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A Lattice Study of the Glue in the Nucleon

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Abstract

By introducing an additional operator into the action and using the Feynman–Hellmann theorem we describe a method to determine both the quark line connected and disconnected terms of matrix elements. As an illustration of the method we calculate the gluon contribution (chromo-electric and chromo-magnetic components) to the nucleon mass.

1 Introduction

One of the earliest experimental indications that the nucleon consists not only of three quarks, but also has a gluonic contribution came from the measurement of the fraction of the nucleon momentum carried by the quarks. That this did not sum up to 1 as is required from the energy–momentum sum rule gave evidence for the existence of the gluon. Denoting $\langle x \rangle_f$ as the fraction of the nucleon momentum carried by parton f we have

$$\sum_q \langle x \rangle_q + \langle x \rangle_g = 1\,, \tag{1}$$

where for the quarks $f \equiv q = u, d, \ldots$ and for the gluon $f \equiv g$. Experimentally $\langle x \rangle_{u+d} \sim 0.4$ so the missing component is large $\sim 50\%$ of the total nucleon momentum. Both $\langle x \rangle_q$ and $\langle x \rangle_g$ have similar definitions and so analogously to the definition of $\langle x \rangle_q$ we have, with $\mathcal{M}$ denoting Minkowski space

$$\langle N(\vec{p})|[\widehat{\mathcal{O}}^{\mathcal{M}(g)\mu_1\mu_2} - \tfrac{1}{4}\eta^{\mu_1\mu_2}\widehat{\mathcal{O}}^{\mathcal{M}(g)\alpha}{}_{\alpha}]|N(\vec{p})\rangle = 2\langle x \rangle_g \left[p^{\mu_1}p^{\mu_2} - \tfrac{1}{4}\eta^{\mu_1\mu_2}m_N^2\right]\,, \tag{2}$$

where

$$O^{\mathcal{M}(g)\mu_1\mu_2} = -\mathrm{tr}_c F^{\mathcal{M}\mu_1\alpha} F^{\mathcal{M}\mu_2}{}_{\alpha}\,, \tag{3}$$

(where $\mathcal{O}(t) = \int d^3x\, O(t,\vec{x})$ and with normalisation $\langle N(\vec{p})|N(\vec{p}\,')\rangle = 2E_N\delta(\vec{p} - \vec{p}\,'))$. Note that we can generalise from a nucleon to an arbitrary hadron (averaging over polarisations if necessary). Higher moments can also be considered, by inserting covariant derivatives between the Fs. These occur when using the Wilson operator product expansion which relates them to moments of structure functions in a twist expansion.

There have been many lattice estimates of the quark momentum fraction $\langle x \rangle_q$ both for the nucleon (see e.g. [1, 2] for a review) and the pion e.g. [3, 4], but few attempts for the gluon part, $\langle x \rangle_g$ [5, 6, 7]. This is due to the fact that a lattice simulation must compute a quark line disconnected term, which is extremely noisy and gives a poor signal. These are direct calculations; in this letter we propose a new method using the Feynman-Hellmann theorem, to determine the gradient of E_N as a function of a parameter of an operator which has been introduced into the action $S \to S(\lambda) = S + \lambda S_O$. An obvious disadvantage of this method is that it requires dedicated simulations for each operator of interest, but the gain, as we shall see, is a much cleaner signal.

While the method is general, we shall demonstrate its practicability here by determining $\langle x \rangle_g$ in the quenched case.

2 The Feynman–Hellmann theorem

We first briefly describe the Feynman–Hellmann theorem, in a Euclidean form that will be useful for the case to be considered here. Let S depend on some parameter λ, so $S \to S(\lambda)$. Now as by definition the (Euclidean) correlation function is given by

$$\langle N(t)\overline{N}(0)\rangle_\lambda \equiv \frac{\int[dU]N(t)\overline{N}(0)e^{-S(\lambda)}}{\int[dU]e^{-S(\lambda)}}\,, \tag{4}$$

