# Diphoton production at Tevatron in the quasi-multi-Regge-kinematics approach 

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#### Abstract

We study the production of prompt diphotons in the central region of rapidity within the framework of the quasi-multi-Regge-kinematics approach applying the hypothesis of quark and gluon Reggeization. We describe accurately and without free parameters the experimental data which were obtained by the CDF Collaboration at the Tevatron Collider. It is shown that the main contribution to studied process is given by the direct fusion of two Reggeized gluons into a photon pair, which is described by the effective Reggeon-Reggeon to particle-particle vertex. The contribution from the annihilation of Reggeized quark-antiquark pair into a diphoton is also considered. At the stage of numerical calculations we use the Kimber-Martin-Ryskin prescription for unintegrated quark and gluon distribution functions, with the Martin-Roberts-Stirling-Thorne collinear parton densities for a proton as input.


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## I. INTRODUCTION

The study of production of photons with large transverse momenta in the hard interaction between two partons in the high-energy hadron collisions, so-called prompt photon production, provides precision tests of perturbative quantum chromodynamics (QCD) as well as information on the parton densities within a hadron. Also, these studies are our potential for the observation of a new dynamical regime, namely the high-energy Regge limit, which is characterized by the following condition $\sqrt{S} \gg \mu \gg \Lambda_{Q C D}$, where $\sqrt{S}$ is the total collision energy in the center of mass reference frame, $\Lambda_{Q C D}$ is the asymptotic scale parameter of QCD, $\mu$ is the typical energy scale of the hard interaction. At this high-energy limit, the contribution from the partonic subprocesses involving $t$-channel parton (quark or gluon) exchanges to the production cross section can become dominant. In the region under consideration, the transverse momenta of the incoming partons and their off-shell properties can no longer be neglected, and we deal with "Reggeized" $t$-channel partons. In this kinematics, so-called quasi-multi-Regge kinematics, the particles (multi-Regge) or the groups of particles (quasi-multi-Regge) produced in the collision are strongly separated in rapidity. In our discussion, the both photons from the photon pair are produced in the central region of rapidity.

The quasi-multi-Regge-kinematics (QMRK) approach [1] is particularly appropriate for this kind of high-energy phenomenology. It is based on an effective quantum field theory implemented with the non-Abelian gauge-invariant action including fields of Reggeized gluons [2] and Reggeized quarks [3]. In the QMRK approach we can do calculations with Reggeized quarks [5, 6] that presents an open question in the other uncollinear factorization scheme, namely $k_{T}$-factorization approach [7], following which we can operate correctly with offshell gluons only. Recently it was shown also, that the calculation in the next to leading order (NLO) in strong coupling constant within the framework of QMRK approach can be done [8].

Our previous studies of charmonium and bottomonium production [4], open charm production 5] and inclusive prompt photon production at the DESY HERA and at the Fermilab Tevatron [6] demonstrated the advantages of the high-energy factorization scheme based on the QMRK approach over the collinear parton model as far as the description of experimental data is concerned.

This paper continues our study of the inclusive prompt photon production at Tevatron which was performed recently [6] in the framework of the the QMRK approach. We consider here the diphoton production at the Fermilab Tevatron in the central region of rapidity applying the hypothesis of quark and gluon Reggeization [1, 3, 9].

It is shown that the main contribution to studied process is given by the direct fusion of two Reggeized gluons into a photon pair, which is described by the effective ReggeonReggeon to particle-particle vertex. The contribution from the annihilation of Reggeized quark - antiquark pair into a diphoton is not small and it is taken into account additionaly. We do not take into consideration the contribution of the fragmentation mechanism of the prompt photon production which is strongly suppressed by the isolation cone condition [6].

At the stage of numerical calculations we use the Kimber-Martin-Ryskin prescription 【10] for unintegrated quark and gluon distribution functions, with the Martin-Roberts-StirlingThorne (MRST) collinear parton densities for a proton as input 11] .

This paper is organized as follows. In Sec. III, the relevant Reggeon-Reggeon to particleparticle effective vertices are presented and discussed. In Sec. III, we describe diphoton production spectra at the Tevatron Collider. In Sec. IV, we summarize our conclusions.

