On Inflation in the Presence of a Gaugino Condensate

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Abstract

We study the effect of inflation on gaugino condensation in supergravity. Unless the Hubble scale H is significantly below the gaugino condensation scale, the gaugino condensate is a dynamical variable which cannot be integrated out. For a sufficiently high H, the gaugino condensate evolves to zero which in turn leads to dilaton/moduli destabilization. In practice, this often occurs at the Hubble rate about an order of magnitude below the gaugino condensation scale. This effect is independent of the specifics of moduli stabilization and thus places model independent constraints on inflationary scenarios. It also applies more generally to any periods of fast expansion in the early Universe.

1   Introduction

Gaugino condensation [1] is arguably the most attractive mechanism for creating the hierarchy between the Planck and electroweak (EW) scales [2]-[5]. Starting with a perturbative gauge coupling at the Planck or string scale, a new scale Λ, at which the corresponding gauge group becomes strongly coupled, is created by dimensional transmutation. If these dynamics break supersymmetry, the EW scale can be generated as $\Lambda^3/M_{\rm Pl}^2$ when supersymmetry breaking is communicated by gravity to the observable sector. This idea finds support in explicit string models which produce the (exact) spectrum of the minimal supersymmetric Standard Model (MSSM) [6].

If gaugino condensation is indeed part of reality, it must be combined with inflation [7], which has by now gathered strong observational evidence. The purpose of this paper is to study the dynamics of the gaugino condensate with the background of inflation or, more generally, in the presence of large positive vacuum energy. Some facets of this problem have been considered before. Gaugino condensation generates a potential for moduli, which is modified in the early Universe during inflation and reheating. To ensure that moduli do not run away, certain constraints on the reheating temperature [8] and the Hubble parameter must be satisfied. In the latter case, the situation is model dependent and only particular (Kachru-Kallosh-Linde-Trivedi-type [9]) scenarios with a specific choice of the inflaton have been studied [10]. In the present work, we approach this problem from a more general perspective based on properties of the gaugino condensate itself and without specializing to a particular inflationary model or moduli stabilization mechanism. This allows us to obtain a (largely) model independent constraint on the Hubble rate during inflation or any period of fast expansion.

2   Veneziano–Yankielowicz potential in supergravity

In the Veneziano–Yankielowicz approach [1], gaugino condensation is described in terms of the chiral superfield $U = {\rm Tr}\, W^\alpha W_\alpha$, with W_α being the gauge multiplet superfield which contains the gaugino as its lowest component. The low energy effective action for this field is derived using symmetries of the system and anomaly cancellation. In supergravity, the

resulting Kähler potential $\mathcal{K}$ and the superpotential W are given by [11]

$$\mathcal{K} = -3\log\left[e^{-K/3} - a(UU^*)^{1/3}\right] ,$$

$$W = \frac{1}{4}\, U\left(fS + \frac{2c}{3}\log(\xi U)\right) + \tilde{W} . \tag{1}$$

Here we have taken the gauge kinetic function to be given by the dilaton S[1]; K and $\tilde{W}$ are the Kähler potential and the superpotential for other fields of the system apart from U; a, f, ξ are constants of order one and c is the beta function coefficient

$$c = \frac{3}{16\pi^2}\, C(G) , \tag{2}$$

with $C(G)$ being the quadratic Casimir of the condensing gauge group G.

We will be studying the dynamics of the condensate in the region of physical interest $U \ll 1$ in Planck units. Changing variables $U \equiv u^3/(3a)^{3/2}$ and expanding the result in powers of u, we get

$$\mathcal{K} = K + e^{K/3}uu^* + \frac{1}{6}\, e^{2K/3}(uu^*)^2 + ... ,$$

$$W = u^3(S + 2c\log u) + \tilde{W} , \tag{3}$$

where, for simplicity, we have set $4(3a)^{3/2} = 1$, $f = 1$ and $\xi = (3a)^{3/2}$.

The supergravity scalar potential for this system is given by

$$V = e^G(G_i G_{\bar{j}} G^{i\bar{j}} - 3) . \tag{4}$$

Here the subscript l ($\bar{l}$) denotes differentiation with respect to the l-th (l-th complex conjugate) scalar field; G is a function of the Kähler potential $\mathcal{K}$ and superpotential W: $G = \mathcal{K} + \ln(|W|^2)$, and $G^{i\bar{j}}$ is the inverse of $G_{\bar{j}i}$.

