DESY-09-118

DESY–09–118

# Regge limit of $R$-current correlators in $AdS$ Supergravity

J. Bartels[1], J. Kotanski[1], A.-M. Mischler[1], V. Schomerus[2]

[1] *II. Institute Theoretical Physics, Hamburg University, Germany*
[2] *DESY Theory Group, Hamburg, Germany*

August 17, 2009

**Abstract**

Four-point functions of $R$-currents are discussed within Anti-de Sitter supergravity. In particular, we compute Witten diagrams with graviton and gauge boson exchange in the high energy Regge limit. Assuming validity of the AdS/CFT correspondence, our results apply to $R$-current four-point functions of $\mathcal{N} = 4$ super Yang-Mills theory at strong coupling.



## 1 Introduction

Studies of the Regge limit for scattering amplitudes go back to the 1960ies when experiments started to explore the high energy regime of quantum field theories. In Quantum Chromodynamics (QCD), the weak coupling limit of the Regge limit turned out to be dominated by the BFKL Pomeron [1, 2, 3] which represents a bound state of two reggeized gluons. More general, high energy scattering amplitudes in QCD can be written in terms of reggeon field theory with reggeized gluons [4] as the fundamental degrees of freedom. The BFKL Pomeron is an intriguing starting point for analyzing both the NLO corrections [5, 6, 7] to the BFKL kernel and for generalizing the BFKL equation to the more complex BKP states in the $t$ channel [8, 9, 10]. In the context of large-$N_c$ limits, the BFKL Pomeron represents the leading approximation of the elastic scattering amplitude (color singlet exchange). The BKP states have been found to be integrable for large $N_c$ [11, 12, 13], and these links of high energy QCD with integrable models have raised hopes for at least partial solutions.

Investigations of the planar Regge limit were mostly performed perturbatively, i.e. for small 't Hooft coupling $\lambda$. Up until 1997, strongly coupled gauge theory has remained largely inaccessible, at least with analytical tools. The situation has changed through the discovery of the AdS/CFT correspondence [14, 15, 16]. It relates many interesting superconformal gauge theories to string theories in Anti-de Sitter backgrounds. The simplest example of such a correspondence involves $\mathcal{N} = 4$ supersymmetric Yang-Mills theory in four space-time dimensions. This theory is an attractive toy model. While being severely constrained by its symmetries, the leading Regge asymptotics is identical to that of QCD. The dual description is given by type IIB string theory on $AdS_5 \times S^5$. Even though the latter is very difficult to solve completely, calculations may be performed for large radius $R$ of $AdS_5$ using the approximate description through classical supergravity. According to the AdS/CFT correspondence, the supergravity limit of string theory is dual to gauge theory at strong coupling $\lambda \to \infty$.

In Quantum Chromodynamics, the scattering of electromagnetic currents provides a reliable environment for studying the Pomeron [17, 18]: at large virtuality of the external photons, the QCD coupling constant is small, and the use of perturbation theory is justified. $\mathcal{N} = 4$ super Yang-Mills theory contains close relatives of electromagnetic current [19], namely the $R$-currents which belong to the global $SU_R(4)$ group. Therefore, is seems natural to further explore the Pomeron, within the AdS/CFT correspondence, by investigating four-point correlators of $R$-currents in $\mathcal{N} = 4$ super

Yang-Mills theory. In the weakly coupled regime, the relevant correlation functions have been investigated [20]. Similar to the QCD case, in the high energy limit the scattering amplitude has the form of a convolution of the two $R$-current impact factors

$$\mathcal{A}_{P_A P_B}(s,t) = is \int \frac{d^2k}{(2\pi)^3} \Phi_{P_A}(Q_A^2; \boldsymbol{k}, \boldsymbol{q}-\boldsymbol{k}) \frac{1}{\boldsymbol{k}^2(\boldsymbol{q}-\boldsymbol{k})^2} \Phi_{P_B}(Q_B^2; \boldsymbol{k}, \boldsymbol{q}-\boldsymbol{k}), \tag{1.1}$$

where $\boldsymbol{k}$, $\boldsymbol{q}$ are two-dimensional transverse momentum vectors with $t = -\boldsymbol{q}^2$, $Q_A^2$ and $Q_B^2$ are the virtualities of the two incoming $R$-currents (for simplicity we take the virtualities of the outgoing currents to be identical to the incoming ones). Helicity is conserved, and $P_A$, $P_B$ denote the polarizations of the two incoming external currents (transverse or longitudinal). The impact factors $\Phi_A$, $\Phi_B$ for the $R$-currents in $\mathcal{N} = 4$ have been calculated explicitly [20]. When including higher order corrections in the leading logarithmic approximation the two gluon propagators are replaced by the BFKL Green's function

$$\begin{aligned} \mathcal{A}_{P_A P_B}(s,t) &= is \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} s^{\omega} \int \frac{d^2k}{(2\pi)^3} \frac{d^2k'}{(2\pi)^3} \\ &\quad \times \Phi_{P_A}(Q_A^2; \boldsymbol{k}, \boldsymbol{q}-\boldsymbol{k}) G(\boldsymbol{k}, \boldsymbol{q}-\boldsymbol{k}; \boldsymbol{k}', \boldsymbol{q}-\boldsymbol{k}'; \omega) \Phi_{P_B}(Q_B^2; \boldsymbol{k}', \boldsymbol{q}-\boldsymbol{k}'), \end{aligned} \tag{1.2}$$

and the leading singularity in the $\omega$ plane is located at $\omega_0 = 4N_c\alpha_s \ln 2/\pi$. It may be worthwhile to recall that the high energy behavior (1.2) also allows to address the short distance behavior of the operator product of the $R$-currents: by considering the limit $Q_A^2 \gg Q_B^2$ (like in deep inelastic electron proton scattering), the BFKL Green's function can be used to derive, in particular, the anomalous dimension of the leading twist two gluon operator expanded around the point $\omega = 0$.

As we recalled in the previous paragraph, the limit of strong coupling is determined by classical supergravity. Techniques for the relevant supergravity computations were developed starting from [15]. They have been applied to calculate many two- and three-point correlation functions [21, 22, 23, 24] as well as four-point correlators [25, 26, 27, 28, 29, 30, 31, 32, 33, 34]. Just as in the case of weak coupling, most of the supergravity amplitudes possess unpleasant divergencies which must be renormalized [35, 36, 37, 38, 39, 40, 41, 42, 43]. In evaluating the Regge limit of four-point amplitudes we actually meet a pleasant surprise. It turns out that UV-divergent terms do not contribute to the high energy Regge limit of scattering amplitudes. The main reason is that, in the Regge limit, the coordinates of the $R$-currents on the boundaries are well separated from each other, thus avoiding the ultraviolet divergencies. Therefore, none of the calculations performed below requires holographic renormalization.

In this paper we calculate the high energy limit for the four-point correlation function of $R$-currents at strong coupling, restricting ourselves to the leading term, i.e. in the supergravity regime. We evaluate contributions from graviton and gauge boson exchange in the bulk and show that the leading Regge asymptotics is determined entirely by the t-channel exchange of bulk gravitons. All other amplitudes, including those from s- and u-channel exchange of gravitons and of gauge bosons, are suppressed by at least one power of the energy. Let us mention that there exist a variety of other approaches to high energy scattering at strong coupling [44, 45, 46, 47, 48], [49, 50, 51] and [52, 53, 54, 55]. Our work has a somewhat different take in that is focuses on $R$-currents. Furthermore, the results are obtained by thoroughly analyzing the underlying Witten diagrams, without any shortcuts or additional assumptions.

Let us briefly describe the content of the following sections. We shall begin with a more expository part in which some of the relevant background material is presented. In order to define the high energy limit in Section 2.1 all relevant propagators must be transformed to momentum space. Section 3 is devoted to a rather detailed calculation of the graviton exchange. The corresponding scattering amplitude is proportional to the square of the total energy, $s^2$ [56]. The properties of the amplitude are investigated in some detail. In particular, after performing an inverse Fourier transform on the transverse momenta of the process, we find a simple closed expression in configuration space. The resulting expression for the Regge limit of the graviton exchange can be written rather compactly in terms of a scalar propagator of $AdS_3$. Moreover, we determine the expansion coefficients for the amplitude written as a series in the exchanged momentum. We also investigate the dependence of the forward scattering amplitude on the virtualities of the process. In Section 4 we calculate the Regge limit of gauge boson exchange for general values of the exchanged momentum. As expected for the exchange of a vector boson, the amplitude is proportional to the total energy $s$.

# 2 Definitions and ingredients from supergravity

In this second section we shall formulate our main task we address and we provide the basic ingredients that are required for its completion. After a short review on the calculation of $R$-current correlators from supergravity, we list all the necessary building blocks. These include the bulk-to-bulk propagators for gravitons and gauge bosons in Anti-de Sitter space.

## 2.1 Formulation of the Problem

We consider $\mathcal{N}=4$ super Yang-Mills (SYM) theory in four dimensional Euclidean space. Let us pick one of its $R$-currents by $J_j$ with $j$ labeling the spacial directions, i.e. $j=1,\ldots,d=4$. $\vec{x}=(x_1,x_2,x_3,x_4)$ denotes the four dimensional Euclidean vector. We are interested in evaluating the Fourier transform of the four-point correlator,

$$i(2\pi)^4\delta(\sum_i \vec{p}_i)A_{j_1j_2j_3j_4}(\vec{p}_i)=\int\left(\prod_{i=1}^{4}d^4x_i\,\mathrm{e}^{-i\vec{p}_i\cdot\vec{x}_i}\right)\langle J_{j_1}(\vec{x}_1)J_{j_2}(\vec{x}_2)J_{j_3}(\vec{x}_3)J_{j_4}(\vec{x}_4)\rangle\,. \tag{2.1}$$

Due to the conservation of the $R$-current, i.e. $\partial_j J^j=0$, the contraction of the quantity $A$ with one of the four external momenta vanishes trivially. We can solve these Ward identities explicitly by projecting the scattering amplitudes,

$$\mathcal{A}_{\lambda_1\lambda_2;\lambda_3\lambda_4}(|\vec{p}_i|;s,t)=\sum_{j_i}\epsilon^{(\lambda_1)}_{j_1}(\vec{p}_1)\epsilon^{(\lambda_2)}_{j_2}(\vec{p}_2)\epsilon^{(\lambda_3)}_{j_3}(\vec{p}_3)^*\epsilon^{(\lambda_4)}_{j_4}(\vec{p}_4)^*A_{j_1j_2j_3j_4}(\vec{p}_i)\,,\quad \lambda_i=L,\pm\,, \tag{2.2}$$

with appropriate polarization vectors $\epsilon^{\lambda_i}_j(\vec{p}_i)$ satisfying $p^j_i\epsilon^{(\lambda_i)}_j(\vec{p}_i)=0$ along with an orthonormality condition. A set of polarization vectors with the required properties is spelled out in appendix A (eq. (A.2)). For any given choice of polarizations, the resulting scattering amplitude can only depend on the two Mandelstam variables $s$ and $t$.

The perturbative computation of the full scattering amplitude in gauge theory and supergravity is possible, though a rather tedious exercise. In this study we shall only be interested in the Regge limit of the scattering amplitude $\mathcal{A}(s,t)$ of the process $1+2\to 3+4$, i.e. in the limit where the total energy is much larger than the momentum transfer and the virtualities of the external currents. In Euclidean notation we have

$$s\;=\;-(\vec{p}_1+\vec{p}_2)^2\,,-t\;=\;(\vec{p}_1+\vec{p}_3)^2\,, \tag{2.3}$$

and $|\vec{p}_1|$, $|\vec{p}_2|$ ($|\vec{p}_3|$ and $|\vec{p}_4|$) are the virtualities of the incoming (outgoing) currents. In order to take the limit we are interested in, namely

$$|\vec{p}_i|^2\,,-t\ll s\,, \tag{2.4}$$

we have to go, via Wick rotation, to the Minkowski space.

Our aim here is to calculate the amplitude (2.1) in the limit of infinite 't Hooft coupling (the weak coupling limit has been addressed in [20]). To this end we make use of the conjectured AdS/CFT correspondence [14] between IIB string theory on $AdS_{d+1}$ space and $\mathcal{N}=4$ $SU(N_c)$ super Yang-Mills theory. An efficient calculation can only be performed in the limit of large $N_c$ (planar limit). At the same time we send the 't Hooft coupling $\lambda=g^2_{YM}N_c$ to infinity. In this regime, the full string theory on $AdS_{d+1}$ is well approximated by classical supergravity.

The AdS/CFT correspondence comes with a prescription to compute correlation functions in the $d$-dimensional quantum field theory [15, 16]. To be more precise, sources $\phi_0$ of operators in super Yang-Mills theory correspond to the boundary values of supergravity fields in $AdS_{d+1}$, i.e $\phi|_{\partial AdS} \sim \phi_0$. For an n-point function we have

$$\langle J(1)J(2)\ldots J(n)\rangle_{CFT} = \omega_n \frac{\delta^n}{\delta\phi_0(1)\ldots\delta\phi_0(n)} \exp(-S_{AdS}[\phi[\phi_0]])\big|_{\phi_0=0}, \tag{2.5}$$

where the factor $\omega_n$ comes from the relative normalization of sources to $\phi_0$ values and the normalization of the action [23]. On the right hand side, $S_{AdS}$ denotes a classical supergravity action that is evaluated with fixed boundary values of $\phi$.

Before we can spell out the supergravity theory we consider, let us briefly fix some conventions concerning the Anti-de Sitter space $AdS_{d+1}$. Its Euclidean continuation is parameterized by $z_0 > 0$ and $\vec{x}$ with coordinates $x_i$ enumerated by the Latin indices $i = 1, \ldots, d$. The metric is given by

$$ds^2 = \frac{1}{z_0^2}(dz_0^2 + d\vec{x}^2), \tag{2.6}$$

where $d\vec{x}^2$ can be related to the metric of Minkowski space by Wick rotation. The boundary of the Anti-de Sitter space is at $z_0 = 0$. Our computations will be performed for $d = 4$, the case that is relevant for QCD.

