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Higgs production as a probe of dark energy interactions

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Abstract. We study Higgs production under the influence of a light, scalar dark energy field with chameleon-like couplings to matter. Our analysis is relevant for hadron colliders, such as the Large Hadron Collider, which are expected to manufacture Higgs particles through weak boson fusion, or associated production with a Z or $W^{\pm}$. We show that the corrections arising in these models are too small to be observed. This result can be attributed to the gauge invariance of the low energy Lagrangian. As a by-product of our analysis, we provide the first microphysical realization of a dark energy model coupled to the electromagnetic field strength. In models where dark energy couples to all matter species in a uniform manner we are able to give a new, stringent bound on its coupling strength.

Keywords: Dark energy theory, Weak interactions beyond the Standard Model, Cosmology of theories beyond the Standard Model

1. Introduction

We now have compelling evidence that the rate of cosmological expansion is accelerating, requiring the universe to be dominated by a form of matter, known as 'dark energy,' characterized by the equation of state $p \approx -\rho$. This dark energy could be a cosmological constant unnaturally tuned at the level of 1 part in 10^{123} [1]. Alternatively, it may be associated with the potential energy of a scalar field [2, 3, 4, 5]. Unravelling the microphysics of dark energy is the outstanding puzzle of contemporary cosmology, but progress may be possible. For example, if a scalar field is the agent of acceleration then its microstates could modify the properties of astrophysical objects, or the phenomenology observed at a particle collider. Failure to observe such effects rules out large regions of parameter space [6, 7, 8, 9, 10].

In a classical approximation, scalar field models are experimentally acceptable provided their mass is determined by a 'chameleon' mechanism [11, 12, 13, 14, 15]. This makes the field heavy in a dense environment, but light in vacuum. A chameleon mechanism makes it mandatory for the scalar field to interact with conventional matter. Therefore, although it is not known whether microphysical, quantum realizations of the chameleon scenario exist, an unambiguous prediction emerges if they do: new, light scalar quanta must be present in the beam pipe of any particle accelerator. In this paper, we study the consequences of such quanta at a hadron collider, such as the *Large Hadron Collider* (LHC) or the *Tevatron*, focusing on the process of Higgs production, which is a key target at both facilities. Our results apply for any light scalars with the requisite chameleon-like couplings, whatever their origin.

We discuss these theories in the framework of quantum mechanics. This is important, because the cosmological constant problem—which is the motivation for scalar field scenarios of dark energy—exists only after quantum effects have been taken into account. There is no profit in replacing the cosmological constant with another theory which is classically acceptable, but unnatural once quantum corrections are included.

The models which furnish chameleon-like scalar particles at low energy are effective theories, valid below some energy scale M. In Ref. [16], corrections to electroweak precision observables from particles with chameleonic or axionic couplings were analysed, without making a commitment to any specific choice of physics at energies above M. For processes involving fermions and $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauge bosons, it was shown that large effects could always be absorbed into renormalizations of the Fermi constant, G_F, and the gauge boson masses. When Higgs processes are included it is no longer clear that large effects can be hidden in this way. Therefore, in principle, Higgs production could function as a diagnostic of dark energy physics.

The details of Higgs production depend on which couplings occur in the theory. To achieve a successful chameleon phenomenology, in which unwanted fifth forces mediated by dark energy are suppressed, we assume conformal couplings to matter.†

† This is not the only possibility which can lead to a successful chameleon phenomenology, but it is the

This has implications for interactions with the gauge boson kinetic terms, which afford an important means of detection in collider experiments. (This will be described in more detail in §4 below.) The Standard Model gauge bosons have conformally invariant kinetic terms, which develop no couplings to a scalar field of this type. In this paper we show that violations of conformal symmetry, arising from couplings to matter species, can be communicated to the kinetic term by quantum corrections. Therefore, a coupling *is* generated at energies below M via loops of charged heavy particles.

What scale, M, should be associated with these new degrees of freedom? A conservative choice would be the GUT scale, $M \sim 10^{16}$ GeV, which may be connected with an early inflationary stage, or a seesaw explanation of neutrino mass [17, 18, 19]. If so, heavy neutrinos of mass M might exist but would be sterile, having no $\mathrm{SU}(2) \times \mathrm{U}(1)$ quantum numbers. Alternatively, if supersymmetry is realized in nature then M could be associated with the scale at which it is spontaneously broken, perhaps of order 1–10 TeV. In any supersymmetric completion of the Standard Model there exist fermionic partners of γ, $W^{\pm}$, Z and the Higgs, known as gauginos and Higgsinos, for a review see [20] and references therein. The mass eigenstates of these particles ("charginos") would naturally be of order M. In this article we remain agnostic about the nature of whatever particles circulate within loops. We give our calculation in a form which can be specialized immediately to the minimal supersymmetric Standard Model (MSSM), but our conclusions are general and do not depend on the details of a specific implementation. In any case, the calculation we describe can be adapted easily to any heavy particle carrying the requisite quantum numbers.

The outline of this paper is as follows. In §2 we introduce our model and show that a coupling between gauge bosons and dark energy is generated at low energy by integrating out heavy particles. In §3 we briefly review Higgs production at a hadron collider, emphasizing the role of vertices between the electroweak gauge bosons and the Higgs. We show that oblique corrections can modify Higgs production in particle colliders, and in §4 we compute these for the low energy theory written down in §2. We show that the effective Lagrangian includes new contact interactions between the gauge bosons and the Higgs. At scales smaller than $1/M$ these contact interactions resolve into heavy particle loops. We compute the correction to Higgs production from both effects. We conclude in §5.

Throughout this paper, we adopt units in which $c = \hbar = 1$. We set the reduced Planck mass, $M_{\mathrm{P}} \equiv (8\pi G)^{-1/2}$, to unity. Our metric convention is $(-,+,+,+)$.

2. A low-energy theory of gauge bosons and scalars

Khoury & Weltman [12, 11] suggested that a scalar field, χ, might evade detection and yet remain light in vacuum if it coupled to each matter species, labelled i,† via a

easiest to motivate from possible ultraviolet completions of the Standard Model, such as string theory.
† If the dark energy couples through a species-independent conformal rescaling of the metric, then the strength of the coupling to each matter species is the same. For phenomenological purposes, however,

conformally rescaled metric,

$$g_{ab} \to g^i_{ab} = f_i(\beta_i \chi) g_{ab}. \tag{1}$$

In this formula, g_{ab} is the spacetime metric, and $\beta_i \equiv 1/M_i$ is a coupling scale. The conformal functions f_i must be chosen to realise a successful chameleon mechanism. Certain classical realizations of this idea have been constructed, but it is not known whether quantum realizations exist, or what condition the f_i must satisfy once quantum corrections are taken into account. In what follows we will neglect the index i for simplicity, but it is not necessary to assume that the dark energy field couples with the same strength to all matter fields.

We work in the Einstein frame, in which the gravitational part of the action is that of general relativity and the scalar field χ appears only in the action for the matter fields. If the scalar field couples conformally with a universal strength, then an equivalent description can be obtained by transforming to the Jordan frame where all effects of the scalar fields are moved to the gravitational sector. We choose the Einstein frame to simplify the calculations that follow, but all physical observables are independent of the choice of frame.

The kinetic terms of spin-1 particles are conformally invariant, and are left inert under the substitution $g_{ab} \to g^i_{ab}$ in Eq. (1). However, particle physics is not conformal; any coupling to other matter species will generically break this invariance. If so, quantum corrections will cause the kinetic terms to depend on g^i_{ab}. Such interactions could potentially explain variation in the fine structure constant, α [21, 22, 23], and would lead to strong laboratory [24, 25, 26, 27] and astrophysical constraints [28, 29, 30, 31, 32]. Certain astronomical observations may be explained by including cosmologically light scalars which couple in this manner [33]. In this section we compute these threshold corrections, leading to an effective theory which describes the interaction of gauge bosons and the dark energy field at low energy.

