## Anomaly-Induced Inflaton Decay and Gravitino-Overproduction Problem

Motoi Endo<sup>1</sup>, Fuminobu Takahashi<sup>1</sup>, and T. T. Yanagida<sup>2,3</sup>

<sup>1</sup> Deutsches Elektronen Synchrotron DESY, Notkestrasse 85, 22607 Hamburg, Germany

<sup>2</sup>Department of Physics, University of Tokyo, Tokyo 113-0033, Japan

<sup>3</sup>Research Center for the Early Universe, University of Tokyo, Tokyo 113-0033, Japan

We point out that the inflaton spontaneously decays into any gauge bosons and gauginos via the super-Weyl, Kähler and sigma-model anomalies in supergravity, once the inflaton acquires a non-vanishing vacuum expectation value. In particular, in the dynamical supersymmetry breaking scenarios, the inflaton necessarily decays into the supersymmetry breaking sector, if the inflaton mass is larger than the dynamical scale. This generically causes the overproduction of the gravitinos, which severely constrains the inflation models.

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Inflation [1] not only solves basic problems in the big bang cosmology such as the horizon and flatness problems, but also provides a natural mechanism to generate density fluctuations necessary to form the present structure of the universe. In fact, the standard slowroll inflation predicts almost scale-invariant power spectrum, which fits the recent cosmic microwave background data [2] quite well. During inflation, the universe is dominated by the potential energy of an inflaton field, and it expands exponentially [1, 3]. After inflation, the inflaton transfers its energy to a thermal plasma by the decay and reheats the universe. It is of great importance to unravel the reheating processes to have a successful thermal history after inflation. Indeed, the reheating is subject to several constraints; the reheating temperature should be high enough to generate the baryon asymmetry, while low enough to avoid the overproduction of unwanted relics.

One usually introduces couplings of the inflaton to the standard-model particles to cause its decay and hence reheating. The stronger couplings result in the higher reheating temperature, and so, the couplings must be so weak to evade the overproduction of the unwanted relics. In supergravity, for instance, gravitinos are overproduced by particle scatterings in thermal plasma, if the reheating temperature is too high [4]. So far, it has been considered that one can avoid the cosmological difficulties associated with the unwanted relics (e.g. gravitinos), by setting the coupling of the inflaton to the visible sector weak enough<sup>a</sup>.

In this letter we show that, once the inflaton acquires a finite vacuum expectation value (VEV), it spontaneously decays into any gauge bosons and gauginos via the quantum effects, anomalies in supergravity. Among the super-Weyl, Kähler, and sigma-model anomalies [8, 9], we will concentrate on the effect of the super-Weyl anomaly, for simplicity. The other Kähler and sigma-model anomalies can affect the inflaton decay at the same order of magnitude<sup>b</sup>. However, the following discussion is essentially unchanged even if these are included<sup>c</sup>.

In the dynamical supersymmetry (SUSY) breaking (DSB) scenarios, SUSY is broken as a result of strong dynamics in a gauge theory. The couplings induced by the super-Weyl anomaly makes it unavoidable for the inflaton to decay into the hidden gauge bosons and gauginos, which subsequently produce gravitinos. The gravitino production turns out to be prevalent in generic DSB models, which tightly constrains both the inflation models and SUSY breaking scenarios. In particular, as we will see, the gravity-mediation scenario is almost excluded, and high-scale inflation models such as hybrid [11] and smooth hybrid [12] inflation models are severely constrained. We stress that the gravitino production from the inflaton decay is almost unavoidable, and that it cannot be solved by taking the reheating temperature low enough. On the contrary, the lower reheating temperature makes the problem even worse.

We recently pointed out that the inflaton decays into matter fields in the visible and the hidden SUSY-breaking sectors through supergravity effects, even without direct couplings between them in the Einstein frame of the supergravity<sup>d</sup>. We call it as a spontaneous decay [7]. (See also Ref. [13] for the non-SUSY case.) Here, we show that the inflaton decays into any gauge bosons and gauginos via the super-Weyl anomaly in the supergravity in

<sup>&</sup>lt;sup>a</sup> It should be noted, however, that Refs. [5, 6, 7] recently pointed out that the inflaton can decay into the gravitinos, which puts severe constraints on both the inflation models and the supersymmetry breaking scenarios.