Our supergravity calculations can be truncated consistently to a theory involving fluctuations of the metric on $AdS_{d+1}$ along with fluctuations of an $U(1)_R$ gauge field $A_\mu$. The latter is related to the gauge theory $R$-currents through the AdS/CFT correspondence. The relevant supergravity action reads

$$S = \frac{1}{2\kappa^2}\int d^{d+1}z\sqrt{g}(-\mathcal{R} + \Lambda) + S_m, \tag{2.7}$$

with $\mathcal{R}$ being the scalar curvature and where the covariant matter action is [21, 23, 57, 33]

$$S_m = \frac{1}{2\kappa^2}\int d^{d+1}z\sqrt{g}\left[\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{ik}{24\sqrt{g}}\varepsilon_{\mu\nu\rho\sigma\lambda}F_{\mu\nu}F_{\rho\sigma}A_\lambda - A_\mu J^\mu + \ldots\right]. \tag{2.8}$$

Here $2\kappa^2 = 15\pi^2 R^3/N_c^2$, $R$ denotes the radius of $AdS_5$, and $F_{\mu\nu}$ is the field strength of the gauge field $A$, as usual. Throughout this note, Greek indices refer to the $(d+1)$-dimensional space, i.e. they take values from 0 to $d$. Latin subscripts, on the other hand, parameterize directions along the Euclidean $d$-dimensional boundary of $AdS_{d+1}$. Repeated indices are always summed over after they have been lowered. To lower indices we use the $d+1$-dimensional metric. The coefficient $k$ of the Chern-Simons is an integer.

Evaluation of the four-point correlation function of $R$-currents using eq. (2.5) along with eq. (2.7) is, in principle, rather straightforward. In practice, we can use a very convenient and intuitive diagrammatic procedure that was first proposed by Witten [15] and then developed further by many other authors. In our case, the computation of the relevant Witten diagrams requires only three basic building blocks. These include the bulk-to-bulk propagators for the graviton and the gauge $R$-bosons as well as the bulk-to-boundary $R$-boson propagator. They are connected by vertices which can be inferred form eqs. (2.7) and (2.8). The diagrams that shall be analyzed below are plotted in Fig. 1. We are only interested in their leading Regge behavior. Since the regime (2.4) is characterized through momenta, it is necessary to transform the various propagators to momentum space. In order to make our presentations reasonably self-contained, we shall list the Fourier transform of the basic building blocks in the following three subsections.

## 2.2 Bulk-to-bulk propagator of the gauge boson

Let us begin by discussing the Fourier transform of the bulk-to-bulk gauge boson propagator. According to [58], its coordinate space representation is given by

$$G_{\mu\nu'}(z,w) = -(\partial_\mu\partial_{\nu'}u)F(u) + \partial_\mu\partial_{\nu'}S(u), \tag{2.9}$$

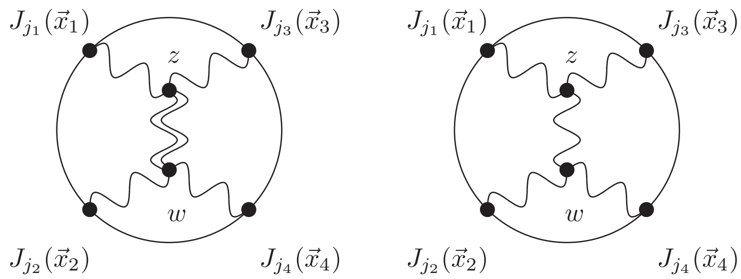


Figure 1: Witten diagrams for the graviton and boson exchange in the $t-$channel, respectively

where $\partial_\mu$ is the derivative with respect to $z_\mu$ and $\partial_{\mu'}$ denotes derivatives with respect to the components $w_{\mu'}$. The propagation of the physical components are described by the massive scalar propagator $F$ with mass parameter $m^2 = -(d-1)$,

$$F(u) = \frac{\Gamma\left(\frac{d-1}{2}\right)}{(4\pi)^{(d+1)/2}}[u(2+u)]^{-(d-1)/2} = \frac{\Gamma\left(\frac{d-1}{2}\right)}{(4\pi)^{(d+1)/2}}\xi^{d-1}\sum_{k=0}^{\infty}\frac{\Gamma(k+\frac{d-1}{2})}{\Gamma(\frac{d-1}{2})\Gamma(k+1)}\xi^{2k}\,. \tag{2.10}$$

The function $S(u)$, on the other hand, is a gauge artifact. We shall not need its form below. The so-called chordal distance variable $u$, finally, is defined by

$$u \equiv \frac{(z-w)^2}{2z_0w_0}\,, \qquad \xi = \frac{1}{1+u} = \frac{2w_0z_0}{z_0^2+w_0^2+(\vec{z}-\vec{w})^2}\,, \tag{2.11}$$

where $(z-w)^2 = \delta_{\mu\nu}(z-w)_\mu(z-w)_\nu$ is the so-called flat Euclidean distance. In spelling out the Fourier transform, we distinguish two different cases according to whether the second index $\nu'$ is parallel or transverse to the boundary of $AdS_{d+1}$. In the first case where $\nu' = j$, the result of the Fourier transform reads

$$\begin{aligned} G_{\mu j}(z_0,w_0,\vec{q}) &= \int d^4x\, \mathrm{e}^{i\vec{q}\cdot\vec{x}}\, G_{\mu j}(z_0,w_0,\vec{x}) = q_\mu q_j \tilde{S} + \sum_{k=0}^{\infty}\frac{2^{1-d}(z_0w_0)^{d/2-1}}{\Gamma(k+d/2)\Gamma(k+1)} \\ \times &\left[\delta_{\mu j}\left(\frac{z_0^2w_0^2\vec{q}^2}{4\varpi_0^2}\right)^{k+d/4-1/2} K_{2k+d/2-1}(|\vec{q}|\varpi_0) - \delta_{\mu 0}\tfrac{i}{2}q_j w_0\left(\frac{z_0^2w_0^2\vec{q}^2}{4\varpi_0^2}\right)^{k+d/4-1} K_{2k+d/2-2}(|\vec{q}|\varpi_0)\right], \end{aligned}$$

where $\vec{x} = \vec{z}-\vec{w}$ and $\tilde{S}$ is the Fourier transform of $S$. Moreover, we introduced the function $\varpi_0 = \varpi_0(z_0,w_0) = \sqrt{w_0^2+z_0^2}$. Here and below, $K_m(x)$ denotes the modified Bessel function.

In the second case, when $\nu' = 0$, the Fourier transform of the gauge boson's bulk-to-bulk propagator reads,

$$\begin{aligned} G_{\mu 0}(z_0,w_0,\vec{q}) &= \int d^4x\, \mathrm{e}^{i\vec{q}\cdot\vec{x}}\, G_{\mu 0}(z_0,w_0,\vec{x}) = q_\mu q_0 \tilde{S} + \sum_{k=0}^{\infty}\frac{2^{1-d}(z_0w_0)^{d/2-1}}{\Gamma(k+d/2)\Gamma(k+1)} \\ \times &\left[\delta_{\mu 0}\frac{\varpi_0^2}{z_0w_0}\left(\frac{z_0^2w_0^2\vec{q}^2}{4\varpi_0^2}\right)^{k+d/4-1/2} K_{2k+d/2-1}(|\vec{q}|\varpi_0) + \delta_{\mu j}\tfrac{i}{2}q_j z_0\left(\frac{z_0^2w_0^2\vec{q}^2}{4\varpi_0^2}\right)^{k+d/4-1} K_{2k+d/2-2}(|\vec{q}|\varpi_0)\right. \\ &\left. -\delta_{\mu 0}(k+\tfrac{d}{2}-1)\left(\frac{z_0^2w_0^2\vec{q}^2}{4\varpi_0^2}\right)^{k+d/4-1} K_{2k+d/2-2}(|\vec{q}|\varpi_0)\right]. \end{aligned}$$

The symbol $q_0$ in the first term on the right hand side denotes the derivative $q_0 \equiv i\partial_0$ with respect to $z_0$.

## 2.3 Bulk-to-boundary propagator of the gauge boson

Following [15], let us now consider the bulk-to-boundary propagator of the gauge boson in Lorentz-like gauge. It is given by the simple expression

$$G_{\mu j}(z,\vec{x}) \;=\; N_d \delta_{\mu j} \frac{z_0^{d-2}}{((\vec{z}-\vec{x})^2+z_0^2)^{d-1}} - N_d \delta_{\mu 0} \frac{1}{2(d-2)} \frac{\partial}{\partial \vec{x}_j} \frac{z_0^{d-3}}{((\vec{z}-\vec{x})^2+z_0^2)^{d-2}}\,, \tag{2.12}$$

where the normalization

$$N_d \;=\; \frac{(d-2)\Gamma(d)}{2\pi^{d/2}(d-1)\Gamma(d/2)}$$

is chosen such that the bulk-to-boundary propagator $G_{jl}(z,\vec{x}) \to \delta_{jl}\delta^{(d)}(\vec{z}-\vec{x})$ in the limit where $z_0$ is sent to $z_0 = 0$. The formula we state here is valid for dimensions $d > 2$.

Performing a Fourier transform in the spacial coordinate $\vec{x}$ on the boundary, we obtain

$$\begin{aligned} G_{\mu j}(z_0,\vec{p}) \;&=\; \int dz\, \mathrm{e}^{i\vec{p}\cdot(\vec{z}-\vec{x})}\, G_{\mu j}(z,\vec{x}) \;=\; N_d \delta_{\mu j} z_0^{d-2} \frac{2\pi^{d/2}}{\Gamma(d-1)} \left(\frac{|\vec{p}|}{2z_0}\right)^{d/2-1} K_{d/2-1}(z_0|\vec{p}|) \\ &\quad - N_d \delta_{\mu 0} i p_j z_0^{d-3} \frac{\pi^{d/2}}{\Gamma(d-1)} \left(\frac{|\vec{p}|}{2z_0}\right)^{d/2-2} K_{d/2-2}(z_0|\vec{p}|)\,. \end{aligned} \tag{2.13}$$

This expression simplifies considerably when we set $d = 4$. In this case, the normalization $N_4$ takes the form $N_4 = 2/\pi^2$ and therefore

$$G_{\mu j}(z_0,\vec{p}) \;=\; z_0 \left[\delta_{\mu j} |\vec{p}| K_1(z_0|\vec{p}|) - i p_j \delta_{\mu 0} K_0(z_0|\vec{p}|)\right]\,. \tag{2.14}$$

## 2.4 Bulk-to-bulk propagator of the graviton

Finally, we turn our attention to the bulk-to-bulk propagator of the graviton. According to [58] this quantity reads as

$$G_{\mu\nu;\mu'\nu'} \;=\; (\partial_\mu \partial_{\mu'} u\, \partial_\nu \partial_{\nu'} u + \partial_\mu \partial_{\nu'} u\, \partial_\nu \partial_{\mu'} u)\, G(u) + g_{\mu\nu} g_{\mu'\nu'}\, H(u) + \ldots \tag{2.15}$$

Here the dots $\ldots$ stand for terms of the form $\partial_\rho X$ where $\rho = \mu, \mu', \nu, \nu'$. These turn out not to contribute to our computation below. Furthermore, $G$ is the massless scalar propagator

$$G(u) \;=\; 2^d \xi^d C_d\, {}_2F_1(d/2,(d+1)/2;d/2+1;\xi^2) = \pi^{-\frac{d+1}{2}} \xi^d \sum_{k=0}^{\infty} \frac{\Gamma\left(\frac{d+1}{2}+k\right)}{2(d+2k)\Gamma(k+1)} \xi^{2k}\,, \tag{2.16}$$

where

$$C_d \;=\; \frac{\Gamma\left(\frac{d+1}{2}\right)}{(4\pi)^{(d+1)/2} d}.$$

The function $H$, finally, is given by the following two explicit formulas

$$\begin{aligned} H(u) \;&=\; -\frac{2G(u)}{(d-1)\xi^2} + \frac{2(d-2)}{\xi(d-1)^2} 2C_d (2\xi)^{d-1}\, {}_2F_1((d-1)/2, d/2; d/2+1; \xi^2) \\ &=\; -\frac{2}{(d-1)} \frac{\xi^{d-2}}{\pi^{\frac{d+1}{2}}} \sum_{k=0}^{\infty} \frac{\Gamma\left(\frac{d+1}{2}+k\right)}{2(d+2k)\Gamma(k+1)} \xi^{2k} + \frac{(d-2)}{(d-1)} \frac{\xi^{d-3}}{\pi^{\frac{d+1}{2}}} \sum_{k=0}^{\infty} \frac{\Gamma\left(\frac{d-1}{2}+k\right)}{2(d+2k)\Gamma(k+1)} \xi^{2k}\,. \end{aligned} \tag{2.17}$$

Before studying the Fourier transform, we evaluate the quantity

$$\partial_\mu \partial_{\mu'} u \;=\; \frac{1}{z_0 w_0} \delta_{\mu\mu'} + \frac{(z-w)_\mu}{z_0 w_0^2} \delta_{\mu' 0} + \frac{(w-z)_{\mu'}}{z_0^2 w_0} \delta_{\mu 0} - \frac{(z-w)^2}{2 z_0^2 w_0^2} \delta_{\mu 0} \delta_{\mu' 0}. \tag{2.18}$$

Let us anticipate that only the first of these terms does actually contribute to the high energy behavior. With this is in mind we focus on the relevant part by defining

$$G^{(1)}_{\mu\nu;\mu'\nu'}(z_0,w_0,\vec{x}) \equiv \left( \frac{1}{(z_0 w_0)^2} \delta_{\mu\mu'} \delta_{\nu\nu'} + \frac{1}{(z_0 w_0)^2} \delta_{\mu\nu'} \delta_{\nu\mu'} \right) G(u). \tag{2.19}$$

Its Fourier transform is easily computed and reads

$$G^{(1)}_{\mu\nu;\mu'\nu'}(z_0,w_0,\vec{q}) = \int d^4x\, \mathrm{e}^{i\vec{q}\cdot\vec{x}}\, G^{(1)}_{\mu\nu;\mu'\nu'}(z_0,w_0,\vec{x}) = (\delta_{\mu\mu'}\delta_{\nu\nu'}+\delta_{\mu\nu'}\delta_{\nu\mu'}) \times \sum_{k=0}^{\infty}\frac{(z_0w_0)^{2k+d-2}}{\Gamma(k+d/2+1)\Gamma(k+1)}\left(\frac{|\vec{q}|^2}{4\varpi_0^2}\right)^{k+d/4} K_{2k+d/2}(\varpi_0|\vec{q}|)\,, \quad (2.20)$$

where $\vec{x}=\vec{z}-\vec{w}$, as before. Subleading terms can be obtained in a similar way, but they will not be needed in the present context. We finally mention that, for large $|q|$, the Fourier transform of the function $G(u)$ goes as $1/|q|^2$ [48].

