dots \\ &= \frac{i}{\not{q}} \sum_{n=0}^{\infty} \left(i \not{q} \Sigma_{f,L}(q^2) \frac{i}{\not{q}} \right)^n = \frac{i}{\not{q} (1 + \Sigma_{f,L}(q^2))}, \\ \overline{\frac{r}{\not{q}}} \longrightarrow &:= \overline{r} \longrightarrow + \overline{r} \longrightarrow \text{1-PI} \longrightarrow + \overline{r} \longrightarrow \text{1-PI} \longrightarrow \text{1-PI} \longrightarrow + \dots \\ &= \frac{i}{\not{q}} \sum_{n=0}^{\infty} \left(i \not{q} \Sigma_{f,R}(q^2) \frac{i}{\not{q}} \right)^n = \frac{i}{\not{q} (1 + \Sigma_{f,R}(q^2))}, \end{aligned} \quad (15)$$

and the vertex-type symbols

$$\begin{aligned} \overline{l} \longrightarrow \odot \longrightarrow r &:= \overline{l} \longrightarrow \bullet \longrightarrow r + \overline{l} \longrightarrow \text{1-PI} \longrightarrow r = i m_{f,0} (\Sigma_{f,S}(q^2) - 1), \\ \overline{r} \longrightarrow \odot \longrightarrow l &:= \overline{r} \longrightarrow \bullet \longrightarrow l + \overline{r} \longrightarrow \text{1-PI} \longrightarrow l = i m_{f,0} (\Sigma_{f,S}(q^2) - 1). \end{aligned} \quad (16)$$

Next, we evaluate the dressed propagator of the left-handed fermion field, including all radiative corrections, as

$$\begin{aligned} S_u^{-1}(q) &= \overline{\frac{l}{\not{q}}} \longrightarrow + \overline{\frac{l}{\not{q}}} \odot \longrightarrow \odot \longrightarrow \overline{\frac{l}{\not{q}}} \longrightarrow + \overline{\frac{l}{\not{q}}} \odot \longrightarrow \odot \longrightarrow \overline{\frac{l}{\not{q}}} \odot \longrightarrow \odot \longrightarrow \overline{\frac{l}{\not{q}}} \longrightarrow + \dots \\ &= \frac{i}{\not{q} (1 + \Sigma_{f,L}(q^2))} \sum_{n=0}^{\infty} \left[i m_{f,0} (\Sigma_{f,S}(q^2) - 1) \frac{i}{\not{q} (1 + \Sigma_{f,R}(q^2))} i m_{f,0} (\Sigma_{f,S}(q^2) - 1) \right. \\ &\quad \times \left. \frac{i}{\not{q} (1 + \Sigma_{f,L}(q^2))} \right]^n \\ &= \frac{i \not{q}}{1 + \Sigma_{f,L}(q^2)} \frac{1}{q^2 - m_{f,0}^2 f(q^2)}, \end{aligned} \quad (17)$$

where

$$f(q^2) = \frac{(1 - \Sigma_{f,S}(q^2))^2}{(1 + \Sigma_{f,L}(q^2))(1 + \Sigma_{f,R}(q^2))}. \quad (18)$$

In a similar way, we find the dressed propagator of the right-handed fermion field, including all radiative corrections, to be

$$S_{rr}^{-1}(q) = \frac{i \not{q}}{1 + \Sigma_{f,R}(q^2)} \frac{1}{q^2 - m_{f,0}^2 f(q^2)}. \quad (19)$$

For completeness, we also resum the loop contributions by which a left-handed field converts into a right-handed one and vice versa. Proceeding similarly as in Eq. (17), we obtain

$$\begin{aligned}
S_{lr}^{-1}(q) &= \overrightarrow{\bullet} \circlearrowleft \overrightarrow{\bullet} + \overrightarrow{\bullet} \circlearrowleft \overrightarrow{\bullet} \circlearrowleft \overrightarrow{\bullet} + \dots \\
&= \frac{i}{q(1 + \Sigma_{f,L}(q^2))} im_{f,0} (\Sigma_{f,S}(q^2) - 1) \frac{i}{q(1 + \Sigma_{f,R}(q^2))} \\
&\quad \times \sum_{n=0}^{\infty} \left[im_{f,0} (\Sigma_{f,S}(q^2) - 1) \frac{i}{q(1 + \Sigma_{f,L}(q^2))} im_{f,0} (\Sigma_{f,S}(q^2) - 1) \right. \\
&\quad \times \left. \frac{i}{q(1 + \Sigma_{f,R}(q^2))} \right]^n \\
&= \frac{im_{f,0}(1 - \Sigma_{f,S}(q^2))}{(1 + \Sigma_{f,L}(q^2))(1 + \Sigma_{f,R}(q^2))} \frac{1}{q^2 - m_{f,0}^2 f(q^2)}. \tag{20}
\end{aligned}$$

Since Eq. (20) is symmetric under the interchange of the indices L and R , we also have

$$S_{rl}^{-1}(q) = S_{lr}^{-1}(q). \tag{21}$$

We now derive the fermion mass counterterm. Writing $m_{f,0} = m_f + \delta m_f$, where m_f is the renormalised mass and δm_f is the mass counterterm, and imposing the on-shell renormalisation condition,

$$S_{ij}(q) u_f(q)|_{q^2=m_f^2} \stackrel{!}{=} 0, \tag{22}$$

where $ij = ll, rr, lr, rl$ and $u_f(q)$ is the spinor of the incoming fermion f , we obtain

$$\frac{\delta m_f}{m_f} = \frac{1}{\sqrt{f(m_f^2)}} - 1. \tag{23}$$

Expanding Eq. (23), we find the explicit one- and two-loop expressions,

$$\frac{\delta m_f^{(1)}}{m_f} = \frac{1}{2} \Sigma_{f,L}^{(1)}(m_f^2) + \frac{1}{2} \Sigma_{f,R}^{(1)}(m_f^2) + \Sigma_{f,S}^{(1)}(m_f^2), \tag{24}$$

$$\begin{aligned}
\frac{\delta m_f^{(2)}}{m_f} &= \frac{1}{2} \Sigma_{f,L}^{(2)}(m_f^2) + \frac{1}{2} \Sigma_{f,R}^{(2)}(m_f^2) + \Sigma_{f,S}^{(2)}(m_f^2) - \frac{1}{8} \left(\Sigma_{f,L}^{(1)}(m_f^2) - \Sigma_{f,R}^{(1)}(m_f^2) \right)^2 \\
&\quad + \Sigma_{f,S}^{(1)}(m_f^2) \frac{\delta m_f^{(1)}}{m_f}. \tag{25}
\end{aligned}$$

The one-loop expression of Eq. (24) is well known (see, e.g., Ref. [6]). The two-loop expression of Eq. (25) agrees with the one obtained in Ref. [25] using an alternative procedure.

Finally, we derive the wave-function renormalisation constants for the left-handed and right-handed fields. Expanding Eqs. (17) and (19)–(21) about $q = m_f$ and taking the limit $q \rightarrow m_f$, we have

$$\begin{aligned} S_{ll/rr}^{-1}(q) &= \frac{iq}{q^2 - m_f^2} \frac{1}{\left(1 + \Sigma_{f,L/R}(m_f^2)\right) \left(1 - m_f^2 \frac{f'(m_f^2)}{f(m_f^2)}\right) + \mathcal{O}(q^2 - m_f^2)} \\ &\xrightarrow{q^2 \rightarrow m_f^2} \frac{iZ_{f,L/R}}{q^2 - m_f^2}, \\ S_{lr/rl}^{-1}(q) &= \frac{im_f}{q^2 - m_f^2} \frac{1}{\sqrt{\left(1 + \Sigma_{f,L}(m_f^2)\right) \left(1 + \Sigma_{f,R}(m_f^2)\right)}} \left(1 - m_f^2 \frac{f'(m_f^2)}{f(m_f^2)}\right) + \mathcal{O}(q^2 - m_f^2) \\ &\xrightarrow{q^2 \rightarrow m_f^2} \frac{im_f \sqrt{Z_{f,L} Z_{f,R}}}{q^2 - m_f^2}, \end{aligned} \quad (26)$$

where

$$Z_{f,L/R} = \frac{1}{\left(1 + \Sigma_{f,L/R}(m_f^2)\right) \left(1 - m_f^2 \frac{f'(m_f^2)}{f(m_f^2)}\right)}. \quad (27)$$

Writing $Z_{f,L/R} = 1 + \delta Z_{f,L/R}$ and performing a loop expansion of Eq. (27), we have

$$\delta Z_{f,L}^{(1)} = -\Sigma_L^{(1)} - \Sigma_L^{(1)\prime} - \Sigma_R^{(1)\prime} - 2\Sigma_S^{(1)\prime}, \quad (28)$$

$$\delta Z_{f,R}^{(1)} = -\Sigma_R^{(1)} - \Sigma_L^{(1)\prime} - \Sigma_R^{(1)\prime} - 2\Sigma_S^{(1)\prime}, \quad (29)$$

$$\begin{aligned} \delta Z_{f,L}^{(2)} &= -\Sigma_L^{(2)} - \Sigma_L^{(2)\prime} - \Sigma_R^{(2)\prime} - 2\Sigma_S^{(2)\prime} + \Sigma_L^{(1)} \left(\Sigma_L^{(1)} + 2\Sigma_L^{(1)\prime} + \Sigma_R^{(1)\prime} + 2\Sigma_S^{(1)\prime}\right) \\ &\quad + \Sigma_R^{(1)} \Sigma_R^{(1)\prime} - 2\Sigma_S^{(1)} \Sigma_S^{(1)\prime} + \left(\Sigma_L^{(1)\prime} + \Sigma_R^{(1)\prime} + 2\Sigma_S^{(1)\prime}\right)^2, \end{aligned} \quad (30)$$

$$\begin{aligned} \delta Z_{f,R}^{(2)} &= -\Sigma_R^{(2)} - \Sigma_L^{(2)\prime} - \Sigma_R^{(2)\prime} - 2\Sigma_S^{(2)\prime} + \Sigma_R^{(1)} \left(\Sigma_R^{(1)} + \Sigma_L^{(1)\prime} + 2\Sigma_R^{(1)\prime} + 2\Sigma_S^{(1)\prime}\right) \\ &\quad + \Sigma_L^{(1)} \Sigma_L^{(1)\prime} - 2\Sigma_S^{(1)} \Sigma_S^{(1)\prime} + \left(\Sigma_L^{(1)\prime} + \Sigma_R^{(1)\prime} + 2\Sigma_S^{(1)\prime}\right)^2. \end{aligned} \quad (31)$$

Here, we used the abbreviations

$$\begin{aligned} \Sigma_X^{(n)} &= \Sigma_{f,X}^{(n)}(m_f^2), \\ \Sigma_X^{(n)\prime} &= m_f^2 \frac{\partial}{\partial q^2} \Sigma_{f,X}^{(n)}(q^2) \Big|_{q^2=m_f^2}, \end{aligned} \quad (32)$$

where $X = L, R, S$. These expressions again agree with Refs. [6,25].

