

# A tetraquark interpretation of the Belle data on the anomalous $\Upsilon(1S)\pi^+\pi^-$ and $\Upsilon(2S)\pi^+\pi^-$ production near the $\Upsilon(5S)$ resonance

Ahmed Ali\* and Christian Hambrock†

*Deutsches Elektronen-Synchrotron DESY, D-22607 Hamburg, Germany*

M. Jamil Aslam‡

*Physics Department, Quaid-i-Azam, University, Islamabad, Pakistan*

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We analyze Belle data [1] on the processes  $e^+e^- \rightarrow \Upsilon(1S)\pi^+\pi^-$ ,  $\Upsilon(2S)\pi^+\pi^-$  near the peak of the  $\Upsilon(5S)$  resonance, which are found to be anomalously large in rates compared to similar dipion transitions between lower  $\Upsilon$  resonances. Assuming these final states arise from the production and decays of the  $J^{PC} = 1^{--}$  state  $Y_b(10890)$ , which we interpret as a bound (diquark-antidiquark) tetraquark state  $[bq][\bar{b}\bar{q}]$ , a dynamical model for the decays  $Y_b \rightarrow \Upsilon(1S)\pi^+\pi^-$ ,  $\Upsilon(2S)\pi^+\pi^-$  is presented. A distinguishing aspect of our approach is that the dipions in these decays receive significant resonant contributions from the scalar  $0^{++}$  low mass tetraquark states,  $f_0(600)$  and  $f_0(980)$ , with only the  $f_0(600)$  allowed kinematically for the state  $\Upsilon(2S)\pi^+\pi^-$ . In addition, the dipion mass spectrum in  $Y_b \rightarrow \Upsilon(1S)\pi^+\pi^-$  also shows a resonating component from the  $2^{++}$   $q\bar{q}$ -meson state  $f_2(1270)$ . Including non-resonant contributions, we get excellent fits for both the invariant dipion mass spectra and the helicity angle distributions for these decays in our approach, strengthening the case for  $Y_b$  as a tetraquark  $b\bar{b}$  bound state.

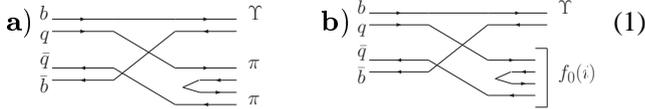
The observation of the  $\Upsilon(1S)\pi^+\pi^-$  and  $\Upsilon(2S)\pi^+\pi^-$  states near the  $\Upsilon(5S)$  resonance peak at  $\sqrt{s} = 10.87$  GeV at the KEKB  $e^+e^-$  collider by the Belle collaboration [1] has received a lot of theoretical attention [2]. The two puzzling features of these data are that the rates for  $e^+e^- \rightarrow \Upsilon(5S) \rightarrow \Upsilon(1S)\pi^+\pi^-$ ,  $\Upsilon(2S)\pi^+\pi^-$  are anomalously larger (by more than two orders of magnitude) than the expectations from scaling the comparable  $\Upsilon(4S)$  decays to the  $\Upsilon(5S)$ , and the shapes of the distributions in the dipion invariant mass  $m_{\pi\pi}$  and the cosine of the helicity angle,  $\cos\theta$ , where  $\theta$  is the angle between the  $\pi^-$  and  $\Upsilon(5S)$  in the dipion rest frame, are not described by the models [3] based on the QCD multipole expansion [4, 5] - a feature also at variance with similar dipion transitions between lower  $\Upsilon$  resonances. A critical observation towards understanding these features is that the final states in question are produced in the process  $e^+e^- \rightarrow Y_b(10890) \rightarrow \Upsilon(1S)\pi^+\pi^-$ ,  $\Upsilon(2S)\pi^+\pi^-$ , with  $Y_b$  a  $1^{--}$  state, having a total decay width  $\Gamma(Y_b) = 55 \pm 9$  MeV [6]. In a closely related recent paper [7], we have analyzed the BaBar data [8] obtained at the SLAC B factory during an energy scan of the  $e^+e^- \rightarrow b\bar{b}$  cross section in the range of the center of mass energy  $\sqrt{s} = 10.54$  to 11.20 GeV, observing that the BaBar data on the  $R_b$ -scan are consistent with the presence of an additional  $b\bar{b}$  state  $Y_{[bq]}$  with a mass of 10.90 GeV and a width of about 30 MeV, apart from the  $\Upsilon(5S)$  and  $\Upsilon(6S)$  resonances. Identifying the  $J^{PC} = 1^{--}$  state  $Y_{[bq]}(10900)$  seen in the energy scan of the  $e^+e^- \rightarrow b\bar{b}$  cross section by BaBar [8] with the state  $Y_b(10890)$  seen by Belle [1], we present a dynamical model based on the tetraquark in-