2.1. Axion-like couplings from heavy particle loops‡

The necessary corrections are depicted in Fig. 1, in which a heavy fermionic particle circulates in the loop. Fermions couple to the metric (1) via a vielbein, e^μ_a, which satisfies

$$g_{ab} = \eta_{\mu\nu} e^\mu_{(a} e^\nu_{b)}. \tag{2}$$

There is an inverse vielbein, e^a_μ, satisfying $e^\mu_a e^a_\nu = \delta^\mu_\nu$ and $e^\mu_a e^b_\mu = \delta^a_b$. The indices μ, ν, ... transform under a rigid Lorentz symmetry and can be coupled to Dirac γ-matrices. The action for a Dirac fermion, λ, with large mass M,§ can be written

$$L = \sqrt{-g} f^2(\beta\chi) \left\{ -\bar{\lambda}(\gamma^\mu e^a_\mu D_a + M)\lambda \right\}, \tag{3}$$

one may wish to relax this restriction. In the remainder of this paper we allow the matter couplings to be distinct, but frequently return to the minimal scenario where all coupling strengths are the same.

‡ We would like to thank D. Shaw for very helpful discussions while preparing the text of this section.

§ Note that M is unrelated to the chameleon mass scale, $\mathcal{M}$, discussed in the Introduction (§1). In the remainder of this paper, M always refers to a heavy fermion mass and never to the chameleon scale.

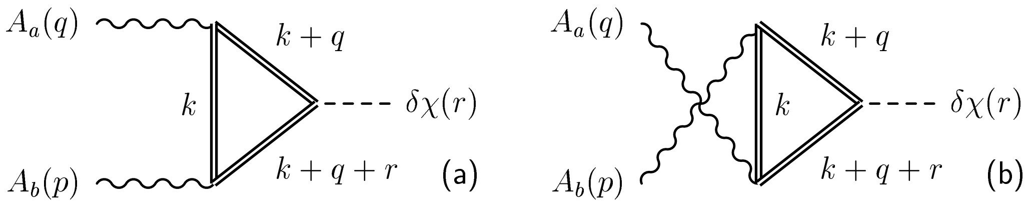


Figure 1. Diagrams contributing to the leading interaction between dark energy and the electroweak gauge bosons, which determine an effective operator acting on $A_a(q)A_b(p)\chi(r)$. Note that the momentum carried by χ is taken to flow into the diagram. Double lines represent a species of heavy fermion charged under $\mathrm{SU}(2)\times\mathrm{U}(1)$.

where D_a is a gauge-covariant derivative and $\bar{\lambda} \equiv \lambda^\dagger\gamma^0$ is the spinor conjugate to λ. We suppose that λ transforms under an Abelian symmetry with gauge coupling constant e, so that

$$D_a \equiv \partial_a + \frac{1}{8}\gamma_{\mu\nu}\omega_a^{\mu\nu} - \mathrm{i}eA_a, \tag{4}$$

where $\gamma_{\mu\nu} = [\gamma_\mu, \gamma_\nu]$. Note that the spin connexion, $\omega_a^{\mu\nu}$, transforms non-trivially under conformal rescalings. The calculation will be generalized to non-Abelian symmetries below Eq. (7). We take the dark energy scalar to have a spatially-independent vacuum expectation value $\langle\chi\rangle = \bar{\chi}$, around which we quantize small fluctuations $\delta\chi$. (For this reason our calculation cannot be applied to very large spacetime volumes in which χ may develop appreciable gradients.) After rescaling the fermion fields to have canonical kinetic terms at leading order, the interaction between λ and these fluctuations can be written

$$L_{\text{eff}} \supseteq -\frac{1}{2}\sqrt{-g}\bar{f}^{-1/2}\bar{f}'\beta M(\bar{\lambda}\lambda)\delta\chi, \tag{5}$$

where $\bar{f} \equiv f(\beta\bar{\chi})$ and a prime $'$ denotes the derivative of a function with respect to its argument. Conformal invariance is broken by the fermion mass, M. Like the mass of any canonical field, M transforms under conformal transformations like $M \to \bar{f}^{-1/2}\bar{f}'M$. If all energy scales ran in the same way with the vacuum expectation value of the scalar field then the effects of this coupling would never be observable, but this is not the case. Both the Planck scale, M_P, and the scale controlling the strength of the scalar coupling to matter, β, are unchanged by variations in the scalar field.

In each diagram of Fig. 1, operators $A_a(q)$, $A_b(q)$ and $\delta\chi(r)$ are inserted on the external legs, with all momenta flowing inwards. Fig. 1(b) depicts the "crossed" diagram, which corresponds to reversing the sense of momentum flow in the fermion loop. It can be obtained from Fig. 1(a) by the simultaneous replacements $p \leftrightarrow q$ and $a \leftrightarrow b$. Accounting for both diagrams we find that the correlation function $\langle A_a(q)A_b(p)\delta\chi(r)\rangle$ can be written

$$\langle A_a(q)A_b(p)\delta\chi(r)\rangle = -(2\pi)^4\delta(p+q+r)\frac{\bar{f}^{3/2}\bar{f}'}{2}e^2\beta M \cdot \delta\chi(r)\cdot A_a(q)A_b(p) \times$$
$$\int \frac{\mathrm{d}^4k}{(2\pi)^4}\operatorname{tr}\left\{\gamma^a\frac{-\mathrm{i}(\not{k}+\not{q})+M}{(k+q)^2+M^2-\mathrm{i}\epsilon}\frac{-\mathrm{i}(\not{k}-\not{p})+M}{(k-p)^2+M^2-\mathrm{i}\epsilon}\gamma^b\frac{-\mathrm{i}\not{k}+M}{k^2+M^2-\mathrm{i}\epsilon}\right\}$$

$$+\begin{pmatrix} p \leftrightarrow q \\ a \leftrightarrow b \end{pmatrix}, \tag{6}$$

where 'tr' denotes a trace over Dirac indices.

The k integral is divergent, and such integrals are not guaranteed to be invariant under the rigid shift $k^a \to k^a + a^a$. In the absence of a shift symmetry, the integral can depend on the labeling of momenta within the loop. We apply dimensional regularization to maintain gauge invariance, after which it can be checked that the result is insensitive to the routing of momentum through the diagram. Eq. (6) also contains a potentially gauge-violating zero derivative term, which would be proportional to $\delta\chi A^a A_a$ in real space. This term vanishes when the k integral is evaluated using any gauge-invariant regulator. Discarding terms which are proportional to the equations of motion, we find that Eq. (6) can be reproduced from an effective Lagrangian of the form

$$L_{\text{eff}} \supseteq \frac{e^2}{3(4\pi)^2}\beta \bar{f}^{3/2}\bar{f}' \cdot \delta\chi F^{ab}F_{ab} + \mathrm{O}(\partial^4), \tag{7}$$

where $\mathrm{O}(\partial^4)$ denotes terms containing four or more derivatives. Eq. (7) applies for each species of heavy fermion in the theory. If there are many such fermions, each contributes with its own β and f. If e, $\bar{f}$ and $\bar{f}'$ are of order unity for all species, the coupling will be dominated by the fermion with largest β. Similar couplings would be induced by scalar particles, such as heavy sleptons.

The physical effect of the coupling in Eq. (7) could also be understood in the Jordan frame. There, one would typically neglect the effect of dark energy because of its tiny coupling, of order M_{P}^{-1}. In the model we are considering, this neglect would be unjustified. The scalar field is more strongly coupled, and its effects would be manifest in the curvature of spacetime caused by particles participating in an interaction. The curvature scale would be associated with energies $M \ll M_{\mathrm{P}}$, which would be larger than that typically associated with gravitational phenomena.