If parity was conserved, we would have $\Sigma_L^f(q^2) = \Sigma_R^f(q^2)$ and thus recover the structure

$$S_f^{-1}(q) \xrightarrow{q^2 \rightarrow m_f^2} \frac{iZ_f}{q - m_f}, \quad (33)$$

which is familiar from quantum electrodynamics.

2.1.3 W -boson mass renormalisation

The amputated one-particle-irreducible self-energy of the W boson can be decomposed into a transverse and a longitudinal part as

$$\begin{array}{c} W_\mu \\ \sim\!\!\! \sim\!\!\! \sim\!\!\! \sim \\ \overrightarrow{q} \end{array} \left(\text{1-PI} \right) \begin{array}{c} W_\nu \\ \sim\!\!\! \sim\!\!\! \sim\!\!\! \sim \\ \end{array} = -i\Pi_W^{\mu\nu}(q) = -i(\Delta^{\mu\nu}\Sigma_{W,T}(q^2) + q^{\mu\nu}\Sigma_{W,L}(q^2)), \quad (34)$$

where

$$\begin{aligned} \Delta^{\mu\nu} &= g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}, \\ q^{\mu\nu} &= \frac{q^\mu q^\nu}{q^2}. \end{aligned} \quad (35)$$

Owing to the loop-induced mixing of the W boson with the charged Higgs-Kibble ghost ϕ , we must also take into account the one-particle-irreducible $W \leftrightarrow \phi$ transition amplitudes and the one-particle-irreducible ϕ -boson self-energy,

$$\begin{array}{c} W_\mu \\ \sim\!\!\! \sim\!\!\! \sim\!\!\! \sim \\ \overrightarrow{q} \end{array} \left(\text{1-PI} \right) \begin{array}{c} \phi \\ \dash\dash\dash \\ \end{array} = iq^\mu\Sigma_{W\phi}(q^2), \\ \begin{array}{c} \phi \\ \dash\dash\dash \\ \overrightarrow{q} \end{array} \left(\text{1-PI} \right) \begin{array}{c} W_\mu \\ \sim\!\!\! \sim\!\!\! \sim\!\!\! \sim \\ \end{array} = -iq^\mu\Sigma_{W\phi}(q^2), \\ \begin{array}{c} \phi \\ \dash\dash\dash \\ \overrightarrow{q} \end{array} \left(\text{1-PI} \right) \begin{array}{c} \phi \\ \dash\dash\dash \\ \end{array} = i\Sigma_\phi(q^2). \end{math>$$

In 't Hooft-Feynman gauge, the bare propagators of the W and ϕ bosons are given by

$$G_W^{\mu\nu}(q^2) = \frac{-ig^{\mu\nu}}{q^2 - M_{W,0}^2}, \quad (37)$$

$$G_\phi(q^2) = \frac{i}{q^2 - M_{W,0}^2}, \quad (38)$$

with a common bare mass $M_{W,0}$. In order to obtain the dressed W -boson propagator, we proceed in two steps. In the first step, we resum the one-particle irreducible self-energies of the W and ϕ bosons separately. In the second step, we systematically combine these results by accommodating all possible $W \leftrightarrow \phi$ transitions.

The resummation of the one-particle irreducible W -boson self-energy leads to

$$\begin{aligned} \sim\!\!\! \sim\!\!\! \sim\!\!\! \sim \stackrel{W}{\sim\!\!\! \sim\!\!\! \sim\!\!\! \sim} &:= \sim\!\!\! \sim\!\!\! \sim\!\!\! \sim + \sim\!\!\! \sim\!\!\! \sim\!\!\! \sim \left(\text{1-PI} \right) \sim\!\!\! \sim\!\!\! \sim + \sim\!\!\! \sim\!\!\! \sim \left(\text{1-PI} \right) \sim\!\!\! \sim\!\!\! \sim \left(\text{1-PI} \right) \sim\!\!\! \sim\!\!\! \sim + \dots \\ &= G_{W,\mu\nu}(q^2) + G_{W,\mu\alpha}(q^2) \left(-i\Pi_W^{\alpha\beta}(q) \right) G_{W,\beta\nu}(q^2) \\ &\quad + G_{W,\mu\alpha}(q^2) \left(-i\Pi_W^{\alpha\beta}(q) \right) G_{W,\beta\gamma}(q^2) \left(-i\Pi_W^{\gamma\delta}(q) \right) G_{W,\delta\nu}(q^2) + \dots \end{aligned} \quad (39)$$

The series in Eq. (39) may be resummed by inserting Eqs. (34) and (37) and exploiting the identities

$$\begin{aligned} \Delta^\mu_{\nu\rho}\Delta^\nu_{\rho} &= \Delta^\mu_{\rho}, \\ \Delta^{\mu\nu}q_{\nu\rho} &= 0, \\ q^{\mu\nu}q_{\nu\rho} &= q^\mu_{\rho}, \end{aligned} \tag{40}$$

as follows

$$\begin{aligned}
& \sim \tilde{\sim}^W = G_{W,\mu\alpha}(q^2) \left[g^\alpha_\nu - \frac{\Delta^\alpha_\nu \Sigma_{W,T}(q^2) + q^\alpha_\nu \Sigma_{W,L}(q^2)}{q^2 - M_{W,0}^2} \right. \\
& \quad \left. + \frac{\Delta^\alpha_\nu (\Sigma_{W,T}(q^2))^2 + q^\alpha_\nu (\Sigma_{W,L}(q^2))^2}{(q^2 - M_{W,0}^2)^2} - \dots \right] \\
& = G_{W,\mu\alpha}(q^2) \left[g^\alpha_\nu + \Delta^\alpha_\nu \sum_{n=1}^{\infty} \left(\frac{-\Sigma_{W,T}(q^2)}{q^2 - M_{W,0}^2} \right)^n + q^\alpha_\nu \sum_{n=1}^{\infty} \left(\frac{-\Sigma_{W,L}(q^2)}{q^2 - M_{W,0}^2} \right)^n \right] \\
& = G_{W,\mu\alpha}(q^2) \left[g^\alpha_\nu + \Delta^\alpha_\nu \left(\frac{1}{1 + \frac{\Sigma_{W,T}(q^2)}{q^2 - M_{W,0}^2}} - 1 \right) + q^\alpha_\nu \left(\frac{1}{1 + \frac{\Sigma_{W,L}(q^2)}{q^2 - M_{W,0}^2}} - 1 \right) \right] \\
& = -i \frac{\Delta_{\mu\nu}}{q^2 - M_{W,0}^2 + \Sigma_{W,T}(q^2)} - i \frac{q_{\mu\nu}}{q^2 - M_{W,0}^2 + \Sigma_{W,L}(q^2)} \\
& = (S_{W,\text{pure}}^{-1})_{\mu\nu}(q). \tag{41}
\end{aligned}$$

The resummation of the one-particle-irreducible ϕ -boson self-energy proceeds in analogy to the Higgs-boson case discussed in Section 2.1.1 and yields

The contribution of unmixed W -boson propagation in Eq. (41) needs to be complemented by the contribution that emerges by combining it with the contribution of unmixed ϕ -boson propagation of Eq. (42) via the one-particle-irreducible $W \leftrightarrow \phi$ transition

amplitudes in all possible ways. This additional contribution is given by

$$\begin{aligned}
(S_{W,\text{mix}}^{-1})_{\mu\nu}(q) &= \overbrace{\text{---}}^W \text{---} \overset{\phi}{\textcircled{1PI}} \text{---} \overbrace{\text{---}}^W \text{---} \overset{\phi}{\textcircled{1PI}} \text{---} \overbrace{\text{---}}^W \text{---} \overset{\phi}{\textcircled{1PI}} \text{---} \overbrace{\text{---}}^W \text{---} \dots \\
&= \frac{q_\mu \Sigma_{W\phi}(q^2)}{q^2 - M_{W,0}^2 + \Sigma_{W,L}(q^2)} \frac{i}{q^2 - M_{W,0}^2 + \Sigma_\phi(q^2)} \\
&\quad \times \sum_{n=0}^{\infty} \left(\frac{q^2 (\Sigma_{W\phi}(q^2))^2}{(q^2 - M_{W,0}^2 + \Sigma_{W,L}(q^2)) (q^2 - M_{W,0}^2 + \Sigma_\phi(q^2))} \right)^n \\
&\quad \times \frac{-q_\nu \Sigma_{W\phi}(q^2)}{q^2 - M_{W,0}^2 + \Sigma_{W,L}(q^2)} \\
&= \frac{-iq_{\mu\nu}}{q^2 - M_{W,0}^2 + \Sigma_{W,L}(q^2)} \frac{-q^2}{q^2 - \frac{(q^2 - M_{W,0}^2 + \Sigma_{W,L}(q^2))(q^2 - M_{W,0}^2 + \Sigma_\phi(q^2))}{(\Sigma_{W\phi}(q^2))^2}}. \quad (43)
\end{aligned}$$