## II. BASIC FORMALISM

In the phenomenology of strong interactions at high energies, it is necessary to describe the QCD evolution of the parton distribution functions of colliding particles starting with some scale $\mu_{0}$, which controls a non-perturbative regime, to the typical scale $\mu$ of the hard-scattering processes, which is typically of the order of the transverse mass $M_{T}=\sqrt{M^{2}+\left|\vec{k}_{T}\right|^{2}}$ of the produced particle with mass $M$ and transverse momentum $\vec{k}_{T}$. In the region of very high energies, in so-called Regge limit, the typical ratio $x=\mu / \sqrt{S}$ becomes very small, $x \ll 1$. That leads to large logarithmic contributions of the type $\left[\alpha_{s} \ln (1 / x)\right]^{n}$ in the resummation procedure, which is described by the Balitsky-Fadin-Kuraev-Lipatov (BFKL) evolution equation [12] or other BFKL-like ones for unintegrated gluon (quark) distribution functions $\Phi_{g, q}\left(x,\left|\mathbf{q}_{T}\right|^{2}, \mu^{2}\right)$. Correspondingly, in the QMRK approach [1], the initial-state $t$-channel gluons and quarks are considered as Reggeons, or Reggeized gluons ( $R$ ) and Reggeized quarks $(Q)$. They are off-mass shell and carry finite transverse two-momenta $\mathbf{q}_{T}$ with respect to the hadron beam from which they stem.

The advantages of the QMRK approach include: first, it uses gauge-invariant amplitudes and is based on a factorization hypothesis that is proven in the leading logarithmic approximation; second, it carries over to non-leading orders in the strong-coupling constant, as recently proven [8]; third, it works both with Reggeized gluons and with Reggeized quarks. The Reggeization of amplitudes provides the opportunity to efficiently take into account large radiative corrections to processes in the Regge limit beyond what is included in the collinear approximation, which is of great practical importance. The particle Reggeization is known effect in the high-energy quantum electrodynamics (QED) for electrons [13], and for gluons [12] and quarks [3, 9] in QCD.

Recently, in Refs. [3, 14], the Feynman rules for the effective theory based on the nonAbelian gauge-invariant action including fields of Reggeized gluons [2] and Reggeized quarks [9] were derived for the induced and some important effective vertices. The effective vertices for the $2 \rightarrow 2$ processes with Reggeized gluons in the initial state only, $C^{R R \rightarrow g g}$ and $C^{R R \rightarrow q \bar{q}}$, were obtained in Refs. [12, 15], the effective vertices $C^{Q \bar{Q} \rightarrow g g}$ and $C^{R Q \rightarrow g q}$ with Reggeized quark and Reggeized gluon in the initial state were obtained in Ref. [3]. For our purposes we need to construct the effective vertices with two photons in the final state, i.e. $C^{Q \bar{Q} \rightarrow \gamma \gamma}$ and $C^{R R \rightarrow \gamma \gamma}$. The first one can be easy obtained from the above mentioned vertices after cutting the contribution from the three-gluon vertex and replacement the quark-gluon vertex by the same quark-photon vertex. The effective vertex $C^{R R \rightarrow \gamma \gamma}$ can be obtained using the Feynman rules [14] for the Reggeized gluons and the well-known fourth-rank vacuum polarization tensor, which corresponds to the set of quark-box diagrams [16].

We perform calculations in the laboratory frame of the Tevatron Collider, where proton and antiproton beams have equal energies, $E_{p}=E_{\bar{p}}=900(980) \mathrm{GeV}$. We introduce two lightcone vectors corresponding to the massless particle momenta as follows: $P_{1}=E_{1}(1,0,0,1)$ and $P_{2}=E_{2}(1,0,0-1)$, where $E_{1}=E_{p}$ and $E_{2}=E_{\bar{p}}$. Also we define two additional four-vectors $\left(n^{+}\right)^{\mu}=P_{2}^{\mu} / E_{2}$ and $\left(n^{-}\right)^{\mu}=P_{1}^{\mu} / E_{1}$. For any arbitrary four-momentum $k^{\mu}$, we define $k^{ \pm}=k \cdot n^{ \pm}=k^{\mu} n_{\mu}^{ \pm}$. We use the vertex functions defined in Ref.[3], which describe transition of the Reggeized quark with the four-momentum $q$ and gluon or photon with the four-momentum $k$ into the on-shell massless quark with four-momentum $k+q$ :

$$
\begin{align*}
& \gamma_{\mu}^{(+)}(k, q)=\gamma_{\mu}+\hat{q} \frac{n_{\mu}^{+}}{k^{+}}=\gamma_{\mu}+\hat{q} \frac{P_{2 \mu}}{P_{2} \cdot k}  \tag{1}\\
& \gamma_{\mu}^{(-)}(k, q)=\gamma_{\mu}+\hat{q} \frac{n_{\mu}^{-}}{k^{-}}=\gamma_{\mu}+\hat{q} \frac{P_{1 \mu}}{P_{1} \cdot k} . \tag{2}
\end{align*}
$$