<sup>&</sup>lt;sup>b</sup> Counter terms [10] may also contribute to the inflaton decay.

<sup>&</sup>lt;sup>c</sup> The total effect depends on the form of the Kähler potential. For instance, in the case of the minimal Kähler potential, the decay rate is proportional to  $(T_G - T_R)^2$  instead of  $b_0^2$ . In the case of the Kähler potential of the sequestered type, the contributions from the Kähler and sigma model anomalies cancel, and the decay is dominantly induced by the super-Weyl anomaly.

<sup>&</sup>lt;sup>d</sup> A part of the decays is thought of as arising from direct couplings in the conformal frame.

addition to the spontaneous decay.

Let us assume that the inflaton does not have any direct couplings to the gauge sector. Then, the Lagrangian of the gauge multiplets is invariant under the super-Weyl transformation at the classical level, and hence the inflaton decay into the gauge sector is prohibited at the tree level [14]. However the symmetry is anomalous at the quantum level. The anomaly not only mediates the SUSY-breaking effects to the visible sector [15] but also enables the inflaton to couple to the gauge supermultiplets. By using the superfield description of the supergravity, the 1PI effective Lagrangian of the super-Weyl anomaly is [8, 9]

$$\Delta \mathcal{L} = \frac{g^2 b_0}{64\pi^2} \int d^2 \Theta \, 2\mathcal{E} W^{\alpha} W_{\alpha} \frac{1}{\partial^2} (\bar{\mathcal{D}}^2 - 8R) \bar{R} + h.c. \quad (1)$$

in the conformal frame and in the Planck units:  $M_P = 1$ . Here g is a gauge coupling constant,  $b_0 = 3T_G - T_R$  is the beta function coefficient, and  $W_{\alpha}$  is a field strength of corresponding gauge supermultiplet. A sum over all matter representations is understood. The chiral density  $\mathcal{E}$ , the  $\Theta$  variable and the covariant derivative  $\mathcal{D}$  are those defined in the supergravity [16]. Note that the inflaton linearly contributes to the R-current as  $b_a \sim \frac{i}{2}(K_{\phi}\partial_a\phi - K_{\phi}^*\partial_a\phi^*)$ , and the superspace curvature  $\bar{R}$  contains  $\bar{R} = -\frac{1}{6}(M^* + \Theta^2(-\frac{1}{2}\mathcal{R} + ie_a^m\mathcal{D}_m b^a) + \cdots)$ , where M is a auxiliary field of the supergravity multiplet, and  $\mathcal{R}$  is the Einstein curvature scalar. Also, the combination  $(K_{\phi}\phi + K_{\phi}^*\phi^*)$  appears in the scalar component of the graviton. The inflaton field then couples to the gauge bosons in the Einstein frame as

$$\Delta \mathcal{L} = \frac{g^2 b_0}{192\pi^2} K_{\phi} \frac{\phi}{M_P} \left( \mathcal{F}_{mn} \mathcal{F}^{mn} - i \mathcal{F}_{mn} \tilde{\mathcal{F}}^{mn} \right) + h.c. \tag{2}$$

where  $\mathcal{F}_{mn}$  is a field strength of the gauge field and  $\tilde{\mathcal{F}}^{mn} = \epsilon^{mnkl} \mathcal{F}_{kl}/2$ . In addition, noting that the F-term satisfies the equation of the motion,  $F^i = -e^{K/2} K^{ij^*} (W_j + K_j W)^*$ , the inflaton couples to the gaugino  $\lambda$  as

$$\Delta \mathcal{L} = \frac{g^2 b_0}{96\pi^2} K_{\phi} \frac{m_{\phi}}{M_P} \phi^* \lambda \lambda + h.c.$$
(3)

in the Einstein frame. Here we assume that the inflaton mass  $m_{\phi}$  is dominated by the supersymmetric mass term, and used  $F^{\phi}_{,\phi^*} \simeq -m_{\phi}$ . Therefore the inflaton field couples to the gauge sector through the super-Weyl anomaly as long as the Kähler potential contains a linear term of the inflaton, which is roughly given by the inflaton VEV  $\langle \phi \rangle^{\rm e}$ . The decay rate becomes