Adding Eqs. (41) and (43), we obtain the fully dressed W -boson propagator as

$$(S_W^{-1})_{\mu\nu}(q) = (S_{W,\text{pure}}^{-1})_{\mu\nu}(q) + (S_{W,\text{mix}}^{-1})_{\mu\nu}(q). \quad (44)$$

Its inverse is found to be

$$S_W^{\mu\nu}(q) = ig^{\mu\nu}(q^2 - M_{W,0}^2) + i\Delta^{\mu\nu}\Sigma_{W,T}(q^2) + iq^{\mu\nu} \left(\Sigma_{W,L}(q^2) - \frac{q^2 (\Sigma_{W\phi}(q^2))^2}{q^2 - M_{W,0}^2 + \Sigma_\phi(q^2)} \right). \quad (45)$$

The on-shell renormalisation condition reads

$$S_W^{\mu\nu}(q^2)\epsilon_{W,\nu}(q)|_{q^2=M_W^2} \stackrel{!}{=} 0, \quad (46)$$

where $\epsilon_W^\mu(q)$ is the polarisation four-vector of an external W boson. Writing $M_{W,0}^2 = M_W^2 + \delta M_W^2$ and exploiting the transversality property $q^\mu\epsilon_{W,\mu}(q) = 0$, we finally have

$$\delta M_W^2 = \Sigma_{W,T}(M_W^2). \quad (47)$$

We note in passing that Eq. (47) is not influenced by $W \leftrightarrow \phi$ mixing.

2.2 External-leg corrections

In this section, we discuss the structure of the amputated matrix element \mathcal{A} for the decay process $H \rightarrow b\bar{b}$ and explain how to obtain from it the transition matrix element \mathcal{T} by incorporating the wave-function renormalisation constants.

The general form of \mathcal{A} reads

$$\begin{aligned}
&-\frac{H}{q_1+q_2} - \text{Amp.} = i\mathcal{A} \\
&= i \left(\mathcal{A}_1 + q_1\mathcal{A}_2 + q_2\mathcal{A}_3 + q_2q_1\mathcal{A}_4 + \gamma_5\mathcal{A}_5 + \gamma_5q_1\mathcal{A}_6 + \gamma_5q_2\mathcal{A}_7 + \gamma_5q_2q_1\mathcal{A}_8 \right), \quad (48)
\end{aligned}$$

where q_1 and q_2 are the four-momenta of the outgoing \bar{b} and b quarks, respectively, and \mathcal{A}_i ($i = 1, \dots, 8$) are scalar form factors. Projecting onto each of these form factors, we observe that, to the orders we consider in this paper, only two of them are independent. In fact, we have

$$\begin{aligned}\mathcal{A}_2 &= -\mathcal{A}_3 = \mathcal{A}_6 = -\mathcal{A}_7, \\ \mathcal{A}_4 &= \mathcal{A}_5 = \mathcal{A}_8 = 0,\end{aligned}\tag{49}$$

so that \mathcal{A} collapses to the simple form

$$\mathcal{A} = \mathcal{A}_A + \mathcal{A}_B \left(q_2 - q_1 \right) \omega_-, \tag{50}$$

where $\mathcal{A}_A = \mathcal{A}_1$ and $\mathcal{A}_B = -2\mathcal{A}_2$.

Then, \mathcal{T} is obtained by dressing \mathcal{A} with the renormalised wave functions of the external legs as

$$\begin{aligned}\mathcal{T} &= \sqrt{Z_H} \left(\sqrt{Z_{b,R}} \bar{u}_r(q_2, r_2) + \sqrt{Z_{b,L}} \bar{u}_l(q_2, r_2) \right) \mathcal{A} \left(\sqrt{Z_{b,R}} v_r(q_1, r_1) + \sqrt{Z_{b,L}} v_l(q_1, r_1) \right) \\ &= \sqrt{Z_H} \bar{u}_b(q_2, r_2) \left(\sqrt{Z_{b,R}} \omega_- + \sqrt{Z_{b,L}} \omega_+ \right) \mathcal{A} \left(\sqrt{Z_{b,R}} \omega_+ + \sqrt{Z_{b,L}} \omega_- \right) v_b(q_1, r_1),\end{aligned}\tag{51}$$

where $v_b(q_1, r_1)$ and $\bar{u}_b(q_2, r_2)$ denote the spinors of the outgoing \bar{b} and b quarks with spins r_1 and r_2 , respectively. Inserting Eq. (50) into Eq. (51), we obtain the master formula

$$\mathcal{T} = \sqrt{Z_H} \left(\sqrt{Z_{b,L} Z_{b,R}} \mathcal{A}_A + m_b Z_{b,L} \mathcal{A}_B \right) \bar{u}_b(q_2, r_2) v_b(q_1, r_1). \tag{52}$$

Note, that the terms involving γ_5 vanish upon application of the Dirac equation.

2.3 Tadpole renormalisation

As is well known (see, for instance, Ref. [26]), one can introduce a so-called tadpole renormalisation in order to avoid the calculation of diagrams containing tadpoles. For the reader's convenience, in this section, we rederive the counterterm vertices of the tadpole renormalisation along with the counterterm vertices of the Higgs-boson mass renormalisation.

The tadpole renormalisation concerns only the Higgs part of the SM Lagrangian,

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) + \mu^2 \Phi^\dagger \Phi - \frac{\lambda}{4} (\Phi^\dagger \Phi)^2, \tag{53}$$

where Φ is a weak-isospin doublet of two complex scalar fields. The free parameters, μ and λ , are chosen in such a way that one stays with a non-vanishing vacuum expectation value v , which is defined by

$$\frac{v^2}{2} = |\langle 0 | \Phi(x) | 0 \rangle|^2 = \frac{2\mu^2}{\lambda}. \tag{54}$$

If we parameterise

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}(v + H(x) + i\chi(x)) \end{pmatrix} \quad (55)$$

and substitute μ and λ by

$$\begin{aligned} t &= v \left(\mu^2 - \frac{\lambda v^2}{4} \right), \\ M_H^2 &= -\mu^2 + \frac{3\lambda v^2}{4}, \end{aligned} \quad (56)$$

Eq. (53) takes the form

$$\begin{aligned} \mathcal{L}_{\text{Higgs}} &= \frac{1}{2}(D_\mu H)(D^\mu H) + \frac{1}{2}(D_\mu \chi)(D^\mu \chi) + (D_\mu \phi^-)(D^\mu \phi^+) + tH - \frac{M_H^2}{2}H^2 \\ &\quad + \frac{t}{2v}(\chi^2 + 2\phi^-\phi^+) - \frac{1}{2v} \left(\frac{t}{v} + M_H^2 \right) H (H^2 + \chi^2 + 2\phi^-\phi^+) \\ &\quad - \frac{1}{8v^2} \left(\frac{t}{v} + M_H^2 \right) (H^2 + \chi^2 + 2\phi^-\phi^+)^2, \end{aligned} \quad (57)$$

where $\phi^- = (\phi^+)^\dagger$. We see that M_H has the physical meaning of the Higgs-boson mass. In this step, we did not exploit Eq. (54), which implies that $t = 0$, so that we could just have emitted all terms containing t . However, as was argued above, it is useful to keep them and to renormalise t along with M_H^2 by substituting

$$\begin{aligned} t &\rightarrow t_0 = 0 + \delta t, \\ M_H^2 &\rightarrow M_{H,0}^2 = M_H^2 + \delta M_H^2 \end{aligned} \quad (58)$$

in Eq. (57). Notice that Eq. (57) represents a bare Lagrangian, so that v , t , and M_H are actually bare parameters. For consistency, we thus also substitute $v \rightarrow v_0$. Then, Eq. (57) becomes

$$\begin{aligned} \mathcal{L}_{\text{Higgs}} &= \frac{1}{2}(D_\mu H)(D^\mu H) + \frac{1}{2}(D_\mu \chi)(D^\mu \chi) + (D_\mu \phi^-)(D^\mu \phi^+) - \frac{M_H^2}{2}H^2 \\ &\quad - \frac{M_H^2}{2v_0}H(H^2 + \chi^2 + 2\phi^-\phi^+) - \frac{M_H^2}{8v_0^2}(H^2 + \chi^2 + 2\phi^-\phi^+)^2 \\ &\quad + \delta t H - \frac{\delta M_H^2}{2}H^2 + \frac{\delta t}{2v_0}(\chi^2 + 2\phi^-\phi^+) - \frac{1}{2v_0} \left(\frac{\delta t}{v_0} + \delta M_H^2 \right) H \\ &\quad \times (H^2 + \chi^2 + 2\phi^-\phi^+) - \frac{1}{8v_0^2} \left(\frac{\delta t}{v_0} + \delta M_H^2 \right) (H^2 + \chi^2 + 2\phi^-\phi^+)^2. \end{aligned} \quad (59)$$

From the terms proportional to δt and δM_H^2 , we can read off the desired counterterm vertices, which we list in Table 1.

Table 1: Counterterm vertices related to the Higgs-boson tadpole and mass renormalisation.