(the unpolarised case for the nucleon and where we make the obvious replacements N by H and $\overline{N}$ by $H^\dagger$ for other hadrons), then we have

$$\frac{\partial}{\partial\lambda}\langle N(t)\overline{N}(0)\rangle_\lambda = -\left\langle N(t)\left(\frac{\partial S(\lambda)}{\partial\lambda} - \langle\frac{\partial S(\lambda)}{\partial\lambda}\rangle_\lambda\right)\overline{N}(0)\right\rangle_\lambda\,. \tag{5}$$

We now use the transfer matrix formalism on both sides of this equation. Ignoring finite size effects this gives

$$\langle N(t)\overline{N}(0)\rangle_\lambda = A_N(\lambda)e^{-E_N(\lambda)t} + \text{exp. smaller terms}\,. \tag{6}$$

so on the LHS of eq. (5),

$$\frac{\partial}{\partial\lambda}\langle N(t)\overline{N}(0)\rangle_\lambda = -\frac{\partial E_N(\lambda)}{\partial\lambda}\langle N(t)\overline{N}(0)\rangle_\lambda\, t + \text{exp. smaller terms}\,. \tag{7}$$

Furthermore, if $\Omega(\tau)$ is any operator (local in time), then using the transfer matrix formalism again the associated 3-point function gives

$$\frac{\langle N(t)\Omega(\tau)\overline{N}(0)\rangle_\lambda}{\langle N(t)\overline{N}(0)\rangle_\lambda} = \begin{cases} \frac{1}{2E_N(\lambda)}\langle N|\widehat{\Omega}|N\rangle_\lambda + \text{exp. small terms} & 0 \ll \tau \ll t \\ \text{exp. small terms} & \text{otherwise} \end{cases}\,. \tag{8}$$

Note that we have inserted a $2E_N$ in the denominator of the RHS to account for the mis-match of normalisations, i.e. to agree with those of eq. (2). Hence summing over τ also gives a linear term in t. Thus from this equation, replacing $\sum_\tau \Omega(\tau)$ by the operator in the RHS of eq. (5), and together with eq. (7) we have the Feynman–Hellmann theorem

$$\frac{\partial E_N(\lambda)}{\partial\lambda} = \frac{1}{2E_N(\lambda)}\left\langle N\left|:\widehat{\frac{\partial S(\lambda)}{\partial\lambda}}:\right|N\right\rangle_\lambda\,, \tag{9}$$

(where $:\ldots:$ means that the vacuum term has been subtracted). Thus by suitably choosing S_O and by identifying numerically the gradient of $E_N(\lambda)$ at $\lambda = 0$ we can determine the desired matrix element.

3 The lattice method

3.1 Gluon operators

Before considering the lattice, let us first Euclideanise the gluon operators[1] to give us an indication of what we might add to the action. Defining

$$O_{\mu\nu} = -\mathrm{tr}_c F_{\mu\alpha} F_{\nu\alpha} \,, \tag{10}$$

($\mathrm{tr}_c F^2 = \frac{1}{2} F^{a\,2}$) this then gives the two obvious operator choices (a) and (b),

$$\begin{aligned} O_{a\,i} &= O_{i4} = \mathrm{tr}_c(\vec{E} \times \vec{B})_i \\ O_b &= O_{44} - \tfrac{1}{3} O_{jj} = \tfrac{2}{3}\mathrm{tr}_c(-\vec{E}^2 + \vec{B}^2) \end{aligned} \tag{11}$$

($O_a^{\mathcal{M}(g)} \to i O_a$ and $O_b^{\mathcal{M}(g)} \to O_b$). The relation to $\langle x \rangle_g$ is given by

$$\begin{aligned} \langle N(\vec{p}) | \widehat{\mathcal{O}}_{a\,i} | N(\vec{p}) \rangle &= -2 i E_N p_i \, \langle x \rangle_g \\ \langle N(\vec{p}) | \widehat{\mathcal{O}}_b | N(\vec{p}) \rangle &= 2(m_N^2 + \tfrac{4}{3}\vec{p}^{\,2}) \, \langle x \rangle_g \,, \end{aligned} \tag{12}$$

with

$$\widehat{\mathcal{O}}_{a\,i} = \mathrm{tr}_c(\vec{\mathcal{E}} \times \vec{\mathcal{B}})_i \,, \qquad \widehat{\mathcal{O}}_b = \tfrac{2}{3}\mathrm{tr}_c(-\vec{\mathcal{E}}^{\,2} + \vec{\mathcal{B}}^{\,2}) \,. \tag{13}$$

Both choices have their difficulties: operator (a) always needs a non-zero momentum $\vec{p}$, while operator (b) requires a delicate subtraction between two terms similar in magnitude.