terpretation of  $Y_b(10890)$  and show that it is in excellent agreement with the measured distributions in the decays  $Y_b \rightarrow \Upsilon(1S)\pi^+\pi^-$ ,  $\Upsilon(2S)\pi^+\pi^-$ .

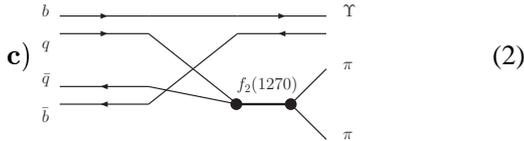
In the tetraquark interpretation,  $Y_{[bq]}$  is a  $J^{PC} = 1^{--}$  bound (diquark-antidiquark) state having the flavor content  $Y_{[bq]} = Q\bar{Q} = [bq][\bar{b}\bar{q}]$  (here  $q = u$  or  $q = d$ ) with the spin and orbital momentum quantum numbers:  $S_Q = 0$ ,  $S_{\bar{Q}} = 0$ ,  $S_{Q\bar{Q}} = 0$ ,  $L_{Q\bar{Q}} = 1$  [9]. Here, the first two quantum numbers are the diquark spin, antidiquark spin, respectively, and the last two denote the spin and the orbital angular quantum numbers of the tetraquarks, with the total spin being  $J = S_{Q\bar{Q}} + L_{Q\bar{Q}} = 1$ . Such spin-0 diquarks are called ‘‘good’’ diquarks [10] and an interpolating diquark operator can be written as  $Q_{i\alpha} = \epsilon_{\alpha\beta\gamma}(\bar{b}_c^\beta \gamma_5 q_i^\gamma - \bar{q}_i^\beta \gamma_5 b^\gamma)$  (with  $q_i = u, d$  for  $i = 1, 2$  and  $\bar{b}_c$  the charge conjugate  $b$ -quark field  $\bar{b}_c = -ib^T \sigma_2 \gamma_5$ ). So defined, the ‘‘good’’ diquark  $Q_{i\alpha}$  is in the attractive anti-triplet ( $\bar{3}$ ) color channel (with the color quantum numbers denoted by the Greek letters). There are two such  $J^{PC} = 1^{--}$  states,  $Y_{[bq]} = ([bq]_{S=0}[\bar{b}\bar{q}]_{S=0})_{P\text{-wave}}$ , with the mass eigenstates, called  $Y_{[b,l]}$  and  $Y_{[b,h]}$  in [7], being orthogonal combinations of  $Y_{[bu]}$  and  $Y_{[bd]}$ . Their mass difference is induced by isospin splitting  $m_d - m_u$  and a mixing angle, and is estimated as  $\Delta M(Y_b) = (5.6 \pm 2.8)$  MeV. Their dominant decays  $Y_{[b,l]} \rightarrow B^{(*)}\bar{B}^{(*)}$  and  $Y_{[b,h]} \rightarrow B^{(*)}\bar{B}^{(*)}$  were calculated in [7]. The decays  $Y_{[b,l]}(Y_{[b,h]}) \rightarrow \Upsilon(1S)\pi^+\pi^-$ ,  $\Upsilon(2S)\pi^+\pi^-$  are subdominant, but Zweig-allowed and involve essentially quark rearrangements shown below. In the following, we will not distinguish between the lighter and the heavier of these

