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Leptogenesis from Quantum Interference in a Thermal Bath

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Thermal leptogenesis explains the observed matter-antimatter asymmetry of the universe in terms of neutrino masses, consistent with neutrino oscillation experiments. We present a full quantum mechanical calculation of the generated lepton asymmetry based on Kadanoff-Baym equations. Origin of the asymmetry is the departure of the statistical propagator of the heavy Majorana neutrino from the equilibrium propagator, together with CP violating couplings. The lepton asymmetry is calculated directly in terms of Green's functions without referring to 'number densities'. A detailed comparison with Boltzmann equations shows that conventional leptogenesis calculations have an uncertainty of at least one order of magnitude. Particularly important is the inclusion of thermal damping rates in the full quantum mechanical calculation.

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Most theories of baryogenesis involve quantum interferences in a thermal bath in a crucial manner [1]. Here we consider thermal leptogenesis [2] which, in its simplest version, is dominated by the CP violating interactions of the lightest of the heavy Majorana neutrinos, the seesaw partners of the ordinary neutrinos. For neutrino masses inferred from neutrino oscillations, leptogenesis is dominated just by decays and inverse decays of the heavy neutrinos in the thermal plasma [3].

Almost all leptogenesis calculations are based on Boltzmann equations. This treatment has a basic conceptual problem: the Boltzmann equations are classical equations for the time evolution of phase space distribution functions; the involved collision terms, however, are usually zero-temperature S-matrix elements which involve quantum interferences. Clearly, a full quantum mechanical treatment is necessary to understand the range of validity of the Boltzmann equations and to determine the size of corrections [4, 5].

Various thermal corrections [6] have been incorporated in the context of Boltzmann equations, and 'quantum Boltzmann equations' have been derived from Kadanoff-Baym equations [7, 8]. In [4], a solution of the Kadanoff-Baym equations for leptogenesis has been found to leading order in a derivative expansion in terms of distribution functions satisfying the Boltzmann equations.

In this Letter we discuss leptogenesis directly in terms of Green's functions which are solutions of the Kadanoff-Baym equations, thus avoiding all approximations necessary to arrive at Boltzmann equations. Our work is based on [9], where the approach to thermal equilibrium has been discussed in terms of Green's functions for a toy model, a scalar field coupled to a large thermal bath. Here we extend this method to leptogenesis, which yields the lepton asymmetry directly in terms of Green's functions. In the following we describe the main result of our work. Detailed derivations will be given in [10].

The interactions of N, the lightest of the heavy Majorana neutrinos, with the Higgs doublet ϕ and lepton doublets l_{Li} is described by the lagrangian (cf. [4]),

$$\mathcal{L} = \bar{l}_{Li}\widetilde{\phi}\lambda^*_{i1}N + N^T\lambda_{i1}Cl_{Li}\phi - \frac{1}{2}MN^TCN + \frac{1}{2}\eta_{ij}l^T_{Li}\phi\, Cl_{Lj}\phi + \frac{1}{2}\eta^*_{ij}\bar{l}_{Li}\widetilde{\phi}\, C\bar{l}^T_{Lj}\widetilde{\phi}\ ; \qquad (1)$$

here C is the charge conjugation matrix, $\widetilde{\phi} = i\sigma_2\phi$, and the coupling

$$\eta_{ij} = \sum_{k>1}\lambda_{ik}\frac{1}{M_k}\lambda^T_{kj} \qquad (2)$$

is obtained by integrating out the heavy Majorana neutrinos $N_{k>1}$ with $M_{k>1} \gg M_1 \equiv M$. We shall consider the case of small Yukawa couplings, $\lambda_{i1} \ll 1$, such that the decay width of N is much smaller than its mass. The Lagrangian (1) represents an effective low-energy theory, valid for momenta up to $M_{k>1}$.

The Boltzmann equations for the time evolution of the distribution functions of heavy neutrinos, Higgs and lepton doublets are well known [11]. In this Letter our main goal is the comparison of Boltzmann and Kadanoff-Baym equations. We therefore focus on the CP-violating source term for the asymmetry and ignore the washout terms and the Hubble expansion, which can be added in a straightforward way [10].