In this calculation, the gauge field A_a was taken to be Abelian. However, it is clear that the same calculation generalizes immediately to non-Abelian fields for which the heavy fermion, λ, transforms in the fundamental representation. Consider any gauge group with generators t_α, so that $A_a = A_a^\alpha t_\alpha$. The t_α may be normalized to satisfy

$$\mathrm{Tr}(t_\alpha t_\beta) = c_1 \delta_{\alpha\beta}, \tag{8}$$

for an arbitrary constant c_1, given that 'Tr' denotes a trace over indices in the gauge group. Eq. (7) therefore applies equally in a non-Abelian theory after the substitution $F^{ab}F_{ab} \to \mathrm{Tr}\, F^{ab}F_{ab}$.

2.2. Constraints on Chameleon couplings

Eq. (7) will be accompanied by more complicated corrections which couple $\mathrm{Tr}\, F^{ab}F_{ab}$ to all powers of $\delta\chi$. Resumming this expansion, we generate a coupling of the form $B(\beta_\gamma\chi)\,\mathrm{Tr}\, F^{ab}F_{ab}$ for some function B and coupling scale β_γ.

Eq. (7) shows that, in a conformally-coupled theory, although contact interactions involving $\mathrm{Tr}\, F^{ab}F_{ab}$ do not arise in the ultra-violet, they will inevitably be generated after passing the mass threshold of any matter species which couples both to χ and the gauge field. Therefore, laboratory and astrophysical bounds cannot be evaded merely by taking the $\delta\chi \cdot \mathrm{Tr}\, F^{ab}F_{ab}$ interaction to be absent, although their interpretation becomes model dependent. We believe this to be the first microphysical derivation of the coupling in Eq. (7).

Under certain circumstances it is possible to translate the stringent bounds obtained from electromagnetic probes, discussed above, into bounds on the matter coupling. Eq. (7) shows that the effective electromagnetic coupling can be written

$$\beta_\gamma \equiv \frac{e^2}{3(4\pi)^2} \beta \bar{f}^{3/2} \bar{f}'. \tag{9}$$

For $\beta\bar{\chi} \lesssim 1$ and gauge coupling $e \approx 0.5$, which is appropriate for the SU(2) and U(1) couplings of the Standard Model, we find

$$\beta_\gamma \approx 10^{-3}\beta. \tag{10}$$

Therefore, constraints on β_γ can be translated to limits on the matter coupling, β. Unfortunately this constraint is highly model-dependent, and can be weakened arbitrarily by decreasing the gauge coupling e. Strong constraints are obtained only when e can be determined by other means.

In a minimal model, the dark energy couples to matter with a uniform strength β, irrespective of species. This coupling is subject to only mild restrictions, depending on the precise self-interaction potential which is chosen for the chameleon field. Even where such restrictions exist, they typically require β to be no smaller than the ordinary scale of nuclear physics, $\beta \lesssim (1 \text{ GeV})^{-1}$ [15]. On the other hand, the strongest bound on β_γ follows from astrophysical tests, which yield $\beta_\gamma \lesssim (10^9 \text{ GeV})^{-1}$ [31]. In these minimal models, it follows that there is a new, stronger bound on the matter coupling,‖

$$\beta \lesssim \frac{1}{10^6 \text{ GeV}} \simeq \frac{10^{13}}{M_{\mathrm{P}}}. \tag{11}$$

3. Higgs production

3.1. Production at particle colliders

Below the electroweak symmetry breaking scale of order 1 TeV, interactions of the lightest neutral Higgs, h, with the Z boson are described by cubic and quartic couplings [34],

$$L_{ZZh} = \int \mathrm{d}^4x \; Z_a Z^a \left\{ \sqrt{2^{1/2} G_F} M_Z^2 h + \sqrt{2} G_F M_Z^2 h^2 \right\}, \tag{12}$$

The Z mass is $M_Z \simeq 91.2$ GeV and the Fermi constant, G_F, is measured experimentally to be $G_F \simeq 1.17 \times 10^{-5}$ GeV^{-2} [35]. Eq. (12) is numerically correct for a minimal

‖ Consistency with our assumptions requires $\beta\bar{\chi} \lesssim 1$, corresponding to $\bar{\chi} \lesssim M_{\mathrm{P}}/10^{10}$, which should be easily satisfied for the chameleon field in all relevant backgrounds.

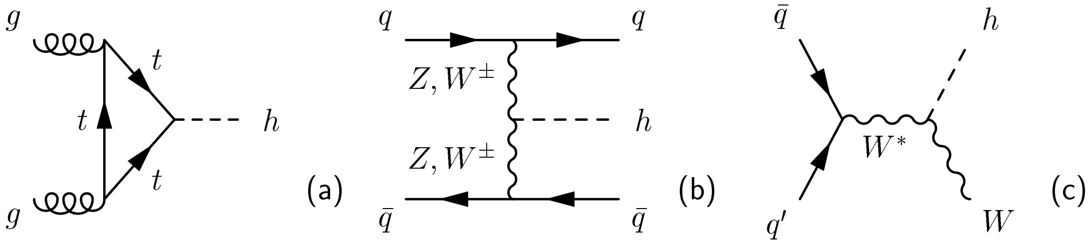


Figure 2. Higgs production channels relevant for LEP, the LHC and the Tevatron. In (a), gluon–gluon fusion is the dominant source of Higgs particles at a hadron collider, such as the LHC, but the decay products of the final Higgs are difficult to distinguish from the background. Since the Higgs couples to particles in proportion to their mass, the dominant diagram has a top quark circulating in the loop. In (b), weak boson fusion is the next-to-leading process at a hadron collider. A $q\bar{q}$ pair undergoes the splitting $q\bar{q} \to q\bar{q}V\bar{V}$, where q is a generic quark species and V is a vector boson. The final state Higgs is radiated via fusion of the intermediate $V\bar{V}$ pair. In (c), associated production occurs when two quarks, q' and $\bar{q}$, fuse to form an off-shell W. The final state is achieved by Higgsstrahlung radiation. At a lepton collider the initial state q', $\bar{q}$ can be replaced by an e^+e^- pair and an intermediate Z^*. The final state Higgs is produced in association with an on-shell Z.

Standard Model Higgs. In a two Higgs doublet model these couplings are shared between the two neutral scalars, leading to suppression by a numerical factor.† Analogous interactions for the $W^\pm$ are obtained by the substitutions $Z_a Z^a \to 2W_a^+ W^{-a}$ and $M_Z \to M_W$.

A summary of the methods for Higgs production at a hadron collider can be found in the recent book by Kilian [36]. (See also Ref. [37].) The dominant mode of Higgs production at the LHC is expected to be gluon–gluon fusion, $gg \to h$, shown in Fig. 2(a). Owing to the large top mass, gluon–gluon fusion is more significant than any other single production channel for a wide range of Higgs masses. Its influence decreases as the Higgs becomes heavier; for a very heavy Higgs it may be subdominant to weak boson fusion, described below. Unfortunately, the decay products from this process may be very difficult to distinguish from the ubiquitous QCD background.

Unwanted backgrounds are smaller, or more easily removed, in other channels. Among the most significant of these is 'weak boson fusion,' $V\bar{V} \to h$, shown in Fig. 2(b), where V is any of the vector bosons $W^\pm, Z$. This process is generally sub-dominant; for a relatively light Higgs, weak boson fusion could contribute $\sim 20\%$ of the leading gluon fusion cross section. The precursor bosons originate within spectator quarks, as shown in Fig. 2(b) and discussed in more detail in §3.2 below. Following a boson fusion event, the spectator quarks are disrupted and initiate transverse hadronic jets. Hadronic activity is weak in the central rapidity region.

† In a supersymmetric Standard Model this factor is $\sin(\beta - \alpha)$, where β parametrizes the ratio of Higgs vacuum expectation values, $\tan\beta \equiv v_2/v_1$, and α is an angle which occurs when diagonalizing the Higgs mass matrix. See, eg., Ref. [20]. In a simple MSSM, $\sin(\beta - \alpha)$ may be near unity.

These properties allow accurate triggering. If the Higgs is sufficiently heavy to decay into vector bosons, then the cleanest signal comes from $h \to ZZ$. For an intermediate mass, with $M_H < 2M_W$, it may be necessary to observe the Higgs in its purely leptonic or photonic final states. If the necessary mass resolution can be achieved, the Higgs couplings could be measured rather precisely, perhaps within 5–20%, making O(1) departures from the Standard Model prediction within the discovery reach of the LHC.

