

# TMD PDFs: a Monte Carlo implementation for the sea quark distribution

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This article gives a concise introduction to transverse momentum dependent (TMD) parton distribution functions and their use in shower Monte Carlo event generators for high-energy hadron collisions, and describes recent progress in the treatment of sea quark effects within a TMD parton-shower framework.

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## 1 Introduction

While QCD factorization methods are well established in the case of scattering observables involving a single large mass scale [1], the treatment of processes with multiple mass scales is rather more subtle. In multi-scale processes, generalized factorization formulas are required [2] if one is to control perturbative large logarithms to higher orders of perturbation theory and to describe appropriately nonperturbative physics in the initial and final states of the collision. Such generalized factorization formulas typically involve transverse-momentum dependent (TMD), or “unintegrated”, parton distribution and parton decay functions [2, 3].

A full treatment of factorization at unintegrated level, valid uniformly over the whole space, is yet to be achieved [4]. Results however exist which apply in specific phase space regions. One such case is given by QCD in the high-energy, or small- $x$ , limit [5], in which factorization of TMD gluon distributions holds [6], in correspondence to perturbatively-resummed coefficient functions. Results based on small- $x$  TMD distributions have for instance been used in analyses of inclusive observables such as deeply inelastic structure functions (see [7] for a review) and in Monte Carlo simulations of the exclusive structure of final states in hadronic collisions (see [8]).

Many aspects of the experimental program at the Large Hadron Collider (LHC) depend on the analysis of processes containing multiple hard scales, and will be influenced by improved formulations of factorization in QCD at unintegrated level. In this article we give a concise overview of recent progress on TMD distributions and their use for simulations of final states in hadronic collisions by shower Monte Carlo event generators. Most of TMD computational tools have so far been developed within a quenched approximation in which only gluon and valence quark effects are taken into account at TMD level. We describe results of recent work [9, 10] to go beyond this approximation by including sea quark effects, and we present first numerical applications to forward  $Z$ -boson production.

The article is organized as follows. In Sec. 2 we recall operator definitions for parton

distribution functions (PDFs) and basic issues associated with their TMD generalization. In Sec. 3 we consider parton branching methods and the role of unintegrated distributions in shower Monte Carlo generators. In Sec. 4 we discuss the approach [9] to incorporate effects from quark emission and flavor-singlet sea-quark contributions in the framework of transverse-momentum dependent parton showers. We summarize in Sec. 5.

## 2 TMD parton distribution functions

In this section we introduce the parton correlation functions used to define parton distributions. In Subsec. 2.1 we consider operator matrix elements for TMD distributions, and discuss current open issues, including lightcone divergences and factorization breaking effects. In Subsec. 3.2 we focus on the case of small  $x$ , related factorization results, and introduce applications which are the subject of the sections that follow.

### 2.1 Operator matrix elements

The relevance of consistent operator definitions for parton  $k_\perp$  distributions was emphasized long ago in the context of Sudakov processes [11], jet physics [12], exclusive production [13], spin physics [14]. The approach commonly used to ensure gauge invariance is to generalize the coordinate-space matrix elements that define ordinary parton distribution functions (pdfs) [15] to the case of field operators at non-lightcone distances. For instance, for the quark distribution one has (Fig. 1)

$$\tilde{f}(y) = \langle P | \bar{\psi}(y) V_y^\dagger(n) \gamma^+ V_0(n) \psi(0) | P \rangle \quad . \quad (1)$$

Here  $\psi$  are the quark fields evaluated at distance  $y = (0, y^-, y_\perp)$ , where  $y_\perp$  is in general nonzero, and  $V$  are eikonal-line operators in direction  $n$ ,

$$V_y(n) = \mathcal{P} \exp \left( i g_s \int_0^\infty d\tau n^\mu A_\mu(y + \tau n) \right) \quad , \quad (2)$$

which we require to make the matrix element gauge-invariant. The unintegrated, or TMD, quark distribution is obtained from the double Fourier transform in  $y^-$  and  $y_\perp$  of  $f$ .

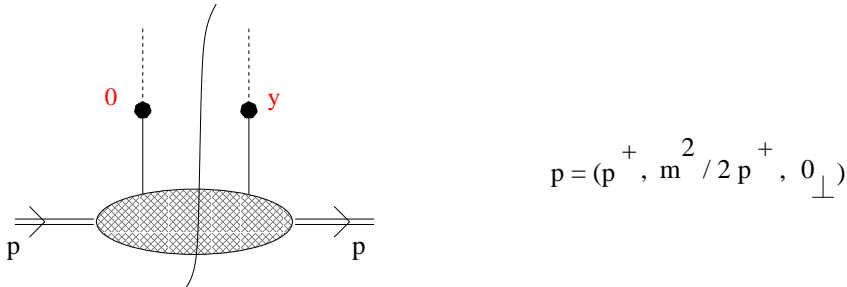


Figure 1: Correlator of two quark fields at distance  $y$ .