Note that, because of Euclideanisation (footnote 1) the *energy* has a negative $\mathcal{E}^2$ term, while the *action* (see section 3.2) has a positive $\mathcal{E}^2$ term.

3.2 The action

We now turn to the lattice. We shall use the Wilson gluonic action

$$S = \tfrac{1}{3}\beta \sum_{x\,\mu<\nu} \mathrm{Re}\,\mathrm{tr}_c \left[1 - U_{\mu\nu}^{\Box}(x) \right] \,, \tag{14}$$

(i.e. sum over plaquettes), with $\beta = 6/g^2$. As

$$\mathrm{Re}\,\mathrm{tr}_c \left[1 - U_{\mu\nu}^{\Box}(x) \right] = \tfrac{1}{4} a^4 g^2 F_{\mu\nu}^a(x)^2 + \ldots \,, \tag{15}$$

[1] Our conventions follow [3]. So $E^{\mathcal{M}i} = F^{\mathcal{M}i0} \to iF_{i4} \equiv iE_i$ and $B^{\mathcal{M}i} = -\frac{1}{2}\epsilon^{ijk} F^{\mathcal{M}}{}_{jk} \to \frac{1}{2}\epsilon_{ijk} F_{jk} \equiv B_i$.

this motivates the simplest definition of electric and magnetic field on each time slice as

$$\begin{aligned}
\tfrac{1}{2}\mathcal{E}^{a\,2}(\tau) &= \tfrac{1}{3}\beta\frac{1}{a}\sum_{\vec{x}\,i} \mathrm{Re}\,\mathrm{tr}_c\left[1 - U^{\Box}_{i4}(\vec{x},\tau)\right] \\
\tfrac{1}{2}\mathcal{B}^{a\,2}(\tau) &= \tfrac{1}{3}\beta\frac{1}{a}\sum_{\vec{x}\,i<j} \mathrm{Re}\,\mathrm{tr}_c\left[1 - U^{\Box}_{ij}(\vec{x},\tau)\right] ,
\end{aligned} \tag{16}$$

respectively. For the action we thus take

$$S(\lambda) = a\sum_{\tau} \tfrac{1}{2}[\mathcal{E}^{a\,2}(\tau) + \mathcal{B}^{a\,2}(\tau)] - \lambda a \sum_{\tau} \tfrac{1}{2}[-\mathcal{E}^{a\,2}(\tau) + \mathcal{B}^{a\,2}(\tau)] , \tag{17}$$

or in terms of the gauge plaquettes

$$\begin{aligned}
S(\lambda) &= \tfrac{1}{3}\beta(1+\lambda)\sum_{i} \mathrm{Re}\,\mathrm{tr}_c\left[1 - U^{\Box}_{i4}(\vec{x},\tau)\right] \\
&\quad + \tfrac{1}{3}\beta(1-\lambda)\sum_{i<j} \mathrm{Re}\,\mathrm{tr}_c\left[1 - U^{\Box}_{ij}(\vec{x},\tau)\right] .
\end{aligned} \tag{18}$$

Of course for $\lambda = 0$, then this reduces to the standard action, eq. (14).

3.3 Gluon moment

Comparing the results of sections 3.1 and 3.2 we see that they can be applied to operator (b) only; operator (a) would require the clover definition of the field strength tensor. Using eq. (11) together with eq. (12) and eq. (9) gives from the Feynman–Hellmann theorem

$$\frac{\partial E_N(\lambda)}{\partial \lambda} = -\frac{1}{2E_N(\lambda)} \langle N(\vec{p})|\tfrac{1}{2}(-\widehat{\mathcal{E}}^{a\,2} + \widehat{\mathcal{B}}^{a\,2})|N(\vec{p})\rangle_\lambda , \tag{19}$$

which leads to

$$\left.\frac{\partial E_N(\lambda)}{\partial \lambda}\right|_{\lambda=0} = -\frac{3}{2E_N} \left(m_N^2 + \tfrac{4}{3}\vec{p}^{\,2}\right) \langle x\rangle_g^{lat} , \tag{20}$$

where the ^{lat} superscript on $\langle x\rangle_g^{lat}$ signifies that it is now the lattice operator.