states and denote them by the common symbol  $Y_b$ . With the  $J^{PC}$  of the  $Y_b$  and  $\Upsilon(nS)$  both  $1^{--}$ , the  $\pi^+\pi^-$  states in the decays  $Y_b \rightarrow \Upsilon(1S) \pi^+\pi^-$ ,  $\Upsilon(2S) \pi^+\pi^-$  are allowed to have the  $0^{++}$  and  $2^{++}$  quantum numbers. There are only three low-lying states in the PDG [11] which can contribute as intermediate states, namely the two  $0^{++}$  states,  $f_0(600)$  and  $f_0(980)$ , which, following [12], we take as the lowest tetraquark states, and the  $2^{++}$   $q\bar{q}$ -meson state  $f_2(1270)$ , all of which decay dominantly into  $\pi\pi$ . For the decay  $Y_b \rightarrow \Upsilon(1S) \pi^+\pi^-$ , all three states contribute. However, kinematics allows only the  $f_0(600)$  in the decay  $Y_b \rightarrow \Upsilon(2S) \pi^+\pi^-$ . In addition, there is a non-resonant contribution in the decays  $Y_b \rightarrow \Upsilon(1S) \pi^+\pi^-$  and  $Y_b \rightarrow \Upsilon(2S) \pi^+\pi^-$ . The dynamical model described below encodes all these features.

We start by showing the relevant diagrams for the decays  $Y_b(q) \rightarrow \Upsilon(p) + \pi^+(k_1) + \pi^-(k_2)$ .



The initial state represents the tetraquark states  $Y_b = [bq][\bar{b}\bar{q}]$ , and  $\Upsilon$  stands for  $\Upsilon(1S)$  and  $\Upsilon(2S)$ . Both diagrams involve the creation of a  $q\bar{q}$  pair from the vacuum, with the diagram on the left resulting into the (non-resonant) final states  $\Upsilon(1S) \pi^+\pi^-$  and  $\Upsilon(2S) \pi^+\pi^-$ , and the diagram shown on the right leading to the final states  $\Upsilon(1S) (f_0(600), f_0(980))$  and  $\Upsilon(2S) f_0(600)$ , with the implied subsequent decays  $(f_0(600), f_0(980)) \rightarrow \pi^+\pi^-$ . The  $2^{++}$  intermediate state  $f_2(1270)$  contributing to the decay  $Y_b \rightarrow \Upsilon(1S) \pi^+\pi^-$  is depicted below.



Writing the Lorentz-invariant amplitudes as

$$\mathcal{M} = \varepsilon_\mu^Y(q) \varepsilon_\nu^\Upsilon(p) \sum_{i=a,b,c} \mathcal{M}_i^{\mu\nu}(p, k_1, k_2), \quad (3)$$

where  $\varepsilon_\mu^Y(q)$  and  $\varepsilon_\nu^\Upsilon(p)$  are the polarization vectors of the  $Y_b$  and  $\Upsilon(nS)$ , respectively, we give below the explicit expressions for  $\mathcal{M}_i^{\mu\nu}(p, k_1, k_2)$ .

The amplitude corresponding to the non-resonant part is written, following Novikov and Shifman in [3], as

$$\mathcal{M}_a^{\mu\nu} = g^{\mu\nu} \frac{F}{F_\pi^2} \left[ m_{\pi\pi}^2 - \beta(\Delta M)^2 \left( 1 + \frac{2m_\pi^2}{m_{\pi\pi}^2} \right) + \frac{3}{2} \beta((\Delta M)^2 - m_{\pi\pi}^2) \left( 1 - \frac{4m_\pi^2}{m_{\pi\pi}^2} \right) (\cos^2\theta - \frac{1}{3}) \right], \quad (4)$$

Here  $\Delta M = M_{Y_b} - M_\Upsilon$ ,  $F_\pi = 130$  MeV is the pion decay constant,  $m_{\pi\pi} = \sqrt{(k_1 + k_2)^2}$  is the invariant mass of the two outgoing pions,  $\beta$  is a model-dependent parameter, which needs to be fitted from the data, and  $\theta$  is the angle between the  $\pi^-$  and  $Y_b$  in the dipion rest frame. We stress that the dynamical quantities  $F$  (a form factor) and  $\beta$  are specific to the decays  $Y_b \rightarrow \Upsilon(1S, 2S) \pi^+\pi^-$ .