While Eq. (1) works at tree level (including an extra gauge link at infinity in the case of physical gauge [16], see also [17]), going beyond tree level requires treating lightcone singularities [11, 18], associated with the  $x \rightarrow 1$  endpoint. The singularity structure at  $x \rightarrow 1$  is different

than for ordinary (integrated) distributions, giving divergences even in dimensional regularization with an infrared cut-off [19]. The singularities can be understood in terms of gauge-invariant eikonal-line matrix elements [19] and related to cusp anomalous dimensions [20, 21, 22].

This can be analyzed explicitly at one loop. Expansion in powers of  $y^2$  of the coordinate-space matrix element (1) at this order gives [19, 23]

$$\begin{aligned} \tilde{f}_1(y) = & \frac{\alpha_s C_F}{\pi} p^+ \int_0^1 dv \frac{v}{1-v} \left\{ [e^{ip \cdot yv} - e^{ip \cdot y}] \Gamma(2 - \frac{d}{2}) \left(\frac{4\pi\mu^2}{\rho^2}\right)^{2-d/2} \right. \\ & \left. + e^{ip \cdot yv} \pi^{2-d/2} \Gamma(\frac{d}{2} - 2) (-y^2 \mu^2)^{2-d/2} + \dots \right\} , \end{aligned} \quad (3)$$

where  $d$  is the number of space-time dimensions,  $\mu$  is the dimensional-regularization scale and  $\rho$  is the infrared mass regulator. The first term in the right hand side of Eq. (3) corresponds to the case of ordinary pdfs. The lightcone singularity  $v \rightarrow 1$ , corresponding to the exclusive boundary  $x = 1$ , cancels in this term, but it is present, even at  $d \neq 4$  and finite  $\rho$ , in subsequent terms.

These endpoint singularities come from gluon emission at large rapidity. They imply that, using the matrix element (1), in momentum space the  $1/(1-x)$  factors from real emission probabilities do not in general combine with virtual corrections to give  $1/(1-x)_+$  distributions, but leave uncanceled divergences at fixed  $k_\perp$ . It is only after supplying the above matrix element with a regularization prescription that the distribution is well-defined.

A possible regularization method for the endpoint is by cut-off, implemented by taking the eikonal line  $n$  in Eq. (2) to be non-lightlike [11, 24], combined with evolution equations in the cut-off parameter  $\eta = (p \cdot n)^2/n^2$  [12, 20]. Then the cut-off in  $x$  at fixed  $k_\perp$  is of order  $1-x \gtrsim k_\perp/\sqrt{4\eta}$ . Monte Carlo event generators that make use of unintegrated pdfs also implement a cut-off. We consider such applications in Sec. 3.

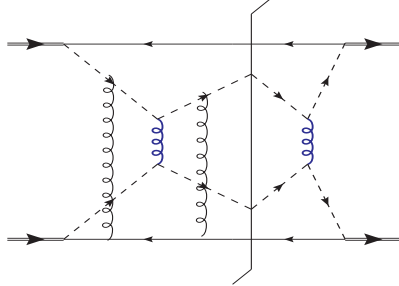


Figure 2: *Soft gluon exchange with spectator partons.*

An alternative method is provided by the subtractive method [25, 26], in which the direction  $n$  in Eq. (2) is kept lightlike but the divergences are canceled by multiplicative, gauge-invariant counterterms given by vacuum expectation values of eikonal operators. This method leads to well-prescribed counterterms [19] for the transverse momentum dependent splitting probabilities, which can be viewed as generalizing the plus-distribution regularization for  $k_\perp \neq 0$ . On one hand, this approach has been used to relate the endpoint behavior at fixed  $k_\perp$  with the cusp anomalous dimension [22] and investigate the role of the Mandelstam-Leibbrandt prescription in lightcone gauges. On the other hand, it can likely be more useful than the cut-off to untangle issues of factorization and non-universality [2, 4, 27, 28] which arise beyond leading order, and investigate the relationship between evolution equations in rapidity and in virtuality [12, 20, 29].

Infrared subtractions analogous to those in the method [25, 26] are also discussed in the context of the soft-collinear effective theory [30] (under the form, however, of counterterms that are not automatically gauge-invariant) in the case of the Sudakov form factor [31] and of initial-state beam functions [32] describing the incoming jet (see also [33]).

As noted earlier, a full treatment of factorization at the level of TMD pdfs is yet to be achieved [2, 4]. In the hadroproduction of nearly back-to-back hadrons, factorization is broken [4] by soft gluons exchanged between subgraphs in different collinear directions (Fig. 2). (See also the analyses [34, 35] for the Drell-Yan case). The underlying dynamics is that of non-abelian Coulomb phase [36], involving interactions with spectator partons [37] (which were treated long ago in [1] limited to the case of fully inclusive Drell-Yan). It was noted in [8] that the factorization-breaking contributions [4, 27] are related to the contributions [38] responsible for the appearance of super-leading logarithms in di-jet cross sections with a gap in rapidity.