For the distribution function of the heavy neutrinos one has [12],

$$\frac{\partial}{\partial t}f_N(t,\omega) = -\frac{2}{\omega}\int_{\mathbf{k},\mathbf{q}}(2\pi)^4\delta^4(k+q-p)\left(\lambda^\dagger\lambda\right)_{11}p\cdot k \times\left[f_N(t,\omega)(1-f_l(k))(1+f_\phi(q)) - f_l(k)f_\phi(q)(1-f_N(t,\omega))\right], \qquad (3)$$

where $\omega = \sqrt{M^2+\mathbf{p}^2}$, k and q are the energies of N, l and ϕ with equilibrium distribution functions f_l and f_ϕ, respectively; the averaged decay matrix element is $|M(N(p)\to l(k)\phi(q))|^2 = 2\left(\lambda^\dagger\lambda\right)_{11}p\cdot k$ (cf. [4]). For the momentum integrations we use the notation

$$\int_{\mathbf{p}} \ldots = \int \frac{d^3p}{(2\pi)^3 2\omega} \ldots \ . \qquad (4)$$

The sum of decay and inverse decay widths, which determines the rate for the approach to equilibrium [13], is given by

$$\Gamma_\beta(\omega) = \left(\lambda^\dagger\lambda\right)_{11} \frac{2}{\omega} \int_{\mathbf{k},\mathbf{q}} (2\pi)^4 \delta^4(k+q-p) \\ p\cdot k \; f_{l\phi}(k,q) \ , \qquad (5)$$

where we have introduced the function

$$f_{l\phi}(k,q) = f_l(k) f_\phi(q) + (1-f_l(k))(1+f_\phi(q)) \\ = 1 - f_l(k) + f_\phi(q) \ . \qquad (6)$$

For the solution of the Boltzmann equation (3) with vacuum initial condition, $f_N(0,\omega) = 0$, one easily obtains ($\Gamma_\beta(\omega) \equiv \Gamma$),

$$f_N(t,\omega) = f_N^{eq}(\omega)\left(1 - e^{-\Gamma t}\right) , \qquad (7)$$

where $f_N^{eq}(\omega) = 1/(e^{\beta\omega}+1)$, and $\beta = 1/T$ is the inverse temperature.

The Boltzmann equation for the lepton distribution function is given by

$$\frac{\partial}{\partial t} f_l(t,k) = -\frac{1}{2k}\int_{\mathbf{q},\mathbf{p}} (2\pi)^4\delta^4(k+q-p) \\ \times \left[|M(l\phi\to N)|^2 f_l(k) f_\phi(q)(1-f_N(t,\omega)) \right. \qquad (8) \\ \left. - \ |M(N\to l\phi)|^2 f_N(t,\omega)(1-f_l(k))(1+f_\phi(q))\right] \ ,$$

where now $\mathcal{O}(\lambda^4)$ corrections to the matrix elements have to be kept. Using Eq. (7) one obtains for the lepton asymmetry $f_{Li} = f_{li} - f_{\bar{l}i}$, with initial condition $f_{Li}(0,k) = 0$,

$$f_{Li}(t,k) = -\epsilon_{ii}\frac{1}{k}\int_{\mathbf{q},\mathbf{p}} (2\pi)^4\delta^4(k+q-p)\; p\cdot k \\ \times f_{l\phi}(k,q) f_N^{eq}(\omega)\frac{1}{\Gamma}\left(1-e^{-\Gamma t}\right) \ , \qquad (9)$$

where

$$\epsilon_{ij} = \frac{3\mathrm{Im}\{\lambda_{i1}^*(\eta\lambda^*)_{j1}\}M}{16\pi} \ . \qquad (10)$$

Summing over all flavours, the generated asymmetry is proportional to the familiar CP-asymmetry: $\epsilon = \sum_i \epsilon_{ii}/\left(\lambda^\dagger\lambda\right)_{11} = 3\mathrm{Im}(\lambda^\dagger\eta\lambda)M/(16\pi\left(\lambda^\dagger\lambda\right)_{11})$ [4].