The amplitude  $\mathcal{M}_b^{\mu\nu}$  coming from the diagram **b** is the resonant part involving the  $0^{++}$  states  $f_0(600)$  and  $f_0(980)$ , and the subsequent decays  $f_0(600), f_0(980) \rightarrow \pi^+\pi^-$ :

$$\mathcal{M}_b^{\mu\nu} = F_{f_0(i)} F_\pi g^{\mu\nu} \frac{g_{f_0(i)} k_1 \cdot k_2}{k^2 - m_{f_0(i)}^2 + i m_{f_0(i)} \Gamma_{f_0(i)}(m_{\pi\pi})}, \quad (5)$$

where  $f_0(i)$  are the two  $0^{++}$  resonances and the various dynamical factors are defined below in terms of the relevant vertices and the propagator:

$$\begin{aligned} Y_b \xrightarrow{\mu} \xrightarrow{q} \Upsilon \xrightarrow{\nu} &\hat{=} F_{f_0(i)} F_\pi g^{\mu\nu}, \\ f_0(i) \xrightarrow{k} \xrightarrow{k_1} \xrightarrow{k_2} &\hat{=} g_{f_0(i)} k_1 \cdot k_2, \\ \frac{1}{k^2 - m_{f_0(i)}^2 + i m_{f_0(i)} \Gamma_{f_0(i)}(m_{\pi\pi})} &\hat{=} \frac{1}{k^2 - m_{f_0(i)}^2 + i m_{f_0(i)} \Gamma_{f_0(i)}(m_{\pi\pi})}, \end{aligned} \quad (6)$$

where  $f_0(i) = f_0(600)$  or  $f_0(980)$ . The propagator of  $f_0(600)$  should not be taken in the minimal width approximation, since the total decay width and the mass are of the same order [11, 13]. Following [14], the width is multiplied by a momentum-dependent factor:

$$\Gamma(m_{\pi\pi}) = \Gamma_{f_0(600)} \frac{m_{f_0(600)} p^*}{m_{\pi\pi} p_0^*}, \quad (7)$$

where  $p_0^* = p^*(m_{f_0(600)})$  and  $p^* = p^*(m_{\pi\pi})$  are the decay momenta measured in the resonance rest frame. The other scalar ( $f_0(980)$ ), having  $\Gamma_{f_0(980)}/m_{f_0(980)} \ll 1$ , is taken in the minimal width approximation, i.e.  $\Gamma(m_{\pi\pi}) = \Gamma_{f_0(980)}$ .

The amplitude  $\mathcal{M}_c^{\mu\nu}$  coming from diagram **c** is

$$\mathcal{M}_c^{\mu\nu} = g^{\mu\nu} A_{f_2(1270)}(m_{\pi\pi}) = g^{\mu\nu} \frac{\sqrt{8\pi(2J+1)}}{\sqrt{m_{\pi\pi}}} Y_2^2 \times \frac{a_{f_2(1270)} \sqrt{m_{f_2(1270)}}}{m_{f_2(1270)}^2 - m_{\pi\pi}^2 - i m_{f_2(1270)} \Gamma_{f_2(1270)}}. \quad (8)$$

For  $f_2(1270)$ ,  $J = 2$  and we have kept only the helicity-2 component of the D wave with  $Y_2^2$  the corresponding spherical harmonics,  $|Y_2^2| = \sqrt{\frac{15}{32\pi}} \sin^2\theta$ . In principle, there is also a helicity-0 component of the D wave  $Y_2^0$

present in the amplitude, but following the high statistics experimental measurement of the process  $\gamma\gamma \rightarrow f_2(1270) \rightarrow \pi^+\pi^-$  by Belle [15], this contribution is small, characterized by the value of  $r_{02}$ , the helicity 0-to helicity 2 ratio in  $f_2(1270) \rightarrow \pi\pi$ ,  $r_{02} = (3.7 \pm 0.3_{-2.9}^{+15.9})\%$ . This can be included as more precise measurements become available.