# 3 Graviton exchange in the high energy limit

The aim of this section is to compute and analyze high energy limit of the scattering amplitude $\mathcal{A}(s,t)$ introduced in the previous section. Since we are interested in the $AdS_5$ case, we shall set $d=4$ from now on. We switch to Minkowski metric $g=\mathrm{diag}(+,-,-,-)$. However, for simplicity we continue using our previous notation: i.e. $\vec{p}_j$ now stands for the four vector $(p_{j;4},p_{j;1},p_{j;2},p_{j;3})$ with $|\vec{p}_j|^2=-p^2_{j;4}+p^2_{j;1}+p^2_{j;2}+p^2_{j;3}=-p_j^2$, Latin indices continue to run from 1 to 4 (where the fourth component denotes Minkowski 'time'), and Greek indices run between 0 and 4.

The first subsection contains the main results on the high energy limit. In the second subsection we make an attempt to re-interpret the result as coming from a correlation function in $AdS_3$. Finally, we investigate in some detail the properties of the forward scattering at infinite 't Hooft coupling.

## 3.1 The graviton exchange

Let us begin by computing the contribution to the four-point function of $R$-currents that is obtained from the exchange of a single graviton in the t-channel (left figure in 1). According to the rules of the AdS/CFT correspondence, this quantity is given by[1]

$$I^{\mathrm{GR}} = \frac{1}{4}\int\frac{d^4z dz_0}{z_0}\int\frac{d^4w dw_0}{w_0}T_{(13)\mu\nu}(z)G_{\mu\nu;\mu'\nu'}(z,w)T_{(24)\mu'\nu'}(w)\,. \quad (3.1)$$

We shall refer to $T_{(ij)}$ as the stress-energy tensor. It is determined through the bulk-to-boundary propagators of the gauge boson and it contains the coupling between gauge bosons and gravitons.

$$T_{(13)\mu\nu} = z_0^2\partial_{[\mu}G_{\lambda]j_1}(z,\vec{x}_1)\partial_{[\nu}G_{\lambda]j_3}(z,\vec{x}_3)+z_0^2\partial_{[\nu}G_{\lambda]j_1}(z,\vec{x}_1)\partial_{[\mu}G_{\lambda]j_3}(z,\vec{x}_3) -\frac{1}{2}z_0^2\delta_{\mu\nu}\partial_{[\alpha}G_{\beta]j_1}(z,\vec{x}_1)\partial_{[\alpha}G_{\beta]j_3}(z,\vec{x}_3)\;. \quad (3.2)$$

Note that $T$ satisfies the four-dimensional Ward identity by construction. We shall see below that only the first two terms of $T_{(13)\mu\nu}$ contribute to the high energy behavior of the amplitude. Performing the Fourier transform of the expression (3.1) we obtain

$$\tilde{I}^{\mathrm{GR}}(\vec{p}_i) = \int\prod_i d^4x_i\,\mathrm{e}^{-i\sum_j\vec{p}_j\cdot\vec{x}_j}\,I^{\mathrm{GR}}(\vec{x}_i) = (2\pi)^4\,\delta^{(4)}(\sum_i\vec{p}_i) \quad (3.3)$$

$$\times\frac{1}{4}\int\frac{dz_0}{z_0}\int\frac{dw_0}{w_0}\tilde{T}_{(13)\mu\nu}(z_0,\vec{p}_1,\vec{p}_3)\tilde{T}_{(24)\mu'\nu'}(w_0,\vec{p}_2,\vec{p}_4)G_{\mu\nu;\mu'\nu'}(z_0,w_0;\vec{p}_1+\vec{p}_3)\,, \quad (3.4)$$

where

$$T_{(13)\mu\nu}=\frac{1}{(2\pi)^8}\int d^4p_1d^4p_3\,\mathrm{e}^{i\vec{p}_1\cdot(\vec{x}_1-\vec{z})}\,\mathrm{e}^{i\vec{p}_3\cdot(\vec{x}_3-\vec{z})}\,\tilde{T}_{(13)\mu\nu}\,. \quad (3.5)$$

[1]The correlation functions and amplitudes are calculated up to multiplicative constants, which can be easily restored from the action (2.7).

Before we analyze which terms give the leading contributions to the high energy behavior, we recall that we still need to contract our amplitude with the appropriate polarization vectors

$$\tilde{\mathcal{I}}^{\rm GR}_{\lambda_1,\lambda_2\lambda_3,\lambda_4} \;=\; \sum\nolimits_{j_i} \epsilon^{(\lambda_1)}_{j_1}(\vec{p}_1)\epsilon^{(\lambda_2)}_{j_2}(\vec{p}_2)\epsilon^{(\lambda_3)}_{j_3}(\vec{p}_3)^*\epsilon^{(\lambda_4)}_{j_4}(\vec{p}_4)^*(\tilde{I}^{\rm GR})_{j_1j_2j_3j_4}\,. \tag{3.6}$$

Since the amplitude (3.4) satisfies the Ward identities associated with the conservation of $R$-currents, we are allowed to shift the polarization vectors (A.2) by the momenta of the corresponding particles. If we allow for this additional freedom, the polarization vectors may be brought into the form displayed in eq. (A.4) of the Appendix. We note that contractions of these shifted polarization vectors with any tensor cannot give additional powers of the energy $s$. Hence, we can determine the dominant terms of the scattering amplitude before we actually switch to the polarization basis. After these remarks let us look back at the form of the stress-energy tensor $T$. Each of the three terms contains two derivatives which are replaced by momenta after Fourier transformation. These are combined with two more momentum components from the second stress-energy tensor. Therefore, we can at most obtain terms which are of the order $s^2$. But this requires that momentum components of $\vec{p}_1$ and $\vec{p}_3$ are contracted with momentum components $\vec{p}_2$ and $\vec{p}_4$. Terms in which $\vec{p}_1$ is contracted with $\vec{p}_3$, on the other hand, are clearly subleading. This implies that we can drop the term in the second line of eq. (3.2) and it explains why we had previously introduced the quantity $G^{(1)}$ in our discussion of the bulk-to-bulk propagator for the graviton. In fact, $G^{(1)}$ contains all terms of the propagator that can contribute to the leading high energy behavior. We can summarize the results of our discussion through the following two formulas

$$\tilde{G}^{(1)}_{ij;\,i'j'}(z_0,w_0,\vec{q}) \approx \frac{4}{s^2(z_0w_0)^2}\left(p_{2\,i}p_{1\,i'}p_{2\,j}p_{1\,j'} + p_{2\,i}p_{1\,j'}p_{2\,j}p_{1\,i'}\right)\tilde{G}\,, \tag{3.7}$$

$$\begin{aligned}\tilde{T}_{(13)\mu\nu}(z_0,\vec{p}_1,\vec{p}_3) \;&\approx\; z_0^4(\delta_{\mu k_1}\delta_{\nu k_3}+\delta_{\nu k_1}\delta_{\mu k_3})p_{1\,k_1}p_{3\,k_3}\;\times\\ &\times\;\left[p_{1\,j_1}p_{3\,j_3}K_0(z_0|\vec{p}_1|)K_0(z_0|\vec{p}_3|) - \delta_{j_1j_3}|\vec{p}_1||\vec{p}_3|K_1(z_0|\vec{p}_1|)K_1(z_0|\vec{p}_3|)\right]\end{aligned} \tag{3.8}$$

for the high energy limit of the graviton bulk-to-bulk propagator and the stress-energy tensor, respectively. Here and throughout the rest of the paper, $\approx$ means equality up to terms that are subleading in the high energy limit. Note that the graviton propagator has exactly the form that is expected in the Regge limit: for the exchange of a spin one gauge boson it is well-know that the leading high energy behavior comes from a particular $t$-channel helicity state. If $j$ ($j'$) denote the upper (lower) Lorentz indices of the $t$-channel exchange propagator and $p_1$ ($p_2$) the large momenta at the upper (lower) vertex, this dominant helicity state contributes through the tensor

$$\frac{2p_{2\,j}p_{1\,j'}}{s}. \tag{3.9}$$

In eq. (3.7) we see that the leading behavior of the graviton exchange can be interpreted as the (symmetrized) tensor product of two spin one bosons. Furthermore, as we will demonstrate below the first and the second term within the square bracket in the last line of eq. (3.8) correspond to longitudinal and transverse polarization of the $R$-boson, respectively.

If we now substitute the two expressions (3.8) and (2.20) back into the amplitude (3.4) and use that $(\vec{p}_1\cdot\vec{p}_2)(\vec{p}_3\cdot\vec{p}_4)\approx s^2/4$, we arrive at the high energy limit of the graviton exchange:

$$\tilde{I}^{\rm GR}_{\rm Regge} = (2\pi)^4\delta^{(4)}(\sum_i \vec{p}_i)\,\frac{s^2}{2}\int dz_0\int dw_0\,\Phi_{j_1j_3}(p_1,p_3;z_0)\Sigma(|\vec{p}_1+\vec{p}_3|,z_0,w_0)\Phi_{j_2j_4}(p_2,p_4;w_0) \tag{3.10}$$

where

$$\begin{aligned}\Sigma(|\vec{p}_1+\vec{p}_3|,z_0,w_0) \;&=\; \sum_{k=0}^{\infty}\frac{z_0^{2k+5}w_0^{2k+5}}{\Gamma(k+1)\Gamma(k+3)}\left(\frac{|\vec{p}_1+\vec{p}_3|^2}{4\varpi_0^2}\right)^{k+1}K_{2k+2}(|\vec{p}_1+\vec{p}_3|\varpi_0)\,, \qquad (3.11)\\ \Phi_{j_1j_3}(\vec{p}_1,\vec{p}_3;z_0) \;&=\; \sum_{m=0,1}\tilde{W}^m_{j_1j_3}(\vec{p}_1,\vec{p}_3)K_m(z_0|\vec{p}_1|)K_m(z_0|\vec{p}_3|)\,,\end{aligned}$$

while

$$\tilde{W}^{m_1}_{j_1 j_3}(\vec{p}_1, \vec{p}_3) = (\delta_{j_1 j_3} |\vec{p}_1||\vec{p}_3| \delta_{m_1,1} - p_{1\,j_1} p_{3\,j_3} \delta_{m_1,0}) \,. \tag{3.12}$$

This formula has reminiscent of eq. (1.1) where $\Phi_{j_a j_b}$ plays a role of an impact factor while $\Sigma(|\vec{p}_1 + \vec{p}_3|, z_0, w_0)$ is the analog of a propagator.

It is convenient to switch to the helicity basis: by contraction with the polarization vectors from eqs. (A.4) and (A.3) and making use of the orthonormality of the transverse polarizations we obtain

$$\begin{aligned} \tilde{\mathcal{W}}^{m_1}_{\lambda_1 \lambda_3}(\vec{p}_1, \vec{p}_3) &= \sum_{j_1, j_3} \epsilon^{(\lambda_1)}_{j_1}(\vec{p}_1) \epsilon^{(\lambda_3)}_{j_3}(\vec{p}_3)^* \tilde{W}^{m_1}_{j_1 j_3}(\vec{p}_1, \vec{p}_3) \\ &\approx |\vec{p}_1||\vec{p}_3| (\delta_{m_1,1} \delta_{\lambda_1,h} \delta_{\lambda_3,h} + \delta_{m_1,0} \delta_{\lambda_1,L} \delta_{\lambda_3,L}) \,, \end{aligned} \tag{3.13}$$

i.e. the first term with $m_1 = 1$ only contributes to the transverse polarizations $h = \pm$, whereas $m_1 = 0$ belongs to the longitudinal polarization. From eq. (3.13) we learn that helicity is conserved: $\lambda_1 = \lambda_3$. As we also see in eq. (3.13), the quantity $\tilde{W}_{\lambda_1, \lambda_3}$ in the helicity basis only depends upon the virtualities of the external currents. Consequently, also the "impact factor" in the helicity basis

$$\Phi_{\lambda_1 \lambda_3}(|\vec{p}_1|, |\vec{p}_3|; z_0) = \sum_{m=0,1} \tilde{\mathcal{W}}^{m}_{\lambda_1 \lambda_3}(\vec{p}_1, \vec{p}_3) K_m(z_0|\vec{p}_1|) K_m(z_0|\vec{p}_3|) \,, \tag{3.14}$$

only depends on the virtualities.

With these expressions our scattering amplitude finally reads:

$$\mathcal{A}^{\mathrm{GR}}_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}(s,t) = \frac{s^2}{2} \int dz_0 dw_0 \Phi_{\lambda_1 \lambda_3}(|\vec{p}_1|, |\vec{p}_3|; z_0) \, \Sigma(|\vec{p}_1 + \vec{p}_3|, z_0, w_0) \, \Phi_{\lambda_2 \lambda_4}(|\vec{p}_2|, |\vec{p}_4|; w_0) \,. \tag{3.15}$$

The amplitude (3.15) is proportional to $s^2$. To be complete, one also has to consider the exchange of the graviton in the $s$- and $u$- channels. In these cases the last term of the stress-energy tensor (3.2) is also important. Counting powers of momenta in the stress-energy tensor one might at first expect to get additional contributions of order $s^2$. But, unlike in the $t$-channel exchange that we have discussed at length, the $s$- and $u$-channel exchanges of the graviton are suppressed by an additional factor $s^{-1}$ that comes in through the graviton propagator itself. Therefore, there is no need to analyze such contributions to the amplitude any further.

Equation (3.15) should be compared with the weak coupling result (1.1). Again we have the structure of two impact factors $\Phi$ which depend upon the virtualities of the external currents, connected by an exchange propagator $\Sigma$ and convoluted by a two-dimensional integration. The power of $s$ reflects the spin of the exchanged graviton. On the gauge theory side, in the weak coupling limit, the amplitude is given by the exchange of two gluons, and higher order corrections in $g^2$ replace the two gluon exchange by the BFKL Green's function, modifying the power of $s$ from 1 to $1 + \omega_0$. On the string side it has been argued that, due to the reggeization of the graviton, the power behavior $s^2$ of the graviton exchange will be modified to $s^{2-\Delta}$ where $\Delta = \mathcal{O}(1/\sqrt{\lambda})$. However, in order to compute $\Delta$, one has to go beyond the supergravity approximation used in this paper.