The effective Reggeon-Reggeon to particle-particle vertex $C_{\mu \nu}^{\bar{Q} Q \rightarrow \gamma \gamma}\left(q_{1}, q_{2}, k_{1}, k_{2}\right)$, which describes the Reggeized quark - Reggeized antiquark annihilation into a photon pair in the process

$$
\begin{equation*}
Q\left(q_{1}\right)+\bar{Q}\left(q_{2}\right) \rightarrow \gamma\left(k_{1}\right)+\gamma\left(k_{2}\right), \tag{3}
\end{equation*}
$$

can be presented as follows:

$$
\begin{align*}
C_{\mu \nu}^{\bar{Q} Q \rightarrow \gamma \gamma}\left(q_{1}, q_{2}, k_{1}, k_{2}\right) & =-e_{q}^{2} e^{2}\left[\gamma_{\nu}^{(+)}\left(k_{2}, q_{2}\right) \frac{\hat{q}_{1}-\hat{k}_{1}}{\left(q_{2}+k_{2}\right)^{2}} \gamma_{\mu}^{(-)}\left(-k_{1}, q_{1}\right)+\right. \\
& \left.+\gamma_{\mu}^{(+)}\left(k_{1}, q_{2}\right) \frac{\hat{q}_{1}-\hat{k}_{2}}{\left(q_{2}+k_{1}\right)^{2}} \gamma_{\nu}^{(-)}\left(-k_{2}, q_{1}\right)+\Delta_{\mu \nu}\left(q_{1},-q_{2}\right)\right] \tag{4}
\end{align*}
$$

where momenta of the initial Reggeized parton are $q_{(1,2)}=x_{(1,2)} P_{(1,2)}+q_{(1,2) T}$, and the induced term $\Delta_{\mu \nu}\left(q_{1}, q_{2}\right)$ has the form

$$
\begin{equation*}
\Delta_{\mu \nu}\left(q_{1}, q_{2}\right)=\hat{q}_{1} \frac{n_{\mu}^{-} n_{\nu}^{-}}{k_{1}^{-} k_{2}^{-}}+\hat{q}_{2} \frac{n_{\mu}^{+} n_{\nu}^{+}}{k_{1}^{+} k_{2}^{+}} \tag{5}
\end{equation*}
$$

The vertex (4) satisfies the gauge-invariant condition, which reads as follows

$$
\begin{equation*}
C_{\mu \nu}^{\bar{Q} Q \rightarrow \gamma \gamma}\left(q_{1}, q_{2}, k_{1}, k_{2}\right) k_{2}^{\mu}=C_{\mu \nu}^{\bar{Q} Q \rightarrow \gamma \gamma}\left(q_{1}, q_{2}, k_{1}, k_{2}\right) k_{1}^{\nu}=0 . \tag{6}
\end{equation*}
$$

The effective gauge-invariant vertex $C_{\mu \nu}^{R R \rightarrow \gamma \gamma}\left(q_{1}, q_{2}, k_{1}, k_{2}\right)$, which describes the direct Reggeized gluon fusion into a diphoton in the process

$$
\begin{equation*}
R\left(q_{1}\right)+R\left(q_{2}\right) \rightarrow \gamma\left(k_{1}\right)+\gamma\left(k_{2}\right), \tag{7}
\end{equation*}
$$

can be presented as follows

$$
\begin{equation*}
C_{\mu \nu}^{R R \rightarrow \gamma \gamma}\left(q_{1}, q_{2}, k_{1}, k_{2}\right)=\Pi_{T}^{(-) \alpha}\left(q_{1}\right) \Pi_{T}^{(+) \beta}\left(q_{2}\right) G_{\alpha \beta \mu \nu}\left(q_{1}, q_{2}, k_{1}, k_{2}\right), \tag{8}
\end{equation*}
$$

$G_{\alpha \beta \mu \nu}\left(q_{1}, q_{2}, k_{1}, k_{2}\right)$ is the vacuum polarization tensor of forth rank corresponding the set of quark-box diagrams, which are taken with the relevant color factor.

It is suitable to define the new csalar vertex $C^{R R \rightarrow \gamma \gamma}\left(q_{1}, q_{2}, k_{1}, k_{2}\right)$, in which the summation over final photon polarizations has been performed:

$$
\begin{equation*}
C^{R R \rightarrow \gamma \gamma}\left(q_{1}, q_{2}, k_{1}, k_{2}\right)=\sum_{\lambda_{1}, \lambda_{2}} \varepsilon^{\mu}\left(k_{1}, \lambda_{1}\right) \varepsilon^{\nu}\left(k_{2}, \lambda_{2}\right) C_{\mu \nu}^{R R \rightarrow \gamma \gamma}\left(q_{1}, q_{2}, k_{1}, k_{2}\right), \tag{9}
\end{equation*}
$$

where $\varepsilon^{\mu}\left(k_{1}, \lambda_{1}\right)$ and $\varepsilon^{\nu}\left(k_{2}, \lambda_{2}\right)$ are the polarization four-vectors of final photons. The vertex (9) is presented as follows