These diagrams yield a coherent amplitude, and the various contributions interfere with each other having non-trivial strong (interaction) phases, which are *a priori* unknown. We treat them as free parameters to be determined by the fits to the Belle data. Combining all three amplitudes, the complete decay amplitudes for  $Y_b \rightarrow \Upsilon(1S)\pi^+\pi^-$ ,  $\Upsilon(2S)\pi^+\pi^-$  are:

$$\begin{aligned} \mathcal{M} &= \varepsilon^Y \cdot \varepsilon^\Upsilon \left[ \sum_{res} \text{diagram} + \text{diagram} \right] \\ &= \varepsilon^Y \cdot \varepsilon^\Upsilon \left[ a_{f_2(1270)} e^{i\varphi_{f_2(1270)}} A_{f_2(1270)}(m_{\pi\pi}) \right. \\ &\quad + \sum_i \frac{a_{f_0(i)} e^{i\varphi_{f_0(i)}} (m_{\pi\pi}^2 - 2m_\pi^2)/2}{m_{\pi\pi}^2 - m_{f_0(i)}^2 + im_{f_0(i)}\Gamma_{f_0(i)}(m_{\pi\pi})} \\ &\quad + \frac{F}{F_\pi^2} \left[ m_{\pi\pi}^2 - \beta(\Delta M)^2 \left(1 + \frac{2m_\pi^2}{m_{\pi\pi}^2}\right) + \right. \\ &\quad \left. \left. \frac{3}{2}\beta((\Delta M)^2 - m_{\pi\pi}^2) \left(1 - \frac{4m_\pi^2}{m_{\pi\pi}^2}\right) (\cos^2\theta - \frac{1}{3}) \right] \right], \end{aligned} \quad (9)$$

where  $a_{f_0(i)} = g_{f_0(i)} F_{f_0(i)} F_\pi$ . The sum over  $i$  runs over all  $0^{++}$  resonances contributing in the given energy range. As already stated, we restrict ourselves to  $f_0(600)$  and  $f_0(980)$ , since the contribution of  $f_0(1370)$  is expected to be small (it may even be forbidden by kinematics, as the measurements of  $f_0(1370)$  in [16] yield a mass  $m_{f_0(1370)} = 1434 \pm 18 \pm 9$  MeV). The couplings  $g_{f_0(600)} = -c_f$  and  $g_{f_0(980)} = \sqrt{2}c_I$  are taken from [12], where  $c_f = 0.02 \pm 0.002$  MeV<sup>-1</sup> and  $c_I = -0.0025 \pm 0.0012$  MeV<sup>-1</sup>. We use the central values for the couplings.

The differential decay width (averaged over the polarizations of the initial  $Y_b$ -meson and summed over polarizations of the final  $\Upsilon$ -meson) is given by

$$d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{32M_{Y_b}^3} |\overline{\mathcal{M}}|^2 dm_{\Upsilon\pi}^2 dm_{\pi\pi}^2, \quad (10)$$

where  $m_{\Upsilon\pi}^2 = (p+k_1)^2$  (the amplitude is symmetric under the interchange of the two pions). The  $\cos\theta$  dependence is given implicitly by  $m_{\Upsilon\pi}$ . By integrating over the phase space, we derive the two distributions in  $m_{\pi\pi}$  and  $\cos\theta$ .

We have undertaken fits of the Belle data [1] with our model (9), normalizing the distributions for the  $\Upsilon(1S)\pi^+\pi^-$  and  $\Upsilon(2S)\pi^+\pi^-$  channels to yield the measured partial decay widths  $\Gamma_{\Upsilon(1S)+2\pi} = 0.59 \pm 0.04 \pm$

$0.09$  MeV and  $\Gamma_{\Upsilon(2S)+2\pi} = 0.85 \pm 0.07 \pm 0.16$  MeV. The input parameters given in Table I are taken from the PDG [11], except for the  $f_0(600)$ , for which we have taken the values from E791 [14].

TABLE I. Input masses and decay widths (in GeV) of the resonances  $f_0(600)$ ,  $f_0(980)$  and  $f_2(1270)$ .