## 3.2 Going back to configuration space

In the previous subsection we determined the leading Regge asymptotics for the Fourier transform of the four-point correlator of $R$-currents. The result was expressed in terms of the four momenta $\vec{p}_i$ and it involved an infinite summation in the construction of the kernel functions $\Sigma$. In principle, one might attempt to perform the inverse Fourier transform and to re-phrase our result as an expression for the four-point correlator of $R$-currents. But the answer turns out to be rather complicated.

We will therefore take a different route and perform an inverse Fourier transform exclusively in the transverse momenta. To make this more precise, we fix a particular frame: working in the Minkowski metric $g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$ we take the large momenta along the 3-axis. We introduce the light-like reference vectors $p_A = (p,0,0,p)$ and $p_B = (p,0,0,-p)$ with $s = 4p^2$, and the transverse momentum vectors $p_{i;\perp} = (0,p_{i;1},p_{i;2},0)$ $(i = 1,...,4)$, $q_\perp = (0,q_1,q_2,0)$ with $p^2_{i;\perp} = -\boldsymbol{p}_i^2$, $q^2 = -\boldsymbol{q}^2$. Throughout this subsection, we adopt the four vector notation $p_i = (p_{i;4},p_{i;1},p_{i;2},p_{i;3})$, and we take the momenta $p_3$ and $p_4$ to be outgoing. Momenta in bold face refer to the 2-dimensional transverse space. For large $s$ we find

$$\begin{aligned}
p_1 &= \left(1+\frac{Q_2^2+p_{2;\perp}^2-\frac{1}{2}(p_{1;\perp}+p_{2;\perp})^2}{s}\right)p_A - \frac{Q_1^2+p_{1;\perp}^2}{s}p_B + p_{1;\perp}\,,\\
p_2 &= -\frac{Q_2^2+p_{2;\perp}^2}{s}p_A + \left(1+\frac{Q_1^2+p_{1;\perp}^2-\frac{1}{2}(p_{1;\perp}+p_{2;\perp})^2}{s}\right)p_B + p_{2;\perp}\,,\\
p_3 &= \left(1+\frac{Q_4^2+p_{4;\perp}^2-\frac{1}{2}(p_{3;\perp}+p_{4;\perp})^2}{s}\right)p_A - \frac{Q_3^2+p_{3;\perp}^2}{s}p_B + p_{3;\perp}\,,\\
p_4 &= -\frac{Q_4^2+p_{4;\perp}^2}{s}p_A + \left(1+\frac{Q_3^2+p_{3;\perp}^2-\frac{1}{2}(p_{3;\perp}+p_{4;\perp})^2}{s}\right)p_B + p_{4;\perp}
\end{aligned}$$

with $p_{1;\perp} - p_{3;\perp} = p_{4;\perp} - p_{2;\perp}$. Here we introduced the virtualities $Q_i^2 = |\vec{p}_i|^2$, and for the momentum transfer $t$ we have

$$t = (p_{1;\perp}-p_{3;\perp})^2 + \frac{(Q_2^2-Q_4^2+p_{2;\perp}^2-p_{4;\perp}^2)(Q_3^2-Q_1^2+p_{3;\perp}^2-p_{1;\perp}^2)}{s} + \ldots \approx -\boldsymbol{q}^2\,. \tag{3.16}$$

We now compute the two-dimensional Fourier transform with respect to $q$:

$$\begin{aligned}
\mathcal{I}^{\rm GR}_{\rm T,Regge} &= \frac{1}{(2\pi)^8}\int\prod_i d^2p_i\prod_i e^{i(\boldsymbol{x}_1\cdot\boldsymbol{p}_1+\boldsymbol{x}_2\cdot\boldsymbol{p}_2-\boldsymbol{x}_3\cdot\boldsymbol{p}_3-\boldsymbol{x}_4\cdot\boldsymbol{p}_4)}\\
&\quad\times(2\pi)^2\delta^{(2)}(\boldsymbol{p}_1+\boldsymbol{p}_2-\boldsymbol{p}_3-\boldsymbol{p}_4)\,\mathcal{A}^{GR}_{\lambda_1\lambda_2\lambda_3\lambda_4}(s,t)\,.
\end{aligned}\tag{3.17}$$

Here, the two dimensional transverse vectors $\boldsymbol{x}_i$ define the positions in transverse space of the scattering $R$-currents. In particular, the difference $\boldsymbol{b} = \frac{1}{2}(\boldsymbol{x}_1+\boldsymbol{x}_3) - \frac{1}{2}(\boldsymbol{x}_2+\boldsymbol{x}_4)$ is the impact parameter. The subscript $T$ indicates that the Fourier transform has been performed in the transverse space only. In calculating the two-dimensional Fourier transform we recall that $s$ and $|\vec{p}_i|$ are kept fixed so that the integration over transverse momenta only effects the arguments of the delta function and the variable $t = -|\vec{p}_1+\vec{p}_3|^2$ in eq. (3.16). The result is

$$\mathcal{I}^{\rm GR}_{\rm T,Regge} \approx \delta^{(2)}(\boldsymbol{x}_1-\boldsymbol{x}_3)\delta^{(2)}(\boldsymbol{x}_2-\boldsymbol{x}_4) \tag{3.18}$$

$$\times\frac{s^2}{2}\int dz_0 z_0^2\int dw_0 w_0^2\Phi_{\lambda_1\lambda_3}(|\vec{p}_1|,|\vec{p}_3|;z_0)G_{\Delta=3,d=2}(\bar{u})\,\Phi_{\lambda_2\lambda_4}(|\vec{p}_2|,|\vec{p}_4|;w_0)\tag{3.19}$$

The variable $\bar{u} = (1-\bar{\xi})/\bar{\xi}$ is related to a 3-dimensional analogue of our variable $\xi$ through

$$\bar{\xi} = \frac{2z_0w_0}{z_0^2+w_0^2+\boldsymbol{b}^2}\,.$$

The function $G$ that appears in formula (3.18) is obtained as a special choice of the scalar Green's function in $AdS_{d+1}$ [59, 60, 61, 62]

$$G_{\Delta,d}(u) = 2^\Delta\frac{\Gamma(\Delta)\Gamma(\Delta-\frac{d}{2}+\frac{1}{2})}{(4\pi)^{(d+1)/2}\Gamma(2\Delta-d+1)}\xi^\Delta\;{}_2F_1(\tfrac{\Delta}{2},\tfrac{\Delta+1}{2};\Delta-\tfrac{d}{2}+1;\xi^2)\,. \tag{3.20}$$

Our parameters $\Delta = 3$ and $d = 2$ are associated with a scalar of mass $m^2 = \Delta(\Delta-d) = 3$ in $AdS_3$. Standard properties of hypergeometric functions allow to simplify the expression for this particular propagator to read

$$G_{\Delta=3,d=2}(\bar{u}) = \frac{2-\bar{\xi}^2-2\sqrt{1-\bar{\xi}^2}}{4\pi\bar{\xi}\sqrt{1-\bar{\xi}^2}}\,. \tag{3.21}$$

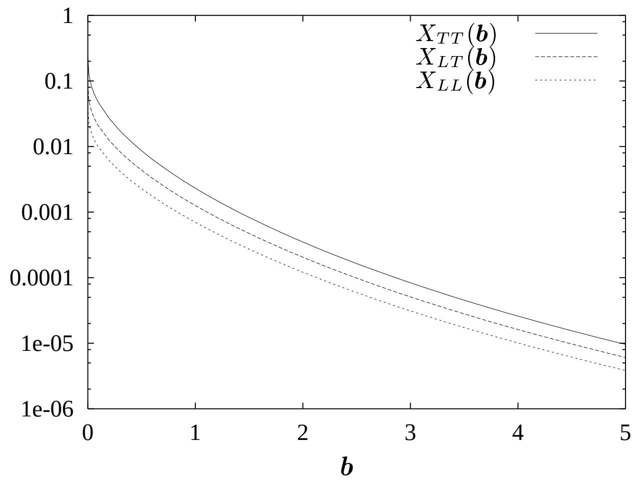


Figure 2: Integrals of the amplitude (3.18) plotted as a function of $\boldsymbol{b}$ and fixed $Q_i = 1$. Different lines correspond to different polarizations of $R$-bosons taken from the Bessel function subscripts, namely $1 \to T$ and $0 \to L$. For $\boldsymbol{b} \to \infty$ the amplitudes $X_{TT}$, $X_{TL} = X_{LT}$, $X_{TT}$ vanish as $\frac{32}{25\pi}\boldsymbol{b}^{-6}$, $\frac{64}{75\pi}\boldsymbol{b}^{-6}$, $\frac{128}{225\pi}\boldsymbol{b}^{-6}$, respectively.

Note that the dependence of the scattering amplitude on the 2-dimensional positions $\boldsymbol{x}_i$ is rather simple, in that the transverse positions of the incoming currents are conserved: $\boldsymbol{x}_1 = \boldsymbol{x}_3$ and $\boldsymbol{x}_2 = \boldsymbol{x}_4$. In our computation, the delta functions appear while taking the high energy limit. Our formula (3.18) can be used to study the b-dependence of the high energy scattering amplitude. For large impact parameter, we expand in inverse powers of $b^2$, and the leading terms has the exponent $\delta = -6$. In the regime in which $b^2$ is small, the scattering amplitude is proportional to $\log b^2$. Numerical curves of the amplitude (3.18) are plotted in Fig. 2.

### 3.3 Investigation of the forward amplitude

Before we spell out and study the formula for the amplitude at $t = 0$ it is useful to re-expand the function $\Sigma$ that was introduced in eq. (3.11) in the vicinity of vanishing $-t = |\vec{q}|^2 = |\vec{p}_1 + \vec{p}_3|^2$. This may be achieved with the help of eq. (B.2). The resulting series

$$\Sigma(|\vec{q}|, z_0, w_0) \quad = \quad \sum_{m=0}^{\infty} |\vec{q}|^{2m} U_m \,, \tag{3.22}$$

is often helpful to read off properties of the Regge limit (2.4). Its expansion coefficients $U_m$ are spelled out in Appendix C in terms of hypergeometric functions. Only the first summand $U_0$ is needed for the forward amplitude:

$$\begin{aligned} \mathcal{A}^{\mathrm{GR}}_{\lambda_1\lambda_2\lambda_3\lambda_4}(s, t=0) \quad &= \quad s^2 \int_0^\infty dz_0 z_0^3 \Phi_{\lambda_1\lambda_3}(|\vec{p}_1|, |\vec{p}_3|; z_0) \\ &\times \int_0^\infty dw_0 w_0^3 \Phi_{\lambda_2\lambda_4}(|\vec{p}_2|, |\vec{p}_4|; w_0) \tfrac{1}{2} G_{\Delta=2,d=0}(\hat{u}) \,, \end{aligned} \tag{3.23}$$

where $\hat{u} = (z_0 - w_0)^2/2z_0w_0$, and the propagator

$$G_{\Delta=2,d=0}(\hat{u}) = \frac{1}{4w_0^2 z_0^2}(\theta(w_0 - z_0)z_0^4 + \theta(z_0 - w_0)w_0^4), \tag{3.24}$$

is again a special case of the scalar propagator in $AdS_{d+1}$ that we had introduced previously in eq. (3.20).

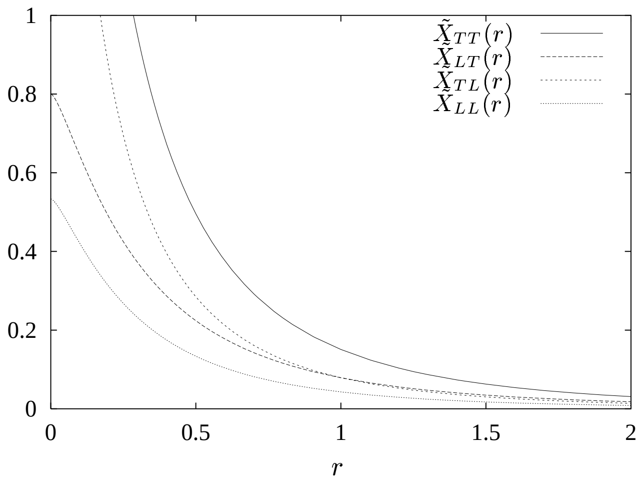


Figure 3: The amplitude functions defined by Eq. (3.26) plotted as a function of $r = Q_A/Q_B$.

In the following evaluation of the forward amplitude we restrict ourselves to two independent virtualities denoted by $Q_A^2 = |\vec{p}_1|^2 = |\vec{p}_3|^2$ and $Q_B^2 = |\vec{p}_2|^2 = |\vec{p}_4|^2$. First we are interested in the behavior of the high energy amplitude as a function of the ratio $r = Q_A/Q_B$. We start from eq. (3.23). From eq. (3.13) we know that helicity is conserved, i.e. we have the four possibilities that the helicities $\lambda_1 = \lambda_3 = P_A$ and $\lambda_2 = \lambda_4 = P_B$ are transverse or longitudinal. Let us denote these cases by $P_A P_B = TT$, $LT$, $TL$, and $LL$, resp. Correspondingly we write:

$$\mathcal{A}^{\rm GR}_{P_A P_B}(s, t=0) = \frac{s^2}{8Q_B^4} \tilde{X}_{P_A P_B} \,, \tag{3.25}$$

where

$$\tilde{X}_{P_A P_B}(r) = r^2 \int_0^\infty dz_B dw_B (\theta(w_B - z_B) z_B^5 w_B + \theta(z_B - w_B) w_B^5 z_B) K^2_{m_A}(z_B r) K^2_{m_B}(w_B) \,, \tag{3.26}$$

and $P_A, P_B$ stand for the two polarizations $T, L$. The index of the Bessel function is given by $m_A = 1$ when $P_A = T$ and by $m_A = 0$ for $P_A = L$. Our integration variables $z_B$ and $w_B$ are related to $z_0$ and $w_0$ through $z_B = z_0 Q_B$ and $w_B = w_0 Q_B$.