The vacuum term which appears in section 2 has been dropped, because

$$\langle 0|\tfrac{1}{2}(-\widehat{\mathcal{E}}^{a\,2} + \widehat{\mathcal{B}}^{a\,2})|0\rangle = 0 . \tag{21}$$

This follows from rotation symmetry. In the Euclidean vacuum the time and space directions are equivalent, so the average trace of the chromo-electric plaquettes, $U^{\Box}_{i4}$, is the same as that of the chromo-magnetic plaquettes, $U^{\Box}_{ij}$, in eq. (16), leading to perfect cancellation in eq. (21).

4 Lattice results

We work with quenched Wilson clover fermions at $\beta = 6.0$, $c_{sw} = 1.769$ and $\kappa = 0.1320, 0.1324, 0.1333, 0.1338, 0.1342$ on a $24^3 \times 48$ lattice with antiperiodic time boundary conditions for the fermion. We have generated O(500) configurations for each ensemble. We use standard nucleon interpolating operators together with Jacobi smeared source/sink as in e.g. [3]. The results were generated using the Chroma program suite, [8]. We have only considered the case $\vec{p} = \vec{0}$ so eq. (20) reduces to

$$\langle x \rangle_g^{lat} = -\frac{2}{3am_N} \left. \frac{\partial am_N(\lambda)}{\partial \lambda} \right|_{\lambda=0} . \tag{22}$$

To estimate the gradient at $\lambda = 0$, we have generated data at $\lambda = -0.03333$, 0.0, 0.03333 which enables us to straddle the $\lambda = 0$ point. The raw data results are given in Table 1.

κ	$\lambda = -0.03333$	$\lambda = 0$	$\lambda = 0.03333$
0.1320	1.0033(29)	0.9772(33)	0.9564(34)
0.1324	0.9537(30)	0.9283(34)	0.9077(36)
0.1333	0.8357(33)	0.8117(40)	0.7923(41)
0.1338	0.7649(38)	0.7413(47)	0.7236(47)
0.1342	0.7044(47)	0.6799(62)	0.6647(55)

Table 1: Nucleon masses, am_N, as a function of λ for five quark masses, κ, calculated on ensembles with fixed $\beta = 6.0$ and $c_{sw} = 1.769$.

In Fig. 1 we plot the nucleon mass, am_N, against λ for the five quark masses. The data show no $O(\lambda^2)$ effects for the λ values chosen. These gradients (at $\lambda = 0$) together with the nucleon masses (again at $\lambda = 0$) determine $\langle x \rangle_g^{lat}$ from eq. (22) which are given in Table 2.

κ	am_π	$\langle x \rangle_g^{lat}$
0.1320	0.55499(48)	0.4826(456)
0.1324	0.51745(49)	0.4985(502)
0.1333	0.42531(52)	0.5383(644)
0.1338	0.36711(55)	0.5620(811)
0.1342	0.31433(62)	0.5893(1062)

Table 2: The pion mass and $\langle x \rangle_g^{lat}$ for the five different quark masses.