$H:$	$i\delta t$	$HHHH: -i\frac{3}{v_0^2} \left(\frac{\delta t}{v_0} + \delta M_H^2 \right)$
$HH:$	$-i\delta M_H^2$	$\chi\chi\chi\chi: -i\frac{3}{v_0^2} \left(\frac{\delta t}{v_0} + \delta M_H^2 \right)$
$\chi\chi:$	$i\frac{\delta t}{v_0}$	$HH\chi\chi: -i\frac{1}{v_0^2} \left(\frac{\delta t}{v_0} + \delta M_H^2 \right)$
$\phi\phi:$	$i\frac{\delta t}{v_0}$	$HH\phi\phi: -i\frac{1}{v_0^2} \left(\frac{\delta t}{v_0} + \delta M_H^2 \right)$
$HHH:$	$-i\frac{3}{v_0} \left(\frac{\delta t}{v_0} + \delta M_H^2 \right)$	$\chi\chi\phi\phi: -i\frac{1}{v_0^2} \left(\frac{\delta t}{v_0} + \delta M_H^2 \right)$
$H\chi\chi:$	$-i\frac{1}{v_0} \left(\frac{\delta t}{v_0} + \delta M_H^2 \right)$	$\phi\phi\phi\phi: -i\frac{2}{v_0^2} \left(\frac{\delta t}{v_0} + \delta M_H^2 \right)$
$H\phi\phi:$	$-i\frac{1}{v_0} \left(\frac{\delta t}{v_0} + \delta M_H^2 \right)$	

The Higgs-boson mass renormalisation condition was already discussed in Section 2.1.1. As a renormalisation condition for δt , we set

$$\delta t \stackrel{!}{=} -T, \quad (60)$$

where T stands for the sum of all amputated one-particle-irreducible tadpole diagrams,

$$--\overset{H}{\circ}--\overset{\text{1-PI}}{\circ}=iT. \quad (61)$$

As can be seen from Table 1, there is a one-point Higgs-boson counterterm vertex, $i\delta t$, that forces a cancellation with all diagrams having a tadpole at its place. Therefore, upon tadpole renormalisation, one does not have to consider tadpole diagrams anymore. However, now one has to take into account all the tadpole counterterm vertices in Table 1, except for the one mentioned above.

3 Results

In this section, we present the details of our actual calculations. After making some general remarks, we describe in Sections 3.1, 3.2, and 3.3 the explicit computation of the decay rate at tree level, at the one-loop order $\mathcal{O}(G_F m_t^2)$, and at the two-loop order

$\mathcal{O}(G_F^2 m_t^4)$, respectively. Section 3.1 also contains the expressions for the renormalisation constants at order $\mathcal{O}(G_F m_t^2)$, which are needed in the one-loop and two-loop calculations.

In order to compute the leading large- m_t contributions of the various two-loop diagrams, we apply the asymptotic-expansion technique (for a careful introduction, see Ref. [27]). However, it turns out that all non-trivial contributions of the self-energy and $Hb\bar{b}$ vertex diagrams (see Figs. 4, 5, and 6), which are of leading order in m_t , cancel among themselves or, in case of the W -boson self-energy, in combination with complete counterterm diagrams arising from the Higgs-boson tadpole and mass renormalisations. Specifically, in Fig. 4, there are non-naive contributions due to the asymptotic expansion of diagrams (i)–(o) that cancel against diagrams (p)–(v); in Fig. 5, the non-naive contributions of diagrams (a) and (t) cancel; and in Fig. 6 those of the diagrams (e) and (i) cancel. After these cancellations, only naive contributions due to diagrams involving top-quark propagators remain. Therefore, we can naively expand in all masses and momenta except for the top-quark mass and retain only the leading terms. Obviously, this requires the Higgs-boson mass to be smaller than the top-quark mass, which is compatible with the intermediate-mass range of the Higgs boson, as mentioned in the Introduction.

The ultraviolet divergences which have to disappear in the final expression for the decay rate are cancelled through the application of the renormalisation procedure, which we carry out in the on-mass-shell renormalisation scheme. This provides a non-trivial check for our calculations. As explained in Section 2.3, we use the counterterm vertices of Table 1 for the Higgs-boson tadpole and mass renormalisations. However, while we renormalise the Higgs-boson mass already at the Lagrangian level, we replace all other bare parameters at the end of the calculations without recourse to any counterterm vertices. This procedure turns out to be most convenient for our purposes.

As a further check on our calculations, we also rederive the correction of order $\mathcal{O}(\alpha_s G_F m_t^2)$. This result is presented in Section 3.4. Finally, we apply a Higgs-boson low-energy theorem [28], which allows for an independent calculation of the various $Hb\bar{b}$ diagrams at order $\mathcal{O}(G_F^2 m_t^4)$. This is explained in Section 4.

3.1 Tree-level result and $\mathcal{O}(G_F m_t^2)$ renormalisation constants

The tree-level diagram is depicted in Fig. 1(a). Using the notation introduced in Eq. (50), the corresponding amputated matrix element is in bare form written as

$$\mathcal{A}_0^{(0)} = \mathcal{A}_{A,0}^{(0)} = -\frac{m_{b,0}}{v_0}. \quad (62)$$

The tree-level transition matrix element is

$$\mathcal{T}^{(0)} = \mathcal{A}_0^{(0)} \bar{u}_b(q_2, r_2) v_b(q_1, r_1), \quad (63)$$

and the decay rate is

$$\Gamma^{(0)} = \frac{\sqrt{2} N_c G_F M_H m_b^2}{8\pi} \left(1 - \frac{4m_b^2}{M_H^2}\right)^{3/2}, \quad (64)$$

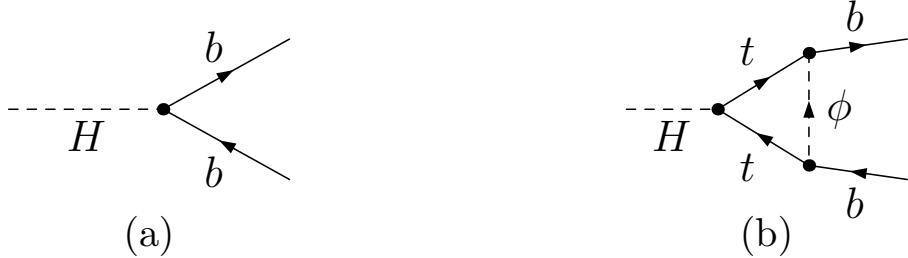


Figure 1: Diagrams contributing to $H \rightarrow b\bar{b}$ at (a) tree level and (b) order $\mathcal{O}(G_F m_t^2)$.

where $N_c = 3$ is the number of quark colours. Furthermore, we have introduced Fermi's constant G_F via the Born relation

$$\frac{1}{v} = 2^{1/4} G_F^{1/2}. \quad (65)$$

In the following, we have to renormalise the vacuum expectation value. Through the order of our calculations, this can be achieved by writing [29]

$$\frac{1}{v_0} = 2^{1/4} G_{F,0}^{1/2}, \quad (66)$$

with

$$G_{F,0} = G_F \frac{M_W^2}{M_{W,0}^2}. \quad (67)$$

Thus, the renormalisation of the vacuum expectation value is reduced to the one of the W -boson mass.

In the remainder of this subsection, we list all relevant renormalisation constants of order $\mathcal{O}(G_F m_t^2)$. They are derived by evaluating the diagrams of Fig. 2 and applying Eqs. (4), (7), (24), (28), (29), (47), and (60). Since we shall compute the correction of order $\mathcal{O}(G_F^2 m_t^4)$, these renormalisation constants are needed through order $\mathcal{O}(\epsilon)$ in the

expansion in ϵ . The results read

$$\delta t^{(1)} = C_{\epsilon,0} x_{t,0} m_{t,0}^2 v_0 N_c \left[\frac{4}{\epsilon} + 4 + (4 + 2\zeta(2))\epsilon + \mathcal{O}(\epsilon^2) \right], \quad (68)$$

$$\delta M_H^{2(1)} = C_{\epsilon,0} x_{t,0} m_{t,0}^2 N_c \left[-\frac{12}{\epsilon} - 4 + (-4 - 6\zeta(2))\epsilon + \mathcal{O}(\epsilon^2) \right], \quad (69)$$

$$\delta Z_H^{(1)} = C_{\epsilon,0} x_{t,0} N_c \left[-\frac{2}{\epsilon} + \frac{4}{3} - \zeta(2)\epsilon + \mathcal{O}(\epsilon^2) \right], \quad (70)$$

$$\frac{\delta m_b^{(1)}}{m_b} = C_{\epsilon,0} x_{t,0} \left[-\frac{3}{2\epsilon} - \frac{5}{4} + \left(-\frac{9}{8} - \frac{3}{4}\zeta(2) \right) \epsilon + \mathcal{O}(\epsilon^2) \right], \quad (71)$$

$$\delta Z_{b,L}^{(1)} = C_{\epsilon,0} x_{t,0} \left[-\frac{1}{\epsilon} - \frac{3}{2} + \left(-\frac{7}{4} - \frac{1}{2}\zeta(2) \right) \epsilon + \mathcal{O}(\epsilon^2) \right], \quad (72)$$

$$\delta Z_{b,R}^{(1)} = 0, \quad (73)$$

$$\frac{\delta m_t^{(1)}}{m_t} = C_{\epsilon,0} x_{t,0} \left[\frac{3}{2\epsilon} + 4 + \left(9 - \frac{5}{4}\zeta(2) \right) \epsilon + \mathcal{O}(\epsilon^2) \right], \quad (74)$$

$$\delta M_W^{2(1)} = C_{\epsilon,0} x_{t,0} M_{W,0}^2 N_c \left[-\frac{2}{\epsilon} - 1 + \left(-\frac{1}{2} - \zeta(2) \right) \epsilon + \mathcal{O}(\epsilon^2) \right], \quad (75)$$

where we use the abbreviations

$$C_\epsilon = \left(\frac{4\pi\mu^2}{m_t^2} e^{-\gamma_E} \right)^\epsilon, \\ x_t = \frac{G_F m_t^2}{8\pi^2 \sqrt{2}}, \quad (76)$$

with γ_E being Euler's constant.