$M_{Y_b}$	10.890	$m_{f_0(600)}$	0.478	$\Gamma_{f_0(600)}$	0.324
$M_{\Upsilon(1S)}$	9.460	$m_{f_0(980)}$	0.980	$\Gamma_{f_0(980)}$	0.07
$M_{\Upsilon(2S)}$	10.023	$m_{f_2(1270)}$	1.270	$\Gamma_{f_2(1270)}$	0.185

The dipion invariant mass distribution  $d\Gamma/dm_{\pi\pi}$  and the angular distribution  $d\Gamma/d\cos\theta$  [GeV] measured by Belle [1] for the final state  $\Upsilon(2S)\pi^+\pi^-$  are shown in Fig. 1, normalized to yield an integrated decay width of  $\Gamma(Y_b \rightarrow \Upsilon(2S)\pi^+\pi^-) = 0.85$  MeV. The shaded histograms are the corresponding theoretical distributions having a  $\chi^2/d.o.f. \approx 9/8$ , with the fit parameters given in Table II. The solid curves are the distributions from the non-resonant part alone (4) for  $\beta = 0$ , which are representative of the models in [3].

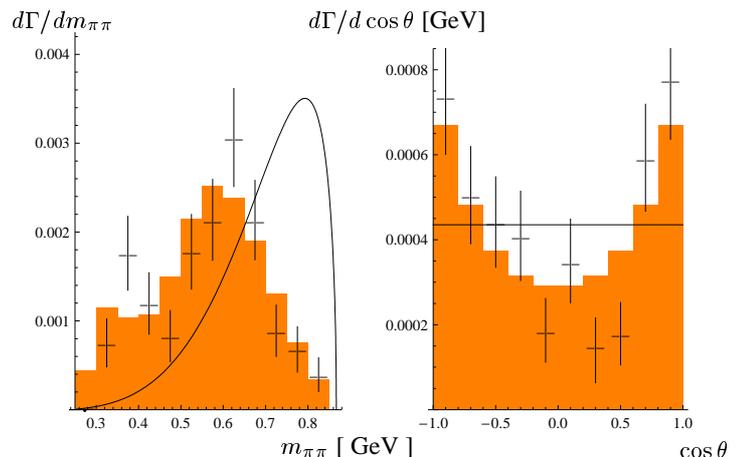


FIG. 1. Dipion invariant mass ( $m_{\pi\pi}$ ) distribution (left frame) and the  $\cos\theta$  distribution (right frame) measured by Belle [1] for the final state  $\Upsilon(2S)\pi^+\pi^-$  (Crosses), and the theoretical distributions based on this work, corresponding to the fit having a  $\chi^2/d.o.f. \approx 9/8$  (histograms). The solid curves indicate the fit where only the diagram given in (4) contributes. The distributions yield an integrated decay width  $\Gamma(Y_b \rightarrow \Upsilon(2S)\pi^+\pi^-) = 0.85$  MeV.

TABLE II. Fit values, yielding  $F = 0.86 \pm 0.34$ ,  $\beta = 0.7 \pm 0.3$  for the non-resonant contribution, and for the parameters entering in the resonant amplitude from  $f_0(600)$  for the decay  $Y_b \rightarrow \Upsilon(2S)\pi^+\pi^-$ .

	$a_{f_0(i)}$	$F_{f_0(i)}$	$F_{f_0(i)}/F$	$\varphi_{f_0(i)}$ (rad.)
$f_0(600)$	$10.89 \pm 2.4$	$4.19 \pm 0.92$	$4.86 \pm 2.18$	$2.76 \pm 0.22$

The measured spectra (in  $m_{\pi\pi}$  and  $\cos\theta$ ) for the final state  $\Upsilon(1S)\pi^+\pi^-$  from Belle [1], normalized to yield an integrated decay width of  $\Gamma(Y_b \rightarrow \Upsilon(1S)\pi^+\pi^-) = 0.66$  MeV, are shown in Fig. 2 together with the theoretical distributions (histograms) having a  $\chi^2/d.o.f. \approx 5/5$ , with the fit parameters given in Table III. The theoretical distributions in this case include the resonant contributions from  $f_0(600)$ ,  $f_0(980)$  and  $f_2(1270)$  as well as the non-resonant contribution, with the complete amplitude displayed in Eq. (9). The solid curves correspond to the non-resonant part alone ((4)) for  $\beta = 0$ .