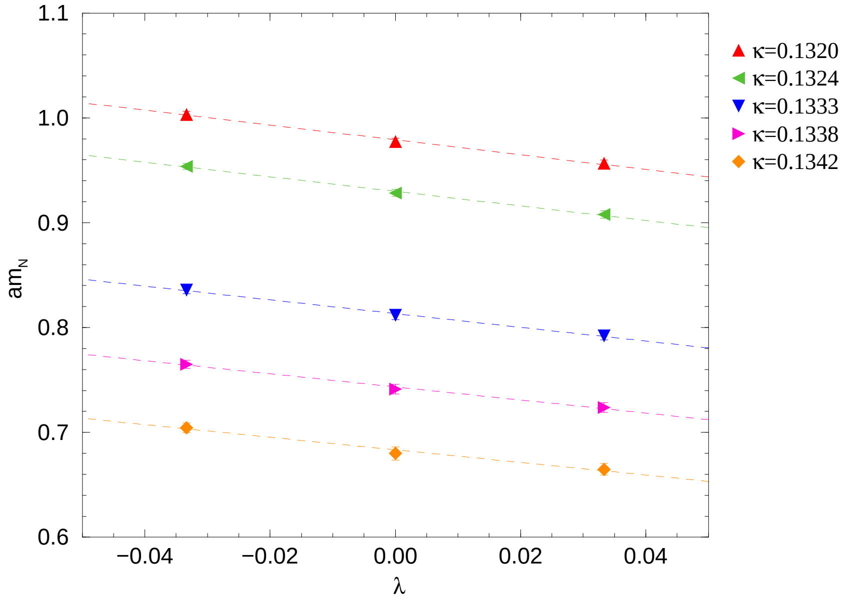

Figure 1: The nucleon mass against λ for the five κ values, together with a linear fit for each κ value.

5 Renormalisation

As gluon operators are singlets, they can mix with the quark singlet. However there exists a combination of singlet operators with vanishing anomalous dimension. (This is due to the conservation of the energy-momentum tensor, eq. (1).) We follow [6] and first write

$$\langle x\rangle_g^{bare} + \sum_q \langle x\rangle_q^{bare} = 1 + O(a^2)\,, \tag{23}$$

where

$$\langle x\rangle_g^{bare} = Z_g \langle x\rangle_g^{lat}\,, \qquad \langle x\rangle_q^{bare} = Z_q \langle x\rangle_q^{lat}\,. \tag{24}$$

Together with the change to a scheme (here taken as $\overline{MS}$)

$$\begin{pmatrix} \langle x\rangle_g^{\overline{MS}}(\mu) \\ \sum_q \langle x\rangle_q^{\overline{MS}}(\mu) \end{pmatrix} = \begin{pmatrix} Z^{\overline{MS}}_{bare\,gg}(\mu) & 1 - Z^{\overline{MS}}_{bare\,qq}(\mu) \\ 1 - Z^{\overline{MS}}_{bare\,gg}(\mu) & Z^{\overline{MS}}_{bare\,qq}(\mu) \end{pmatrix} \begin{pmatrix} \langle x\rangle_g^{bare} \\ \sum_q \langle x\rangle_q^{bare} \end{pmatrix}, \tag{25}$$

this completes the renormalisation procedure. As we are considering quenched QCD only there is a simplification as $Z^{\overline{MS}}_{bare\,gg} = 1$,

$$\begin{aligned} \langle x\rangle_g^{\overline{MS}}(\mu) &= \langle x\rangle_g^{bare} + [1 - Z^{\overline{MS}}_{bare\,qq}(\mu)] \sum_q \langle x\rangle_q^{bare} \\ \langle x\rangle_q^{\overline{MS}}(\mu) &= Z^{\overline{MS}}_{bare\,qq}(\mu) \langle x\rangle_q^{bare}\,, \end{aligned} \tag{26}$$

($Z^{\overline{MS}}_{bare\,qq}(\mu)$ is common for all the quarks). We thus need to determine Z_g, Z_q and $Z^{\overline{MS}}_{bare\,qq}(\mu)$. We can find Z_g by following [10] in considering an alternative interpretation of the action (18). We motivated this action by adding a multiple of the gluon x operator to the standard action, but we could also write the action as

$$S = \tfrac{1}{3}\beta_t \sum_i \mathrm{Re}\,\mathrm{tr}_c \left[1 - U^{\square}_{i4}(\vec{x},\tau)\right] + \tfrac{1}{3}\beta_s \sum_{i<j} \mathrm{Re}\,\mathrm{tr}_c \left[1 - U^{\square}_{ij}(\vec{x},\tau)\right] . \qquad (27)$$

