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Medium-modified average multiplicity and multiplicity fluctuations in jets

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Abstract: The energy evolution of average multiplicities and multiplicity fluctuations in jets produced in heavy-ion collisions is investigated from a toy QCD-inspired model. In this model, we use modified splitting functions accounting for medium-enhanced radiation of gluons by a fast parton which propagates through the quark gluon plasma. The leading contribution of the standard production of soft hadrons is enhanced by a factor $\sqrt{N_s}$ while next-to-leading order (NLO) corrections are suppressed by $1/\sqrt{N_s}$, where the parameter $N_s > 1$ accounts for the induced-soft gluons in the medium. Our results for such global observables are cross-checked and compared with their limits in the vacuum.

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Recent experiments at the Relativistic Heavy Ion Collider (RHIC) have established a phenomenon of strong high-transverse momentum hadron suppression [1], which supports the picture that hard partons going through dense matter suffer a significant energy loss prior to hadronization in the vacuum (for recent review see [2]).

Predictions concerning multi-particle production in nucleus-nucleus collisions can be carried out by using a toy QCD-inspired model introduced by Borghini and Wiedemann in [3]; it allows for analytical computations and may capture some important features of a more complete QCD description. In this model, the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) splitting functions $q \to g\bar{q}$ and $g \to gg$ [4] of the QCD evolution equations were distorted so that the role of soft emissions was enhanced by multiplying the infra-red singular terms by the medium factor N_s. The model [3] was further discussed and used on the description of final states hadrons produced in heavy-ion collisions [5].

Within the model, we make predictions for the medium-modified average multiplicity N_A in quark and gluon jets ($A = q, g$) produced in such reactions, for the ratio $r = N_g/N_q$ and finally for the second multiplicity correlators $\langle N_A(N_A-1)\rangle/N_A^2$, which determines the width of the multiplicity distribution.

The starting point of our analysis is the NLO or Modified-Leading-Logarithmic-Approximation (MLLA) master evolution equation for the generating functional [4] which determine the jet properties at all energies together with the initial conditions at threshold at small x, where x is the fraction of the outgoing jet energy carried away by a single gluon. Their solutions with medium-modified splitting functions can be resummed in powers of $\sqrt{\alpha_s/N_s}$ and the leading contribution can be represented as an exponential of the medium-modified anomalous dimension which takes into account the N_s-dependence:

$$N_A \simeq \exp\left\{\int^Y \gamma_{\text{med}}\left(\alpha_s(Y)\right) dY\right\}, \qquad (1)$$

where $\gamma_{\text{med}}(\alpha_s)$ can be expressed as a power series of $\sqrt{\alpha_s/N_s}$ in the symbolic form:

$$\gamma_{\text{med}}\left(\alpha_s\right) \simeq \sqrt{N_s} \times \sqrt{\alpha_s}\left(1+\sqrt{\frac{\alpha_s}{N_s}}+\mathcal{O}\left(\frac{\alpha_s}{N_s}\right)\right).$$

Within this logic, the leading double logarithmic approximation (DLA, $\mathcal{O}(\sqrt{N_s\alpha_s})$), which resums both soft and collinear gluons, and NLO (MLLA, $\mathcal{O}(\alpha_s)$), which resums hard collinear partons and accounts for the running of the coupling constant α_s, are complete. The choice $dY = d\Theta/\Theta$, where $\Theta \ll 1$ is the angle between outgoing couples of partons in independent partonic emissions, follows from Angular Ordering (AO) in intra-jet cascades [4]. In order to obtain the hadronic spectra, we advocate for the Local Parton Hadron Duality (LPHD) hypothesis [6]: global and differential partonic observables can be normalized to the corresponding hadronic observables via a certain constant $\mathcal{K}$ that can be fitted to the data, i.e. $N_{g,q}^h = \mathcal{K} \times N_{g,q}$.