For later comparison with solutions of the Kadanoff-Baym equations, it is convenient to rewrite (9) in the form

$$f_{Li}(t,k) = -\epsilon_{ii}\frac{16\pi}{k}\int_{\mathbf{q},\mathbf{p},\mathbf{q}',\mathbf{k}'} k\cdot k' \\ \times (2\pi)^4\delta^4(k+q-p)(2\pi)^4\delta^4(k'+q'-p) \\ \times f_{l\phi}(k,q) f_N^{eq}(\omega)\frac{1}{\Gamma}\left(1-e^{-\Gamma t}\right) \ . \qquad (11)$$

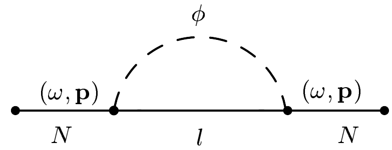

FIG. 1: 1-loop contribution to the self-energies $\Sigma^\pm_{\mathbf{p}}$ of the Majorana neutrino N.

Note that the integrand is now proportional to the averaged matrix element $|M(l\phi\to\bar{l}\bar{\phi})|^2 = 2k\cdot k'(\lambda^\dagger\lambda)_{11}/M^2$ (cf. [4]), which involves the product of the 4-vectors k and k'. At low temperatures, $T \ll M$, the integrand falls off like $e^{-\beta\omega} < e^{-\beta M}$, i.e., the generated lepton asymmetry is strongly suppressed.

Let us now consider spectral function and statistical propagator (cf. [14]) for the heavy Majorana neutrino,

$$G^-_{\alpha\beta}(x_1,x_2) = i\langle\{N_\alpha(x_1), N_\beta(x_2)\}\rangle \ , \qquad (12)$$

$$G^+_{\alpha\beta}(x_1,x_2) = \frac{1}{2}\langle[N_\alpha(x_1), N_\beta(x_2)]\rangle \ , \qquad (13)$$

which satisfy the Kadanoff-Baym equations [4, 10]

$$C(i\gamma^0\partial_{t_1} - \mathbf{p}\boldsymbol{\gamma} - M)G^-_{\mathbf{p}}(t_1-t_2) = \\ -\int_{t_2}^{t_1} dt'\Sigma^-_{\mathbf{p}}(t_1-t')G^-_{\mathbf{p}}(t'-t_2) \ , \qquad (14)$$

$$C(i\gamma^0\partial_{t_1} - \mathbf{p}\boldsymbol{\gamma} - M)G^+_{\mathbf{p}}(t_1,t_2) = \\ +\int_0^{t_2} dt'\Sigma^+_{\mathbf{p}}(t_1-t')G^-_{\mathbf{p}}(t'-t_2) \\ -\int_0^{t_1} dt'\Sigma^-_{\mathbf{p}}(t_1-t')G^+_{\mathbf{p}}(t',t_2) \ . \qquad (15)$$

Here we have assumed spatial homogeneity and performed a Fourier transform. The 1-loop contribution to the self-energies $\Sigma^\pm_{\mathbf{p}}$ is shown in Fig. 1. For small couplings, $\lambda \ll 1$, leading to a small width $\Gamma \ll M$, explicit solutions of the Kadanoff-Baym equations can be found in the Breit-Wigner approximation [10],

$$G^-_{\mathbf{p}}(y) = \left(i\gamma_0\cos(\omega y) + \frac{M - \mathbf{p}\boldsymbol{\gamma}}{\omega}\sin(\omega y)\right) \\ \times e^{-\Gamma|y|/2}C^{-1} \ , \qquad (16)$$

$$G^+_{\mathbf{p}}(t,y) = -\left(i\gamma_0\sin(\omega y) - \frac{M - \mathbf{p}\boldsymbol{\gamma}}{\omega}\cos(\omega y)\right) \\ \times\left[\frac{\tanh\left(\frac{\beta\omega}{2}\right)}{2}e^{-\Gamma|y|/2} + f_N^{eq}(\omega)e^{-\Gamma t}\right]C^{-1}, \qquad (17)$$