The issue of factorization depends on developing a systematic treatment, capable of handling overlapping divergences in infrared regions for complex observables that involve color charges in both initial and final states. This motivates the subtraction techniques quoted above, in the version [25, 26] or in the SCET version [30, 31, 32, 33, 34, 35].

Factorization involves in general, besides TMD pdfs, a nonperturbative soft factor, also characterized in terms of operator matrix elements, see [25]. The first paper in Ref. [26] analyzed the possibility of reabsorbing the soft factor into a redefinition of the TMD pdfs (and analogous final-state fragmentation functions), using an explicit construction at one loop in the case of initial-state and final-state parton showers in DIS. More recently, an explicit procedure to reabsorb the soft factor into redefined TMD pdfs has been presented in Ref. [2] in the context of Drell-Yan. An analogous redefinition is implied by the Drell-Yan analyses in [34, 35]. It remains to be seen how generally this can be done. This will influence future programs of phenomenological determinations of TMD pdfs, see [3].

## 2.2 TMD formulation at small $x$

The case of back-to-back di-hadron or di-jet hadroproduction [4, 27] illustrates that, due to the difficulty in disentangling soft and collinear gluon correlations between initial and final states, a general TMD factorization formula is still lacking. In the case of small  $x$ , however, a TMD factorization result holds [6] owing to the dominance of single gluon helicity at high energy. In this case, a TMD gluon distribution can be defined gauge-invariantly from the high-energy pole in physical cross sections. See [39, 40, 41, 42] for recent discussions of unintegrated pdfs based on small  $x$ .

The main reason why such a definition for TMD pdfs can be constructed in the high-energy limit is that one can relate directly (up to perturbative corrections) the cross section for a *physical* process, e.g. photoproduction of a heavy-quark pair [43], to an unintegrated, transverse momentum dependent gluon distribution (Fig. 3). This is quite like what one does for deeply inelastic scattering, in the conventional parton picture, in terms of ordinary (integrated) parton distributions. On the other hand, the difficulties in defining a TMD distribution in the general case, over the whole phase space, can be associated with the fact that it is not obvious how to determine one such relationship for general kinematics.

The evolution equations obeyed by TMD distributions defined from the small- $x$  limit are evolution equations in energy [5], with corrections down by logarithms of energy rather than powers of momentum transfer. Using this evolution, the high-energy factorization [6] can be related, order-by-order in  $\alpha_s$  [44], with the renormalization-group factorization [1, 15, 45]. This

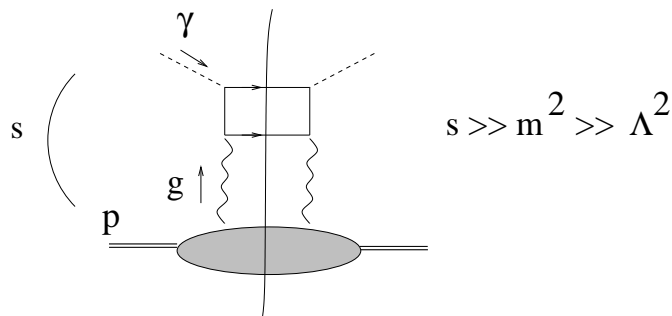


Figure 3: *TMD factorization at high energy for heavy quark photoproduction.*

allows one to describe in this framework the ultraviolet region of arbitrarily high  $k_{\perp}$ , and in particular re-obtain the structure of QCD logarithmic scaling violations, see e.g. [7, 46].

The above observation justifies the use of this approach for hard production physics. In particular, it is the basis for using Monte Carlo implementations of transverse momentum dependent parton showers (see e.g. [8, 47] and references therein) to treat multi-scale hard processes at the LHC. Parton-shower applications are discussed in Sec. 3. Such applications have so far focused on TMD gluon distributions. Ongoing work on the generalizations needed to include quark channels is discussed in Sec. 4.

Let us finally recall that extensions of the factorization results above are required if one is to take into account the nonlinear effects that are expected to arise in the small  $x$  region from high parton densities. Recent work in this direction, focusing on multiple scattering effects in dense targets and nuclei, may be found in [42, 48, 49]. In this respect, we note that techniques such as those in [50] have been proposed to incorporate the treatment of multiple-gluon rescattering graphs at small  $x$  starting from the operator matrix elements [11, 15] for parton distributions. They may thus be helpful for extensions to the high density region that are aimed at retaining accuracy also in the treatment of contributions from high  $p_T$  processes.