To understand the Veneziano–Yankielowicz result, let us start with $\tilde{W} = 0$. At $u \ll 1$, the dominant contribution to the potential is given by

$$V \simeq e^G|G_u|^2 G^{uu^*} . \tag{5}$$

The stationary points of this function are at $W = 0, W_u = 0$ and $W_{uu} = 0$. The usual Veneziano–Yankielowicz solution corresponds to $W_u = 0$:

$$u_{\min} = e^{-\frac{S}{2c}-\frac{1}{3}} , \tag{6}$$

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[1]We are neglecting threshold corrections to the gauge coupling.

which describes a supersymmetric vacuum with massive excitations. (More precisely, for an $SU(N)$ group there are N vacua which differ by a phase factor in u; this, however, is not important for our purposes.) The solution to $W_{uu} = 0$ is a local maximum at

$$u_{\max} = e^{-\frac{S}{2c} - \frac{5}{6}} \, . \tag{7}$$

Finally, $u = 0$ formally corresponds to another supersymmetric chirally invariant vacuum, Fig. 1. However, the Veneziano–Yankielowicz potential cannot be trusted at $u \to 0$. The existence of a supersymmetric chirally invariant vacuum is not allowed by general considerations [12] and also inconsistent with the Witten index theorem [13] (see also [14]). The interpretation of the state at $u = 0$ remains controversial and it has been conjectured that it corresponds to a non-supersymmetric (unstable) state [15][2]. In any case, the Veneziano–Yankielowicz potential is trustable around the SUSY minimum (6) and since the local maximum is close to it, $u_{\max} = u_{\min}/\sqrt{e}$, the existence of a potential barrier between the SUSY vacuum and some other state at $u \ll u_{\min}$ is also expected to be reliable.

In the SUSY vacuum (6), the gaugino condensate corresponds to a heavy field and can be integrated out. This creates an effective superpotential for the dilaton, which is a necessary ingredient for addressing the problem of dilaton/moduli stabilization. However, in the early Universe this procedure is not always consistent: if the expansion rate of the Universe is close to or greater than the gaugino condensation scale, the condensate remains a dynamical field whose evolution has to be taken into account. To address this issue, in the next section we study the behavior of the condensate in the presence of large vacuum energy.

3   Inclusion of an inflaton

Consider the system of the dilaton, gaugino condensate and an extra field ϕ which generates large vacuum energy ("inflaton"). This system can be described by Eq.(1), and consequently Eq.(3), with

$$
\begin{aligned}
K &= K(S) + K(\phi) \, , \\
\tilde{W} &= \tilde{W}(\phi) \, ,
\end{aligned}
\tag{8}
$$

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[2]We are grateful to M. Shifman for clarifying this point.

where $\tilde{W}(\phi) \gg u^3(S + 2c \log u)$. Since $u \ll 1$, one can expand the scalar potential in powers of u. Including terms up to forth order, we get

$$
\begin{aligned}
V &= V_0 + \frac{2}{3} e^{K/3} V_0 \, uu^* \\
&+ \left(e^K \tilde{W}^* \left[K_{S\bar{S}}^{-1} K_{\bar{S}} - \frac{2c}{3} \left(K_{S\bar{S}}^{-1} |K_S|^2 + K_{\phi\bar{\phi}}^{-1} |K_\phi|^2 + K_{\phi\bar{\phi}}^{-1} K_\phi W_\phi^* / \tilde{W}^* - 3 \right) \right] u^3 + \text{h.c.} \right) \\
&+ e^{2K/3} \left(\left| u^2 (3S + 6c \log u + 2c) \right|^2 + \frac{1}{3} V_0 \, (uu^*)^2 \right) + ...
\end{aligned} \tag{9}
$$

Here the vacuum energy is given by

$$
V_0 = e^K \left(K_{S\bar{S}}^{-1} |\tilde{W} K_S|^2 + K_{\phi\bar{\phi}}^{-1} |\tilde{W} K_\phi + W_\phi|^2 - 3|\tilde{W}|^2 \right) . \tag{10}
$$

We see that the condensate receives mass of order the Hubble scale ($V_0 = 3H^2$) as expected from general considerations [16]. The $\mathcal{O}(u^3)$ contribution is an analog of the A–term, while the $\mathcal{O}(u^4)$ terms include the Veneziano–Yankielowicz potential $|W_u|^2$ and an extra contribution proportional to V_0. Note that the potential for the canonically normalized condensate is obtained by the rescaling $u = e^{-K/6} \tilde{u}$.