The functions (3.26) are plotted in Fig. 3 as a function of the ratio $r = Q_A/Q_B$, for all choices of polarizations. When $r \to 0$, the integrals $\tilde{X}_{LL}$ and $\tilde{X}_{LT}$ go to the finite values

$$\tilde{X}_{LL}(0) = \tfrac{8}{15} \qquad \text{and} \qquad \tilde{X}_{LT}(0) = \tfrac{8}{10} \quad .$$

The other two integrals $\tilde{X}_{TL}$ and $\tilde{X}_{TT}$, on the other hand, diverge logarithmically, i.e.

$$\tilde{X}_{TL}(r) \sim \tfrac{8}{15} \ln r^{-2} \qquad \text{and} \qquad \tilde{X}_{TT}(r) \sim \tfrac{8}{10} \ln r^{-2}$$

for small $r \sim 0$. The behavior of the amplitudes at large $r$ can easily be obtained using the symmetry with respect to the exchange of $p_A$ and $p_B$. Our definition of the quantities $\tilde{X}$ respects the exchange symmetry only up to an overall factor $r^4$, i.e.

$$\tilde{X}_{P_A P_B}(1/r) \;=\; r^4 \tilde{X}_{P_B P_A}(r) \,. \tag{3.27}$$

From this we can immediately read off the asymptotic behavior of $\tilde{X}$ at large values of $r$. The functions $\tilde{X}_{LL}$ and $\tilde{X}_{TL}$ vanish like $r^{-4}$ while $\tilde{X}_{LT}$ and $\tilde{X}_{TT}$ behave as $r^{-4} \ln r^2$.

The region of large $r = Q_A/Q_B$ corresponds to the limit of 'deep inelastic scattering' of the upper $R$-current on the lower $R$-current, i.e. the upper current plays the role of the photon, the lower one that of the target. Since in the Witten diagram approximation our scattering amplitude has no imaginary part, we cannot compute a cross section; nevertheless, it may be instructive to analyze the power behavior in $r$. Introducing the scaling variable $x_{bj} \approx Q_A^2/s$, and combining, in eq. (3.25), the power $r^{-4}$ of $\tilde{X}_{P_A P_B}(r)$ with the factor $s^2$ in front of the integral, we find for large $r$ the leading behavior

$$\mathcal{A}^{GR}_{P_A P_B} \sim \left(\frac{1}{x_{bj}}\right)^2 , \tag{3.28}$$

for $(P_A P_B)=$ LL and TL, whereas

$$\mathcal{A}^{GR}_{P_A P_B} \sim \left(\frac{1}{x_{bj}}\right)^2 \ln r^2 , \tag{3.29}$$

for the cases LT and TT. As we have indicated after eq. (1.1), the limit $r^2 = Q_A^2/Q_B^2 \to \infty$ is connected with the short distance behavior of the product of the $R$-currents $J(\vec{x}_1)$ and $J(\vec{x}_3)$.

It may be of interest to compare this behavior with the large $r = Q_A/Q_B$ behavior of the weak coupling limit of the forward scattering amplitude in $\mathcal{N} = 4$ SYM in eq. (1.1). For large $r = Q_A/Q_B$, the maximal power of logarithms in $r$ comes from the region of strongly ordered transverse momenta: $Q_A^2 \gg \boldsymbol{k}^2 \gg Q_B^2$. In addition, the impact factors $\Phi_A$ and $\Phi_B$, depending upon the polarization of the external currents, may contain logarithms of the type $\ln Q_A^2/\boldsymbol{k}^2$ and $\ln \boldsymbol{k}^2/Q_B^2$. From [17, 18] we take:

$$\begin{aligned}
\Phi_{A,P_A}(Q_A^2, \boldsymbol{k}^2) &\sim \begin{cases} \frac{Q_A^2}{\boldsymbol{k}^2} \ln \frac{Q_A^2}{\boldsymbol{k}^2} & : P_A = T \\ \frac{\boldsymbol{k}^2}{Q_A^2} & : P_A = L \end{cases} \\
\Phi_{B,P_B}(Q_B^2, \boldsymbol{k}^2) &\sim \begin{cases} \ln \frac{\boldsymbol{k}^2}{Q_B^2} & : P_B = T \\ \text{const} & : P_B = L \end{cases} .
\end{aligned} \tag{3.30}$$

This leads to

$$\mathcal{A}_{P_A P_B} \sim \begin{cases} c_{TT} \cdot \ln^3 r^2 & : (P_A P_B) = (TT) \\ c_{TL} \cdot \ln^2 r^2 & : (P_A P_B) = (TL) \\ c_{LT} \cdot \ln^2 r^2 & : (P_A P_B) = (LT) \\ c_{LL} \cdot \ln r^2 & : (P_A P_B) = (LL) \end{cases} , \tag{3.31}$$

and the constants are composed of the coefficient functions and anomalous dimensions. Returning to the graviton exchange amplitude, it is suggestive to associate the $\ln r^2$ modification also with anomalous dimensions: it would be interesting to perform a systematic operator analysis of the strong coupling limit.

In the last part of this section we would like to determine the region in the integration over $z_0$ and $w_0$ from which the four amplitudes $\tilde{X}$ receive their dominant contributions. Let us consider the integrands of the functions $\tilde{X}_{PP'}$ which we now write as

$$\tilde{X}_{PP'} = \int_0^\infty dz_B \int_0^\infty dw_B \tilde{J}_{PP'} . \tag{3.32}$$

As one can see from eq. (3.26), the integrand is finite at $z_B = 0$ and $w_B = 0$, and due to the Bessel functions it falls off exponentially at infinity. One therefore expects, for $Q_A \sim Q_B$, the main contribution to the integrals to come from the region where $z_B$ and $w_B$ are of order unity. There are two kinds of maxima: first, there is a 'ridge' along the diagonal line $z_B = w_B$, resulting from $\theta$-functions in eq. (3.26). Second, there are local maxima away from the diagonal. Maxima along the ridge are found from the condition

$$\partial_x \tilde{J}_{P_A P_B}(z_B = w_B = x)|_{x = x_{P_A P_B}} = 0, \tag{3.33}$$

which has the form:

$$0 = 2(m_A + m_B + 3) K_{m_A}(rx) K_{m_B}(x) - 2v K_{m_A}(rx) K_{m_B+1}(x) - 2rx K_{m_A+1}(rx) K_{m_B}(x) , \tag{3.34}$$

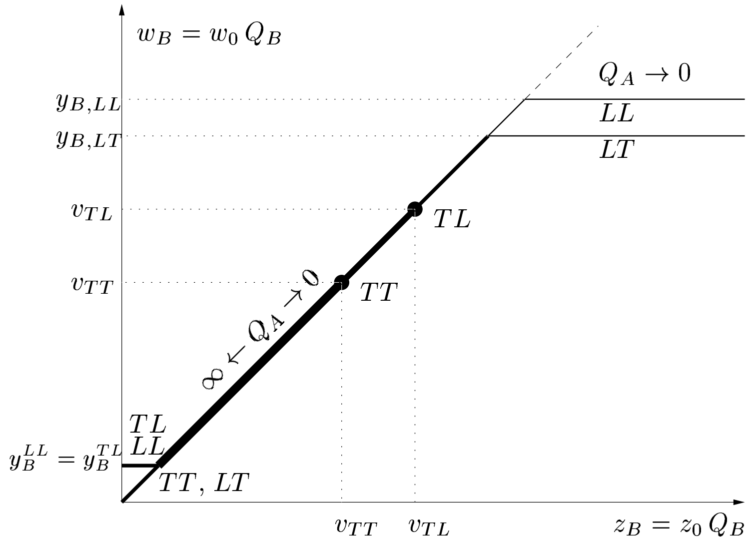


Figure 4: The position of the integrand maximum in the $(z_0, w_0)-$space as a function of $Q_A$ and with $Q_B =$ fixed for different $R$-boson polarizations. The black circles denote points corresponding to $Q_A \to 0$ for polarizations $TT$ and $TL$.

whereas the other extrema satisfy the conditions

$$\begin{aligned}
\partial_{w_B} \tilde{J}_{P_A P_B} &= r^2 w_B^4 z_B K_{m_A}(r z_B)^2 K_{m_B}(w_B) \left((2m_B + 5) K_{m_B}(w_B) - 2 w_B K_{m_B+1}(w_B)\right) = 0\,, \\
\partial_{z_B} \tilde{J}_{P_A P_B} &= r^2 w_B^5 K_{m_A}(r z_B) K_{m_B}(w_B)^2 \left((2m_A + 1) K_{m_A}(r z_B) - 2 r z_B K_{m_A+1}(r z_B)\right) = 0\,.
\end{aligned}$$

We remind that $m_A$ ($m_B$) refer to the helicities of the upper (lower) $R$-currents, and $m = 1$ ($m = 0$) belongs to transverse (longitudinal) polarizations. In the following we describe the results of our analysis which are illustrated in Fig. 4.

Let us begin with the special case in which the two virtualities $Q_A$ and $Q_B$ are equal, i.e. $r = 1$. The maximum of the integrand lies on the diagonal line line at $z_B = w_B = x_{P_A P_B}$: in Fig. 4 these points lie to the left of the points labeled by 'TT'. The numerical values of $v_{TT} \approx 1.3316$ and $v_{TL} \approx 1.5527$, as expected, are close to unity. This implies that, in the graviton exchange Witten diagram in Fig.1, the coordinates $z_0$ and $w_0$ of the interaction vertices are both of the order $1/Q_B$.

Next, let us fix $Q_B$ and consider the amplitude as a function of $r = Q_A/Q_B$. When $r$ decreases, the maxima on the ridge at $z_B = w_B = x_{P_A P_B}$ move upwards along the diagonal. For the polarizations TT and TL, they reach, for $r = 0$, their final values at the points marked by 'TT' and 'TL', resp. For the other two cases, LL and LT, the maxima continue to move along the diagonal ray until they reach, for some value $r^{(l)}$, their turning values denoted by $y_B^{LL}$ and $y_B^{LT}$. After that they continue along the horizontal lines to the right of these turning points. For small $r$, their $z_B$ coordinate grows as $1/r$.

Next we consider the limit $r \to \infty$. Beginning again at $r = 1$ on the diagonal line and increasing $r$, the maxima $z_B = w_B = x_{P_A P_B}$ now move in the opposite direction, towards the origin. For the cases TT and LT they stay on the diagonal until they reach the origin, and for large $r$ the values $x_{P_A P_B}$ decrease as $1/r$. For the polarizations LL and TL, there are again turning points at $y_{B,LL} = y_{B,LT}$, and beyond these points the maxima move on the horizontal line towards the $w_B$ axis.[2]

Let us visualize these results in the graviton exchange diagram in Fig.1. When $Q_A$ becomes large (at fixed $Q_B$), we have found that $z_0 \sim 1/Q_A$ becomes small, i.e. the vertex where the

[2]These conclusions for the large-$r$ behavior can be obtained from the observation that the integrands $I_{P_A P_B}$ are invariant when replacing $r$ by $1/r$, i.e. $\tilde{I}_{P_A P_B}(1/r) \leftrightarrow r^2 \tilde{I}_{P_B P_A}(r)$. This symmetry can then be used to evaluate the amplitudes for large values of the ratio $r$.

exchanged graviton couples to the gauge boson moves close to the boundary, for both polarizations $P_A$ =T and $P_A$ =L. For the lower vertex we see a difference between transverse and longitudinal polarization of the target current: in the former case, $w_0 \sim 1/Q_A$ becomes small, in the latter case $w_0$ remains constant of the order $1/Q_B$.

# 4 Gauge boson exchange in the high energy limit

In the following pages we shall briefly analyze contributions from gauge boson exchange in the bulk. In analogy to the case of graviton exchange, the leading contribution arises from the $t$-channel exchange of the gauge boson. Not surprisingly, this leading term is linear in $s$ and, hence, subleading when compared to the graviton exchange.

We restrict ourselves to the abelian part of the $SU(4)$ group. The coupling of the external gauge boson to the exchanged bulk boson then proceeds only through the Chern-Simons interaction term

$$\int d^d z dz_0 \varepsilon_{\mu\nu\rho\sigma\lambda} \partial_\mu A_\nu(z) \partial_\rho A_\sigma(z) A_\lambda(z) \,. \tag{4.1}$$

Using once more the standard rules of the AdS/CFT correspondence, we can determine the amplitude for boson exchange in $t-$channel to be of the form

$$\begin{aligned} I^{\rm CS} \quad &= \quad \int d^4 z dz_0 \int d^4 w dw_0 \varepsilon_{\mu_1\mu_2\mu_3\mu_4\mu_5} \varepsilon_{\nu_1\nu_2\nu_3\nu_4\nu_5} \partial_{[\mu_1} G_{\mu_2]j_1}(z,\vec{x}_1) \partial_{[\mu_3} G_{\mu_4]j_3}(z,\vec{x}_3) \\ &\qquad \times G_{\mu_5\nu_5}(z,w) \partial_{[\nu_1} G_{\nu_2]j_2}(w,\vec{x}_2) \partial_{[\nu_3} G_{\nu_4]j_4}(w,\vec{x}_4) \,, \end{aligned} \tag{4.2}$$

where the partial derivatives $\partial$ acting on $z$ and $w$, respectively. There are similar amplitudes including derivatives on the bulk-to-bulk propagator. But all these contributions to the full $t$-channel exchange turn out to be identical, as a result of the Bianchi identity. As in the case of the graviton exchange, we perform the Fourier transform, i.e.

$$I^{\rm CS} = \frac{1}{(2\pi)^{16}} \int d^4 p_1 d^4 p_2 d^4 p_3 d^4 p_4 \, \mathrm{e}^{i\vec{p}_1\cdot\vec{x}_1} \, \mathrm{e}^{i\vec{p}_2\cdot\vec{x}_2} \, \mathrm{e}^{i\vec{p}_3\cdot\vec{x}_3} \, \mathrm{e}^{i\vec{p}_4\cdot\vec{x}_4} \, \tilde{I}^{\rm CS} \,. \tag{4.3}$$