3.2 Correction of order $\mathcal{O}(G_F m_t^2)$

At order $\mathcal{O}(G_F m_t^2)$, only the one diagram depicted in Fig. 1(b) contributes. Using the notation of Eq. (50), we obtain for the expansion in ϵ through order $\mathcal{O}(\epsilon)$:

$$\begin{aligned} \mathcal{A}_{A,0}^{(1)} &= C_{\epsilon,0} x_{t,0} \frac{m_{b,0}}{v_0} \left[-\frac{2}{\epsilon} + 2 + (2 - \zeta(2))\epsilon + \mathcal{O}(\epsilon^2) \right] \\ \mathcal{A}_{B,0}^{(1)} &= C_{\epsilon,0} x_{t,0} \frac{1}{v_0} \left(-1 - \frac{3}{2}\epsilon + \mathcal{O}(\epsilon^2) \right). \end{aligned} \quad (77)$$

Expanding Eq. (52) and replacing the bare masses by the renormalised ones plus their counterterms in Eq. (62), we find the transition matrix element to be

$$\mathcal{T}^{(1)} = \mathcal{A}_{A,0}^{(1)} + m_b \mathcal{A}_{B,0}^{(1)} + \mathcal{A}_0^{(0)} \left(\delta_u^{(1)} + \frac{\delta m_b^{(1)}}{m_b} + \frac{1}{2} \delta Z_{b,L}^{(1)} + \frac{1}{2} \delta Z_{b,R}^{(1)} \right), \quad (78)$$

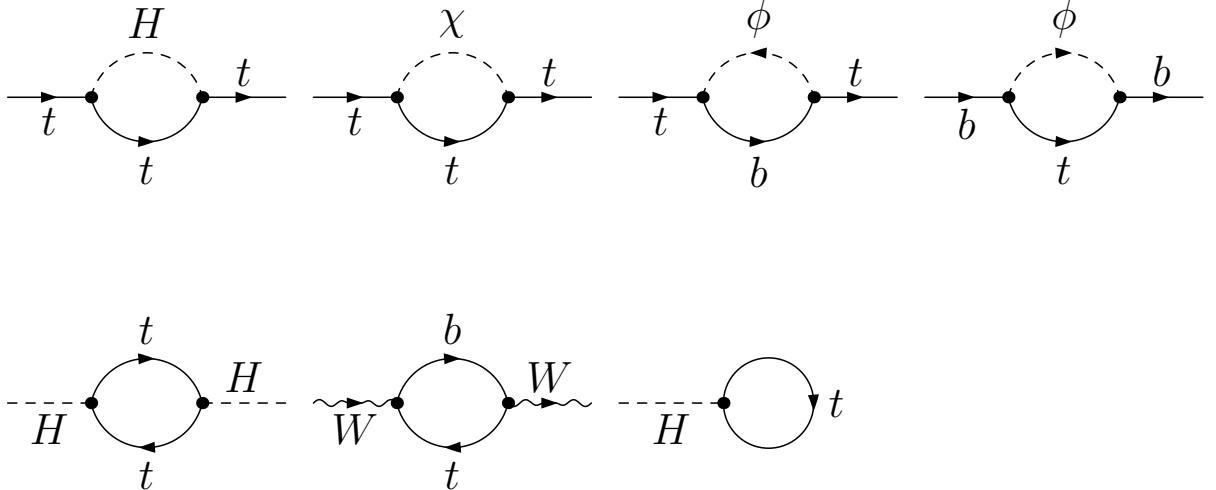


Figure 2: One-loop self-energy and tadpole diagrams contributing at order $\mathcal{O}(G_F m_t^2)$.

where $\mathcal{A}^{(0)}$ is the amputated matrix element of Eq. (62) and

$$\delta_u^{(1)} = \frac{1}{2} \delta Z_H^{(1)} - \frac{1}{2} \frac{\delta M_W^{2(1)}}{M_W^2} \quad (79)$$

is the one-loop contribution to the universal counterterm δ_u , which exhausts the full $\mathcal{O}(G_F m_t^2)$ corrections for Higgs-boson decays to fermion-antifermion pairs, except for those into $t\bar{t}$ and $b\bar{b}$ pairs. For simplicity, we omitted the spinors on the right-hand side of Eq. (78); we shall also do this in the following. $\delta_u^{(1)}$ and $\mathcal{T}^{(1)}$ are ultraviolet finite and read

$$\begin{aligned} \delta_u^{(1)} &= x_t N_c \frac{7}{6} \\ &= x_t \frac{7}{2}, \end{aligned} \quad (80)$$

$$\mathcal{T}^{(1)} = \mathcal{T}^{(0)} x_t \left(-3 + N_c \frac{7}{6} \right). \quad (81)$$

The $\mathcal{O}(G_F m_t^2)$ correction to the decay rate thus becomes

$$\begin{aligned} \frac{\Gamma^{(1)}}{\Gamma^{(0)}} &= x_t \left(-6 + N_c \frac{7}{3} \right) \\ &= x_t, \end{aligned} \quad (82)$$

where $\Gamma^{(0)}$ is given in Eq. (64). The results of this subsection are in accordance with Ref. [6].

3.3 Correction of order $\mathcal{O}(G_F^2 m_t^4)$

Expanding Eq. (52) up to the two-loop order and replacing all bare masses in the tree-level and one-loop amputated matrix elements by the renormalised masses plus the corresponding counterterms, we find the following master formula for the transition matrix element

$$\begin{aligned} \mathcal{T}^{(2)} = & \mathcal{A}_{A,0}^{(2)} + m_b \mathcal{A}_{B,0}^{(2)} + \mathcal{A}_{A,0}^{(1)} \left(\frac{\delta m_b^{(1)}}{m_b} + \frac{1}{2} \delta Z_{b,L}^{(1)} + \frac{1}{2} \delta Z_{b,R}^{(1)} \right) + m_b \mathcal{A}_{B,0}^{(1)} \delta Z_{b,L}^{(1)} \\ & + \left(\mathcal{A}_{A,0}^{(1)} + m_b \mathcal{A}_{B,0}^{(1)} \right) \left[\delta_u^{(1)} + 2(1-\epsilon) \frac{\delta m_t^{(1)}}{m_t} - \frac{\delta M_W^{2(1)}}{M_W^2} \right] \\ & + \mathcal{A}_0^{(0)} \left[\delta_u^{(2)} + \frac{\delta m_b^{(2)}}{m_b} + \frac{1}{2} \delta Z_{b,L}^{(2)} + \frac{1}{2} \delta Z_{b,R}^{(2)} + \delta_u^{(1)} \left(\frac{\delta m_b^{(1)}}{m_b} + \frac{1}{2} \delta Z_{b,L}^{(1)} + \frac{1}{2} \delta Z_{b,R}^{(1)} \right) \right. \\ & \left. + \frac{1}{2} \frac{\delta m_b^{(1)}}{m_b} \left(\delta Z_{b,L}^{(1)} + \delta Z_{b,R}^{(1)} \right) - \frac{1}{8} \left(\delta Z_{b,L}^{(1)} - \delta Z_{b,R}^{(1)} \right)^2 \right], \end{aligned} \quad (83)$$

where

$$\delta_u^{(2)} = \frac{1}{2} \delta Z_H^{(2)} - \frac{1}{2} \frac{\delta M_W^{2(2)}}{M_W^2} - \frac{1}{8} \left(\delta Z_H^{(1)} \right)^2 - \frac{1}{4} \delta Z_H^{(1)} \frac{\delta M_W^{2(1)}}{M_W^2} + \frac{3}{8} \left(\frac{\delta M_W^{2(1)}}{M_W^2} \right)^2 \quad (84)$$

is the universal counterterm.

3.3.1 Universal counterterm

Let us first calculate the universal counterterm. To this end, we need the two-loop expressions for δZ_H and δM_W^2 . The unrenormalised expressions are obtained by evaluating the diagrams in Figs. 3 and 4 and applying Eqs. (8) and (47), the results being

$$\begin{aligned} \delta Z_{H,0}^{(2)} &= C_{\epsilon,0}^2 x_{t,0}^2 N_c \left[\frac{3}{\epsilon^2} - \frac{11}{2\epsilon} - \frac{17}{12} + 5\zeta(2) + N_c \left(\frac{4}{\epsilon^2} - \frac{16}{3\epsilon} + \frac{16}{9} + 4\zeta(2) \right) + \mathcal{O}(\epsilon) \right], \\ \delta M_{W,0}^{2(2)} &= C_{\epsilon,0}^2 x_{t,0}^2 M_{W,0}^2 N_c \left(\frac{3}{\epsilon^2} + \frac{3}{2\epsilon} - \frac{69}{4} + 17\zeta(2) + \mathcal{O}(\epsilon) \right), \end{aligned} \quad (85)$$

in accordance with Ref. [18]. In addition, there are contributions from the renormalisations of the bare parameters in Eqs. (70) and (75), so that

$$\begin{aligned} \delta Z_H^{(2)} &= \delta Z_{H,0}^{(2)} + \delta Z_H^{(1)} \left[2(1-\epsilon) \frac{\delta m_t^{(1)}}{m_t} - \frac{\delta M_W^{2(1)}}{M_W^2} \right], \\ \delta M_W^{2(2)} &= \delta M_{W,0}^{2(2)} + 2(1-\epsilon) \frac{\delta m_t^{(1)}}{m_t} \delta M_W^{2(1)}. \end{aligned} \quad (86)$$