### 3 Parton-branching applications

In this section we move to applications of the TMD formalism to parton branching methods. The main role of TMD splitting functions and distributions in this context is that they serve to take into account coherence effects of multiple gluon radiation for small longitudinal momentum fractions  $x$  in the initial state parton cascade. In Subsec. 3.1 we briefly recall the motivation for angular-ordered parton showers, and in Subsec. 3.2 we discuss the treatment of gluon coherence at small  $x$  and corrections to angular ordering.

#### 3.1 Collinear showering and soft gluon coherence

Branching algorithms in standard shower Monte Carlo generators [51] are based on collinear evolution of the jets, both time-like and space-like, developing from the hard event. The branching probability is given in terms of splitting functions  $P$  and form factors  $\Delta$  (Fig. 4) as

$$d\mathcal{P} = \int \frac{dq^2}{q^2} \int dz \alpha_S(q^2) P(z) \Delta(q^2, q_0^2) . \quad (4)$$

The theoretical basis for the branching approach is the factorizability of universal splitting functions in QCD cross sections in the collinear limit [1, 52], which justifies the probabilistic picture.

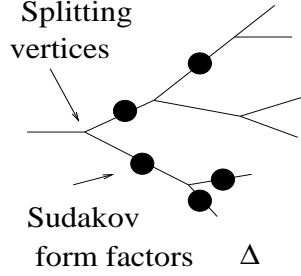


Figure 4: *Parton branching in terms of splitting probabilities and form factors.*

Besides small-angle, incoherent parton emission, many of the current shower generators also take into account further radiative contributions from emission of soft gluons, which are essential for realistic phenomenology [51]. To incorporate these in a probabilistic framework, one appeals to properties of coherence of color radiation [52, 53, 54]. Soft-gluon emission amplitudes factorize in terms of eikonal currents [55, 56]

$$\mathbf{J}_\mu^a = \sum_{i=1}^n \mathbf{Q}_i^a \frac{p_{i\mu}}{p_i \cdot q} \quad , \quad (5)$$

where  $p_i$  are the emitters' momenta,  $q$  is the soft momentum, and the color charge operators  $\mathbf{Q}_i^a$  are associated with the emission of gluon  $a$  from parton  $i$ . In general, interferences are expected to contribute to the radiative terms relating the  $(n+1)$ -parton process to the  $n$ -parton process. Nevertheless, a probabilistic branching-like picture can be recovered [57, 58, 59] by exploiting soft-gluon coherence. This is illustrated in Fig. 5 [47] for the case of two-gluon emission.

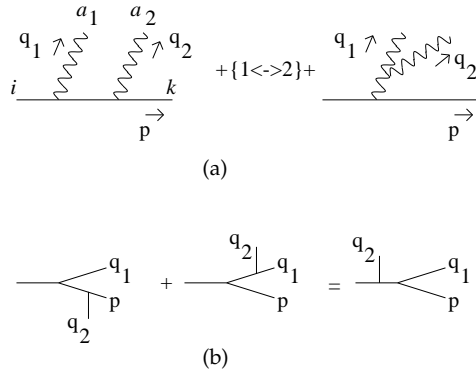


Figure 5: (a) *Two gluon emission from a fast quark;* (b) *coherence of soft gluon emission at large angle.*

In Fig. 5a two soft gluons with momenta  $q_1$  and  $q_2$  are produced from a fast parton with momentum  $p$ . Suppose  $q_2^0 \ll q_1^0$ . We distinguish two angular regions for the softest gluon

$q_2$  [47]. i) When  $q_2$  is at small angle from  $p$  ( $q_1$ ), then the amplitude can be seen as the sequential emission of  $q_1$  from  $p$  and of  $q_2$  from  $p$  ( $q_1$ ). This corresponds to the standard bremsstrahlung picture based on radiation cones centered around  $p$  and  $q_1$ . ii) When  $q_2$  is at large angle,  $\theta_{pq_2} \gg \theta_{pq_1}$ , then the directions of  $p$  and  $q_1$  can be identified and the two emission amplitudes act coherently to give (Fig. 5b) what can be seen as the sequential emission of  $q_2$  from  $p$  and of  $q_1$  from  $p$ . The reversed order of the emissions compared to case i) reflects the fact that the radiated gluon sees the total color charge of the emitting jet. Fig. 5b illustrates that contributions of different emitters combine to give an effective contribution in which the emissions are ordered in angle, so that angular ordering [57, 58, 59] replaces energy ordering.

The above framework of collinear showering supplemented with phase space constraints designed to implement the angular ordering [51] forms the basis of standard shower Monte Carlo event generators. We next discuss the modifications of this framework that are required to treat parton showers at increasingly high energies.