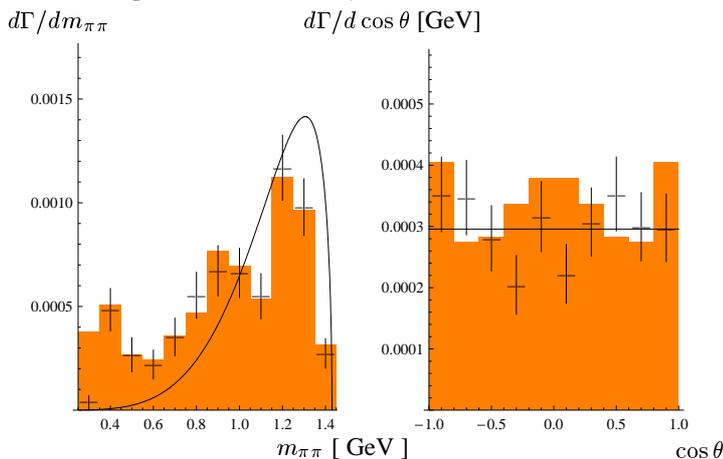


FIG. 2. The distributions measured by Belle [1] for the final state  $\Upsilon(1S)\pi^+\pi^-$  (Crosses), and the theoretical distributions based on this work, corresponding to the fit having a  $\chi^2/d.o.f \approx 5/5$  (histograms). The solid curves indicate the fit where only the diagram given in (4) contributes. The distributions yield an integrated decay width  $\Gamma(Y_b \rightarrow \Upsilon(2S)\pi^+\pi^-) = 0.66$  MeV.

TABLE III. Fit values, yielding  $F = 0.19 \pm 0.03$ ,  $\beta = 0.54 \pm 0.12$  for the non-resonant contribution,  $a_{f_2(1270)} = 0.5 \pm 0.16$ ,  $\varphi_{f_2(1270)} = 3.33 \pm 0.06$  for  $f_2(1270)$ , and for the parameters entering in the resonant amplitude from  $f_0(600)$  and  $f_0(980)$  for the decay  $Y_b \rightarrow \Upsilon(1S)\pi^+\pi^-$ .

	$a_{f_0(i)}$	$F_{f_0(i)}$	$F_{f_0(i)}/F$	$\varphi_{f_0(i)}$ (rad.)
$f_0(600)$	$3.6 \pm 0.7$	$1.38 \pm 0.27$	$7.34 \pm 1.94$	$1.14 \pm 0.14$
$f_0(980)$	$0.47 \pm 0.02$	$1.02 \pm 0.04$	$5.42 \pm 1.0$	$4.12 \pm 0.3$

We also remark that using the fits of the data for the decay  $Y_b \rightarrow \Upsilon(1S)\pi^+\pi^-$  presented here, we are able to explain the decay width for the decay  $Y_b \rightarrow \Upsilon(1S)K^+K^-$ , measured by Belle [1], with the decay width (and hence the dikaon invariant mass spectrum) dominated by the  $0^{++}$  tetraquark state  $f_2(980)$ . Details will be published elsewhere. The dynamical model presented here will be tested in great detail with improved data, which we expect in the near future from Belle. As we have argued here, the decays  $Y_b \rightarrow \Upsilon(1S, 2S)\pi^+\pi^-$  are radically different than the similar dipion transitions measured in the  $\Upsilon(4S)$  and lower mass Quarkonia. The dipion mass distributions in

$Y_b \rightarrow \Upsilon(1S, 2S)\pi^+\pi^-$  can be used to improve our knowledge of the low mass scalars,  $f_0(600)$  and  $f_0(980)$ .

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\* ahmed.ali@desy.de

† christian.hambrock@desy.de

‡ muhammadjamil.aslam@gmail.com

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