which is the standard way of writing a gluon action on an anisotropic asymmetric lattice, with differing spatial and temporal lattice spacings, $a_s \neq a_t$. This action has been studied in detail, in particular the way in which the anisotropy $\xi = a_s/a_t$ depends on β_s and β_t is known both perturbatively and non-perturbatively [11]. At tree-level the anisotropy is given by $\xi^2_{tree} = \beta_t/\beta_s$. Z_g can be found by comparing the anisotropy actually produced by splitting β_s and β_t with this tree-level value. The result is $Z_g = 1 - \frac{g^2}{2}(c_\sigma - c_\tau)$ where the anisotropy coefficients c_σ and c_τ are defined in [11]. Using the perturbative values for $c_{\sigma,\tau}$ [12] yields $Z_g = 1 - 0.16677g^2 + \cdots$ as the 1-loop perturbative Z_g. In [9] this result was combined with non-perturbative determinations of $c_{\sigma,\tau}$, [11], to give a Padé expression

$$Z_g = \frac{1 - 1.0225g^2 + 0.1305g^4}{1 - 0.8557g^2}\,, \qquad \beta \geq 5.7\,, \qquad (28)$$

(with an error of $\sim 1\%$). So for $\beta = 6.0$ this gives $Z_g = 0.748$.

To estimate Z_q we use the results for $\langle x\rangle^{lat}_g$ from Table 2 together with those for $\langle x\rangle^{lat}_u$, $\langle x\rangle^{lat}_d$ from [13] (i.e. v_{2b}) together with eqs. (23) and (24). In Fig. 2 we plot[2] $\langle x\rangle^{lat}_u + \langle x\rangle^{lat}_d$ against $\langle x\rangle^{lat}_g$. From eq. (23) we would expect that the y-intercept is given by $1/Z_g$ and the x-intercept is given by $1/Z_q$. At present we do not have enough results for a determination, so we shall just check for consistency by fixing the y-intercept as 1/0.748 and the x-intercept as 1, [6]. This gives consistency so we shall adopt here $Z_q = 1$ together with a 10% error.

Also from [13], we have for $\mu = 2\,\mathrm{GeV}$,

$$\begin{aligned} Z^{\overline{MS}}_{bare\,qq}(\mu = 2\,\mathrm{GeV})Z_q &= Z^{RGI}_{v_{2b}} \times [\Delta Z^{\overline{MS}}_{v_2}(\mu = 2\,\mathrm{GeV})]^{-1} \\ &= 1.45 \times 0.732(9) = 1.06(1)\,, \end{aligned} \qquad (29)$$

where the second equation uses the notation of that article (the non-perturbative $RI-MOM$ scheme is converted to an RGI form and then back to the $\overline{MS}$ scheme). Further values of $\Delta Z^{\overline{MS}}_{v_2}(\mu)$ are also given in [13]. With Z_q this then gives $Z^{\overline{MS}}_{bare\,qq}$.

[2] The total contribution to $\langle x\rangle_q$ from sea quarks has the form $N_f \times$ (disconnected term). So, even though the disconnected loop term is itself non-zero, we do not need to consider it because its coefficient vanishes if we work consistently in the quenched approximation.