### 3.2 Space-like parton shower at high energies

New effects arise if one is to extend the picture of the above subsection to the case of very high energies, where processes with multiple hard scales become significant. The first new effect is that soft-gluon insertion rules [53, 58] for  $n$ -parton scattering amplitudes  $M^{(n)}$  can still be given in terms of real and virtual soft-gluon currents  $\mathbf{J}^{(R)}$  and  $\mathbf{J}^{(V)}$ ,

$$|M^{(n+1)}(k, p)|^2 = \left\{ [M^{(n)}(k+q, p)]^\dagger [\mathbf{J}^{(R)}]^2 M^{(n)}(k+q, p) - [M^{(n)}(k, p)]^\dagger [\mathbf{J}^{(V)}]^2 M^{(n)}(k, p) \right\}, \quad (6)$$

but the currents are modified in the high-energy, multi-scale region by terms that depend on the total transverse momentum transmitted down the initial-state parton decay chain [43, 60, 61]. For this reason the physically relevant distribution to describe space-like showers at high energies is not an ordinary parton density but rather a TMD parton density.

The next point concerns the structure of virtual corrections. Besides Sudakov form-factor contributions included in standard shower algorithms [51, 52], one needs in general virtual-graph terms to be incorporated in transverse-momentum dependent (but universal) splitting functions [60]. These allow one to take account of gluon coherence not only for collinear-ordered emissions but also in the non-ordered region that opens up at high  $\sqrt{s}/p_\perp$ , where  $\sqrt{s}$  is the total center-of-mass energy and  $p_\perp$  is the typical transverse momentum of a produced jet.

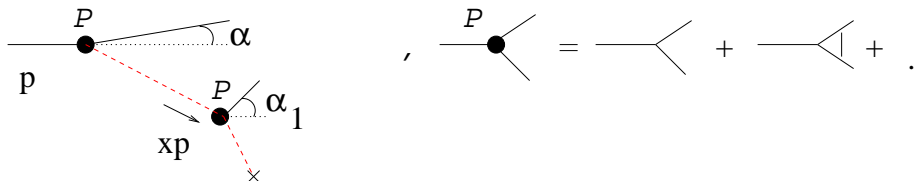


Figure 6: (left) Coherent radiation in the space-like parton shower for  $x \ll 1$ ; (right) the unintegrated splitting function  $\mathcal{P}$ , including small- $x$  virtual corrections.

The resulting structure of the parton branching, depicted in Fig. 6, differs from that in Eq. (4): the branching probability is  $k_\perp$ -dependent, and part of the virtual corrections are associated to the (unintegrated) splitting functions. Schematically one has, using the recursion relation (6),

$$\begin{aligned} \mathcal{G}(x, k_\perp, \mu) &= \mathcal{G}_0(x, k_\perp, \mu) + \int \frac{dz}{z} \int \frac{dq^2}{q^2} \Theta(\mu - zq) \\ &\times \Delta(\mu, zq) \mathcal{P}(z, q, k_\perp) \mathcal{G}\left(\frac{x}{z}, k_\perp + (1-z)q, q\right) \quad , \end{aligned} \quad (7)$$

where  $\mathcal{G}$  is the unintegrated gluon distribution,  $\Delta$  is the form factor, and  $\mathcal{P}$  is the unintegrated splitting function (Fig. 6). The kernels  $\mathcal{P}$  depend on transverse momenta and include part of the virtual corrections, in such a way as to avoid double counting with the Sudakov form factor  $\Delta$ , while reconstructing color coherence not only at large  $x$  but also at small  $x$  in the angular region (Fig. 6)

$$\alpha/x > \alpha_1 > \alpha \quad , \quad (8)$$

where the angles  $\alpha$  for the partons radiated from the initial-state shower are taken with respect to the initial beam jet direction, and increase with increasing off-shellness.

In terms of high-energy power-counting, the effects of region (8) are potentially enhanced by terms

$$\alpha_s^k \ln^{k+m} \sqrt{s}/p_\perp \quad . \quad (9)$$

In inclusive processes, coherence leads to strong cancellations between reals and virtuals so that terms with  $m \geq 1$  in Eq. (9) drop out, and high-energy corrections are at most single-logarithmic [44, 62, 63]. For exclusive jet distributions such cancellations are not present and one may expect stronger enhancements. The implementation of coherent-branching effects associated with high-energy logarithms is, from the point of view of jet physics, the main motivation for developing the formalism of unintegrated parton distributions and implementing it in shower Monte Carlo event generators. See [64] for recent phenomenological investigations of such effects on the structure of angular correlations and multiplicity distributions in multi-jet final states, and [65] for applications to the jet structure associated with heavy mass states.

Monte Carlo implementations of the modified branching described in this section include the parton-shower event generators CASCADE [66], LDCMC [67]. See [8, 47, 68] for further references. These implementations take into account gluon coherence effects and include TMD gluon distributions. An extension to valence quark distributions is given in [69]. In the next section we describe recent work to go beyond this approximation and include sea quark contributions.