Another interesting mechanism is 'associated production,' shown in Fig. 2(c), in which two quarks fuse to form a W resonance that subsequently decays via "Higgsstrahlung" into an on-shell W and a Higgs, $q'\bar{q} \to W^* \to Wh$. Leptonic decays of the W provide a trigger, allowing this mode to be discriminated from the background. If the Higgs is not sufficiently heavy to decay into a top quark then its primary decays will be to $b\bar{b}$ pairs. This would allow backgrounds to be reduced further, using vertex-tagging for the bs.

Associated production with Zs may occur, although the cross-section is smaller. If the Z decays leptonically to electrons or muons then Higgs events can be identified cleanly, using the invariant mass of the lepton pair to tag the outgoing Z. The event rate for Z decay into neutrinos is substantially larger, but suffers from a significant background from QCD and other events. These backgrounds can be largely subtracted by triggering on outgoing hadronic jets, formed from decay of the Higgs, and demanding a large ($\gtrsim M_Z$) missing mass. At an e^+e^- collider such as LEP, associated production with a Z is known as the Bjorken process, $e^+e^- \to Z^* \to Zh$.

Weak boson fusion and associated production rely on couplings of the Higgs to two vector bosons. These vertices are present in the Standard Model and in many theories of beyond-the-Standard-Model physics, although in certain cases they can be tuned to be absent. In this paper, we study the possibility that corrections due to dark energy can change the relationship of these vertices to the other measurable parameters of the Lagrangian. Our analysis applies for a large class of models in which these two-boson couplings are represented by Eq. (12) and its generalization to W^+W^- interactions. We ignore gluon–gluon fusion. Gluon fusion may receive corrections from interacting dark energy, but a quantitative understanding depends on presently incalculable details of the strong interaction. We defer this investigation to future work.

3.2. The effective W approximation.

The processes of weak boson fusion and associated production can be studied using conventional perturbation theory, but alternative methods exist. An especially useful tool is the *effective W approximation* (see, eg., Ref. [36]). For weak boson fusion, this means that the vector boson precursors are taken to be on-shell partons within colliding hadrons. In the parton picture, a hadron consists of three valence quarks which are surrounded by a sea of virtual particles. A probe which samples this sea at sufficiently high resolution has a chance to resolve the virtual quanta, rather than valence quarks. Since quarks participate in the electroweak interaction, Z and $W^{\pm}$ bosons will be found within the virtual sea. Its precise composition can be determined by solving a system of DGLAP-like equations, which can be thought of as an approximate Boltzmann hierarchy [38]. In this picture, calculations involving hadron collisions with vector boson intermediate states simplify considerably. In the remainder of this paper we will work within the parton picture and the effective W approximation.

The key component of a parton description is the 'parton distribution function', $f_{p/V_i}(x)$, which denotes the probability density to find a vector boson, of species V_i, within a proton p, carrying momentum fraction between x and $x + \mathrm{d}x$ of its parent. These distribution functions can be calculated using the conventional Feynman rules to obtain the distribution of vector bosons within a quark, $f_{q/V_i}(x)$, and convolving these with the measured distribution of sea quarks within a proton [39, 37]. The outcome of this procedure is the probability density near momentum x, $F_i(x)$, for a vector boson of species i within a proton.

The utility of this description is that a complex cross-section can be factorized into a sequence of simpler subprocesses [40, 41]. (For a discussion of factorization generally, see, for example, Ref. [42].) For example, dropping the contribution from Z bosons, the effective cross-section for weak boson fusion can be written [37, 36]

$$\sigma_{\text{eff}} = \int \frac{\mathrm{d}x_1}{x_1} \frac{\mathrm{d}x_2}{x_2} \sum_{\lambda \in \pm, L} \Gamma(W^+_\lambda W^-_\lambda \to h)_{x_1 x_2 s} F_{W^+_\lambda}(x_1) F_{W^-_\lambda}(x_2), \tag{13}$$

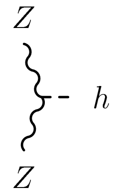
where $\pm, L$ are the transverse and longitudinal polarization modes, and s is the centre of momentum energy of the collision. It is crucial that Eq. (13) depends on the on-shell rate $\Gamma(W^+W^- \to h)$, taken to occur at centre of momentum energy $x_1 x_2 s$. In what follows, we will concentrate on modifications to this quantity.

The effective W approximation is widely used and its accuracy has been carefully tested. In general, its predictions agree with those of exact parton-model calculations within a factor of roughly two. Since our intention is only to diagnose the possible appearance of large modifications to the rate of Higgs production, we believe this approximation to be acceptable.

3.3. New physics and the W^+W^-h, ZZh vertices

The rate $\Gamma(V\bar{V} \to h)$ depends on the amplitude of a Green's function describing effective ZZH and WWH vertices. This Green's function captures the appearance of new physics in Eq. (13). In many theories of physics beyond the Standard Model, the form of the cubic and quartic vertices in Eq. (12) are unmodified. In these theories, corrections to the self-energy diagrams of Z, $W^\pm$, γ and H particles disrupt tree-level relationships between the particle masses $\{M_Z, M_W, M_H\}$, the fine structure constant α, and the Fermi constant G_F. This disruption can be summarized using *oblique parameters* introduced by Peskin & Takeuchi [43, 44] and refined by Maksymyk, London & Burgess [45, 46]. In our model there are both oblique and non-oblique corrections. To summarize our results we use the oblique notation of Refs. [45, 46] and note differences explicitly where they occur.

Consider the Green's function describing an effective ZZh vertex, from which the WWh result can be derived after trivial modifications. The S-matrix element must depend on



$$\left| \quad \right|^2 \propto M_Z^4 G_F \times \text{wavefunction renormalizations,} \tag{14}$$

where the wavefunction renormalizations arise on transition to the S-matrix. We work in unitarity gauge, where the Goldstone modes associated with the SU(2) Higgs doublet are absorbed as longitudinal polarizations of Z and $W^\pm$. Accounting for oblique corrections, the Z propagator is

$$\langle Z^a(k_1) Z^b(k_2) \rangle = -\mathrm{i}(2\pi)^4 \delta(k_1 + k_2) \left(\eta^{ab} + \frac{k^a k^b}{M_Z^2} \right) \Delta'(k^2), \tag{15}$$

where $\Delta'(k^2)$ takes the form

$$\Delta'(k^2)^{-1} \equiv k^2 + M_Z^2 - \Pi_{ZZ}(k^2). \tag{16}$$

In Eqs. (15), k stands for either k_1 or k_2. The quantity Π_{ZZ} is defined as follows. We choose $\mathrm{i}\Pi_{ZZ}^{ab}(k^2)$ to be the sum of all one-particle-irreducible graphs connecting an ingoing and an outgoing Z. In vacuum this has a unique tensorial decomposition,

$$\Pi_{ZZ}^{ab}(k^2) \equiv \eta^{ab} \Pi_{ZZ}^{(0)}(k^2) + k^a k^b \Pi_{ZZ}^{(2)}(k^2). \tag{17}$$

We neglect the term involving $\Pi_{ZZ}^{(2)}$ and can therefore drop superscript '0's without ambiguity, so that $\Pi_{ZZ}^{(0)} \to \Pi_{ZZ}$. It is this quantity which appears in Eq. (16). If the mass of external fermions is at most $\sim M_f$, this neglect is equivalent to dropping powers of M_f/M_Z. It is likely to be a good approximation provided the Higgs is not too heavy: for a Higgs lighter than the top mass, $M_t \simeq 173$ GeV, decay into a top quark is kinematically forbidden. Therefore, M_f/M is at most of order 10^{-1} to 10^{-2}.