$$
\begin{equation*}
C^{R R \rightarrow \gamma \gamma}\left(q_{1}, q_{2}, k_{1}, k_{2}\right)=\Pi_{T}^{(-) \alpha}\left(q_{1}\right) \Pi_{T}^{(+) \beta}\left(q_{2}\right) \tilde{G}_{\alpha \beta}\left(q_{1}, q_{2}, k_{1}, k_{2}\right), \tag{10}
\end{equation*}
$$

where

$$
\tilde{G}_{\alpha \beta}\left(q_{1}, q_{2}, k_{1}, k_{2}\right)=\sum_{\lambda_{1}, \lambda_{2}} \varepsilon^{\mu}\left(k_{1}, \lambda_{1}\right) \varepsilon^{\nu}\left(k_{2}, \lambda_{2}\right) G_{\alpha \beta \mu \nu}\left(q_{1}, q_{2}, k_{1}, k_{2}\right) .
$$

The exact expression for the tensor $\tilde{G}_{\alpha \beta}\left(q_{1}, q_{2}, k_{1}, k_{2}\right)$ have been obtained in the Ref. [16]. We use massless four-quark scheme to calculate this tensor.

To obtain the vertex (10) we operate with projectors on the Reggeized gluon states which can be presented in the two equivalent forms $\Pi_{T}^{(+) \nu}\left(q_{2}\right)=\frac{q_{2 T}^{\nu}}{\left|\vec{q}_{2 T}\right|}$ or $\Pi_{T}^{(+) \nu}\left(q_{2}\right)=-\frac{x_{2} E_{2}\left(n^{+}\right)^{\nu}}{\left|\vec{q}_{2 T}\right|}$ and $\Pi_{T}^{(-) \nu}\left(q_{1}\right)=\frac{q_{1 T}^{\nu}}{\left|\vec{q}_{1 T}\right|}$ or $\Pi_{T}^{(-) \nu}\left(q_{1}\right)=-\frac{x_{1} E_{1}\left(n^{-}\right)^{\nu}}{\left|\vec{q}_{1 T}\right|}$. On the contrary to the definition used in Ref. [3], we do not include on-shell quark spinors in our equations for the effective vertex (4). We also used different normalization for the projector $\Pi_{T}^{( \pm) \mu}$, which reads accordingly Ref.[3, 14] as $\Pi^{( \pm) \mu}=\left(n^{ \pm}\right)^{\mu}$. Our definition implies that the squared Reggeized amplitude in the QMRK approach is normalized to the squared parton amplitude for on-shell quarks and gluons when $\vec{q}_{1 T}=\vec{q}_{2 T}=0$.

At the next step we write the squared matrix elements of the above mentioned Reggeized parton processes taking into account kinematical conditions of the Tevaron Collider.

The squared matrix element for the direct diphoton production via annihilation (3) of Reggeized quark from a proton ( $q_{1}=x_{1} P_{1}+q_{1 T}$ ) and Reggeized antiquark from a antiproton $\left(q_{2}=x_{2} P_{2}+q_{2 T}\right)$ is obtained from the effective vertex (4) and it is presented as follows

$$
\begin{equation*}
\overline{\left|M\left(Q_{p} \bar{Q}_{\bar{p}} \rightarrow \gamma \gamma\right)\right|^{2}}=\frac{32}{3} \pi^{2} e_{q}^{4} \alpha^{2} \frac{x_{1} x_{2}}{a_{1} a_{2} b_{1} b_{2} S \hat{t} \hat{u}}\left(w_{0}+w_{1} S+w_{2} S^{2}+w_{3} S^{3}\right), \tag{11}
\end{equation*}
$$

where $a_{1}=2 k_{1} \cdot P_{2} / S, a_{2}=2 k_{2} \cdot P_{2} / S, b_{1}=2 k_{1} \cdot P_{1} / S, b_{2}=2 k_{2} \cdot P_{1} / S, S=2 P_{1} \cdot P_{2}$, $t_{1}=-q_{1 T}^{2}, t_{2}=-q_{2 T}^{2}, \hat{s}=\left(q_{1}+q_{2}\right)^{2}, \hat{t}=\left(q_{1}-k_{1}\right)^{2}, \hat{u}=\left(q_{1}-k_{2}\right)^{2}$, and we apply that $\hat{s}+\hat{t}+\hat{u}=-t_{1}-t_{2}, x_{1}=a_{1}+a_{2}, x_{2}=b_{1}+b_{2}$,

$$
\begin{equation*}
w_{0}=t_{1} t_{2}\left(t_{1}+t_{2}\right)-\hat{t} \hat{u}(\hat{t}+\hat{u}), \tag{12}
\end{equation*}
$$