## 4 Sea quark distribution and Drell Yan production

Monte Carlo calculations based on TMD approaches have so far been carried out mostly in a quenched approximation in which only gluon and valence quarks are included. In terms of the parton branching kernels, this corresponds to taking into account only contributions from the splitting vertices (a) and (b) in Fig. 7. In this section we briefly describe the work [9] to go beyond this quenched approximation and treat TMD sea quark contributions. The main focus of this work is to take into account effects of the splitting process in Fig. 7(c) at TMD level.

Let us recall that early attempts [70, 71] to treat the unintegrated pdf evolution beyond the quenched approximation include quarks via splitting probabilities to lowest order of perturbation theory, neglecting any transverse momentum dependence in the branching. In [72]



$k_{\perp}$ -dependent kinematic corrections are included, while the splitting kernels are still taken in lowest order.

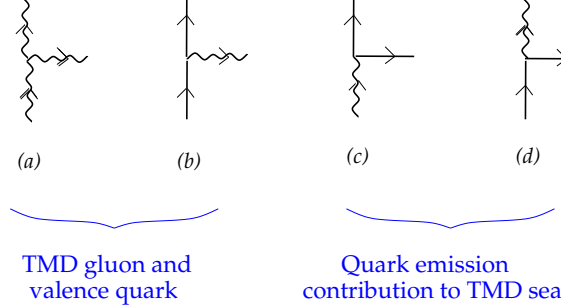


Figure 7: *Splitting vertices for TMD pdf evolution.*

Also, a program to perform shower Monte Carlo evolution at unintegrated level has recently been proposed [73] based on the expansion [45] in two-particle irreducible (2PI) kernels. This program is formulated at the next-to-leading order. However, this is limited at present to the flavor nonsinglet sector, and does not include small- $x$  effects which are present beyond NLO in flavor singlet distributions.

The approach taken in [9] is based on the high-energy form of the (off-shell) TMD quark Green function defined in [44]. This is obtained by generalizing to finite transverse momenta, in the high-energy region, the 2PI expansion [45]. The  $k_{\perp}$ -dependent gluon-to-quark splitting kernel is depicted in Fig. 8(b). Thus, unlike the approaches in [70, 71, 72], on one hand, and in [73], on the other hand, Ref. [9] constructs TMD quark distributions by using the  $k_{\perp}$ -dependent gluon-to-quark splitting function, which governs sea quark evolution to all orders in perturbation theory in the small- $x$  limit. Refs. [9, 74] investigate the phenomenological relevance of the finite  $k_{\perp}$  terms for Drell-Yan production (Fig. 8(a)).

Schematically, the TMD gluon-to-quark kernel has the form

$$\mathcal{P}_{g \rightarrow q}(z; q_{\perp}, k_{\perp}) = P_{qg}^{(0)}(z) \left( 1 + \sum_{n=1}^{\infty} b_n(z) (k_{\perp}^2 / q_{\perp}^2)^n \right), \quad (10)$$

where  $P^{(0)}$  is the lowest-order DGLAP splitting function,  $z$  is the longitudinal momentum transfer,  $k_{\perp}$  and  $q_{\perp}$  are respectively the gluon and quark transverse momenta in Fig. 8 (b), and all coefficients  $b_n(z)$  of the finite terms in  $k_{\perp}$  are known from [44]. Although it is evaluated off-shell, the splitting probability in Eq. (10) is universal [44, 75]. The approaches in [70] and [71, 72] only include the term  $P^{(0)}$  in Eq. (10), while Ref. [9] includes the full series. The finite terms in  $k_{\perp}$  are the ones responsible for determining small- $x$  logarithmic corrections to flavor-singlet quark evolution to all orders in  $\alpha_s$ .

In Ref. [10] contributions from the TMD flavor-singlet quark distribution are implemented in the parton shower Monte Carlo event generator CASCADE [66]. This constitutes a starting point to systematically include quark emissions in the parton shower at TMD level. At present the implementation is done in such a way that the shower couples to quarks only once, while a complete implementation will allow this to occur arbitrarily many times. Nevertheless, already the present implementation incorporates for the first time the small- $x$  dynamics encoded in

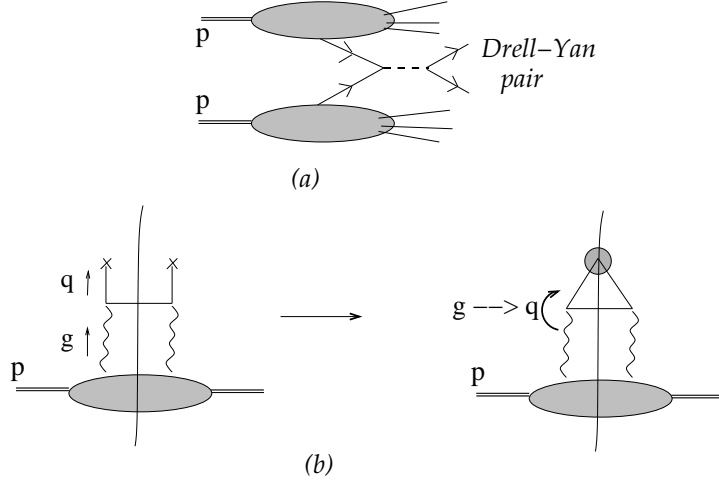


Figure 8: (a)  $\bar{q}q$  Drell-Yan production; (b)  $g \rightarrow q$  splitting contribution to sea quark distribution.