The evolution of a jet of energy E and half-opening angle Θ involves the DLA anomalous dimension γ_0 related to the coupling constant α_s through $\gamma_0^2 = 2N_c\alpha_s/\pi$, with $\alpha_s = 2\pi/4N_c\beta_0(Y+\lambda)$, where $Y = \ln(Q/Q_0)$ ($Q = E\Theta$ is the hardness or maximum transverse momentum of the jet), $\lambda = \ln(Q_0/\Lambda_{_{\rm QCD}})$ is a parameter associated with hadronization (Q_0 is the collinear cut-off parameter, $k_T > Q_0$, and $\Lambda_{_{\rm QCD}}$ is the intrinsic QCD scale) and $\beta_0 = \frac{1}{4N_c}\left(\frac{11}{3}N_c - \frac{4}{3}T_R\right)$, where $T_R = n_f/2$, n_f being the number of active flavors. At MLLA, as a consequence of angular ordering in parton cascading, the average multiplicity inside a gluon and a quark jet, $N_{g,q}$, obey the system of two-coupled evolution equations [7]

(the subscript $_Y$ denotes d/dY)

$$N_{g_Y} = \int_0^1 dx\, \gamma_0^2 \left[\Phi_g^g \left(N_g(x) + N_g(1-x) - N_g\right) + n_f \Phi_g^q \left(N_q(x) + N_q(1-x) - N_g\right)\right], \tag{2}$$

$$N_{q_Y} = \int_0^1 dx\, \gamma_0^2 \big[\Phi_q^g \left(N_g(x) + N_q(1-x) - N_q\right)\big], \tag{3}$$

which follow from the MLLA master evolution equation for the generating functional; $N_{g,q} \equiv N_{g,q}(Y)$, $N_{g,q}(x) \equiv N_{g,q}(Y + \ln x)$, $N_{g,q}(1-x) \equiv N_{g,q}(Y + \ln(1-x))$, Φ_A^B denotes the medium-modified DGLAP splitting functions:

$$\begin{aligned}
\Phi_g^g(x) &= \frac{N_s}{x} - (1-x)[2 - x(1-x)], \\
\Phi_g^q(x) &= \frac{1}{4N_c}[x^2 + (1-x)^2], \\
\Phi_q^g(x) &= \frac{C_F}{N_c}\left(\frac{N_s}{x} - 1 + \frac{x}{2}\right),
\end{aligned} \tag{4}$$

which accounts for parton energy loss in the medium by enhancing the singular terms like $\Phi \approx N_s/x$ as $x \ll 1$ as proposed in the Borghini-Wiedemann model [3]. Thus, when N_s increases the DLA becomes dominant and energy-momentum conservation plays a less important role.

For $Y \gg \ln x \sim \ln(1-x)$, $N_{g,q}(x)$ ($N_{g,q}(1-x)$) can be replaced by $N_{g,q}$ in the hard partonic splitting region $x \sim 1-x \sim 1$ (non-singular or regular parts of the splitting functions), while the dependence at small $x \ll 1$ is kept in the singular term $\Phi(x) \approx N_s/x$ as done in the vacuum. Furthermore, the integration over x can be replaced by the integration over $Y(x) = \ln\left(\frac{xE\Theta}{Q_0}\right)$. Thus, one is left with the approximate system of two-coupled equations,

$$\frac{d^2}{dY^2} N_g(Y) = \gamma_0^2 \left(N_s - a_1 \frac{d}{dY}\right) N_g(Y), \tag{5}$$

$$\frac{d^2}{dY^2} N_q(Y) = \frac{C_F}{N_c}\gamma_0^2 \left(N_s - \tilde{a}_1 \frac{d}{dY}\right) N_g(Y), \tag{6}$$

with the initial conditions at threshold $N_A(0) = 1$ and $N_A^{'}(0) = 0$ and the hard constants:

$$a_1 = \frac{1}{4N_c}\left[\frac{11}{3}N_c + \frac{4}{3}T_R\left(1 - 2\frac{C_F}{N_c}\right)\right], \quad \tilde{a}_1 = 3/4.$$

The quantum corrections $\propto a_1,\ \tilde{a}_1$ in (5,6) arise from the integration over the regular part of the splitting functions, they are $\mathcal{O}(\sqrt{\alpha_s/N_s})$ suppressed and *partially* account for energy conservation as happens in the vacuum.