$$
\begin{gather*}
-w_{1}=t_{1} t_{2}\left(a_{1}-a_{2}\right)\left(b_{1}-b_{2}\right)+t_{2} x_{1}\left(b_{2} \hat{t}+b_{1} \hat{u}\right)+ \\
+t_{1} x_{2}\left(a_{1} \hat{t}+a_{2} \hat{u}\right)+\hat{t} \hat{u}\left(a_{1} b_{1}+2 a_{2} b_{1}+2 a_{1} b_{2}+a_{2} b_{2}\right),  \tag{13}\\
-w_{2}=b_{1} b_{2} x_{1}^{2} t_{2}+a_{1} a_{2} x_{2}^{2} t_{1}+a_{1} b_{2} \hat{t}\left(x_{1} b_{1}+a_{2} b_{2}\right)+a_{2} b_{1} \hat{u}\left(a_{1} b_{1}+a_{2} x_{2}\right),  \tag{14}\\
\quad-w_{3}=a_{1} a_{2} b_{1} b_{2}\left(a_{1} b_{2}\left(\frac{\hat{t}}{\hat{u}}\right)+a_{2} b_{1}\left(\frac{\hat{u}}{\hat{t}}\right)\right) . \tag{15}
\end{gather*}
$$

When we consider the collinear approximation, in which one has $a_{1}=-\frac{\hat{u}}{x_{2} S}, a_{2}=-\frac{\hat{t}}{x_{2} S}$, $b_{1}=-\frac{\hat{t}}{x_{1} S}, b_{2}=-\frac{\hat{u}}{x_{1} S}, x_{1} x_{2}=\frac{\hat{s}}{S}$, and $t_{1}=t_{2}=0$, the well-known answer for the squared on-shell parton amplitude is obtained

$$
\begin{equation*}
\overline{|M(\bar{q} q \rightarrow \gamma \gamma)|^{2}}=\frac{32}{3} \pi^{2} e_{q}^{4} \alpha^{2}\left(\frac{\hat{t}}{\hat{u}}+\frac{\hat{u}}{\hat{t}}\right) \tag{16}
\end{equation*}
$$

The squared matrix element for the symmetric subprocess $\bar{Q}_{p} Q_{\bar{p}} \rightarrow \gamma \gamma$ is treated very similarly, one can be obtained from (11) by the replacements: $t_{1} \leftrightarrow t_{2}, \hat{t} \leftrightarrow \hat{u}, a_{1} \leftrightarrow b_{1}$, $a_{2} \leftrightarrow b_{2}$, and $x_{1} \leftrightarrow x_{2}$. It is easy to see that these replacements do not change formula (11).

The squared matrix element $\overline{|M(R R \rightarrow \gamma \gamma)|^{2}}$ for the direct diphoton production via the fusion of Reggeized gluons (7) is obtained from the effective vertex (10). The analytical answer for the squared matrix element is known, see Ref. [17], in the following approximation: $m_{q}=0$ and $t_{1,2} \ll \hat{s}, \hat{t}, \hat{u}$, where $m_{q}$ is the mass of an quark in the relevant amplitude. It can be presented in the following form:

$$
\begin{equation*}
\overline{|M(R R \rightarrow \gamma \gamma)|^{2}}=2 \alpha^{2} \alpha_{s}^{2}\left(\sum_{i=1}^{n_{f}} e_{q}^{2}\right)^{2}\left(f_{1}+f_{2}+f_{3}\right) \tag{17}
\end{equation*}
$$