Including oblique corrections, the Z mass becomes

$$M_Z^2 = \tilde{M}_Z^2 \left(1 - \frac{\Pi_{ZZ}(-M_Z^2)}{M_Z^2} \right). \tag{18}$$

A similar formula can be written for the W propagator, making the replacements $M_Z \to M_W$ and $\Pi_{ZZ} \to \Pi_{WW}$. Quantities with a tilde, such as $\tilde{M}_Z$ and $\tilde{G}_F$, refer to the value of these parameters in the absence of oblique corrections. With the same conventions, the Fermi constant satisfies

$$G_F = \tilde{G}_F \left(1 + \frac{\Pi_{WW}(0)}{M_W^2} \right). \tag{19}$$

G_F parametrizes the strength of the weak force near zero momentum transfer. Allowing for these shifts, the decay rate $\Gamma(ZZ \to h)$ is related to the pure Standard Model rate by the rule

$$\begin{aligned} \frac{\Gamma(ZZ \to h)}{\tilde{\Gamma}(ZZ \to h)} &= 1 + 2\frac{\Pi_{ZZ}(-M_Z^2)}{M_Z^2} - \frac{\Pi_{WW}(0)}{M_W^2} + 2\Pi'_{ZZ}(-M_Z^2) + \Pi'_{HH}(-M_H^2) \\ &= 1 + \alpha(2V + R). \end{aligned} \tag{20}$$

For W bosons the decay rate is

$$\begin{aligned}\frac{\Gamma(WW \to h)}{\tilde{\Gamma}(WW \to h)} &= 1 + 2\frac{\Pi_{WW}(-M_W^2)}{M_W^2} - \frac{\Pi_{WW}(0)}{M_W^2} + 2\Pi'_{WW}(-M_W^2) + \Pi'_{HH}(-M_H^2) \\ &= 1 + \alpha(2W + R), \end{aligned} \tag{21}$$

where, as above, $\alpha \approx 1/137$ is the fine structure constant. Eqs. (20) and (21) have been written in terms of the conventional oblique quantities V and W, which are defined to satisfy [45]

$$\alpha V \equiv \frac{\mathrm{d}}{\mathrm{d}k^2}\,\Pi_{ZZ}(k^2)\big|_{k^2=-M_Z^2} - \frac{\Pi_{ZZ}(0) - \Pi_{ZZ}(-M_Z^2)}{M_Z^2}, \tag{22}$$

$$\alpha W \equiv \frac{\mathrm{d}}{\mathrm{d}k^2}\,\Pi_{WW}(k^2)\big|_{k^2=-M_W^2} - \frac{\Pi_{WW}(0) - \Pi_{WW}(-M_W^2)}{M_W^2}. \tag{23}$$

In addition, we have introduced a new quantity R which is a measure of the Higgs' wavefunction renormalization,

$$\alpha R \equiv \frac{\mathrm{d}}{\mathrm{d}k^2}\,\Pi_{HH}(k^2)\big|_{k^2=-M_H^2} + \frac{\Pi_{ZZ}(0)}{M_Z^2}. \tag{24}$$

If the dark energy coupling scale is greater than the typical scale of electroweak processes, $M_{\mathrm{EW}} \sim 1$ TeV, we expect V and W to be negligible [16]. The impact of new physics is therefore contained entirely in R.

4. Corrections from a dark energy scalar

Eqs. (20)–(21) determine the sensitivity of weak boson fusion and Higgsstrahlung to new physics. This sensitivity is measured by the Higgs oblique parameter, R. In this section we make a quantitative estimate of its magnitude. To do so, we must be precise about the corrections Π_{ZZ} and Π_{WW} which modify the Standard Model prediction. In §4.1 we determine these quantities in a low energy chameleon-type model coupled to the gauge bosons. We calculate oblique corrections to the production rate, and show that they are sensitive to the high energy completion of the theory. In §4.2 we compute non-oblique corrections generated by integrating out heavy fermions. These are described by a new quartic coupling between the Higgs field and the gauge bosons, and are important to correctly interpret the sensitivity of Higgs production to high energy behaviour.

4.1. Oblique corrections in the low-energy theory

A dark energy field induces both straight and oblique corrections to the vacuum polarizations of the Higgs and gauge bosons. In Ref. [16] it was argued that the straight corrections effectively divide into processes involving "chameleonstrahlung," where dark energy particles are produced but escape the detector, and a collection of "bridges," "daisies" and "rainbows" which dress the bare processes of the Standard Model. At leading order, these dressings are momentum independent. On the other hand, chameleonstrahlung was shown to give constraints roughly comparable to those arising from oblique corrections.

After electroweak symmetry breaking, we can parameterize the interactions of Eq. (7) by adopting an effective Z boson Lagrangian of the form employed in Ref. [16],

$$S = -\frac{1}{4}\int \mathrm{d}^4x \ \left\{B(\beta\chi)(\partial^a Z^b - \partial^b Z^a)(\partial_a Z_b - \partial_b Z_a) + 2B_H(\beta_H\chi)M_Z^2 Z^a Z_a\right\}. \tag{25}$$

The functions B and B_H should satisfy $B(0) = B_H(0) = 1$, but depend on the details of ultra-violet physics. More precisely, they are derived from Eq. (7) and similar higher-order diagrams involving more powers of $\delta\chi$. Likewise, the couplings β and β_H are inherited from whatever heavy particles are integrated out to generate this interaction. Working with a sharp momentum cutoff, the Z vacuum polarization was determined in Ref. [16] and found to be

$$\begin{aligned}\Pi_{ZZ}(k^2) = \frac{\beta^2}{8\pi^2}\frac{\bar{B}'^2}{\bar{B}}\int_0^1 \mathrm{d}x \, \Bigg\{ &\frac{2k^2+\gamma^2 M_Z^2}{4}\left[\Lambda^2 - \frac{\Lambda^2}{2}\frac{\Lambda^2}{\Lambda^2+\Sigma_Z^2} - \Sigma_Z^2 \ln\left(1+\frac{\Lambda^2}{\Sigma_Z^2}\right)\right] \\ &+ (xk^2+\gamma M_Z^2)^2\left[-\frac{1}{2}\frac{\Lambda^2}{\Lambda^2+\Sigma_Z^2} + \frac{1}{2}\ln\left(1+\frac{\Lambda^2}{\Sigma_Z^2}\right)\right] \\ &- \frac{\Omega}{2}(k^2+\epsilon M_Z^2)\left[\frac{\Lambda^2}{2} - \frac{M_\chi^2}{2}\ln\left(1+\frac{\Lambda^2}{M_\chi^2}\right)\right]\Bigg\}, \end{aligned} \tag{26}$$

where Ω satisfies

$$\Omega \equiv \frac{\bar{B}''\bar{B}}{\bar{B}'^2} \tag{27}$$

and $\bar{B} \equiv B(\beta\bar{\chi})$. The parameters ϵ and γ are defined by

$$\epsilon = \frac{B_H''}{B''}\frac{\beta_H^2}{\beta^2} \tag{28}$$

$$\gamma = \frac{B_H'}{B'}\frac{\beta_H}{\beta}. \tag{29}$$

Also, Σ_Z^2 represents

$$\Sigma_Z^2 \equiv x(1-x)k^2 + (1-x)M_Z^2 + xM_\chi^2, \tag{30}$$

where M_χ is the mass of the dark energy fluctuation $\delta\chi$. Near $k^2 \approx 0$, $\Pi_{ZZ}(k^2)$ has the approximate form

$$\Pi_{ZZ}(k^2) \approx \frac{\beta_H^2\Lambda^2}{32\pi^2}\frac{\bar{B}_H'^2}{\bar{B}}\left(\frac{1}{2} - \frac{\bar{B}_H''\bar{B}}{\bar{B}_H'^2}\right) + \mathrm{O}\left(\beta_H^2 M_{\mathrm{EW}}^2\right). \tag{31}$$

We must determine the vacuum polarization of the Higgs. As above, we assume this to follow from a conformal coupling to χ. In the Standard Model, this would give the coupling