$$\Gamma(\phi \to gauge) \simeq \frac{N\alpha^2 b_0^2}{4608\pi^3} |K_{\phi}|^2 \frac{m_{\phi}^3}{M_P^2},$$
 (4)

where N is the number of the generators of the gauge group,  $\alpha$  is defined as  $g^2/4\pi$ , and we assume the canonical normalization of the inflaton and gauge fields. Here we notice that the half of the decay rate comes from the decay into the two gauge bosons and the other half from that into the gaugino pair.

It is interesting to compare the anomaly-induced decay to the recently observed spontaneous decay which occurs at the tree level [7]. It was shown that the inflaton decays into the matter fields in the visible and/or hidden sectors, if the inflaton acquires a finite VEV. The decay proceeds via both the Yukawa interactions (with 3-body final states) and the mass terms in the superpotential, even when there are no direct interaction terms in the Einstein frame. Thus, for a generic Kähler potential (including the minimal one), the inflaton decay may be dominated by the spontaneous decay via the top Yukawa coupling. However, the anomaly-induced decay rate can be comparable to that of the spontaneous decay, although it arises at the 1-loop level. This is because the latter rates are suppressed either by the phase space of the 3-body final states, or by the mass ratio squared  $(M/m_{\phi})^2$  as in the case of the decay into the right-handed (s)neutrinos with a Majorana mass M satisfying  $2M < m_{\phi}$ .

In the DSB scenarios, SUSY is spontaneously broken as a result of non-perturbative dynamics in a gauge theory, in which case the beta function coefficient is positive,  $b_0 > 0$ . The dynamical scale of the hidden gauge interactions is related to the SUSY breaking scale as  $\Lambda = x\sqrt{m_{3/2}M_P}$ , where  $m_{3/2}$  denotes the gravitino mass, and  $x \gtrsim 1$  represents a model-dependent numerical factor. The gauge bosons and gauginos have masses of  $O(\Lambda)$  due to the strong couplings below the DSB scale. Therefore, when the inflaton mass,  $m_{\phi}$ , is larger than the DSB scale, the inflaton decays into the SUSY breaking sector via the super-Weyl anomaly<sup>f</sup>, since the decay is kinematically allowed and the mass of the hidden (s)quarks are much smaller than  $m_{\phi}$  at the decay vertex.

Let us consider how the decay proceeds. First, the inflaton decays into a pair of the hidden gauge bosons or gauginos, flying away to the opposite directions. Then each of them interacts with the hidden (s)quarks and hadronizes due to the strong coupling, followed by cascade decays of the heavy hidden hadrons into lighter ones. The number of the hidden hadrons produced from each jet, which we call here as the multiplicity  $N_H$ , depends on the detailed structure of the hidden sector such as the gauge groups, the number of the matter multiplets, and the mass spectrum of the hidden hadrons. We expect, however, that  $N_H$  is in the range of  $O(1-10^2)$ . The hid-

<sup>&</sup>lt;sup>e</sup> The anomaly-induced decay is suppressed if  $\langle K_{\phi} \rangle = 0$ , while the anomaly itself does not vanish in this case.

<sup>&</sup>lt;sup>f</sup> In addition, there may exists a decay into a messenger sector, due to additional gauge groups introduced to mediate the SUSY breaking effects.

den hadrons should eventually decay and release their energy into the visible sector. The gravitinos are likely to be produced in the decays of the hidden hadrons [17, 18, 19] as well as in the cascade decay processes in jets<sup>g</sup>. We denote the averaged number of the gravitinos produced per each jet as  $N_g$ . Here we assume each hidden hadron produces one gravitino in the end, and use the relation  $N_g \sim N_H^{\rm h}$ . The gravitino abundance is therefore<sup>i</sup>

$$Y_{3/2} = 2N_g \frac{\Gamma_H}{\Gamma_\phi} \frac{3T_{rh}}{4m_\phi},$$
  

$$\simeq 3 \times 10^{-7} \xi \left(\frac{m_\phi}{10^{12} \,\text{GeV}}\right)^2 \left(\frac{10^6 \,\text{GeV}}{T_{rh}}\right), \quad (5)$$