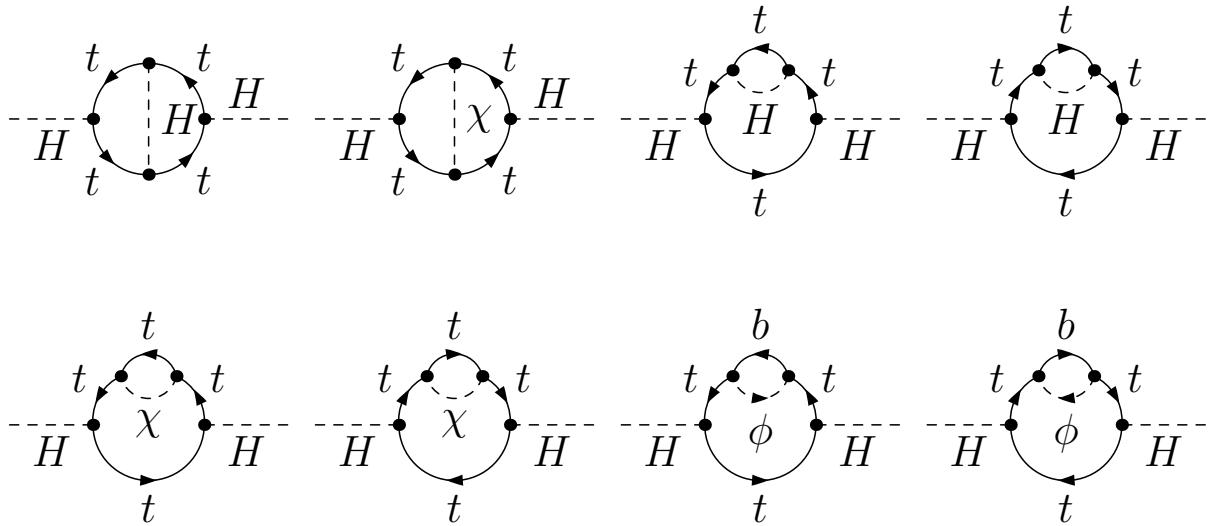


Figure 3: Higgs-boson self-energy diagrams contributing at order $\mathcal{O}(G_F^2 m_t^4)$.

We are now in a position to specify the universal counterterm at order $\mathcal{O}(G_F^2 m_t^4)$ as defined in Eq. (84). The result is

$$\begin{aligned}\delta_u^{(2)} &= x_t^2 N_c \left(\frac{29}{2} - 6\zeta(2) + N_c \frac{49}{24} \right) \\ &= x_t^2 \left(\frac{495}{8} - 3\pi^2 \right).\end{aligned}\quad (87)$$

If we convert Eq. (87) to a mixed renormalisation scheme which uses on-shell definitions for the particle masses and the definitions of the modified minimal-subtraction ($\overline{\text{MS}}$) scheme for all other basic parameters, then we find agreement with Eq. (15) for $x = 0$ in the paper by Djouadi et al. [18]. However, the corresponding result for the pure on-shell scheme presented in their Eq. (27) for $x = 0$ disagrees with our Eq. (87). We can trace this discrepancy to the absence in their Eq. (25) of the additional finite term $\hat{\delta}_u^{(1)} \Delta\rho^{(1)}$ which arises from the renormalisation of the one-loop result in their Eq. (7) according to the prescription in their Eq. (18).

3.3.2 Complete transition matrix element

Having provided $\delta_u^{(2)}$, we now turn to the residual ingredients entering the transition matrix element of Eq. (83). Evaluating the $Hb\bar{b}$ diagrams shown in Fig. 5, we find the form factors in Eq. (50) at order $\mathcal{O}(G_F^2 m_t^4)$ to be

$$\begin{aligned}\mathcal{A}_{A,0}^{(2)} &= \frac{m_{b,0}}{v_0} x_{t,0}^2 C_{\epsilon,0}^2 \left[\frac{1}{\epsilon^2} - \frac{5}{\epsilon} - 5 + 7\zeta(2) + N_c \left(\frac{2}{\epsilon^2} - \frac{2}{\epsilon} - 14 - 2\zeta(2) \right) + \mathcal{O}(\epsilon) \right], \\ \mathcal{A}_{B,0}^{(2)} &= \frac{1}{v_0} x_{t,0}^2 C_{\epsilon,0}^2 \left[\frac{2}{\epsilon} + 1 + N_c \left(\frac{2}{\epsilon} + 9 \right) + \mathcal{O}(\epsilon) \right].\end{aligned}\quad (88)$$

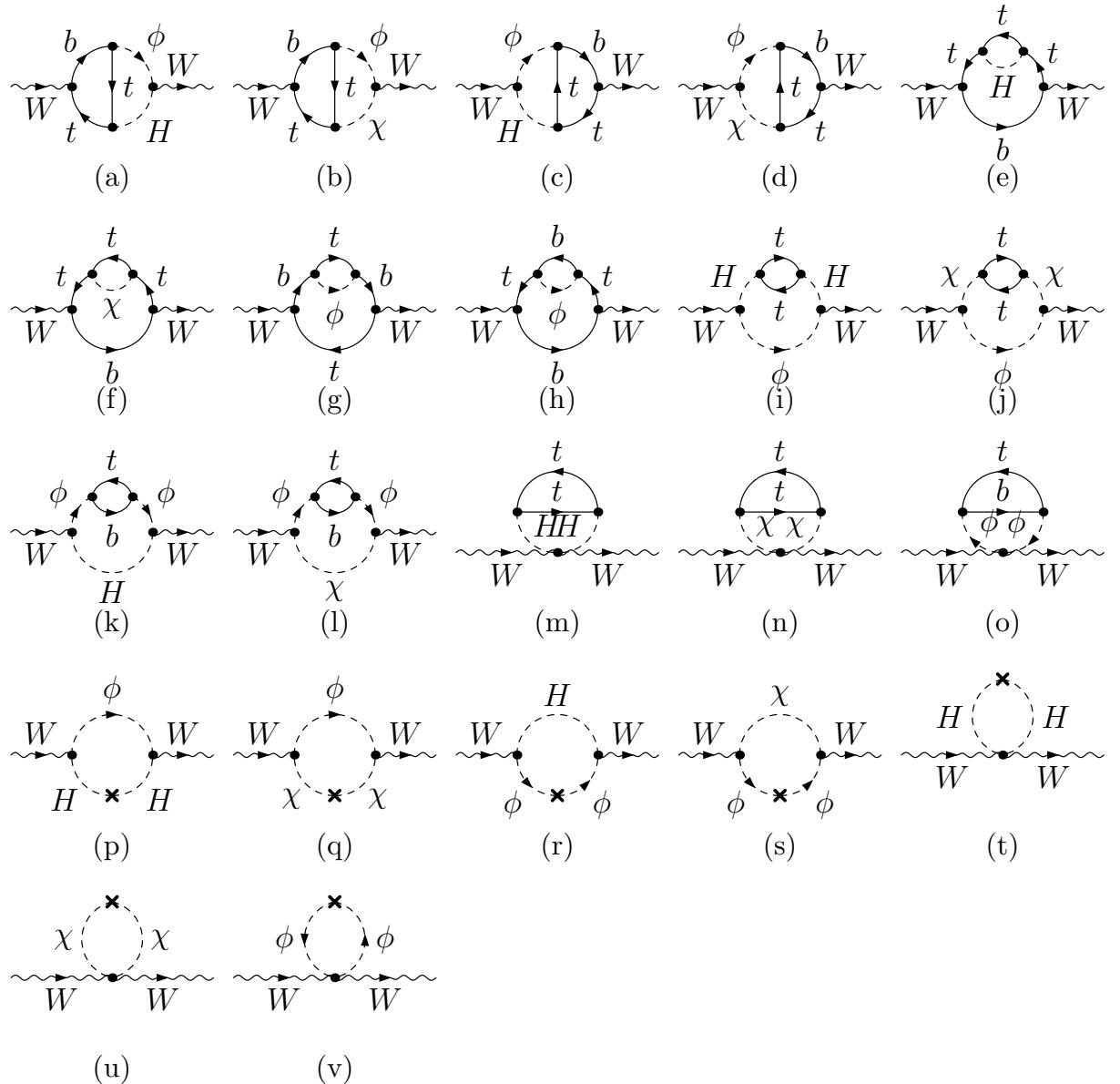


Figure 4: W -boson self-energy diagrams contributing at order $\mathcal{O}(G_F^2 m_t^4)$. Insertions of $-i\delta M_H^2$ in Higgs-boson lines and of $i\delta t/v_0$ in ϕ - or χ -boson lines are indicated by crosses.