For a sufficiently large H, the mass term will dominate and the condensate will quickly evolve to zero, $\tilde{u} \sim e^{-Ht} \tilde{u}_0$. This is intuitively clear since the Hubble expansion is analogous to "heating up" the condensate to temperature of order H (see, e.g. [17]). To determine the critical expansion rate, we need to find V_0 at which the Veneziano–Yankielowicz minimum disappears. A sufficient condition for the absence of local extrema (apart from $u = 0$) is that the curvature of the potential in the u, u^* direction be non-negative,

$$
V_{u\bar{u}}^2 - V_{uu} V_{\bar{u}\bar{u}} \geq 0 . \tag{11}
$$

We are interested in the case when the vacuum energy V_0 is dominated by the inflaton F-term,

$$
F^\phi \gg F^S , \tag{12}
$$

where $F^i = e^{G/2} K^{i\bar{j}} G_{\bar{j}}$, which corresponds to domination of the second term in (10). Then the $\mathcal{O}(u^3)$ contribution in Eq.(9) is (up to a phase)

$$
-\frac{2c}{3} \sqrt{V_0} \, e^{K/2} K_{\phi\bar{\phi}}^{-1/2} K_\phi \, u^3 . \tag{13}
$$

Consider first the case $K_{\phi\bar{\phi}}^{-1/2} K_\phi \leq \mathcal{O}(1)$. Then the cubic term is suppressed by the loop factor c. Further, the $\mathcal{O}(u^4)$ term proportional to V_0 is small compared to the Veneziano–Yankielowicz piece $|W_u|^2$ and can be neglected. As a result, the non-negative curvature

condition amounts approximately to

$$\frac{2}{3} e^{K/3} V_0 + e^{2K/3} |W_{uu}|^2 - e^{2K/3} |W_{uuu} W_u| \geq 0 \ . \tag{14}$$

Although this inequality cannot be solved exactly, one can estimate the critical $V_0 = 3H_{\mathrm{crit}}^2$ by requiring non-negative curvature at the local maximum of the Veneziano–Yankielowicz potential u_{max}, where $W_{uu} = 0$. For a canonically normalized $\tilde{u} = e^{K/6} u$, we then have

$$H_{\mathrm{crit}} \sim c \ |\tilde{u}_{\mathrm{max}}| \ . \tag{15}$$

This agrees with our numerical results. Note that the cubic term in (9) is at most $\mathcal{O}(cH\tilde{u}^3)$, while the quadratic and the Veneziano–Yankielowicz pieces around u_{max} are $\mathcal{O}(H^2\tilde{u}^2)$ and $\mathcal{O}(c^2\tilde{u}^4)$, respectively, such that for $H > c \ |\tilde{u}_{\mathrm{max}}|$ the quadratic term dominates.

For $K_{\phi\bar{\phi}}^{-1/2} K_\phi \gg 1$, the cubic term is important and Eq.(11) gives

$$H_{\mathrm{crit}} \sim c \ \left| K_{\phi\bar{\phi}}^{-1/2} K_\phi \ \tilde{u}_{\mathrm{max}} \right| \ . \tag{16}$$

The potential for the gaugino condensate in the presence of positive vacuum energy is illustrated in Fig. 1.
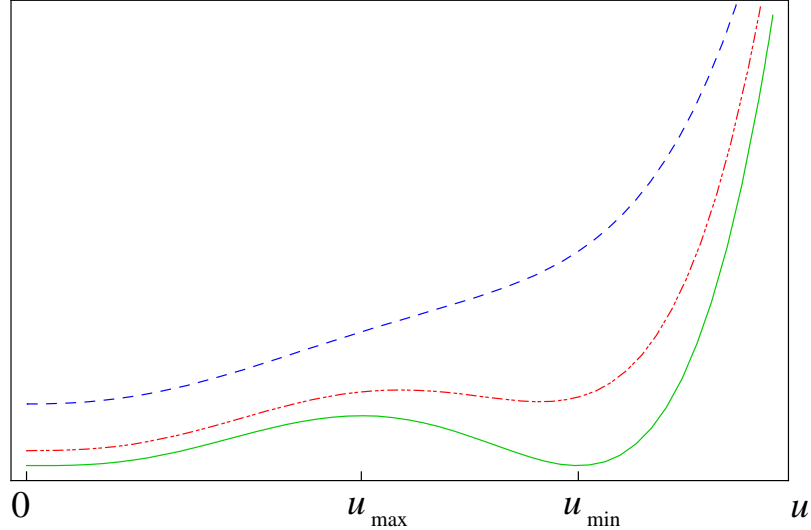