Eq. (10) in a parton shower, and makes it possible to treat in this framework hard processes induced by sea quarks on the same footing as processes induced by gluons.

Refs. [9, 10] present results within this framework for Drell Yan production processes. This is phenomenologically relevant, because Drell Yan production at the LHC (Fig. 8(a)) receives large contributions from sea quark scattering at small  $x$  [68]. Such contributions affect many aspects of LHC physics, as Drell-Yan processes are instrumental in precision electroweak measurements, in luminosity monitoring and pdf determinations, and in new physics searches. Partial results on high-energy Drell-Yan at TMD level have so far been obtained in the  $qg^*$  channel [76], and in the associated production channel  $Z/W +$  heavy quarks [77]. The  $qg^*$  channel is of direct phenomenological relevance for forward Drell-Yan production. Ref. [9] evaluates the perturbative coefficients for the coupling of the TMD sea quark distribution to Drell-Yan by using the “reggeized quark” calculus [78]. This extends the high-energy effective action formalism [79], currently explored at NLO [80], to amplitudes with quark  $t$ -channel exchange [81, 82]. In [9, 10] the method is employed to investigate predictions for Drell-Yan production in the forward region (Fig. 9).

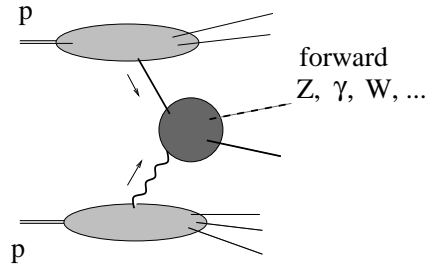


Figure 9: Forward Drell-Yan production.

In Figs. 10,11 we report numerical results for  $Z$ -boson rapidity and transverse momentum spectra. We present results for two possible choices of the scale  $\mu$  in the shower evolution equation (7): one in which  $\mu$  is set equal to the hard scale, defined by

$$\mu^2 = p_{\perp}^2 + M_Z^2 \quad , \quad (11)$$

where  $p_{\perp}$  and  $M_Z$  are the transverse momentum and mass of the  $Z$  boson; another in which  $\mu$  is set equal to the maximum-angle scale determined by the angular ordering kinematics, given by

$$\mu^2 = \frac{q_{\perp}^2 + (1-z)k_{\perp}^2}{(1-z)^2} \quad , \quad (12)$$

where the variables are as specified below Eq. (10). Since we are working in the forward region, we also include results obtained from the  $qg^*$  matrix element.

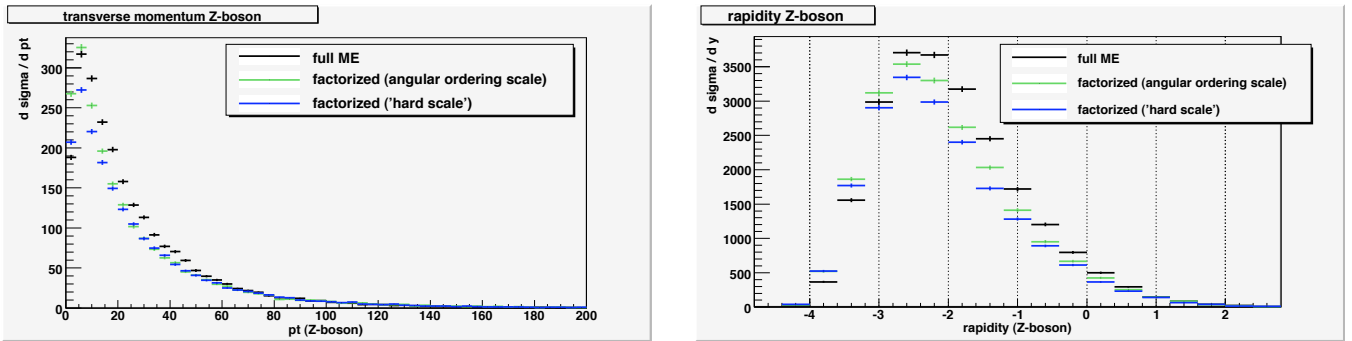


Figure 10:  $Z$ -boson transverse-momentum and rapidity spectra.