where  $\Gamma_H$  is the partial decay rate into the hidden gauge sector given by Eq. (4), and  $\Gamma_{\phi}$  denotes the total decay rate of the inflaton, related to the reheating temperature as  $T_{rh} \equiv (\pi^2 g_*/10)^{-\frac{1}{4}} \sqrt{\Gamma_{\phi} M_P}$ . Here  $g_*$  counts the relativistic degrees of freedom, and we have substituted  $g_* = 228.75$  in the second equality of Eq. (5). We also defined  $\xi \equiv N \alpha^2 b_0^2 N_g$ , where N and  $b_0$  depend on the SUSY breaking scenarios. For instance, in the IYIT model [21] with a SU(2) gauge group and four doublet chiral superfields, we have N = 3, and  $b_0 = 4$ .

It should be noted that the gravitino abundance (5) is inversely proportional to the reheating temperature [5]. That is, for the lower reheating temperature, more gravitinos are produced. This should be contrasted to the thermally produced gravitinos, whose abundance is proportional to the reheating temperature. For the rest of the paper, we regard the reheating temperature as a free parameter by introducing appropriate direct couplings of the inflaton to the visible sector. We will take the maximal value allowed by cosmological constraints to give the most conservative estimates on the gravitino abundance.

The inflaton does not decay into the SUSY breaking sector if  $m_{\phi} \lesssim \Lambda$ . However, the gravitino pair production then becomes important [5]. The gravitino pair production rate is [22]

$$\Gamma_{3/2}^{\text{pair}} \simeq \frac{\eta}{96\pi} |\nabla_{\phi} G_z|^2 \frac{m_{\phi}^3}{M_P^2} \tag{6}$$

with  $\eta = (m_z/m_{\phi})^4$  for  $m_{\phi} > m_z$  and  $\eta = 1$  for  $m_{\phi} < m_z$ , where  $G \equiv K + \ln |W|^2$ , and  $m_z$  is the mass of the SUSY breaking field z with non-vanishing F-term. Also  $\nabla_{\phi}G_z$  is defined by  $\nabla_{\phi}G_z \equiv G_{\phi z} - \Gamma^k_{\phi z}G_k$  with the connection  $\Gamma_{ij}^k \equiv G^{k\ell^*}G_{ij\ell^*}$ . The gravitino abundance is then given by

$$Y_{3/2} = 2 \frac{\Gamma_{3/2}^{\text{pair}}}{\Gamma_{\phi}} \frac{3T_{rh}}{4m_{\phi}} \simeq 7 \times 10^{-11} \eta \left(\frac{\langle \phi \rangle}{10^{15} \,\text{GeV}}\right)^2 \times \left(\frac{m_{\phi}}{10^{12} \,\text{GeV}}\right)^2 \left(\frac{10^6 \,\text{GeV}}{T_{rh}}\right), \quad (7)$$

where we have assumed the minimal Kähler potential in the last equality. Although we have neglected the VEV of z in (6), including the finite VEV that arises below the dynamical scale can make the rate even higher [19]. Indeed, taking account of the mixings and couplings between the  $\phi$  and z which are radiatively induced for  $m_{\phi} < \Lambda$ , the above gravitino abundance increases. To put it concretely, such an operator as  $|\phi|^2 zz$  in the Kähler potential is radiatively induced, and it additionally contributes to the gravitino production for  $m_{\phi} > m_z$ . The mixings in the Kähler potential becomes important especially for  $m_{\phi} < m_z$ .