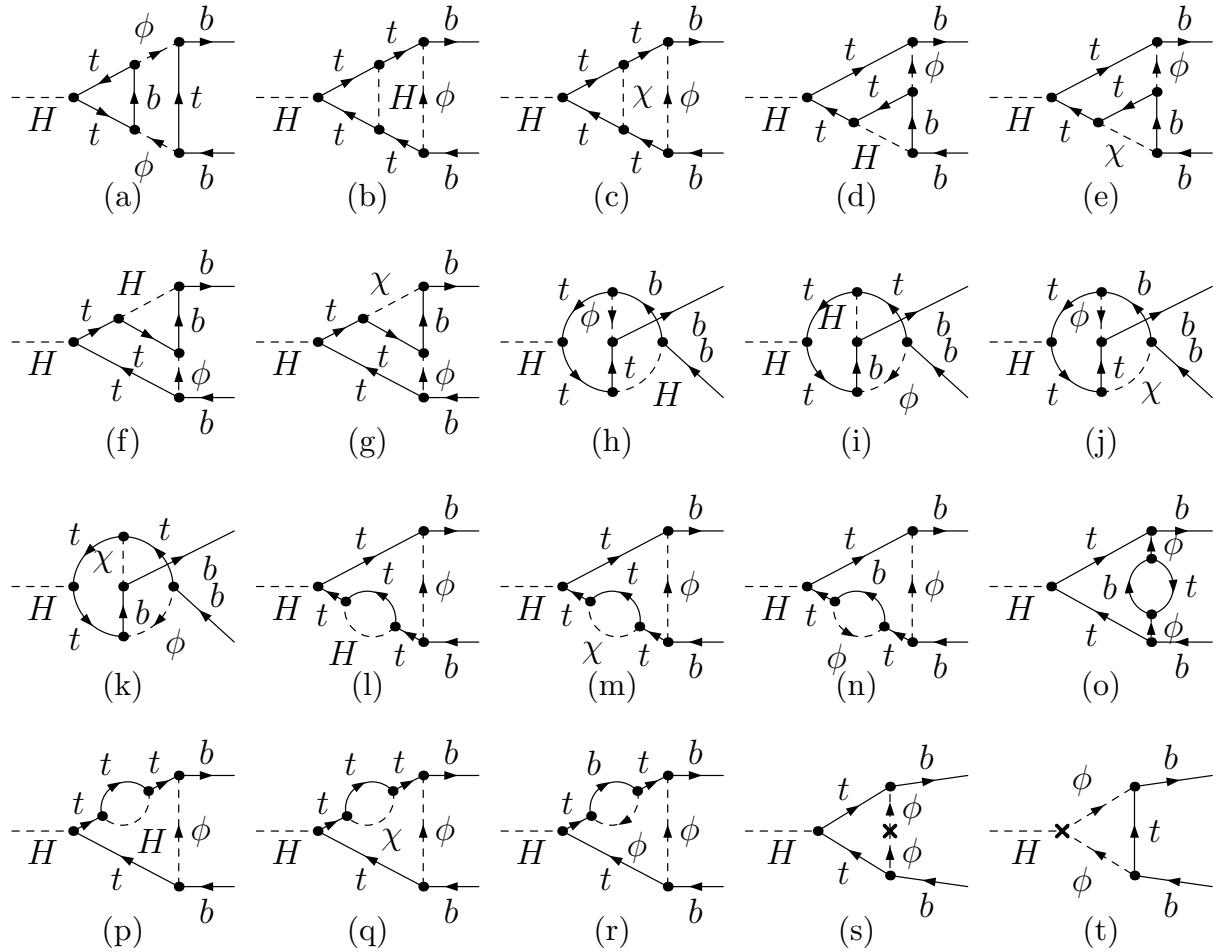


Figure 5: Diagrams contributing to $H \rightarrow b\bar{b}$ at order $\mathcal{O}(G_F^2 m_t^4)$. Insertions of $i\delta t/v_0$ in ϕ -boson lines and of $-i(\delta t/v_0 + \delta M_H^2)/v_0$ in $H\phi\phi$ vertices are indicated by crosses.

Evaluating the diagrams depicted in Fig. 6 and using Eqs. (25), (30), and (31), we obtain the bottom-quark mass and wave-function renormalisation constants at order $\mathcal{O}(G_F^2 m_t^4)$. The renormalisation constants in terms of bare parameters read

$$\begin{aligned} \frac{\delta m_{b,0}^{(2)}}{m_b} &= C_{\epsilon,0}^2 x_{t,0}^2 \left[\frac{27}{8\epsilon^2} + \frac{31}{8\epsilon} + \frac{13}{32} + \frac{59}{8}\zeta(2) + N_c \left(\frac{3}{2\epsilon^2} + \frac{15}{4\epsilon} + \frac{55}{8} - \frac{3}{2}\zeta(2) \right) + \mathcal{O}(\epsilon) \right], \\ \delta Z_{b,L,0}^{(2)} &= C_{\epsilon,0}^2 x_{t,0}^2 \left[\frac{2}{\epsilon^2} + \frac{7}{2\epsilon} + 1 + 6\zeta(2) + N_c \left(\frac{1}{\epsilon^2} + \frac{9}{2\epsilon} + \frac{25}{4} - \zeta(2) \right) + \mathcal{O}(\epsilon) \right], \\ \delta Z_{b,R,0}^{(2)} &= 0. \end{aligned} \quad (89)$$

Additional contributions arise from the replacement of the bare t -quark and W -boson

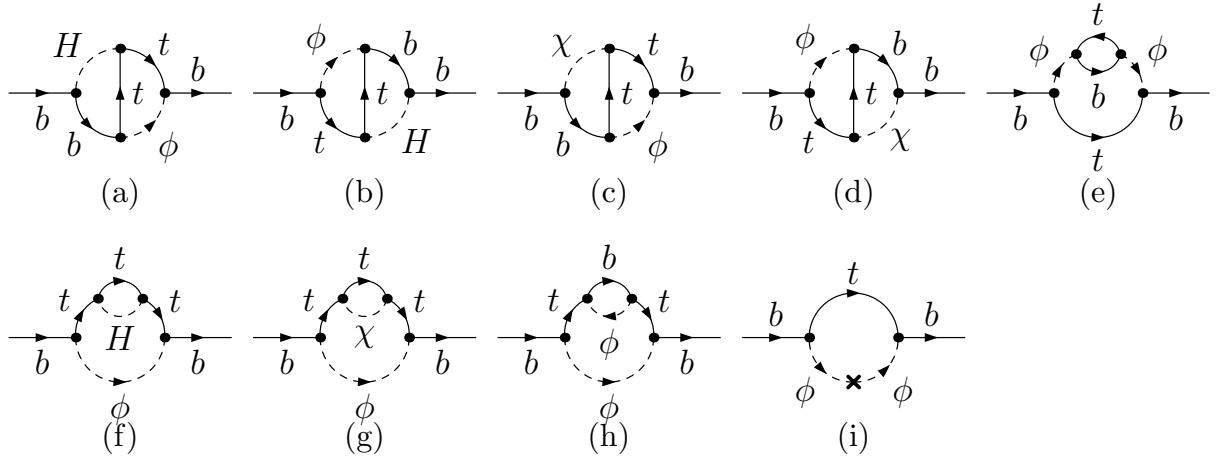


Figure 6: b -quark self-energy diagrams contributing at order $\mathcal{O}(G_F^2 m_t^4)$. Insertions of $i\delta t/v_0$ in ϕ -boson lines are indicated by crosses.

masses in Eqs. (71), (72), and (73), so that

$$\begin{aligned} \frac{\delta m_b^{(2)}}{m_b} &= \frac{\delta m_{b,0}^{(2)}}{m_b} + \frac{\delta m_b^{(1)}}{m_b} \left[2(1-\epsilon) \frac{\delta m_t^{(1)}}{m_t} - \frac{\delta M_W^{2(1)}}{M_W^2} \right], \\ \delta Z_{b,L}^{(2)} &= \delta Z_{b,L,0}^{(2)} + \delta Z_{b,L}^{(1)} \left[2(1-\epsilon) \frac{\delta m_t^{(1)}}{m_t} - \frac{\delta M_W^{2(1)}}{M_W^2} \right], \\ \delta Z_{b,R}^{(2)} &= 0. \end{aligned} \quad (90)$$

Now all ingredients for the evaluation of the renormalised transition matrix element of order $\mathcal{O}(G_F^2 m_t^4)$ according to Eq. (83) are available. We find

$$\mathcal{T}^{(2)} = \mathcal{T}^{(0)} x_t^2 \left[-\frac{29}{2} + N_c (18 - 6\zeta(2)) + N_c^2 \frac{49}{24} \right]. \quad (91)$$

Adding Eqs. (63), (81), and (91), squaring, and extracting the $\mathcal{O}(G_F^2 m_t^4)$ term, we have

$$\begin{aligned} \frac{\Gamma^{(2)}}{\Gamma^{(0)}} &= x_t^2 \left[-20 + N_c (29 - 12\zeta(2)) + N_c^2 \frac{49}{9} \right] \\ &= x_t^2 (116 - 6\pi^2). \end{aligned} \quad (92)$$

3.4 Correction of order $\mathcal{O}(\alpha_s G_F m_t^2)$

As a by-product of our analysis, we can also compute the $\mathcal{O}(\alpha_s G_F m_t^2)$ correction to the $H \rightarrow b\bar{b}$ decay width. The comparison of our result with the literature [13,14,15] provides a partial check of our $\mathcal{O}(G_F^2 m_t^4)$ results. Note, however, that the calculation considerably

simplifies as one passes from order $\mathcal{O}(G_F^2 m_t^4)$ to order $\mathcal{O}(\alpha_s G_F m_t^2)$. Using our tools, we indeed recover the well-known $\mathcal{O}(\alpha_s G_F m_t^2)$ results for the universal correction [13] and the correction to the $H \rightarrow b\bar{b}$ decay width [14,15],

$$\begin{aligned}\delta_u^{(X_t \alpha_s)} &= X_t \frac{\alpha_s}{\pi} C_F N_c \left(-\frac{3}{4} - \frac{\zeta(2)}{2} \right), \\ \frac{\Gamma^{(X_t \alpha_s)}}{\Gamma^{(0)}} &= X_t \frac{\alpha_s}{\pi} C_F \left[-12 + 9 \ln \frac{M_H^2}{M_b^2} + N_c \left(\frac{15}{4} - \zeta(2) - \frac{7}{2} \ln \frac{M_H^2}{M_b^2} \right) \right],\end{aligned}\quad (93)$$