Figure 1: The gaugino condensate potential as a function of vacuum energy. The solid (green) curve represents the Veneziano–Yankielowicz potential ($H = 0$), while the dotted (red) and the dashed (blue) curves correspond to $H < H_{\mathrm{crit}}$ and $H \gtrsim H_{\mathrm{crit}}$, respectively.

4 Discussion and generalizations

We find that, unless $K_{\phi\bar{\phi}}^{-1/2} K_\phi \gg 1$, the Veneziano–Yankielowicz minimum disappears at the Hubble rate a loop factor below the gaugino condensation scale. This is natural as the features of the Veneziano–Yankielowicz potential are due to the loop–induced term $2c \log u$ and, consequently, the curvature of the potential at the maximum is loop–suppressed. In phenomenologically interesting cases, the condensing gauge group is of intermediate size, e.g. SU(5), such that $c = \mathcal{O}(10^{-1})$ and the critical Hubble rate is about an order of magnitude below the gaugino condensation scale.

This means that for $H > H_{\text{crit}}$ the condensate will evolve to zero within a few Hubble times. Consequently, the dilaton superpotential approaches a constant and the dilaton potential attains a run–away form

$$e^K \times \text{const} \tag{17}$$

with the constant of order H^2. The dilaton will thus quickly (within a few Hubble times) evolve to weak coupling $S \gg 1$. Needless to say, this scenario is phenomenologically unacceptable and the constraint $H < H_{\text{crit}}$ must be satisfied.[3]

This conclusion applies regardless of the specifics of the dilaton stabilization mechanism. Indeed, the essential ingredient for dilaton stabilization is the superpotential due to gaugino condensation which becomes unavailable at $H > H_{\text{crit}}$. Consider, for example, the Kähler stabilization scheme [18],[19] with $K(S) = -\log(S + \bar{S}) + \Delta K_{\text{np}}(S)$. The local minimum in S is obtained due to cancellations between the Kähler corrections and the gaugino condensation superpotential. Since the latter is not available as the gaugino condensate evaporates, generically no local minima appear during inflation and the dilaton runs away. Further, in the racetrack models [20],[21] one sums over different condensates in Eq.(1):

$$a(UU^*)^{1/3} \rightarrow \sum_i a_i (U_i U_i^*)^{1/3}$$

$$\frac{1}{4} U \left(fS + \frac{2c}{3} \log(\xi U) \right) \rightarrow \frac{1}{4} \sum_i U_i \left(f_i S + \frac{2c_i}{3} \log(\xi_i U_i) \right) . \tag{18}$$

Each one of them attains mass of order H^2 during inflation and is destabilized at the Hubble rate greater than the largest H_{crit} (in fact, a local minimum in S disappears even if only

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[3]Note also that as H decreases, u will settle in the "wrong" vacuum which is separated from the Veneziano–Yankielowicz minimum by a large potential barrier.

some of the condensates evaporate). The potential becomes $\sim H^2/(S + \bar{S})$ and the dilaton runs away.

It is important to note that our result applies not only to inflation but more generally to any periods of fast expansion in the early Universe. This is because the slow roll condition is not essential and the time scale for the evolution of u and S is given by a few Hubble times. Also, an extension to multiple inflatons $K(\phi) \to K(\phi_i), \tilde{W}(\phi) \to \tilde{W}(\phi_i)$ is straightforward.

Finally, we have taken the gauge kinetic function to be given by the dilaton. This can readily be generalized to other cases, e.g. in KKLT-type models [9] one replaces $S \to T$ with T being the Kähler modulus,

$$
K(T) = -3 \, \log(T + \bar{T}) \, ,
$$
$$
W = u^3(T + 2c \log u) + \tilde{W} \, , \tag{19}
$$

where $\tilde{W} = W_0 + \tilde{W}(\phi)$ and W_0 is the constant superpotential used to stabilize T. To have low energy supersymmetry, this constant must be adjusted to be very small, $\mathcal{O}(10^{-13})$. Again, for large positive vacuum energy, the gaugino condensate acquires mass and quickly evolves to zero, which leads to disastrous consequences. This happens regardless of the details of the "uplifting" mechanism which adjusts the vacuum energy after inflation.