where $t = (t_1+t_2)/2$, $y = t_1 - t_2$ and $\Gamma = \Gamma_\beta(\omega)$ (5). For large t, $G^+_{\mathbf{p}}(t,y)$ approaches the equilibrium solution $G^{+eq}_{\mathbf{p}}(y)$, and for small temperatures, i.e., large β, it becomes the vacuum solution $G^{+vac}_{\mathbf{p}}(y)$. Note that the solution (17) for $G^+_{\mathbf{p}}(t,y)$ satisfies the initial condition

$$G^+_{\mathbf{p}}(0,0) = G^{+vac}_{\mathbf{p}}(0) \ , \qquad (18)$$

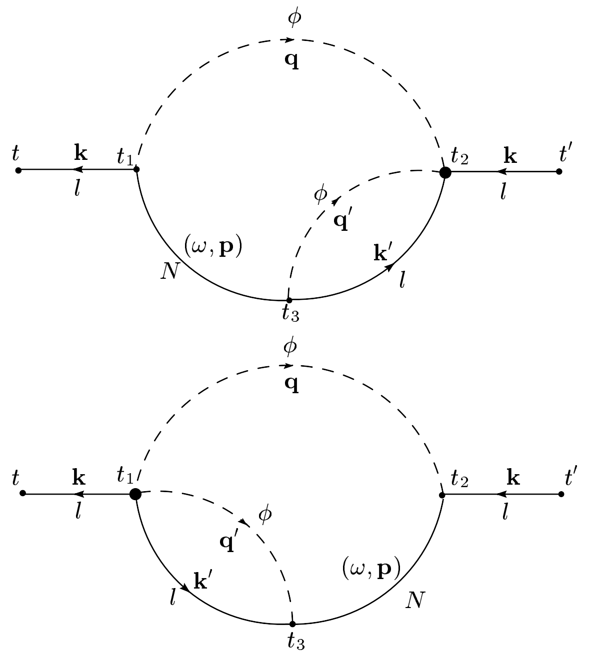

FIG. 2: 2-loop contributions to the lepton self-energies $\Pi^{\pm}_{\mathbf{k}}$, which lead to non-zero lepton number densities.

which is the analogue of the initial condition $f_N(0,\omega) = 0$ for the distribution function. For spectral function and statistical propagator of lepton and Higgs fields we shall use the free equilibrium expressions $\hat{S}^{\pm}_{\mathbf{k}}(y)$ and $\hat{\Delta}^{\pm}_{\mathbf{q}}(y)$, respectively. The thermal equilibrium is assumed to be established by Standard Model interactions.

We are now ready to calculate the lepton asymmetry which is generated during the approach of the right-handed neutrino N to equilibrium. The 'lepton number matrix' is obtained from the statistical propagator of the lepton fields,

$$L_{\mathbf{k}ij}(t,t') = -\mathrm{tr}[\gamma^0 S^{+}_{\mathbf{k}ij}(t,t')] \ . \tag{19}$$

One easily verifies that for free fields in equilibrium, $L_{\mathbf{k}ii}|_{t=t'} = f_{li}(k) - \bar{f}_{li}(k)$, which vanishes for zero chemical potential. To leading order in λ, a flavour non-diagonal asymmetry is generated by the 2-loop self-energies shown in Fig. 2 (cf. [4]). Solving the Kadanoff-Baym equation for $S^{+}_{\mathbf{k}}$ to first order in the self-energy $\Pi^{\pm}_{\mathbf{k}}$, one finds after some algebra

$$\begin{aligned} L_{\mathbf{k}ij}(t,t) = -i\int_0^t dt_1 \int_0^t dt_2\ \mathrm{tr}[&\hat{S}^{+}_{\mathbf{k}}(t_2-t_1)\Pi^{-}_{\mathbf{k}ij}(t_1,t_2) \\ &- \hat{S}^{-}_{\mathbf{k}}(t_2-t_1)\Pi^{+}_{\mathbf{k}ij}(t_1,t_2)] \ . \end{aligned} \tag{20}$$