respectively, where $X_t = G_F M_t^2 / (8\pi^2 \sqrt{2})$ and $C_F = (N_c^2 - 1)/(2N_c)$. In Eq. (93), the bottom- and top-quark masses are denoted with capital letters, M_b and M_t , respectively, to indicate that they are pure on-shell masses, i.e. they are defined in the on-shell scheme also with regard to quantum chromodynamics (QCD). The obvious disadvantage of this choice is the appearance of large logarithms of the type $\ln(M_H^2/m_b^2)$ starting already in order $\mathcal{O}(\alpha_s)$, which spoil the convergence behaviour of the perturbation expansion. As is well known [5], these logarithms can be resummed into the running bottom-quark mass, if m_b appearing in Eq. (64) is QCD-renormalised in the $\overline{\text{MS}}$ scheme at scale $\mu = M_H$, by substituting $m_b = \overline{m}_b(M_H)$. For consistency with the $\mathcal{O}(G_F m_t^2)$ and $\mathcal{O}(G_F^2 m_t^4)$ results presented above, which all refer to the electroweak on-shell scheme, we continue our discussion in a mixed renormalisation scheme where the on-shell definition of bottom-quark mass is adopted for electroweak corrections and the $\overline{\text{MS}}$ one for QCD corrections. Since we wish to treat the masses of the top and bottom quarks on the same footing, we adopt this mixed scheme for the top-quark mass as well. Furthermore, the analysis at order $\mathcal{O}(\alpha_s^2 G_F m_t^2)$ [16,17] reveals that Eq. (93) is further improved according to the renormalisation group if m_t and α_s are taken to be $m_t = \overline{m}_t(m_t)$ and $\alpha_s = \alpha_s^{(n_f)}(m_t)$ with $n_f = 6$ quark flavours, respectively. In this improved renormalisation scheme, Eq. (93) takes the form

$$\begin{aligned}\delta_u^{(x_t \alpha_s)} &= x_t \frac{\alpha_s}{\pi} C_F N_c \left(\frac{19}{12} - \frac{\zeta(2)}{2} \right) \\ &= x_t \frac{\alpha_s}{\pi} \left(\frac{19}{3} - \frac{\pi^2}{3} \right), \\ \frac{\Gamma^{(x_t \alpha_s)}}{\Gamma^{(0)}} &= x_t \frac{\alpha_s}{\pi} C_F \left[-36 + N_c \left(\frac{157}{12} - \zeta(2) \right) \right] \\ &= x_t \frac{\alpha_s}{\pi} \left(\frac{13}{3} - \frac{2}{3} \pi^2 \right).\end{aligned}\quad (94)$$

To the order considered here, we have

$$m_t = M_t \left(1 - \frac{\alpha_s^{(6)}(M_t)}{\pi} C_F \right). \quad (95)$$

4 Low-energy theorem

In this section, we present an alternative way of calculating all but one of the $Hb\bar{b}$ diagrams at order $\mathcal{O}(G_F^2 m_t^4)$ which is based on the Higgs-boson low-energy theorem [28]. In fact, the $Hb\bar{b}$ diagrams of Fig. 5, with the exception of diagram (t), can be generated from the bottom-quark self-energy diagrams of Fig. 6 by in turn attaching an external Higgs-boson line to each of the top-quark lines. Diagrammatically, this can be represented as follows:

$$\begin{array}{ccc} t(q) & \longrightarrow & t(q) \stackrel{|}{\longrightarrow} t(q) \\ \text{---} \nearrow \quad & & \text{---} \nearrow \quad \text{---} \nearrow \\ \frac{i}{q - m_{t,0}} & \longrightarrow & \frac{i}{q - m_{t,0}} \frac{-im_{t,0}}{v_0} \frac{i}{q - m_{t,0}}. \end{array} \quad (96)$$

Here, we also made use of the fact that, in the large- m_t approximation, the external Higgs boson does not carry any four-momentum into the respective diagram. Thanks to the identity

$$\frac{i}{q - m_{t,0}} \frac{-im_{t,0}}{v_0} \frac{i}{q - m_{t,0}} = \frac{m_{t,0}}{v_0} \frac{\partial}{\partial m_{t,0}} \left(\frac{i}{q - m_{t,0}} \right), \quad (97)$$

the amputated matrix element of $H \rightarrow b\bar{b}$ is in the large- m_t limit related to the bottom-quark self-energy as

$$\mathcal{A}_0 = \frac{m_{t,0}}{v_0} \frac{\partial}{\partial m_{t,0}} \Sigma_b, \quad (98)$$

where it is understood that the differential operator only acts on masses which stem from propagators, not to those occurring in vertices, and that all quantities in Eq. (98) are taken to be bare. Exploiting the structures underlying Eqs. (9) and (50), Eq. (98) can be decomposed into two scalar equations. Identifying the four-momentum q in Eq. (9) with q_2 in Eq. (50) and noticing that $q_2 = -q_1$ in the soft-Higgs limit, we have

$$\begin{aligned} \mathcal{A}_{A,0} &= m_{b,0} \frac{m_{t,0}}{v_0} \frac{\partial}{\partial m_{t,0}} \Sigma_{b,S}, \\ \mathcal{A}_{B,0} &= \frac{1}{2} \frac{m_{t,0}}{v_0} \frac{\partial}{\partial m_{t,0}} \Sigma^{b,L}. \end{aligned} \quad (99)$$

The fact that the $H \rightarrow b\bar{b}$ amplitude does not contain a term proportional to $(q_2 - q_1)\omega_+$ is reflected by the fact that the right-handed part of the bottom-quark self-energy, $\Sigma_{b,R}$, vanishes to the orders considered in this paper.

The results for $\mathcal{A}_{A,0}$ and $\mathcal{A}_{B,0}$ obtained through Eq. (99) indeed agree with the direct evaluation of the respective diagrams in Fig. 5.

5 Numerical results

Finally, we explore the phenomenological implications of our results. Adopting from Ref. [30] the values $G_F = 1.16637 \times 10^{-5}$ GeV $^{-2}$, $\alpha_s^{(5)}(M_Z) = 0.1176$, $M_Z = 91.1876$ GeV,

Table 2: Numerical values of the relative corrections $\Delta_l^{(x)}$, $\Delta_q^{(x)}$, and $\Gamma^{(x)}/\Gamma^{(0)}$ to the $H \rightarrow l^+l^-$, $H \rightarrow q\bar{q}$, and $H \rightarrow b\bar{b}$ decay widths, respectively, at orders $x = G_F m_t^2$, $G_F^2 m_t^4$, and $\alpha_s G_F m_t^2$.

Order x	$\Delta_l^{(x)}$	$\Delta_q^{(x)}$	$\Gamma^{(x)}/\Gamma^{(0)}$
$\mathcal{O}(G_F m_t^2)$	+2.021%	+2.021%	+0.289%
$\mathcal{O}(G_F^2 m_t^4)$	+0.064%	+0.064%	+0.047%
$\mathcal{O}(\alpha_s G_F m_t^2)$	+0.060%	+0.452%	-0.022%

and $M_t = 174.2$ GeV for our input parameters, so that $\alpha_s^{(6)}(m_t) = 0.1076$ and $m_t = 166.2$ GeV, we evaluate the relative corrections $\Gamma^{(x)}/\Gamma^{(0)}$ to the $H \rightarrow b\bar{b}$ decay width to orders $x = G_F m_t^2$, $G_F^2 m_t^4$, and $\alpha_s G_F m_t^2$. For comparison, we also evaluate the relative corrections to the $H \rightarrow l^+l^-$ and $H \rightarrow q\bar{q}$ decay widths, where $l = e, \mu, \tau$ and $q = u, d, s, c$, which, to the orders considered here, are given by

$$\begin{aligned}\Delta_l &= (1 + \delta_u)^2 - 1 \\ &= 2\delta_u^{(1)} + 2\delta_u^{(2)} + (\delta_u^{(1)})^2 + 2\delta_u^{(x_t \alpha_s)}, \\ \Delta_q &= (1 + \Delta_{\text{QCD}})(1 + \delta_u)^2 - 1 \\ &= \Delta_{\text{QCD}} + 2\delta_u^{(1)} + 2\delta_u^{(2)} + (\delta_u^{(1)})^2 + 2\delta_u^{(x_t \alpha_s)} + 2\Delta_{\text{QCD}}\delta_u^{(1)},\end{aligned}\quad (100)$$

where [5]

$$\Delta_{\text{QCD}} = \frac{\alpha_s}{\pi} C_F \frac{17}{4} \quad (101)$$

is the $\mathcal{O}(\alpha_s)$ correction in the limit $m_q \ll M_H$, with $m_q = \overline{m}_q(M_H)$.

The results are listed in Table 2. We observe that the $\mathcal{O}(G_F^2 m_t^4)$ correction to $\Gamma^{(0)}$ increases the enhancement due to the $\mathcal{O}(G_F m_t^2)$ one by about 16% and has more than twice the magnitude of the negative $\mathcal{O}(\alpha_s G_F m_t^2)$ one.

6 Conclusions

We analytically calculated the dominant electroweak two-loop correction, of order $\mathcal{O}(G_F^2 m_t^4)$, to the $H \rightarrow b\bar{b}$ decay width of an intermediate-mass Higgs boson, with $M_H \ll m_t$.

We performed various checks for our analysis. The ultraviolet divergences cancelled through genuine two-loop renormalisation. Our final result is devoid of infrared divergences related to infinitesimal scalar-boson masses. We reproduced those $Hb\bar{b}$ triangle diagrams where the external Higgs boson is coupled to an internal top-quark line, which we had computed directly, through application of a low-energy theorem. After switching to a hybrid renormalisation scheme, our $\mathcal{O}(G_F^2 m_t^4)$ result for the universal correction δ_u agrees with Ref. [18]. Using our techniques, we also recovered the $\mathcal{O}(\alpha_s G_F m_t^2)$ correction to the $H \rightarrow b\bar{b}$ decay width as well as the universal correction δ_u in this order.

The $\mathcal{O}(G_F^2 m_t^4)$ correction to the $H \rightarrow b\bar{b}$ decay width amplifies the familiar enhancement due to the $\mathcal{O}(G_F m_t^2)$ correction by about +16% and thus more than compensates the screening by about -8% through QCD effects of order $\mathcal{O}(\alpha_s G_F m_t^2)$.

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