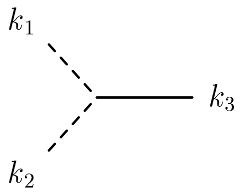
$$S \supseteq -\frac{1}{2}\int \mathrm{d}^4x \left\{B_H(\beta_H\chi)|(\partial_a + \mathrm{i}\vec{A}_a\cdot\vec{t} - \mathrm{i}B_a y)H|^2 - C_H(\beta_H\chi)\mu^2 H^\dagger H + \mathrm{O}\Big([H^\dagger H]^2\Big)\right\}, \tag{32}$$

where $\vec{A}$ and B are the gauge fields of the unbroken SU(2) and $\mathrm{U}(1)_Y$ symmetries, respectively; $\vec{t}$ are a set of appropriately normalized generators of SU(2); and y is the generator of U(1). H is an SU(2) Higgs doublet, and μ is a standard parameter of the quartic Higgs potential, related to the Higgs mass by the rule $M_H^2 = 2\mu^2$. In many

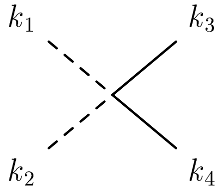
Figure 3. Processes contributing to the self-energy of the Higgs boson. An initial Higgs boson state, represented by a dashed line, radiates into scalar quanta χ (represented by a solid line) which are eventually re-absorbed to yield a final state characterized by the same quantum numbers and momentum as the initial state.

models the phenomenological couplings B, B_H and C_H will be closely related, but for the present we leave them arbitrary. If no relationship exists between the couplings, we find that unitarity is not respected at tree level in two-body scattering of gauge bosons [47, 48, 49, 50] at energy scales above $[G_F|B(\beta\bar{\chi}) - B_H(\beta_H\bar{\chi})]^{-1/2}$.† In models containing more than one Higgs doublet we assume that Eq. (32) continues to give a good approximation to the couplings of the lightest neutral Higgs.

The Higgs vacuum polarization $\Pi_{HH}(k^2)$ can be computed. The one-loop contributions are shown in Fig. 3, and depend on the following vertices:



$$\rightarrow \frac{\beta_H}{2}\Big\{\bar{B}'_H(k_2 \cdot k_3) - \bar{C}'_H M_H^2\Big\}, \tag{33}$$



$$\rightarrow \frac{\beta_H^2}{4}\Big\{\bar{B}''_H(k_2 \cdot k_3) - \bar{C}''_H M_H^2\Big\}, \tag{34}$$

The diagram in Fig. 3(a) corresponds to a vacuum polarization

$$\begin{aligned}\Pi_{HH}(k^2) = \frac{\beta_H^2}{8\pi^2}\int_0^1 \mathrm{d}x \int_0^\Lambda \frac{\kappa^3\,\mathrm{d}\kappa}{(\kappa^2+\Sigma_H^2)^2} \\ \times \left[\bar{B}'^2_H\left(\frac{k^2\kappa^2}{4} + x^2k^4\right) + \bar{C}'_H M_H^2(\bar{C}'_H M_H^2 + 2\bar{B}'_H k^2 x)\right],\end{aligned} \tag{35}$$

where x is a Feynman parameter and we have rotated to Euclidean signature. In analogy with Eq. (30), Σ_H is defined so that

$$\Sigma_H^2 = x(1-x)k^2 + (1-x)M_H^2 + xM_\chi^2. \tag{36}$$

The diagram in Fig. 3(b) contributes

$$\Pi_{HH}(k^2) = -\frac{\beta_H^2}{16\pi^2}\int_0^\Lambda \frac{\kappa^3\,\mathrm{d}\kappa}{\kappa^2+M_\chi^2}(\bar{B}''_H k^2 + \bar{C}''_H M_H^2). \tag{37}$$

† Additional violations of perturbative unitarity near the chameleon scale β^{-1} may arise from new dark energy exchange diagrams in two-body scattering.

Carrying out the κ integrals, we find

$$\Pi_{HH}(k^2) = \frac{\beta_H^2}{8\pi^2}\frac{1}{\bar{B}_H}\int_0^1 \mathrm{d}x \left\{ \frac{\bar{B}_H'^2 k^2}{4}\left[\Lambda^2 - \frac{\Lambda^2}{2}\frac{\Lambda^2}{\Lambda^2+\Sigma_H^2} - \Sigma_H^2 \ln\left(1+\frac{\Lambda^2}{\Sigma_H^2}\right)\right] \right.$$
$$+ \left\{ B_H'^2 x^2 k^4 + M_H^2 \bar{C}_H'(2\bar{B}_H' x k^2 + M_H^2 \bar{C}_H') \right\}\left[-\frac{1}{2}\frac{\Lambda^2}{\Lambda^2+\Sigma_H^2} + \frac{1}{2}\ln\left(1+\frac{\Lambda^2}{\Sigma_H^2}\right)\right]$$
$$\left. - \frac{\bar{B}_H}{2}(\bar{B}_H'' k^2 + M_H^2 \bar{C}_H'')\left[\frac{\Lambda^2}{2} - \frac{M_\chi^2}{2}\ln\left(1+\frac{\Lambda^2}{M_\chi^2}\right)\right] \right\}. \tag{38}$$

From Eqs. (26) and (38) it is possible to compute R, the parameter which summarizes the oblique dependence of the rates $\Gamma(ZZ \to h)$ and $\Gamma(W^+W^- \to h)$. We find

$$\alpha R = \frac{\beta_H^2 \Lambda^2}{32\pi^2}\frac{\bar{B}_H'^2}{\bar{B}}\left[\frac{1}{2}\left(1+\frac{\bar{B}}{\bar{B}_H}\right) - 2\frac{\bar{B}_H''\bar{B}}{\bar{B}_H'^2}\right] + \text{finite terms of order O}\,(\beta_H^2 M_{\mathrm{EW}}^2). \tag{39}$$

At leading order in the divergence, it is independent of C_H. We note, however, that a dependence on C_H persists among those terms which are finite in the limit $\Lambda \to \infty$. These finite terms are of order $\beta_H^2 M_{\mathrm{EW}}^2$ and can be neglected when $\beta_H \ll (1\ \mathrm{TeV})^{-1}$. We conclude that the oblique correction is very small, unless the divergent part of Eq. (39) can contribute a significant effect.

What is the meaning of the divergent term in Eq. (39)? One must be wary when reasoning with power-law divergences, because they can be ascribed no invariant significance. For example, they are absent in dimensional regularization. In Ref. [16] it was found that similar divergences could be absorbed in renormalizations of G_F and the vector boson masses, M_Z and M_W. The divergence in Eq. (39) cannot be absorbed in this way. It expresses a sensitivity to whatever physics completes the low-energy theory containing the Standard Model and the effective interaction, Eq. (7). The same divergence arises in all models with this low-energy limit, irrespective of what physics takes place at high energy. To fix its value with confidence, we must know the details of the high energy completion. In the next section we compute its value for a model in which the low energy theory is obtained by integrating out heavy fermions with SUSY-like couplings.

4.2. Higgs couplings from heavy charged particles

In the low energy effective theory, Eq. (7) will be accompanied by other interactions which cannot be neglected. These will lead to non-oblique corrections to Higgs production. The most important is a new contact interaction between gauge bosons and the Higgs field.

Gauge invariance constrains which operators can appear in the low-energy theory. In the minimal scenario we are considering, the Higgs field is in the fundamental **2** representation of SU(2). By construction, a field strength term such as $F^{ab}F_{ab}$ transforms in the adjoint representation. Therefore the lowest order non-trivial interaction with the Higgs must involve $H^\dagger H$, making $H^\dagger H\,\mathrm{Tr}(F^{ab}F_{ab})$ a gauge invariant dimension-six operator.