After tedious calculations one finds that, in the high energy limit, the main contribution comes from Eq. (4.2) with both nonzero $\mu_5 = m \neq 0$ and $\nu_5 = n \neq 0$, i.e.

$$\begin{aligned} I^{\rm CS}_{\rm Regge} \quad &= \quad 4 \int d^4 z dz_0 \int d^4 w dw_0 \varepsilon_{0\mu_2\mu_3\mu_4 m} \varepsilon_{0\nu_2\nu_3\nu_4 n} \partial_{[0} G_{\mu_2]j_1}(z,\vec{x}_1) \partial_{[\mu_3} G_{\mu_4]j_3}(z,\vec{x}_3) \\ &\qquad \times \, G_{mn}(z,w) \partial_{[0} G_{\nu_2]j_2}(w,\vec{x}_2) \partial_{[\nu_3} G_{\nu_4]j_4}(w,\vec{x}_4) \\ &\quad + \begin{pmatrix} \vec{x}_1 \leftrightarrow \vec{x}_3 \\ j_1 \leftrightarrow j_3 \end{pmatrix} + \begin{pmatrix} \vec{x}_2 \leftrightarrow \vec{x}_4 \\ j_2 \leftrightarrow j_4 \end{pmatrix} + \begin{pmatrix} \vec{x}_1 \leftrightarrow \vec{x}_3 ; \vec{x}_2 \leftrightarrow \vec{x}_4 \\ j_1 \leftrightarrow j_3 ; j_2 \leftrightarrow j_4 \end{pmatrix} . \end{aligned} \tag{4.4}$$

The Fourier transform of this expression is given by

$$\begin{aligned} \tilde{I}^{\rm CS}_{\rm Regge} \quad &= \quad (2\pi)^4 \delta^{(4)}(\vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \vec{p}_4) \tilde{W}^{\rm CS}_{j_1 j_3 j_2 j_4} \\ &\quad \times \Bigg\{ \int dz_0 z_0^3 \int dw_0 w_0^3 |\vec{p}_3| K_1(z_0|\vec{p}_3|) K_0(z_0|\vec{p}_1|) \\ &\quad \times \sum_{k=0}^{\infty} \frac{K_{2k+1}(|\vec{q}|\varpi_0)}{\Gamma(k+2)\Gamma(k+1)} \left( \frac{z_0^2 w_0^2 |\vec{q}|^2}{4\varpi_0^2} \right)^{k+1/2} |\vec{p}_4| K_1(w_0|\vec{p}_4|) K_0(w_0|\vec{p}_2|) \Bigg\} \\ &\quad + \begin{pmatrix} \vec{p}_1 \leftrightarrow \vec{p}_3 \\ j_1 \leftrightarrow j_3 \end{pmatrix} + \begin{pmatrix} \vec{p}_2 \leftrightarrow \vec{p}_4 \\ j_2 \leftrightarrow j_4 \end{pmatrix} + \begin{pmatrix} \vec{p}_1 \leftrightarrow \vec{p}_3 ; \vec{p}_2 \leftrightarrow \vec{p}_4 \\ j_1 \leftrightarrow j_3 ; j_2 \leftrightarrow j_4 \end{pmatrix} , \end{aligned} \tag{4.5}$$

where the polarization tensor $\tilde{W}$ takes following form

$$
\begin{aligned}
\tilde{W}^{\mathrm{CS}}_{j_1 j_3 j_2 j_4} &\approx tsp_{1\,j_1}p_{2\,j_2}\delta_{j_4 j_3} - s|\vec{p}_1|^2|\vec{p}_2|^2(\delta_{j_2 j_3}\delta_{j_1 j_4} - \delta_{j_1 j_2}\delta_{j_3 j_4}) + sp_{1\,j_1}p_{2\,j_2}q_{j_3}q_{j_4} \\
&\quad - s|\vec{p}_1|^2 p_{2\,j_2}(\delta_{j_1 j_4}q_{j_3} - \delta_{j_3 j_4}q_{j_1}) - s|\vec{p}_2|^2 p_{1\,j_1}(\delta_{j_3 j_4}q_{j_2} - \delta_{j_2 j_3}q_{j_4}) \qquad (4.6)
\end{aligned}
$$

with $\vec{q} = \vec{p}_1 + \vec{p}_3$, as before. Going to the polarization base (A.4) the tensor $\tilde{W}^{\mathrm{CS}}$ gets replaced by

$$
\begin{aligned}
\tilde{\mathcal{W}}^{\mathrm{CS}}_{\lambda_1\lambda_3,\lambda_2\lambda_4}(\vec{p}_i) &= \sum_{j_1,j_2,j_3,j_4} \epsilon^{(\lambda_1)}_{j_1}(\vec{p}_1)\epsilon^{(\lambda_2)}_{j_2}(\vec{p}_2)\epsilon^{(\lambda_3)}_{j_3}(\vec{p}_3)^*\epsilon^{(\lambda_4)}_{j_4}(\vec{p}_4)^*\tilde{W}^{\mathrm{CS}}_{j_1 j_3 j_2 j_4}(\vec{p}_i) \qquad (4.7) \\
&\approx -s|\vec{p}_1|^2|\vec{p}_2|^2((\vec{\epsilon}^{(\lambda_1)}_T\cdot\vec{\epsilon}^{(\lambda_4)*}_T)(\vec{\epsilon}^{(\lambda_2)}_T\cdot\vec{\epsilon}^{(\lambda_3)*}_T) - (\vec{\epsilon}^{(\lambda_3)*}_T\cdot\vec{\epsilon}^{(\lambda_4)*}_T)(\vec{\epsilon}^{(\lambda_1)}_T\cdot\vec{\epsilon}^{(\lambda_2)}_T)) \\
&= -s|\vec{p}_1|^2|\vec{p}_2|^2(2\delta^{\lambda_1,\lambda_2} - 1)\delta^{\lambda_1\lambda_3}\delta^{\lambda_2\lambda_4}\,,
\end{aligned}
$$

where $\lambda_i$ are possible transverse polarizations. Putting this back into our expression (4.6) we have now expressed the Regge limit in terms of the kinematical invariants $s, t, |\vec{p}_i|$. The whole amplitude is proportional to $s$, as we anticipated at the beginning of this section. Therefore, it is subleading when compared to the contribution from $t$-channel graviton exchange. We also note that only the transverse polarization contributes to the Regge limit of the $t$-channel exchange of gauge bosons. Furthermore, the helicity is conserved in the high energy limit.

As in the case of graviton scattering (we can perform two-dimensional Fourier transform in the transverse momenta to get

$$
\begin{aligned}
\mathcal{I}^{\mathrm{CS}}_{\mathrm{T,Regge}} &= \delta^{(2)}(\boldsymbol{x}_1 - \boldsymbol{x}_3)\delta^{(2)}(\boldsymbol{x}_2 - \boldsymbol{x}_4)\mathcal{W}^{\mathrm{CS}}_{\lambda_1\lambda_3\lambda_2\lambda_4} \\
&\quad\times \int dz_0 z_0^2 \int dw_0 w_0^2 |\vec{p}_3| K_1(z_0|\vec{p}_3|)K_0(z_0|\vec{p}_1|)\; G_{\Delta=2,d=2}(\tilde{u})\; |\vec{p}_4|K_1(w_0|\vec{p}_4|)K_0(w_0|\vec{p}_2|) \\
&\quad + \begin{pmatrix}\vec{p}_1 \leftrightarrow \vec{p}_3 \\ \lambda_1 \leftrightarrow \lambda_3\end{pmatrix} + \begin{pmatrix}\vec{p}_2 \leftrightarrow \vec{p}_4 \\ \lambda_2 \leftrightarrow \lambda_4\end{pmatrix} + \begin{pmatrix}\vec{p}_1 \leftrightarrow \vec{p}_3; p_2 \leftrightarrow \vec{p}_4 \\ \lambda_1 \leftrightarrow \lambda_3; \lambda_2 \leftrightarrow \lambda_4\end{pmatrix} \qquad (4.8)
\end{aligned}
$$

with $W^{\mathrm{CS}}_{j_1 j_3 j_2 j_4} \sim s$ defined similarly to $W^{m_1}_{j_1 j_3}$ and the scalar propagator $G_{\Delta,d}$ that was spelled out in eq. (3.20). Finally, we would like to provide an expression for the forward limit, i.e. when $|\vec{p}_1 + \vec{p}_3| = |\vec{q}| \to 0$. In this limit, the terms in the curly brackets of eq. (4.5) give

$$
\begin{aligned}
\{\ldots\} &= -\tfrac{1}{2}\int dz_0 \int dw_0 |\vec{p}_3|K_1(z_0|\vec{p}_3|)K_0(z_0|\vec{p}_1|)\; (\theta(w_0 - z_0)z_0^4 w_0^2 + \theta(z_0 - w_0)z_0^2 w_0^4) \\
&\quad \times\; |\vec{p}_4|K_1(w_0|\vec{p}_4|)K_0(w_0|\vec{p}_2|)\,. \qquad (4.9)
\end{aligned}
$$

Thus, for $\vec{p}_1 = -\vec{p}_3$ $\vec{p}_2 = -\vec{p}_4$, we are left with

$$
\begin{aligned}
\tilde{I}^{\mathrm{CS}}_{\mathrm{forward}} &= (2\pi)^4\delta^{(4)}(\vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \vec{p}_4)(8\;\vec{p}_1\cdot\vec{p}_2)\,(\delta_{j_1 j_2}\delta_{j_3 j_4} - \delta_{j_2 j_3}\delta_{j_1 j_4}) \\
&\times \int dz_0 \int dw_0 z_0^3 w_0^3\; |\vec{p}_1|^3 K_0(z_0|\vec{p}_1|)K_1(z_0|\vec{p}_1|)\; G_{\Delta=1,d=0}(\hat{u})\; |\vec{p}_2|^3 K_0(w_0|\vec{p}_2|)K_1(w_0|\vec{p}_2|) \qquad (4.10)
\end{aligned}
$$

The scalar propagator $G$ in the second line is defined through eq. (3.20). It assumes the following form

$$
G_{\Delta=1,d=0}(\hat{u}) = \frac{1}{2z_0 w_0}(\theta(w_0 - z_0)z_0^2 + \theta(z_0 - w_0)w_0^2)\,. \qquad (4.11)
$$

We conclude that the exchange of the gauge bosons in the $t-$channel can give contributions to the scattering amplitude at most proportional to $s$, and it is of the same order as subleading terms of the graviton exchange.

# 5 Summary

The aim of this work was to calculate in the Regge limit the scattering amplitude of $R$-currents for $\mathcal{N}=4$ SYM theory at strong coupling. We make use of the AdS/CFT conjecture which maps this amplitude on four-point correlation functions of $R$-bosons in supergravity which live in the Anti-de Sitter space. We have computed Witten diagrams of graviton and boson exchanges in the $t-$channel. Similar to gauge theories, the exchange of highest spin dominates, i.e. the leading contribution comes from the (real-valued) Witten diagram with graviton $t$-channel exchange. This confirms the duality between the two-gluon exchange in $\mathcal{N}=4$ SYM and the graviton exchange in the strong coupling region. On the gauge theory side, the sum of leading energy logarithms replaces the two gluon exchange by the BFKL Pomeron. On the strong coupling side, it has been argued that the $s^2$ behavior of the graviton exchange is replaced by $s^{2-\Delta}$ where $\Delta = \mathcal{O}(1/\sqrt{\lambda})$: however, the computation of this correction $\Delta$ cannot been done in the supergravity approximation used in the present paper. For spin-1 boson exchange, which is due to the Chern-Simons vertices, we found the expected high behavior proportional to $s$; helicity is conserved, and amplitude with longitudinally polarized bosons are subleading for large $s$.

For the graviton exchange we have found that, in the transverse $(2+1)-$dimensional configuration space, one can re-formulate the scattering amplitude as the exchange of an effective field, build from a scalar field with dimension $\Delta = 3$.

We have also analyzed how the graviton exchange amplitude depends upon the virtualities of the external currents. For large $r^2 = Q_A^2/Q_B^2$, the power behavior is the same as the leading twist behavior on the weak coupling side, and the appearance of a logarithm, $\ln r^2$, hints at the presence of an anomalous dimension in a short distance expansion. A systematic study should be done in a separate paper.

The virtualities of the external currents determine the distance of the vertices away from the boundary. In particular, we have analyzed in some detail the limit $r \to \infty$, i.e. the analogue of 'deep inelastic scattering' where the virtuality of the upper $R$-current is much larger than the lower one: in this case, the distance of the upper 'impact factor' from the boundary is small, of the order $1/r$. A similar result has been found in [49, 50, 51].

We view our results as a first step towards a systematic calculation of the Regge limit of the scattering amplitude at strong coupling. In particular, in order to obtain a nonzero imaginary part, one has to go beyond the tree approximation of Witten diagrams. In [44, 45, 46, 47] a semiclassical approximation of string theory has been proposed. Another possible line of investigation might follow the classical paper of Amati et. al [63] in which string theory in flat space has been investigated.

## Acknowledgments

We are grateful for discussions with A. H. Mueller, L. Motyka and in particular with I. Papadimitriou. This work was supported by the grant of SFB 676, Particles, Strings and the Early Universe: "the Structure of Matter and Space-Time" and the grant of the Foundation for Polish Science.