where $n_{f}=4$ is the number of active quark flavors,

$$
\begin{aligned}
f_{1} & =\frac{1}{8}\left[\left(\frac{\hat{s}^{2}+\hat{t}^{2}}{\hat{u}^{2}} \log ^{2}\left(\frac{-\hat{s}}{\hat{t}}\right)+2 \frac{\hat{s}-\hat{t}}{\hat{u}} \log \left(\frac{-\hat{s}}{\hat{t}}\right)\right)^{2}+\right. \\
& +\left(\frac{\hat{s}^{2}+\hat{u}^{2}}{\hat{t}^{2}} \log ^{2}\left(\frac{-\hat{s}}{\hat{u}}\right)+2 \frac{\hat{s}-\hat{u}}{\hat{t}} \log \left(\frac{-\hat{s}}{\hat{u}}\right)\right)^{2}+ \\
& \left.+\left(\frac{\hat{u}^{2}+\hat{t}^{2}}{\hat{s}^{2}}\left(\log ^{2}\left(\frac{\hat{t}}{\hat{u}}\right)+\pi^{2}\right)+2 \frac{\hat{t}-\hat{u}}{\hat{s}} \log \left(\frac{\hat{t}}{\hat{u}}\right)\right)^{2}\right] \\
f_{2} & =\frac{1}{2}\left[\frac{\hat{s}^{2}+\hat{t}^{2}}{\hat{u}^{2}} \log ^{2}\left(\frac{-\hat{s}}{\hat{t}}\right)+\frac{\hat{s}^{2}+\hat{u}^{2}}{\hat{t}^{2}} \log ^{2}\left(\frac{-\hat{s}}{\hat{u}}\right)+\right. \\
& +\frac{\hat{s}-\hat{t}}{\hat{u}} \log \left(\frac{-\hat{s}}{\hat{t}}\right)+\frac{\hat{s}-\hat{u}}{\hat{t}} \log \left(\frac{-\hat{s}}{\hat{u}}\right)+ \\
& \left.+\frac{\hat{t}^{2}+\hat{u}^{2}}{\hat{s}^{2}}\left(\log ^{2}\left(\frac{\hat{t}}{\hat{u}}\right)+\pi^{2}\right)+2 \frac{\hat{t}-\hat{u}}{\hat{s}} \log \left(\frac{\hat{t}}{\hat{u}}\right)\right] \\
f_{3} & =\frac{\pi^{2}}{2}\left[\left(\frac{\hat{s}^{2}+\hat{t}^{2}}{\hat{u}^{2}} \log \left(\frac{-\hat{s}}{\hat{u}}\right)+\frac{\hat{s}-\hat{t}}{\hat{u}}\right)^{2}+\left(\frac{\hat{s}^{2}+\hat{u}^{2}}{\hat{t}^{2}} \log \left(\frac{-\hat{s}}{\hat{t}}\right)+\frac{\hat{s}-\hat{u}}{\hat{t}}\right)^{2}\right]+4 .
\end{aligned}
$$

The formula (17) formally coincides with the result, which is obtained in the collinear parton model. Any way, we incorporate the (17) with the off-shell kinematics for the initial Reggeized-gluons. Of course, the exact squared matrix elements with the off-shell initial particles is needed. Our relevant calculations for arbitrary values of $t_{1}$ and $t_{2}$ are in the progress.

## III. PROMPT DIPHOTON PRODUCTION AT TEVATRON

During last decade the CDF and D0 Collaborations at Tevatron obtained experimental data for the production of prompt photons: inclusively [18, 19], in association with a jet [20], in association with a heavy quark $(c, b)[21]$, and in pair [22]. The inclusive prompt photon production was studied in the QMRK approach in the Ref.[6]. It was shown that the main mechanism of the inclusive prompt photon production in $p \bar{p}$ collisions is the fusion of Reggeized quark and Reggeized antiquark into a photon, via the effective Regeon-Reggeon to particle (to photon) vertex $C^{Q \bar{Q} \rightarrow \gamma}$. We have described well the inclusive photon transverse momentum spectra measured by the CDF and D0 Collaborations [18, 19] in the wide region of the photon transverse momentum and pseudorapidity.

Here we consider the production of prompt photon pair or diphoton in the framework of the QMRK approach and compare our predictions with the CDF data [22]. The cross sections are measured differentially as a function of the diphoton transverse momentum $\left(p_{T}\right)$, the diphoton invariant mass $(M)$, and the azimuthal angle between the two photons $(\Delta \varphi)$. In this experimental analysis, the isolation condition required that the transverse energy sum in a cone of radius $R=0.4$ (in $\varphi-\eta$ space) about the photon direction, minus the photon energy, be less than 1 GeV . It is important to note, that the both photons in the CDF experiment [22] are produced in the central region of rapidity (pseudorapidity) $\left|\eta_{1,2}\right|<0.9$. We can consider such photons as a quasi-multi-Regge group of particles.

There are two mechanisms of prompt photon production: the production of direct photons and the production of photons in fragmentation of produced quarks and gluons into photons. We have obtained [6] that the contribution of fragmentation mechanism is strongly suppressed by the isolation cone condition, which is applied to the experimental data for the inclusive photon production [18, 19]. We estimate one as small, about $5 \%$, and do not take into account this contribution in the presented analysis too.

The leading contributions in diphoton production are originating from the $2 \rightarrow 2$ processes (3) and (7). The first one is order of $\alpha^{2}$, the second one is order of $\alpha^{2} \alpha_{s}^{2}$.