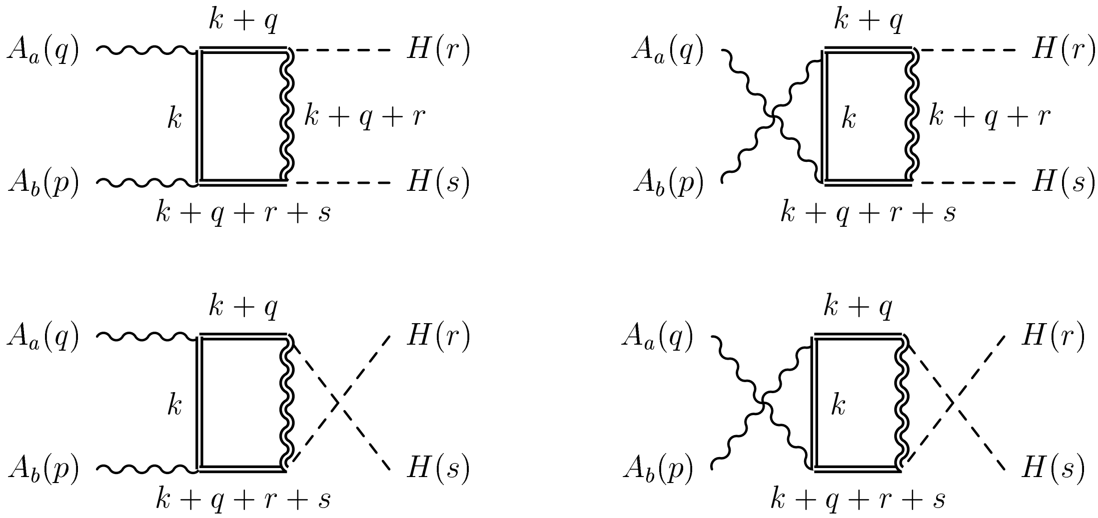
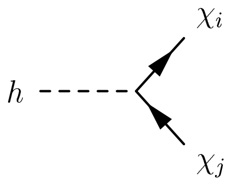


Figure 4. Leading interactions between the Higgs field and electroweak gauge bosons. The interactions are mediated by two species of chargino, λ (straight doubled lines) and λ' (wiggly doubled lines), of masses M and M', respectively.

If this operator is present in the low-energy effective theory, it will modify our expectation for Higgs production. Accordingly, we must evaluate its coefficient. The prediction is model-dependent. Consider a minimal scenario, where the heavy charged particles are fermions which have vertices with the lightest neutral Higgs of the form



$$\to \mathrm{i}g \left(C_{ij} L + C^*_{ji} R \right), \tag{40}$$

where i and j label different species of fermion and the strength of the coupling is parametrized by g. The operator $L \equiv (1+\gamma_5)/2$ projects onto the left-chirality half of a Dirac spinor, and $R \equiv (1-\gamma_5)/2$ is its conjugate. The C_{ij} should be chosen real and symmetric if CP violation is to be avoided.

In a supersymmetric Standard Model, the χ_i will be charginos and neutralinos. These have couplings to the lightest neutral Higgs of the form (40), with C_{ij} determined by the various factors which diagonalize the chargino and neutralino mass matrices. Explicit expressions can be found in §A.7 of Ref. [37]. In this case, the chargino and neutralino loops would be accompanied by heavy slepton loops which we do not calculate. There is no reason to expect the slepton contribution to be larger than the neutralino or chargino terms, so we anticipate that the fermion contribution alone is representative.

Eq. (40) gives rise to effective operators depicted in Fig. 4. We denote a fermion of species i by a straight doubled line, and species j by a wiggly doubled line. These diagrams must be summed over all i and j. The correlation function

$\langle A_a(q) A_b(p) h(r) h(s) \rangle$ satisfies

$$\langle A_a(q) A_b(p) h(r) h(s) \rangle = (2\pi)^4 \delta(p+q+r+s) e^2 g^2 h(r) h(s) A_a(q) A_b(p) \int \frac{\mathrm{d}^4 k}{(2\pi)^4}$$
$$\times \operatorname{tr} \sum_{ij} \left\{ \gamma^a \frac{-\mathrm{i}(\not{k} + \not{q}) + M_i}{(k+q)^2 + M_i^2 - \mathrm{i}\epsilon} (C_{ij} L + C_{ji}^* R) \frac{-\mathrm{i}(\not{k} + \not{q} + \not{r}) + M_j}{(k+q+r)^2 + M_j^2 - \mathrm{i}\epsilon} \right.$$
$$\left. \times (C_{ji} L + C_{ij}^* R) \frac{-\mathrm{i}(\not{k} + \not{q} + \not{r} + \not{s}) + M_i}{(k+q+r+s)^2 + M_i^2 - \mathrm{i}\epsilon} \gamma^b \frac{-\mathrm{i}\not{k} + M_i}{k^2 + M_i^2 - \mathrm{i}\epsilon} \right\}$$
$$+ \begin{pmatrix} p \leftrightarrow q \\ a \leftrightarrow b \end{pmatrix} + (r \leftrightarrow s), \tag{41}$$

where M_i is the mass of species i and the symmetrizations are nested, so that $r \leftrightarrow s$ is carried out after the simultaneous exchanges $p \leftrightarrow q$ and $a \leftrightarrow b$. As above, the k integral is divergent and potentially dependent on the labeling of momenta inside the loop, but once the integral has been regularized it can be checked that this dependence vanishes.

After a tedious calculation, we find that Eq. (41) can be reproduced using the following effective Lagrangian,

$$L_{\text{eff}} \supseteq \sum_{ij} \frac{g^2}{3!(4\pi)^2} \frac{2}{18 M_i^2 (1 - x_{ij}^2)^5} \Big\{ e^2 \zeta_1(x_{ij}) H^\dagger H \operatorname{Tr} F^{ab} F_{ab} + \zeta_2(x_{ij}) |D^2 H|^2$$
$$+ \zeta_3(x_{ij}) (D_a D_b H)^\dagger (D^a D^b H) \Big\} \tag{42}$$

where $x_{ij} \equiv M_j / M_i$ and the functions $\{\zeta_1, \zeta_2, \zeta_3\}$ satisfy

$$\zeta_1(x_{ij}) \equiv \Sigma_{ij} \left[8 - 49x_{ij}^2 + 99x_{ij}^4 - 71x_{ij}^6 + 13x_{ij}^8 - 12x_{ij}^4(3 - 7x_{ij}^2 + 2x_{ij}^4) \ln x_{ij} \right]$$
$$- 2x_{ij}\Gamma_{ij} \left[4 - 6x_{ij}^2 - 9x_{ij}^4 + 14x_{ij}^6 - 3x_{ij}^8 + 6x_{ij}^2(3 - 6x_{ij}^2 + x_{ij}^4) \ln x_{ij} \right], \tag{43}$$
$$\zeta_2(x_{ij}) \equiv 2\Sigma_{ij} \left[7 - 47x_{ij}^2 + 63x_{ij}^4 - 25x_{ij}^6 + 2x_{ij}^8 - 12x_{ij}^4(6 - 5x_{ij}^2 + x_{ij}^4) \ln x_{ij} \right]$$
$$- 4x_{ij}\Gamma_{ij} \left[2 + 9x_{ij}^2 - 18x_{ij}^4 + 7x_{ij}^6 + 6x_{ij}^2(3 - x_{ij}^4) \ln x_{ij} \right], \tag{44}$$
$$\zeta_3(x_{ij}) \equiv -2\Sigma_{ij} \left[4 - 23x_{ij}^2 + 63x_{ij}^4 - 49x_{ij}^6 + 5x_{ij}^8 + 12x_{ij}^6(5 - x_{ij}^2) \ln x_{ij} \right]$$
$$+ 2x_{ij}\Gamma_{ij} \left[1 - 9x_{ij}^2 - 9x_{ij}^4 + 17x_{ij}^6 - 12x_{ij}^4(3 + x_{ij}^2) \ln x_{ij} \right]. \tag{45}$$

The quantities Σ_{ij} and Γ_{ij} are defined by

$$\Sigma_{ij} \equiv C_{ij} C_{ij}^* + C_{ji} C_{ji}^* \qquad \text{(no sum on } i \text{ or } j) \tag{46}$$
$$\Gamma_{ij} \equiv C_{ij} C_{ji} + C_{ij}^* C_{ji}^* \qquad \text{(no sum on } i \text{ or } j) \tag{47}$$

To exhibit its gauge invariance, Eq. (42) has been written in terms of a conventionally normalized SU(2) doublet H, which coincides with the field-space orientation of the lightest neutral Higgs. It will be accompanied by higher derivative terms, represented by $\mathrm{O}(\partial^4)$, which have been neglected. There is also a term of the form $ch^2 A_a A^a$, with c a divergent constant, whose role is to renormalize the charge e. We discard this term and take e to be the renormalized charge. At higher order in H, arbitrary powers of $H^\dagger H$ may be generated by diagrams similar to Fig. 5. This will lead to higher-dimension operators which couple polynomials of H and its derivatives to the gauge field, but at leading order we can restrict our attention to Eq. (42).