# A Euclidean polarization vectors

The polarization vectors $\vec{\epsilon}_k^{(i)}(\vec{p}_j)$ should satisfy

$$\vec{p}_j \cdot \vec{\epsilon}^{(i)}(\vec{p}_j) = 0\,, \qquad \vec{\epsilon}^{(i_1)}(\vec{p}) \cdot \vec{\epsilon}^{(i_2)}(\vec{p})^* = (-)^{L_{i_1,i_2}} \delta^{i_1 i_2}\,, \tag{A.1}$$

where $L_{i_1,i_2} = 1$ when $i_j$ is longitudinal ($L$) and 0 when $i_j$ is transverse ($h = \pm$). The *star* denotes complex conjugation. One of the possible solutions reads as

$$\begin{aligned}
\vec{\epsilon}^{(L)}(\vec{p}_1) &= \tfrac{1}{|\vec{p}_1|}(\vec{p}_1 + \tfrac{2|\vec{p}_1|^2}{s}\vec{p}_2)\,, & \vec{\epsilon}^{(h)}(\vec{p}_1) &= \vec{\epsilon}_T^{(h)} + \frac{2\vec{p}_1 \cdot \vec{\epsilon}_T^{(h)}}{s}\vec{p}_2\,,\\
\vec{\epsilon}^{(L)}(\vec{p}_2) &= \tfrac{1}{|\vec{p}_2|}(\vec{p}_2 + \tfrac{2|\vec{p}_2|^2}{s}\vec{p}_1)\,, & \vec{\epsilon}^{(h)}(\vec{p}_2) &= \vec{\epsilon}_T^{(h)} + \frac{2\vec{p}_2 \cdot \vec{\epsilon}_T^{(h)}}{s}\vec{p}_1\,,\\
\vec{\epsilon}^{(L)}(\vec{p}_3) &= \tfrac{1}{|\vec{p}_3|}(-\vec{p}_1 - \tfrac{2|\vec{p}_3|^2}{s}\vec{p}_2 + \vec{q})\,, & \vec{\epsilon}^{(h)}(\vec{p}_3) &= \vec{\epsilon}_T^{(h)} + \frac{2(\vec{p}_1 - \vec{q}) \cdot \vec{\epsilon}_T^{(h)}}{s}\vec{p}_2\,,\\
\vec{\epsilon}^{(L)}(\vec{p}_4) &= \tfrac{1}{|\vec{p}_4|}(-\vec{p}_2 - \tfrac{2|\vec{p}_4|^2}{s}\vec{p}_1 - \vec{q})\,, & \vec{\epsilon}^{(h)}(\vec{p}_4) &= \vec{\epsilon}_T^{(h)} + \frac{2(\vec{p}_2 + \vec{q}) \cdot \vec{\epsilon}_T^{(h)}}{s}\vec{p}_1\,,
\end{aligned} \tag{A.2}$$

plus subleading terms which do not contribute. We denote the transfered momenta $\vec{q} = \vec{p}_1 + \vec{p}_3$ while

$$\epsilon_T^{(\pm)} = \frac{1}{\sqrt{2}}(0, 1, \pm i, 0)\,. \tag{A.3}$$

In Eq. (A.2) we have shown only the leading terms in $s$. Using the Ward identity one can shift the polarization vectors to

$$\begin{aligned}
\vec{\epsilon}^{(L)}(\vec{p}_1) &= \tfrac{2|\vec{p}_1|}{s}\vec{p}_2\,, & \vec{\epsilon}^{(h)}(\vec{p}_1) &= \vec{\epsilon}_T^{(h)} + \frac{2\vec{p}_1 \cdot \vec{\epsilon}_T^{(h)}}{s}\vec{p}_2\,,\\
\vec{\epsilon}^{(L)}(\vec{p}_2) &= \tfrac{2|\vec{p}_2|}{s}\vec{p}_1 & \vec{\epsilon}^{(h)}(\vec{p}_2) &= \vec{\epsilon}_T^{(h)} + \frac{2\vec{p}_2 \cdot \vec{\epsilon}_T^{(h)}}{s}\vec{p}_1\,,\\
\vec{\epsilon}^{(L)}(\vec{p}_3) &= -\tfrac{2|\vec{p}_3|}{s}\vec{p}_2\,, & \vec{\epsilon}^{(h)}(\vec{p}_3) &= \vec{\epsilon}_T^{(h)} + \frac{2(\vec{p}_1 - \vec{q}) \cdot \vec{\epsilon}_T^{(h)}}{s}\vec{p}_2\,,\\
\vec{\epsilon}^{(L)}(\vec{p}_4) &= -\tfrac{2|\vec{p}_4|}{s}\vec{p}_1\,, & \vec{\epsilon}^{(h)}(\vec{p}_4) &= \vec{\epsilon}_T^{(h)} + \frac{2(\vec{p}_2 + \vec{q}) \cdot \vec{\epsilon}_T^{(h)}}{s}\vec{p}_1\,.
\end{aligned} \tag{A.4}$$

The resulting vectors (A.4) do not satisfy eq. (A.1). For amplitudes which satisfies the Ward identity the contraction with polarization vectors of eq. (A.4) gives the same results as with eq. (A.2). Moreover, since their contraction with other tensors can not produce additional powers of $s$, they are much simpler in use. Changing the sign of the metric and performing the Wick rotation one can get the similar polarization vectors in the Minkowski space.

# B Momentum space

Following [23], Feynman integrals $I$ in momentum space can be calculated using the Schwinger representation, i.e.

$$\begin{aligned}
I_m(x_0, \vec{q}_1) &= \int d^d\vec{x}_1\, \mathrm{e}^{i\vec{q}_1\vec{x}_1}\, \frac{1}{(x_0^2 + \vec{x}_1^2)^{m+1}} = \int d^d\vec{x}_1\, \mathrm{e}^{i\vec{q}_1\vec{x}_1} \left(\frac{1}{\Gamma(m+1)} \int_0^\infty d\tau \tau^m\, \mathrm{e}^{-\tau(\vec{x}_1^2 + x_0^2)}\right)\\
&= \frac{\pi^{d/2}}{\Gamma(m+1)} \int_0^\infty d\tau \tau^{m-d/2}\, \mathrm{e}^{-\tau x_0^2}\, \mathrm{e}^{-\frac{\vec{q}_1^2}{4\tau}}\\
&= \frac{2\pi^{d/2}}{\Gamma(m+1)} \left(\frac{|\vec{q}_1|}{2x_0}\right)^{m+1-d/2} K_{m+1-d/2}(x_0|\vec{q}_1|)\,.
\end{aligned} \tag{B.1}$$

For integer positive $\nu$ the modified Bessel function reads as

$$\begin{aligned}
K_\nu(u) &= 1/2(u/2)^{-\nu} \sum_{k=0}^{\nu-1} \frac{(\nu-k-1)!}{k!}(-u^2/4)^k + (-1)^{\nu+1} \ln(u/2)(u/2)^\nu \sum_{k=0}^{\infty} \frac{(u^2/4)^k}{k!(\nu+k)!}\\
&\quad + (-1)^\nu 1/2(u/2)^\nu \sum_{k=0}^{\infty} (\psi(k+1) + \psi(\nu+k+1)) \frac{(u^2/4)^k}{k!(\nu+k)!}\,.
\end{aligned} \tag{B.2}$$

## C The resummed contribution to the graviton propagator

The sum defined in eq. (3.11) with $q = |\vec{p}_1 + \vec{p}_3|$ and $\varpi_0 = \sqrt{w_0^2 + z_0^2}$ has a following form

$$
\begin{aligned}
\Sigma(z_0, w_0) &= \sum_{m=0}^{\infty} q^{2m} U_m = \sum_{k=0}^{\infty} \frac{z_0^{2k+5} w_0^{2k+5}}{\Gamma(k+1)\Gamma(k+3)} \left(\frac{q^2}{4\varpi_0^2}\right)^{k+1} K_{2k+2}(q\varpi_0) \\
&= \frac{z_0^5 w_0^5}{2\varpi_0^4} \sum_{m=0}^{\infty} \frac{(-1)^{2m} q^{4m} \varpi_0^{4m}}{2^{4m}\Gamma(2m+1)} \sum_{k=m}^{\infty} \frac{\Gamma(2k+2-2m)}{\Gamma(k+1)\Gamma(k+3)} \frac{z_0^{2k} w_0^{2k}}{\varpi_0^{4k}} \\
&\quad - \frac{q^2 z_0^5 w_0^5}{2^3 \varpi_0^2} \sum_{m=0}^{\infty} \frac{(-1)^{2m} q^{4m} \varpi_0^{4m}}{2^{4m}\Gamma(2m+2)} \sum_{k=m}^{\infty} \frac{\Gamma(2k+1-2m)}{\Gamma(k+1)\Gamma(k+3)} \frac{z_0^{2k} w_0^{2k}}{\varpi_0^{4k}} \\
&\quad - \ln(q\varpi_0/2) \frac{z_0^5 w_0^5 q^4}{2^4} \sum_{m=0}^{\infty} \frac{(q/2)^{4m}}{\Gamma(3+2m)} \sum_{k=0}^{m} \frac{\varpi_0^{4m-4k}}{\Gamma(2m-2k+1)} \frac{(-1)^{2k} z_0^{2k} w_0^{2k}}{\Gamma(k+1)\Gamma(k+3)} \\
&\quad - \ln(q\varpi_0/2) \frac{z_0^5 w_0^5 q^6}{2^6} \sum_{m=0}^{\infty} \frac{(q/2)^{4m}}{\Gamma(4+2m)} \sum_{k=0}^{m} \frac{\varpi_0^{4m-4k+2}}{\Gamma(2m-2k+2)} \frac{(-1)^{2k} z_0^{2k} w_0^{2k}}{\Gamma(k+1)\Gamma(k+3)} \\
&\quad + \frac{q^4 z_0^5 w_0^5}{2^5} \sum_{m=0}^{\infty} \frac{(q/2)^{4m}}{\Gamma(2m+3)} \sum_{k=0}^{m} \frac{\varpi_0^{4m-4k}}{\Gamma(2m-2k+1)} \frac{(-1)^{2k} z_0^{2k} w_0^{2k}}{\Gamma(k+1)\Gamma(k+3)} \\
&\quad \times (\psi(2m-2k+1) + \psi(2m+3)) \\
&\quad + \frac{q^6 z_0^5 w_0^5}{2^7} \sum_{m=0}^{\infty} \frac{(q/2)^{4m}}{\Gamma(2m+4)} \sum_{k=0}^{m} \frac{\varpi_0^{4m-4k+2}}{\Gamma(2m-2k+2)} \frac{(-1)^{2k} z_0^{2k} w_0^{2k}}{\Gamma(k+1)\Gamma(k+3)} \\
&\quad \times (\psi(2m-2k+2) + \psi(2m+4)) . \\
&= \sum_{m=0}^{\infty} q^{4m} \left( T_m^{(1)} + q^2 T_m^{(2)} + q^4 T_m^{(3)} + q^6 T_m^{(4)} + q^4 T_m^{(5)} + q^6 T_m^{(6)} \right) \qquad \text{(C.1)}
\end{aligned}
$$

where

$$
T_m^{(1)} = 2^{-4m} \frac{w_0^{2m+5} z_0^{2m+5}}{2\left(w_0^2 + z_0^2\right)^2} \frac{{}_3F_2\left(1, 1, \frac{3}{2}; m+1, m+3; \frac{4w_0^2 z_0^2}{\left(w_0^2+z_0^2\right)^2}\right)}{\Gamma(m+1)\Gamma(m+3)\Gamma(2m+1)} , \qquad \text{(C.2)}
$$

and

$$
T_m^{(2)} = -2^{-4m-2} \frac{w_0^{2m+5} z_0^{2m+5}}{2(w_0^2 + z_0^2)} \frac{{}_3F_2\left(\frac{1}{2}, 1, 1; m+1, m+3; \frac{4w_0^2 z_0^2}{\left(w_0^2+z_0^2\right)^2}\right)}{\Gamma(m+1)\Gamma(m+3)\Gamma(2m+2)} . \qquad \text{(C.3)}
$$

Moreover,

$$
T_m^{(3)} = -2^{-4m-4} \frac{(w_0 z_0)^{2m+5}}{2\Gamma(2m+3)^2} \ln\left(\left(w_0^2 + z_0^2\right) q^2/4\right) C_{2m}^{(-2m-2)} \left(\frac{w_0^2 + z_0^2}{2 z_0 w_0}\right) , \qquad \text{(C.4)}
$$

and

$$
T_m^{(4)} = 2^{-4m-6} \frac{(w_0 z_0)^{2m+6}}{2\Gamma(2m+4)^2} \ln\left(\left(w_0^2 + z_0^2\right) q^2/4\right) C_{2m+1}^{(-2m-3)} \left(\frac{w_0^2 + z_0^2}{2 w_0 z_0}\right) , \qquad \text{(C.5)}
$$

where $C_{2m}^{(k)}$ are Gegenbauer C polynomials. The last terms use

$$
T_m^{(5)} = \sum_{k=0}^{m} \frac{(w_0^2 + z_0^2)^{2m-2k}}{\Gamma(2m-2k+1)} \frac{(-1)^{2k} z_0^{2k+5} w_0^{2k+5}}{\Gamma(k+1)\Gamma(k+3)} \frac{(\psi(2m-2k+1) + \psi(2m+3))}{2^5 \Gamma(2m+3)} , \qquad \text{(C.6)}
$$

and

$$T_m^{(6)} = \sum_{k=0}^{m} \frac{(w_0^2+z_0^2)^{2m-2k+1}}{\Gamma(2m-2k+2)} \frac{(-1)^{2k} z_0^{2k+5} w_0^{2k+5}}{\Gamma(k+1)\Gamma(k+3)} \frac{(\psi(2m-2k+2)+\psi(2m+4))}{2^7\Gamma(2m+4)}\,. \quad \text{(C.7)}$$

Performing the sum over $k$ in eqs. (C.1) and (3.22) one can calculate the first few terms of the expansion in $q$, i.e.

$$U_0 = T_0^{(1)} = \theta(z_0-w_0)\frac{1}{4}z_0 w_0^5 + \theta(w_0-z_0)\frac{1}{4}w_0 z_0^5\,, \quad \text{(C.8)}$$

which determines the forward limit. Similarly,

$$U_1 = T_0^{(2)} = \theta(z_0-w_0)\frac{w_0^5 z_0}{48}(w_0^2-3z_0^2) + \theta(w_0-z_0)\frac{w_0 z_0^5}{48}(z_0^2-3w_0^2) \quad \text{(C.9)}$$

reproduces the next-to-forward contribution which appears at order $q^2$.