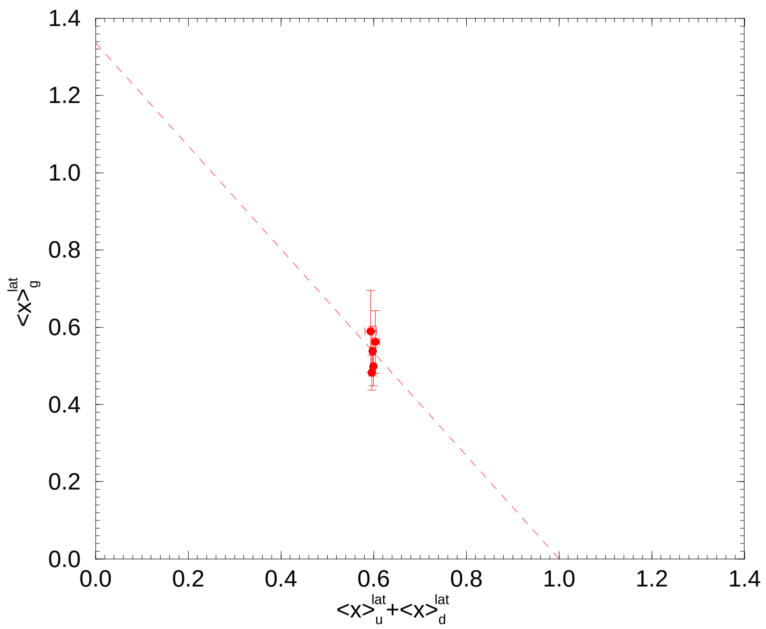

Figure 2: $\langle x\rangle_u^{lat} + \langle x\rangle_d^{lat}$ against $\langle x\rangle_g^{lat}$ for the five κ values, together with the line $y = (1 - x)/0.748$.

6 Results and conclusion

We are now in a position to determine $\langle x\rangle_g^{\overline{MS}}(\mu = 2\,\mathrm{GeV})$. Using the first equation in eq. (26) together with eq. (28) (evaluated at $\beta = 6.0$) and eq. (29) gives $\langle x\rangle_g^{\overline{MS}}(\mu = 2\,\mathrm{GeV})$. In Fig. 3 we plot using eq. (26), $\langle x\rangle_g^{\overline{MS}}(\mu = 2\,\mathrm{GeV})$ versus $(am_\pi)^2$. This gives a value for $\langle x\rangle_g^{\overline{MS}}(\mu = 2\,\mathrm{GeV})$ of

$$\langle x\rangle_g^{\overline{MS}}(\mu = 2\,\mathrm{GeV}) = 0.43(7)(5)\,, \tag{30}$$

as our final result, where the first error is in the determination of $\langle x\rangle_g^{lat}$ and the second is due to the renomalisation procedure. This is a significant improvement of our previous estimate 0.53(23) based on generating $O(5000)$ configurations, [5] (with error given just for $\langle x\rangle_g^{lat}$).

Direct measurements of gluonic expectation values are notoriously plagued by noise problems, because the gluons are bosonic fields. We have seen here that a cheaper alternative, modifying the gluon action and using the Feynman-Hellmann theorem to find expectation values from mass measurements, works well. Here we have performed a test calculation in the quenched case. The method is a generalisation of that used to determine the sigma term (see e.g. [14] and references therein), β-function, e.g. [15], or singlet terms, e.g. [16]. It is clearly interesting to repeat this with dynamical fermions.

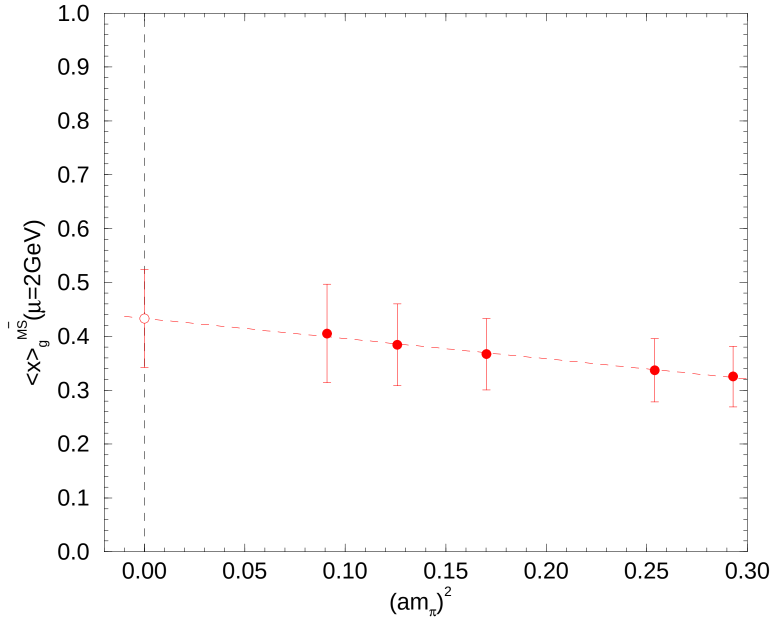

Figure 3: $\langle x\rangle_g^{\overline{MS}}(\mu = 2\,\mathrm{GeV})$ versus $(am_\pi)^2$ for the five κ values, together with a linear chiral extrapolation.

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