Using (5) and (7), we can constrain the inflation models. The results are summarized in Fig. 1, in which we take the maximal value of  $T_{rh}$  allowed by the cosmological constraints. For lower  $T_{rh}$ , the bounds on  $\langle \phi \rangle$  become severer as  $\propto T_{rh}^{1/2}$  for a fixed  $m_{\phi}$ . We also set x = 1,  $N_g = 10, N = 3, \alpha = 0.1, b_0 = 4, \langle K_{\phi} \rangle = \langle \phi^* \rangle, m_z = \Lambda$ as reference values. We consider the following four cases. In the cases A and B, we set  $m_{3/2} = 1 \text{ TeV}$ , assuming the gravitino is unstable. The hadronic branching ratio is given by  $B_h = 1(10^{-3})$  for the case A (B). The gravitino abundance in these cases are severely constrained by BBN [26]. In the case C,  $m_{3/2} = 1$  GeV, and the gravitino is stable. In the case D, we take  $m_{3/2} = 100 \text{ TeV}$ with the wino LSP of a mass given by  $2.7 \times 10^{-3} m_{3/2}$  [15]. The constraints on  $T_{rh}$  and  $Y_{3/2}$  come from the requirement that the abundance of the gravitino (or the winos produced by the gravitino decay) should not exceed the present dark matter abundance [27]. From the figure, one can see that the high-scale inflation models such as the hybrid inflation model are severely constrained, while the new inflation models may escape the bounds if  $B_h$  is suppressed even for  $m_{3/2} = 1$  TeV.

In this letter we have shown that the inflaton decays into any gauge bosons and gauginos via the super-Weyl anomaly in supergravity, once the inflaton acquires a nonzero VEV. In particular, the inflaton necessarily decays into the SUSY breaking sector when the inflaton mass is larger than the DSB scale. This subsequently produces the gravitinos, and therefore the gravitino overproduction problem prevails among the DSB scenarios and most inflation models.

Let us mention that the anomaly-induced decay process and the associated gravitino problem shown above can be avoided in the following cases. In the chaotic inflation model with an approximate  $Z_2$  symmetry [25, 28], the VEV of the inflaton is so suppressed that both the

<sup>&</sup>lt;sup>g</sup> In particular, this is the case if the SUSY breaking field is a bound state of the hidden (s)quarks.

<sup>&</sup>lt;sup>h</sup> In a class of the gauge-mediation models of SUSY breaking, the particles in the hidden sector may dominantly decay into the standard-model particles [20].

<sup>&</sup>lt;sup>i</sup> If the inflaton spontaneously decay into the hidden sector at the tree level [7], more gravitinos will be produced.



FIG. 1: Constraints from the gravitino production by the inflaton decay, for  $m_{3/2} = 1 \text{ TeV}$  with  $B_h = 1 \pmod{3}$  (case A),  $m_{3/2} = 1 \text{ TeV}$  with  $B_h = 10^{-3}$  (case B),  $m_{3/2} = 100 \text{ TeV}$  (case C), and  $m_{3/2} = 1 \text{ GeV}$  (case D). The region above the solid (red) line is excluded for each case. For  $m_{\phi} \gtrsim \Lambda$ , we used the anomaly-induced inflaton decay into the hidden gauge/gauginos to estimate the gravitino abundance (5), while the gravitino pair production (7) was used for  $m_{\phi} \lesssim \Lambda$ . The typical values of  $\langle \phi \rangle$  and  $m_{\phi}$  for the single-field new [23], multi-field new [24], hybrid [11] and smooth hybrid [12], and chaotic [25] inflation models are also shown. Note that we adopt the chaotic inflation model without discrete symmetries, in which case  $\langle K_{\phi} \rangle$  is expected to be around the Planck scale.

anomaly-induced decay and the spontaneous decay are suppressed. Similar arguments also apply to inflation models in the no-scale supergravity [29].

An interesting application of the anomaly-induced inflaton decay can be found in the case with the Kähler potential of the sequestered type:  $K = -3\ln[1 - (|\phi|^2 +$  $|Q|^2$ , where Q collectively denotes the matter multiplets [14]. Since there are no direct couplings of the inflaton to the matter fields in the conformal frame, the possible decay processes are those mediated by the supergravity multiplet. Then, only such an operator that violates the conformal symmetry induces the inflaton decay, and so, the decay via the Yukawa couplings does not occur at the tree level. On the contrary, the anomaly-induced decay is not suppressed even in this case. Also, the inflaton decays into the right-handed (s) neutrinos, since the righthanded Majorana mass violates the conformal invariance. This may naturally generates the baryon asymmetry via leptogenesis scenario [30].

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