Thus, summing all terms suppressed by $q^4$ one gets

$$\begin{aligned} U_2 &= T_1^{(1)} + T_0^{(3)} + T_0^{(5)} = \theta(z_0-w_0)\frac{w_0^5 z_0}{2^8 6}\left(w_0^4 - 8z_0^2 w_0^2 + 18 z_0^4 - 12 z_0^4 \ln\left(z_0^2 q^2\, \mathrm{e}^{2\gamma_E}/4\right)\right) \\ &+\theta(w_0-z_0)\frac{z_0^5 w_0}{2^8 6}\left(z_0^4 - 8w_0^2 z_0^2 + 18 w_0^4 - 12 w_0^4 \ln\left(w_0^2 q^2\, \mathrm{e}^{2\gamma_E}/4\right)\right)\,. \end{aligned} \quad \text{(C.10)}$$

Terms suppressed by $q^6$ read as follows

$$\begin{aligned} U_3 &= T_1^{(2)} + T_0^{(4)} + T_0^{(6)} = \theta(z_0-w_0)\frac{z_0 w_0^5}{2^{10}90}\left(w_0^6 - 15 z_0^2 w_0^4 + 90 z_0^4 w_0^2 + 170 z_0^6\right. \\ &\left. -60 z_0^4\left(z_0^2+w_0^2\right)\ln\left(z_0^2 q^2\, \mathrm{e}^{2\gamma_E}/4\right)\right) \\ &+ \theta(w_0-z_0)\frac{w_0 z_0^5}{2^{10}90}\left(z_0^6 - 15 z_0^4 w_0^2 + 90 w_0^4 z_0^2 + 170 w_0^6\right. \\ &\left. -60 w_0^4\left(w_0^2+z_0^2\right)\ln\left(w_0^2 q^2\, \mathrm{e}^{2\gamma_E}/4\right)\right)\,. \end{aligned} \quad \text{(C.11)}$$

Similarly, terms suppressed by $q^8$ give

$$\begin{aligned} U_4 &= T_2^{(1)} + T_1^{(3)} + T_1^{(5)} = \theta(z_0-w_0)\frac{z_0 w_0^5}{2^{16}3^3 5}\left(w_0^8 - 24 z_0^2 w_0^6 - 270 z_0^4 w_0^4 + 1420 z_0^6 w_0^2 + 645 z_0^8\right. \\ &\left. -60 z_0^4\left(3z_0^4 + 8w_0^2 z_0^2 + 3w_0^4\right)\ln\left(z_0^2 q^2\, \mathrm{e}^{2\gamma_E}/4\right)\right) \\ &+\theta(w_0-z_0)\frac{w_0 z_0^5}{2^{16}3^3 5}\left(z_0^8 - 24 w_0^2 z_0^6 + 270 w_0^4 z_0^4 + 1420 w_0^6 z_0^2 + 645 w_0^8\right. \\ &\left. -60 w_0^4\left(3w_0^4 + 8z_0^2 w_0^2 + 3z_0^4\right)\ln\left(w_0^2 q^2\, \mathrm{e}^{2\gamma_E}/4\right)\right)\,. \end{aligned} \quad \text{(C.12)}$$

# References


[1] E. A. Kuraev, L. N. Lipatov, and Victor S. Fadin. Multi - Reggeon Processes in the Yang-Mills Theory. *Sov. Phys. JETP*, 44:443–450, 1976.

[2] E. A. Kuraev, L. N. Lipatov, and Victor S. Fadin. The Pomeranchuk Singularity in Nonabelian Gauge Theories. *Sov. Phys. JETP*, 45:199–204, 1977.

[3] I. I. Balitsky and L. N. Lipatov. The Pomeranchuk Singularity in Quantum Chromodynamics. *Sov. J. Nucl. Phys.*, 28:822–829, 1978.

[4] V. N. Gribov. A Reggeon diagram technique. *Sov. Phys. JETP*, 26:414–422, 1968.

[5] Victor S. Fadin and L. N. Lipatov. BFKL pomeron in the next-to-leading approximation. *Phys. Lett.*, B429:127–134, 1998.

[6] Marcello Ciafaloni and Gianni Camici. Energy scale(s) and next-to-leading BFKL equation. *Phys. Lett.*, B430:349–354, 1998.

[7] G. Camici and M. Ciafaloni. Irreducible part of the next-to-leading BFKL kernel. *Phys. Lett.*, B412:396–406, 1997.

[8] Jochen Bartels. High-Energy Behavior in a Nonabelian Gauge Theory. 2. First Corrections to $T(n \to m)$ Beyond the Leading LNS Approximation. *Nucl. Phys.*, B175:365, 1980.

[9] J. Kwiecinski and M. Praszalowicz. Three Gluon Integral Equation and Odd c Singlet Regge Singularities in QCD. *Phys. Lett.*, B94:413, 1980.

[10] T. Jaroszewicz. Infrared Divergences and Regge Behavior in QCD. *Acta Phys. Polon.*, B11:965, 1980.

[11] L. N. Lipatov. High-energy asymptotics of multicolor QCD and exactly solvable lattice models. *DFPD-93-TH-70*, 1993.

[12] L. N. Lipatov. Asymptotic behavior of multicolor QCD at high energies in connection with exactly solvable spin models. *JETP Lett.*, 59:596–599, 1994.

[13] L. D. Faddeev and G. P. Korchemsky. High-energy QCD as a completely integrable model. *Phys. Lett.*, B342:311–322, 1995.

[14] Juan Martin Maldacena. The large N limit of superconformal field theories and supergravity. *Adv. Theor. Math. Phys.*, 2:231–252, 1998.

[15] Edward Witten. Anti-de Sitter space and holography. *Adv. Theor. Math. Phys.*, 2:253–291, 1998.

[16] S. S. Gubser, Igor R. Klebanov, and Alexander M. Polyakov. Gauge theory correlators from non-critical string theory. *Phys. Lett.*, B428:105–114, 1998.

[17] Jochen Bartels, A. De Roeck, and H. Lotter. The gamma* gamma* total cross section and the BFKL pomeron at e+ e- colliders. *Phys. Lett.*, B389:742–748, 1996.

[18] S. J. Brodsky, F. Hautmann, and D. E. Soper. Virtual photon scattering at high energies as a probe of the short distance pomeron. *Phys. Rev.*, D56:6957–6979, 1997.

[19] Simon Caron-Huot, Pavel Kovtun, Guy D. Moore, Andrei Starinets, and Laurence G. Yaffe. Photon and dilepton production in supersymmetric Yang- Mills plasma. *JHEP*, 12:015, 2006.

[20] J. Bartels, A.-M. Mischler, and M. Salvadore. Four point function of R-currents in N=4 SYM in the Regge limit at weak coupling. *Phys. Rev.*, D78:016004, 2008.

[21] Daniel Z. Freedman, Samir D. Mathur, Alec Matusis, and Leonardo Rastelli. Correlation functions in the CFT($d$)/AdS($d+1$) correspondence. *Nucl. Phys.*, B546:96–118, 1999.

[22] Eric D'Hoker, Daniel Z. Freedman, Samir D. Mathur, Alec Matusis, and Leonardo Rastelli. Extremal correlators in the AdS/CFT correspondence. *In *Shifman, M.A. (ed.): The many faces of the superworld* 332-360*, 1999.

[23] Gordon Chalmers, Horatiu Nastase, Koenraad Schalm, and Ruud Siebelink. R-current correlators in N = 4 super Yang-Mills theory from anti-de Sitter supergravity. *Nucl. Phys.*, B540:247–270, 1999.

[24] Massimo Bianchi, Maurizio Prisco, and Wolfgang Mueck. New results on holographic three-point functions. *JHEP*, 11:052, 2003.

[25] Daniel Z. Freedman, Samir D. Mathur, Alec Matusis, and Leonardo Rastelli. Comments on 4-point functions in the CFT/AdS correspondence. *Phys. Lett.*, B452:61–68, 1999.

[26] Eric D'Hoker and Daniel Z. Freedman. Gauge boson exchange in AdS(d+1). *Nucl. Phys.*, B544:612–632, 1999.

[27] Eric D'Hoker and Daniel Z. Freedman. General scalar exchange in AdS(d+1). *Nucl. Phys.*, B550:261–288, 1999.

[28] Eric D'Hoker, Daniel Z. Freedman, Samir D. Mathur, Alec Matusis, and Leonardo Rastelli. Graviton exchange and complete 4-point functions in the AdS/CFT correspondence. *Nucl. Phys.*, B562:353–394, 1999.

[29] Eric D'Hoker, Daniel Z. Freedman, and Leonardo Rastelli. AdS/CFT 4-point functions: How to succeed at z-integrals without really trying. *Nucl. Phys.*, B562:395–411, 1999.

[30] Eric D'Hoker, Johanna Erdmenger, Daniel Z. Freedman, and Manuel Perez-Victoria. Near-extremal correlators and vanishing supergravity couplings in AdS/CFT. *Nucl. Phys.*, B589:3–37, 2000.

[31] R. R. Metsaev and A. A. Tseytlin. Supersymmetric D3 brane action in AdS(5) x S**5. *Phys. Lett.*, B436:281–288, 1998.

[32] Wolfgang Mueck and K. S. Viswanathan. Conformal field theory correlators from classical field theory on anti-de Sitter space. II: Vector and spinor fields. *Phys. Rev.*, D58:106006, 1998.

[33] G. Arutyunov and S. Frolov. Four-point functions of lowest weight CPOs in N = 4 SYM(4) in supergravity approximation. *Phys. Rev.*, D62:064016, 2000.

[34] G. Arutyunov and Emery Sokatchev. On a large N degeneracy in N = 4 SYM and the AdS/CFT correspondence. *Nucl. Phys.*, B663:163–196, 2003.

[35] Massimo Bianchi, Daniel Z. Freedman, and Kostas Skenderis. How to go with an RG flow. *JHEP*, 08:041, 2001.

[36] Massimo Bianchi, Daniel Z. Freedman, and Kostas Skenderis. Holographic renormalization. *Nucl. Phys.*, B631:159–194, 2002.

[37] Oliver DeWolfe, Daniel Z. Freedman, and Hirosi Ooguri. Holography and defect conformal field theories. *Phys. Rev.*, D66:025009, 2002.

[38] D. Z. Freedman, S. S. Gubser, K. Pilch, and N. P. Warner. Renormalization group flows from holography supersymmetry and a c-theorem. *Adv. Theor. Math. Phys.*, 3:363–417, 1999.

[39] Oliver DeWolfe and Daniel Z. Freedman. Notes on fluctuations and correlation functions in holographic renormalization group flows. *MIT-CTP-2955*, 2000.

[40] Massimo Bianchi, Oliver DeWolfe, Daniel Z. Freedman, and Krzysztof Pilch. Anatomy of two holographic renormalization group flows. *JHEP*, 01:021, 2001.

[41] Kostas Skenderis. Lecture notes on holographic renormalization. *Class. Quant. Grav.*, 19:5849–5876, 2002.

[42] Ioannis Papadimitriou and Kostas Skenderis. Correlation functions in holographic RG flows. *JHEP*, 10:075, 2004.

[43] Ioannis Papadimitriou and Kostas Skenderis. AdS / CFT correspondence and geometry. *Published in *Strasbourg 2003, AdS/CFT correspondence* 73-101*, 2004.

[44] Joseph Polchinski and Matthew J. Strassler. The string dual of a confining four-dimensional gauge theory. *IAS-TH-00-18, NSF-ITP-00-16*, 2000.

[45] Joseph Polchinski and Matthew J. Strassler. Hard scattering and gauge/string duality. *Phys. Rev. Lett.*, 88:031601, 2002.

[46] Joseph Polchinski and Matthew J. Strassler. Deep inelastic scattering and gauge/string duality. *JHEP*, 05:012, 2003.

[47] Richard C. Brower, Joseph Polchinski, Matthew J. Strassler, and Chung-I Tan. The Pomeron and Gauge/String Duality. *JHEP*, 12:005, 2007.

[48] Richard C. Brower, Matthew J. Strassler, and Chung-I Tan. On the Eikonal Approximation in AdS Space. *JHEP*, 03:050, 2009.

[49] Y. Hatta, E. Iancu, and A. H. Mueller. Deep inelastic scattering off a N=4 SYM plasma at strong coupling. *JHEP*, 01:063, 2008.

[50] Y. Hatta, E. Iancu, and A. H. Mueller. Deep inelastic scattering at strong coupling from gauge/string duality : the saturation line. *JHEP*, 01:026, 2008.

[51] Yoshitaka Hatta. Relating $e^+e^-$ annihilation to high energy scattering at weak and strong coupling. *JHEP*, 11:057, 2008.

[52] Lorenzo Cornalba, Miguel S. Costa, Joao Penedones, and Ricardo Schiappa. Eikonal approximation in AdS/CFT: Conformal partial waves and finite N four-point functions. *Nucl. Phys.*, B767:327–351, 2007.

[53] Lorenzo Cornalba, Miguel S. Costa, Joao Penedones, and Ricardo Schiappa. Eikonal approximation in AdS/CFT: From shock waves to four-point functions. *JHEP*, 08:019, 2007.

[54] Lorenzo Cornalba, Miguel S. Costa, and Joao Penedones. Eikonal Approximation in AdS/CFT: Resumming the Gravitational Loop Expansion. *JHEP*, 09:037, 2007.

[55] Lorenzo Cornalba, Miguel S. Costa, and Joao Penedones. Eikonal Methods in AdS/CFT: BFKL Pomeron at Weak Coupling. *JHEP*, 06:048, 2008.

[56] Gerard 't Hooft. Graviton Dominance in Ultrahigh-Energy Scattering. *Phys. Lett.*, B198:61–63, 1987.

[57] M. Cvetic et al. Embedding AdS black holes in ten and eleven dimensions. *Nucl. Phys.*, B558:96–126, 1999.

[58] Eric D'Hoker, Daniel Z. Freedman, Samir D. Mathur, Alec Matusis, and Leonardo Rastelli. Graviton and gauge boson propagators in AdS(d+1). *Nucl. Phys.*, B562:330–352, 1999.

[59] C. Fronsdal. Elementary particles in a curved space. ii. *Phys. Rev.*, D10:589–598, 1974.

[60] C. P. Burgess and C. A. Lutken. Propagators and effective potentials in Anti-de Sitter space. *Phys. Lett.*, B153:137, 1985.

[61] Takeo Inami and Hirosi Ooguri. One loop effective potential in Anti-de Sitter space. *Prog. Theor. Phys.*, 73:1051, 1985.

[62] Christopher J. C. Burges, Daniel Z. Freedman, S. Davis, and G. W. Gibbons. Supersymmetry in anti-de Sitter space. *Ann. Phys.*, 167:285, 1986.

[63] D. Amati, M. Ciafaloni, and G. Veneziano. Superstring Collisions at Planckian Energies. *Phys. Lett.*, B197:81, 1987.