The factorization formulas for the hadron cross sections in the QMRK approach are the following:

$$
d \sigma(p \bar{p} \rightarrow \gamma \gamma X)=\int \frac{d x_{1}}{x_{1}} \int \frac{d^{2} q_{1 T}}{\pi} \int \frac{d x_{2}}{x_{2}} \int \frac{d^{2} q_{2 T}}{\pi} d \hat{\sigma}(Q \bar{Q} \rightarrow \gamma \gamma) \times
$$

$$
\begin{align*}
& \left.\times\left[\Phi_{Q / p}\left(x_{1}, t_{1}, \mu^{2}\right) \Phi_{\bar{Q} / \bar{p}}\left(x_{2}, t_{2}, \mu^{2}\right)+\Phi_{\bar{Q} / p}\left(x_{1}, t_{1}, \mu^{2}\right) \Phi_{Q / \bar{p}}\left(x_{2}, t_{2}, \mu^{2}\right)\right] 18\right) \\
d \sigma(p \bar{p} \rightarrow \gamma \gamma X)= & \int \frac{d x_{1}}{x_{1}} \int \frac{d^{2} q_{1 T}}{\pi} \int \frac{d x_{2}}{x_{2}} \int \frac{d^{2} q_{2 T}}{\pi} d \hat{\sigma}(R R \rightarrow \gamma \gamma) \times \\
& \times \Phi_{R / p}\left(x_{1}, t_{1}, \mu^{2}\right) \Phi_{R / \bar{p}}\left(x_{2}, t_{2}, \mu^{2}\right) \tag{19}
\end{align*}
$$

where $\Phi_{Q, R / p}\left(x, t, \mu^{2}\right)$ are the unintegrated parton distribution functions, $d \hat{\sigma}(R R, Q \bar{Q} \rightarrow \gamma \gamma)$ are the Reggeized parton cross sections. The unintegrated distribution functions and the corresponding collinear $F_{q, g / p}\left(x, \mu^{2}\right)$ distribution functions are connected by the normalization condition

$$
\begin{equation*}
x F_{q, g / p}\left(x, \mu^{2}\right)=\int_{0}^{\mu^{2}} \Phi_{Q, R / p}\left(x, t, \mu^{2}\right) d t \tag{20}
\end{equation*}
$$

that ensures the correct transition to the collinear-parton limit of equations (18) and (19). The Reggeized parton cross sections are defined in the following manner

$$
\begin{equation*}
d \hat{\sigma}(R R, Q \bar{Q} \rightarrow \gamma \gamma)=(2 \pi)^{4} \delta^{(4)}\left(q_{1}+q_{2}-k_{1}-k_{2}\right) \frac{\overline{|M(R R, Q \bar{Q} \rightarrow \gamma \gamma)|^{2}}}{I_{R R, Q \bar{Q}}} \prod_{i=1}^{2} \frac{d^{3} k_{i}}{(2 \pi)^{3} 2 k_{i}^{0}} \tag{21}
\end{equation*}
$$

where $I_{R R, Q \bar{Q}}=2 x_{1} x_{2} S$ is the flux factor of the incoming particles, $k_{1}^{\mu}=\left(k_{1}^{0}, \vec{k}_{1 T}, k_{1}^{z}\right)$ and $k_{2}^{\mu}=\left(k_{2}^{0}, \vec{k}_{2 T}, k_{2}^{z}\right)$ are the four-momenta of produced photons.

Using the formulas (18), (19) and (21), we obtain the differential cross sections for the diphoton production, which can be written in case of the process (7) as follows:

$$
\begin{align*}
\frac{d \sigma(p \bar{p} \rightarrow \gamma \gamma X)}{d(\triangle \varphi)}= & \frac{1}{16 \pi^{3}} \int d t_{1} \int d \varphi_{1} \int d k_{1 T} \int d \eta_{1} \int d k_{2 T} \int d \eta_{2} \frac{k_{1 T} k_{2 T} \overline{\left.M(R R \rightarrow \gamma \gamma)\right|^{2}}}{\left(x_{1} x_{2} S\right)^{2}} \\
& \times \Phi_{R / p}\left(x_{1}, t_{1}, \mu^{2}\right) \Phi_{R / \bar{p}}\left(x_{2}, t_{2}, \mu^{2}\right)  \tag{22}\\
\frac{d \sigma(p \bar{p} \rightarrow \gamma \gamma X)}{d p_{T}}= & \frac{p_{T}}{16 \pi^{3}} \int d t_{1} \int d \varphi_{1} \int d k_{1 T} \int d \eta_{1} \int d k_{2 T} \int d \eta_{2} \frac{\overline{|M(R R \rightarrow \gamma \gamma)|^{2}}}{\left(x_{1} x_{2} S\right)^{2}|\sin (\triangle \varphi)|} \\
& \times \Phi_{R / p}\left(x_{1}, t_{1}, \mu^{2}\right) \Phi_{R / \bar{p}}\left(x_{2}, t_{2}, \mu^{2}\right)  \tag{23}\\
\frac{d \sigma(p \bar{p} \rightarrow \gamma \gamma X)}{d M}= & \frac{M}{16 \pi^{3}} \int d t_{1} \int d \varphi_{1} \int d k_{1 T} \int d \eta_{1} \int d k_{2 T} \int d \eta_{2} \frac{\overline{\mid M\left(\left.R R \rightarrow \gamma \gamma\right|^{2}\right.}}{\left(x_{1} x_{2} S\right)^{2}|\sin (\triangle \varphi)|} \\
& \times \Phi_{R / p}\left(x_{1}, t_{1}, \mu^{2}\right) \Phi_{R / \bar{p}\left(x_{2}, t_{2}, \mu^{2}\right)} \tag{24}
\end{align*}
$$