Let us now discuss our main assumptions. To establish evaporation of the gaugino condensate at high H, we have relied (1) on the shape of the Veneziano–Yankielowicz potential around the SUSY minimum, which is quite reliable, and (2) on the Kähler potential of the form $(UU^*)^{1/3}$ for small U. The latter is in fact not necessary and our conclusion would hold more generally for Kähler potentials which can be brought to the canonical form by a change of variables $U \to f(U)$ with $f(0) = 0$ (and non-singular scalar potential). In this case, the inflation–induced mass term is positive and the condensate evolves to zero. This fits the intuitive picture that the gaugino condensate vanishes at high de Sitter temperature.

We have also assumed that inflation is driven by the inflaton ϕ and the dilaton does not play any significant role in it. If this is not the case, $F^\phi \sim F^S$, the $\mathcal{O}(u^3)$ term proportional to $K_{S\bar{S}}^{-1}K_{\bar{S}}$ in Eq.(9) becomes important and can generate a local minimum at $u > 0$ during inflation. Then the gaugino condensate will evaporate only for $H \gg u_{\max}$. In this case, inflation does not amount to a background for the evolution of the condensate since there is significant superpotential interaction between the dilaton and the condensate.

Let us remark that realistic vacua with broken supersymmetry after inflation are possible due to the presence of extra fields in Eq.1. In this case, the constraints of Ref. [22]

can be satisfied, for example, when SUSY breaking is dominated by matter–like fields [23]. (Alternatively, one can use non-perturbative corrections to the Kähler potential [18],[19].) Once a dilaton stabilization mechanism is employed, the VY vacuum corresponds to a local minimum in the $S - U$ plane, so the usual procedure of integrating out the condensate is justified. The critical Hubble rate for a specific model depends on which field gets destabilized first. For instance, S can run away to weak coupling $S \to \infty$ due to positive vacuum energy. Thus, during inflation, the dilaton may be destabilized before U is, depending on the size of the barrier separating the local minimum in the S direction from the run-away minimum. For instance, in the Kallosh-Linde model [10] this barrier is large, so U is destabilized before S is, while in the usual racetrack model it is the dilaton that gets destabilized first. In any case, the model–independent bound derived in this paper applies regardless of the moduli stabilization mechanism.

Finally, a comment on loop corrections is in order. In the vacuum, supersymmetry (and R-symmetry) is broken, so one expects SUSY breaking loop corrections. These are governed by $m_{3/2}^2/16\pi^2$ and can only be relevant to the dilaton direction, whose mass is $\geq \mathcal{O}(m_{3/2})$ (see e.g. [8]). Such corrections are subleading and since the precise value of the dilaton mass is unimportant for our purposes, we neglect these effects.

5 Conclusion

We have studied the behavior of the gaugino condensate in the presence of large vacuum energy. If the expansion rate of the Universe is close to or higher than the gaugino condensation scale, the condensate cannot be integrated out. We find that for Hubble rates above a critical value, the gaugino condensate evolves to zero which leads to dilaton/moduli destabilization. When the vacuum energy is dominated by the inflaton (ϕ) other than the field (S) producing the gauge coupling for the condensing gauge group, the critical Hubble rate is given by

$$H_{\mathrm{crit}} \sim \max\{1, \left|K_{\phi\bar{\phi}}^{-1/2} K_\phi\right|\} \, c \, |\tilde{u}_{\mathrm{max}}| \tag{20}$$

with $\tilde{u}_{\mathrm{max}} = e^{K/6 - S/(2c) - 5/6}$ and c being the one loop beta function coefficient. Thus, it is typically an order of magnitude below the corresponding gaugino condensation scale. This result is independent of the specifics of moduli stabilization and thus provides a useful constraint on inflationary models. It also applies more generally to any periods of fast expansion in the early Universe.

It would be interesting to further study the cosmological evolution of the dilaton-gaugino condensate system including a background matter or radiation component, to determine which initial conditions lead to dilaton stabilization [24].

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