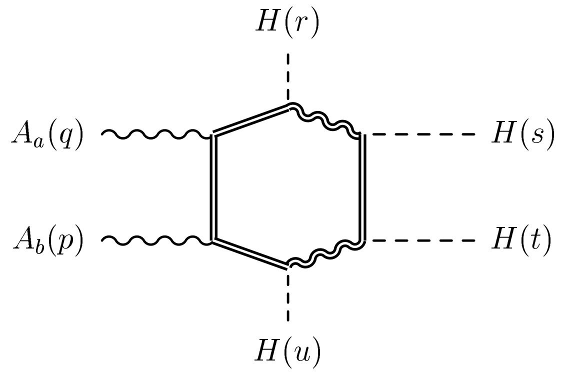


Figure 5. Next-order interactions between the Higgs field and electroweak gauge bosons.

To determine the correct order of magnitude for Λ, the cutoff used in the calculation of the oblique corrections in §4.1, it is safest to match to whatever theory controls physics in the ultra-violet [51]. In the present case, this is summarized by Eq. (42). Using the effective interactions in Eq. (42) it is possible to compute the enhancement to Higgs production. As a reasonable approximation, we take the SU(2) doublet, H, to develop a vacuum expectation value of order the Standard Model scale, $G_F^{-1/2}$. We take neutral excitations around this condensate to be representative of the interaction with the lightest neutral Higgs. Choosing the gauge field to be the Abelian vector associated with the Z, the resulting effective Lagrangian is

$$L_{\text{eff}} \supseteq \frac{e^2 g^2}{2592\pi^2} \sum_{ij} \frac{(\sqrt{2}G_F)^{-1}}{M_i^2(1-x_{ij}^2)^5} \Big\{ [2\zeta_1(x_{ij}) + 2\zeta_3(x_{ij})] h(\partial_a Z_a \partial^a Z^b - \partial_a Z_b \partial^b Z^a) - [\zeta_2(x_{ij}) + \zeta_3(x_{ij})] \partial^2 h Z^a Z_a + \zeta_3(x_{ij}) h Z^b \partial^2 Z_b \Big\}. \quad (48)$$

In the limit where the relative velocity of the colliding vector bosons goes to zero,‡ and defining

$$\hat{\zeta}_m \equiv \sum_{ij} \frac{\zeta_m(x_{ij})}{M_i^2(1-x_{ij}^2)^5}, \quad (49)$$

we find

$$\frac{\delta\Gamma(ZZ\to h)}{\Gamma(ZZ\to h)} = \left(\frac{e^2 g^2}{2592\sqrt{3}\pi^2 G_F}\right)^2 \Big\{ 28\hat{\zeta}_1^2 - 32\hat{\zeta}_1\hat{\zeta}_2 - 28\hat{\zeta}_1\hat{\zeta}_3 + 25\hat{\zeta}_2\hat{\zeta}_3 + 35\hat{\zeta}_3^2 \Big\}. \quad (50)$$

Despite appearances Eq. (50) is dimensionless, because each $\hat{\zeta}_i$ has the same dimension as G_F, $[\text{mass}]^{-2}$. The magnitude of this correction varies with the mass ratio, x_{ij}, approaching zero as $x_{ij} \to 1$ but asymptoting to an approximate constant for large or small ratios. In a typical supersymmetric Standard Model the chargino and neutralino

‡ The calculation does not need to be restricted to this kinematic limit, but it leads to simpler final expressions. Since the vector bosons are taken to be on-shell in the effective W approximation, the invariant magnitude of any momenta will be of order $\sim M_W$. This implies that although the result may be modified by factors of O(1), it is unlikely that we commit a gross error by specializing to the zero-velocity limit.

masses are undetermined, but provided there is not total mass degeneracy we expect that this threshold correction generates a contribution represented by a cutoff of order

$$\Lambda \simeq \beta_H^{-1} \times \frac{e^2 g^2}{M^2 G_F}. \tag{51}$$

This is typically a rather small number, leading to an essentially negligible enhancement in the Higgs production rate.

5. Conclusions

In this paper we have derived the low energy effective theory which governs interactions between the gauge bosons of the electroweak sector and a dark energy scalar field. The dark energy is taken to have conformal couplings to the matter species of the Standard Model. It is possible that couplings of this type allow so-called "chameleon" behaviour, in which the field dynamically adjusts its mass to be large in regions of high average density, and small elsewhere. If such theories exist then they would lead to an unambiguous prediction of light dark energy quanta interacting in the beam pipe of any particle accelerator. Our low energy theory applies strictly in any model containing heavy charged fermions, but a very similar effective Lagrangian would apply for a model containing heavy charged particles of any spin. As a specific example, any supersymmetric Standard Model must contain charginos and neutralinos. These carry the quantum numbers of the $\mathrm{SU}(2) \times \mathrm{U}(1)$ gauge group and have masses of order the supersymmetry-breaking scale. However, our calculation is not restricted to the supersymmetric case.

Using this low energy theory we have computed the oblique corrections to the Higgs, Z and $W^\pm$ propagators at energies below the mass, M, of the heavy charged fermions. Such corrections modify the rates $\Gamma(ZZ \to h)$ and $\Gamma(W^+W^- \to h)$, where h is the lightest neutral Higgs, and in principle could change the rate of production of this particle at a hadron collider. In practice we find these corrections to be of order $\beta_H^2 M_{\mathrm{EW}}^2$, making them negligible in the experimentally-allowed region $\beta_H \ll \mathrm{TeV}^{-1}$.

Other contributions exist, generated by processes taking place at high energy, which are integrated out of the low-energy description. We determine these "threshold corrections" by integrating out heavy fermion loops which mediate interactions between the lightest neutral Higgs and the gauge bosons. It corresponds to the use of a cutoff in the oblique calculation, of order $\Lambda \sim \beta_H^{-1} e^2 g^2 / M^2 G_F^2$, and therefore leads to at most small effects. When M is much larger than the Standard Model scale $G_F^{-1/2}$ it is entirely negligible.

In an unconstrained theory, we might anticipate a cutoff of order $\Lambda \sim \beta_H^{-1}$, because at this scale the effective dark energy theory becomes invalid. If this were true, it would be possible to contemplate corrections to the Higgs production rate of O(1) or larger, which would lie within the discovery reach of the LHC. Unfortunately, the corrections are much smaller. Large terms could only arise from the relevant operator $HZ^a Z_a$, but its appearance is forbidden by gauge invariance above the scale of electroweak symmetry breaking. Therefore, we expect the coefficient of this term to be at most ${G_F}^{-1/2}$, rather than β_H^{-1}. Instead, the leading correction comes from the operators $H^\dagger H \operatorname{Tr} F^{ab} F_{ab}$, $|D^2 H|^2$ and $(D_a D_b H)^\dagger (D^a D^b H)$. These are dimension-six operators, because of the SU(2) nature of the Higgs doublet. Accordingly the cutoff is suppressed by $(M^2 G_F)^{-1}$, making it small. We conclude that corrections to the Higgs production rate from conformally coupled dark energy are unlikely to be observable at the LHC.

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