All three curves in Figs. 10,11 correspond to calculations which are applicable at forward rapidity. The shape of the rapidity spectrum in Fig. 10, falling off for central production, can thus be regarded as defining the kinematic region of applicability. The transverse momentum spectrum in Fig. 10 indicates that the three calculations differ at low  $p_t$ , but they all converge for large enough  $p_t$ . This is illustrated in more detail in Fig. 11. The difference between the matrix element and factorized calculations in the small- $p_t$  region is mostly due to quark  $s$ -channel contributions. This region is dominated by Sudakov form factor effects, giving the turn-over in the  $p_t$  spectrum, which are sensitive to the different choices of scales in Eqs. (11),(12). The difference between the curves disappears in the large  $p_t$  region as the quark  $t$ -channel contribution dominates, and contributions from the parton showers also become relatively insensitive to the different scale choices, because transverse momentum ordering sets in driven by the high  $p_t$ .

The numerical results in Figs. 10,11 can be seen as a consistency check on the approach based on TMD parton branching as regards forward Drell-Yan production. The result [9] for the unintegrated flavor-singlet quark distribution is however more general, and is a necessary ingredient to develop a description of vector boson production and associated final states throughout the rapidity range of forward and central regions. This is the subject of forthcoming work.

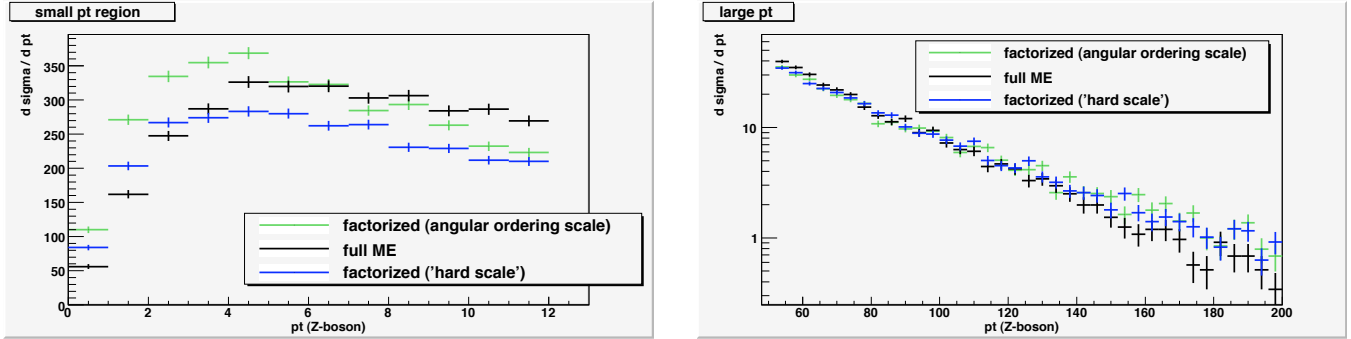


Figure 11: *Numerical comparison in the small- $p_T$  and large- $p_T$  regions.*

## 5 Summary

We have discussed recent progress on transverse momentum dependent parton distributions. Relying on TMD factorization results at small  $x$ , we have discussed applications to branching Monte Carlo methods for initial-state parton showers in high-energy collisions. Calculations based on these methods have mostly been developed within a quenched approximation in which only gluon and valence quark effects are taken into account at unintegrated level. We have reported work to go beyond this approximation by including sea quark contributions.

The method is based on taking into account flavor-singlet quark evolution via the transverse momentum dependent gluon-to-quark splitting kernel in Eq. (10). We have used this along with the Monte Carlo implementation [66] of the small- $x$  coherent branching equation given in Eq. (7). Compared to previous approaches to treat the TMD sea quark distribution, the main feature of this approach is that it includes, in addition to the lowest-order splitting function, the full series of finite- $k_\perp$  terms in the gluon-to-quark two-particle irreducible kernel. This allows one to sum small- $x$  logarithmic corrections to flavor-singlet observables to all orders in  $\alpha_s$ .

We have used this framework to obtain numerical predictions for forward  $Z$ -boson production. To this end, the perturbative coefficients for coupling the TMD sea quark distribution to Drell-Yan production have been determined by using the formalism [78] for high-energy quark  $t$ -channel exchange. We have investigated the dependence of the results on the shower evolution scale.

The method proposed in this work is implemented at present by letting quarks interact with the shower only once. Using the ingredients discussed in this work, this could however be extended by constructing a fully coupled shower evolution in the flavor-singlet sector. Also, only one coupling of the sea quark to vector bosons is evaluated at TMD level at present. This should be extended in order to treat Drell-Yan throughout the rapidity range of forward and central regions. Nevertheless, the results presented here, being the first implementation of small- $x$  sea-quark dynamics in a parton shower, constitute a starting point to include quark emissions systematically and to treat hard processes initiated by sea quarks on the same footing as processes initiated by gluons.

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