A non-zero asymmetry is generated by the departure of $G^{+}_{\mathbf{p}}$ from equilibrium. Due to the chiral couplings only a chiral projection of $G^{+}_{\mathbf{p}}$ contributes,

$$\begin{aligned} \tilde{G}_{\mathbf{p}}(t,y)P_L &= P_L\left(G^{+}_{\mathbf{p}}(t,y) - G^{+eq}_{\mathbf{p}}(y)\right)P_L \ , \\ \tilde{G}_{\mathbf{p}}(t,y) &= \frac{M}{\omega}\cos(\omega y) f^{eq}_N(\omega) e^{-\Gamma t} \ . \end{aligned} \tag{21}$$

After a lengthy calculation one obtains for the lepton number matrix to leading order in the width Γ [10]:

$$\begin{aligned} L_{\mathbf{k}ij}(t,t) = &-\epsilon_{ij}\ 8\pi \int_{\mathbf{q},\mathbf{q}'} \frac{k\cdot k'}{kk'\omega} \\ &\times \frac{\frac{1}{2}\Gamma}{((\omega-k-q)^2+\frac{\Gamma^2}{4})((\omega-k'-q')^2+\frac{\Gamma^2}{4})} \\ &\times f_{l\phi}(k,q) f_{l\phi}(k',q') f^{eq}_N(\omega) \\ &\times \big(\cos[(k+q-k'-q')t] + e^{-\Gamma t} \\ &-(\cos[(\omega-k-q)t] + \cos[(\omega-k'-q')t])e^{-\frac{\Gamma t}{2}}\big) \ , \end{aligned} \tag{22}$$

where $\mathbf{p} = \mathbf{q} + \mathbf{k} = \mathbf{q}' + \mathbf{k}'$. This expression for the lepton asymmetries, generated by quantum interference in a thermal bath, is the main result of this Letter.

The expression (22) contains a logarithmic divergence $\mathcal{O}(\lambda^4)$. It has to be combined with other divergent terms which have been neglected in (22) since they do not contribute to leading order in Γ. The total divergence will be subtracted by a counter term. The dominant finite contribution to the integral stems from momenta $k+q \sim k'+q' \sim \omega$. In the zero-width limit $\Gamma \to 0$, with Γt fixed, the integrand is $\mathcal{O}(1/\Gamma)$. Since $\Gamma \propto \lambda^2$, this dominant contribution to the integral is $\mathcal{O}(\lambda^2)$.

It is interesting to compare the diagonal elements of the lepton number matrix $L_{\mathbf{k}ij}(t,t)$ with the distribution functions $f_{Li}(t,k)$ given in (11). As expected, the same CP asymmetries ϵ_{ii} appear, whereas the dependence of the integrands on time and temperature is different. The reason for the different temperature dependence is the fact that the matrix elements in the Boltzmann equations were calculated at zero temperature. Hence, the factor $f_{l\phi}(k',q')$ is missing in (11). Since Boltzmann equations are local in time whereas Kadanoff-Baym equations contain 'memory effects', the different time dependence of the asymmetries is also expected. One consequence is that $\partial_t f_{Li}(t,k)|_{t=0} \neq 0$, whereas $\partial_t L_{\mathbf{k}ij}(t,t)|_{t=0} = 0$. Particularly important are off-shell effects in (22), which lead to terms oscillating in time.

The additional thermal correction $f_{l\phi}(k',q')$ is linear in the distribution functions f_l and f_ϕ, in contrast to results obtained in the first two papers in [6], but in agreement with [7] and [8]. The memory and off-shell effects found in [7] are qualitatively different from our result (22).