where $k_{1,2 T}=\left|\vec{k}_{1,2 T}\right|, \eta_{1,2}$ are the photon pseudorapidities, $\varphi_{1}$ is the azimuthal angle between $\vec{k}_{1 T}$ and $\vec{q}_{1 T}, x_{1}=\left(k_{1}^{0}+k_{2}^{0}+k_{1}^{z}+k_{2}^{z}\right) / \sqrt{S}, x_{2}=\left(k_{1}^{0}+k_{2}^{0}-k_{1}^{z}-k_{2}^{z}\right) / \sqrt{S}, k_{1,2}^{0}=\frac{k_{1,2 T}}{2}\left(e^{\eta_{1,2}}+e^{-\eta_{1,2}}\right)$, $k_{1,2}^{z}=\frac{k_{1,2 T}}{2}\left(e^{\eta_{1,2}}-e^{-\eta_{1,2}}\right)$. The differential cross sections for the diphoton production in case of the process (3) are written similarly. To perform the multi-dimensional integration we use the REWIAD code from the CERNLIB program library and we control the accuracy at the level of 2-3 percent.

We compare the results of our calculations with the experimental data from the CDF Collaboration [22]. The kinematic region under consideration is defined by the following conditions $\sqrt{S}=1960 \mathrm{GeV},\left|\eta_{1,2}\right|<0.9, k_{1 T}>14 \mathrm{GeV}$ and $k_{2 T}>13 \mathrm{GeV}$. The results of our calculation are shown in the Figs. [173. The description of the data can be considered as good at the whole kinematic region of variables $M, p_{T}$ and $\Delta \varphi$. It is very interesting to compare the relative weight of different processes in connection with the $\alpha_{s}$ order which they have. So, the contribution of $R R \rightarrow \gamma \gamma$ process, which is order of $\alpha^{2} \alpha_{s}^{2}$, is greater, by factor $5-10$, than the contribution of $Q \bar{Q} \rightarrow \gamma \gamma$ process, which is order of $\alpha^{2}$. This fact demonstrates a dominance of the gluon induced interactions at high energies. We see that suppression by factor $\alpha_{s}^{2}$ is smaller that enhancement of the gluon density over the quark density in a proton at the small $x$. The contributions of the $2 \rightarrow 3$ processes $(Q R \rightarrow \gamma \gamma q$ and $Q \bar{Q} \rightarrow \gamma \gamma g)$, which are order of $\alpha^{2} \alpha_{s}$, would be unimportant for the theoretical approach used here. At first, the events when the diphoton and the quark (gluon) jet are produced with close (pseudo)rapidities are suppressed by the isolation cone condition. At second, the events when the diphoton and the quark (gluon) jet are produced with the large (pseudo)rapidity gap are included in the presented consideration via the effective vertices for Reggeized parton interactions in the uncollinear factorization scheme, in which we use unintegrated over the transverse momentum parton distribution functions.

## IV. CONCLUSIONS

We have shown that it is possible to describe data for the prompt diphoton production in high-energy $p \bar{p}$ collisions at the Tevatron Collider in the leading order of the QMRK approach. The scheme of our calculation bases on the hypothesis of quark and gluon Reggeization in the hard processes at high energy. Our results demonstrate that the QMRK approach is a powerful tool in the high-energy phenomenology. It is evidently that the QMRK approach can be used also for description of the photon plus jet production [20] and the photon plus heavy quark production [21] at Tevatron and the LHC Collider. Our results for these processes will be presented in the future publications [23].

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FIG. 1: The diphoton $p_{T}$ distribution from the CDF Collaboration [22]. The curve 1 is the contribution of the $Q \bar{Q} \rightarrow \gamma \gamma$ process, 2 is the contribution of the $R R \rightarrow \gamma \gamma$ process, and 3 is their


FIG. 2: The diphoton mass distribution from the CDF Collaboration [22]. The curves are the same as in Fig. 1.


FIG. 3: The diphoton $\triangle \varphi$ distribution from the CDF Collaboration [22]. The curves are the same as in Fig. 1.


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