It is instructive to consider the approximations, even if they might not be justified, which lead from (22) to the result of Boltzmann equations. Neglecting off-shell effects, i.e., imposing $\omega = k+q = k'+q'$, the cosines are replaced by one; performing then the zero-width approximation $\Gamma \to 0$, with Γt fixed, the integral (22) becomes

$$\begin{aligned} L^{os}_{\mathbf{k}ij}(t,t) = &-\epsilon_{ij}\frac{16\pi}{k}\int_{\mathbf{q},\mathbf{q}',\mathbf{p},\mathbf{k}'} k\cdot k' \\ &\times (2\pi)^4\delta^4(k+q-p)(2\pi)^4\delta^4(k'+q'-p) \\ &\times f_{l\phi}(k,q) f_{l\phi}(k',q') f^{eq}_N(\omega) \\ &\times \frac{1}{\Gamma}\left(1-e^{-\frac{\Gamma t}{2}}\right)^2 \ . \end{aligned} \tag{23}$$

Except for the factor $f_{l\phi}(k',q')$, the only difference compared to the solution (11) of the Boltzmann equations is the time dependence. It is obvious from Fig. 2 and Eq. (20) that in the quantum theory the generation of the lepton asymmetry is nonlocal in time. This leads to the square of the exponential fall-off in (23). On the contrary, in the Boltzmann equations the asymmetry is generated locally in time yielding a simple exponential behaviour. The difference can be numerically important at cosmologically relevant times $t_L \sim 1/\Gamma$.

The calculations leading to Eq. (22) also demonstrate that the result for the lepton asymmetry will be significantly modified by the thermal damping rates for lepton and Higgs fields in the plasma. These thermal widths (cf. [15]) are known to be much larger than the decay width of the Majorana neutrino: $\Gamma_l \sim \Gamma_\phi \sim g^2 T \gg \lambda^2 M$ for $M \lesssim T$. For quantum interferences the thermal damping rates are qualitatively more important than thermal masses which, for simplicity, we ignore in the following. Including naively thermal widths for lepton and Higgs fields in the self-energy $\Pi^+_{\mathbf{k}}$, one obtains instead of (22) to leading order in these widths ($\Gamma_{l\phi} = \Gamma_l + \Gamma_\phi$)

$$\begin{aligned}\tilde{L}_{\mathbf{k}ij}(t,t) &= -\epsilon_{ij}\ 16\pi \int_{\mathbf{q},\mathbf{q}'} \frac{k\cdot k'}{kk'\omega} \\ &\times \frac{\frac{1}{4}\Gamma_{l\phi}\Gamma_\phi}{((\omega-k-q)^2+\frac{1}{4}\Gamma^2_{l\phi})((\omega-k'-q')^2+\frac{1}{4}\Gamma^2_\phi)} \\ &\times f_{l\phi}(k,q) f_{l\phi}(k',q') f_N^{eq}(\omega) \\ &\times \frac{1}{\Gamma}\left(1-e^{-\Gamma t}\right). \end{aligned} \qquad (24)$$

Note that the factors oscillating in time have disappeared. The thermal widths of lepton and Higgs fields have led to a behaviour which is local in time. In the zero-width limit one now obtains the result (11) of the Boltzmann equations except for the thermal correction factor $f_{l\phi}(k',q')$. We emphasize that (24) is speculative at present, and it remains to be seen whether it follows from a solid calculation which includes gauge interactions in a systematic way.

We have studied the generation of a lepton asymmetry at constant temperature as the heavy Majorana neutrino approaches thermal equilibrium. For cosmological leptogenesis one has to calculate the lepton asymmetry in the case of decreasing temperature, which is caused by the expansion of the universe. Analyses based on Boltzmann equations suggest that both asymmetries are of comparable size for $T \sim M$ [3]. A more detailed discussion will be given in [10].

In this Letter we have compared lepton asymmetries calculated on the basis of Boltzmann equations with those obtained from Kadanoff-Baym equations. Our discussion illustrates, that even ignoring spectator and flavour effects [16], current leptogenesis calculations have an uncertainty of at least one order of magnitude. Particularly urgent is the inclusion of gauge interactions with the thermal bath in a full quantum calculation.

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