These equations can be solved by applying the inverse Mellin transform to the self-contained gluonic equation (5), which leads to

$$N_g^h(Y) \simeq \mathcal{K} \times \int_C \frac{d\omega}{2\pi i} \omega^{\frac{a_1}{\beta_0} - 2} \exp\left[\omega(Y + \lambda) + \frac{N_s}{\beta_0 \omega}\right], \tag{7}$$

where the contour C lies to the right of all singularities of $N_g(\omega)$ in the complex plane. Since we are concerned with the asymptotic solution of the equation as $Y \gg 1$ ($E\Theta \gg Q_0$), that is the high-energy limit, the inverse Mellin transform (7) can be estimated by the steepest descent method. Indeed, the large

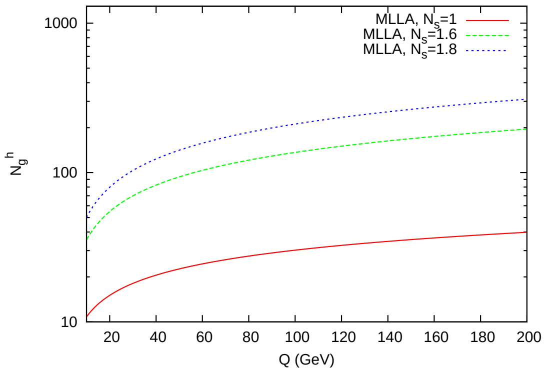

Figure 1: MLLA (8) medium-modified average multiplicity as a function of $Q = E\Theta$ in the vacuum ($N_s = 1$) and in the medium ($N_s = 1.6$ and $N_s = 1.8$) for $n_f = 3$.

parameter is Y and the function in the exponent presents a saddle point at $\omega_0 = \sqrt{N_s/\beta_0(Y+\lambda)}$, such that the asymptotic solution reads

$$N_g^h(Y) \simeq \mathcal{K} \times (Y+\lambda)^{-\frac{\sigma_1}{\beta_0}} \exp\sqrt{\frac{4N_s}{\beta_0}(Y+\lambda)}, \tag{8}$$

where $\sigma_1 = \frac{a_1}{2} - \frac{\beta_0}{4}$. The constant σ_1 is N_s-independent because it resums vacuum corrections. Therefore, the production of soft gluons in the medium becomes $\exp\left[2(\sqrt{N_s}-1)\sqrt{(Y+\lambda)/\beta_0}\right]$ higher than the standard production of soft gluons in the vacuum [4]. From (1) and (8) one obtains the medium-modified MLLA anomalous dimension $\gamma_{\rm med} = \frac{1}{N_g}\frac{dN_g}{dY} = \sqrt{N_s}\gamma_0 - \sigma_1\gamma_0^2$, which is nothing but the MLLA rate of multi-particle production with respect to the *evolution-time* variable Y in the dense medium. In Fig. 1, we display the medium-modified average multiplicity (8) with predictions in the vacuum ($N_s = 1$) in the range $10 \leq Q(\text{GeV}) \leq 200$; we set $Q_0 = \Lambda_{_{\rm QCD}} = 0.23$ GeV in the limiting spectrum approximation [7], and take $\mathcal{K} = 0.2$ from [7]. The values $N_s = 1.6$ and $N_s = 1.8$ in the medium may be realistic for RHIC and LHC phenomenology [3, 5]; the jet energy subrange $10 \leq Q(\text{GeV}) \leq 50$ displayed in Fig. 1 has been recently considered by the STAR collaboration, which reported the first measurements of charged hadrons and particle-identified fragmentation functions from p+p collisions [8] at $\sqrt{s_{\rm NN}} = 200$ GeV. Finally, the whole jet energy range in the same figure, in particular for those values at $Q \geq 50$ GeV, will be reached at the LHC, i.e $Q = 100$ GeV is an accessible value in this experiment (see [3] and references therein).

We find, as expected, that the production of soft hadrons increases as $N_s > 1$: the available phase space for the production of harder collinear hadrons is restricted as the model itself states. The medium-modified MLLA gluon to quark average multiplicity ratio, $r = N_g/N_q = N_g^h/N_q^h$, following from (8) and (3) reads

$$r = r_0\left[1 - r_1\frac{\gamma_0}{\sqrt{N_s}} + \mathcal{O}\left(\frac{\gamma_0^2}{N_s}\right)\right], \; r_0 = \frac{N_c}{C_F}, \tag{9}$$

where we introduced the coefficient $r_1 = a_1 - \tilde{a}_1$ in the term suppressed by $\gamma_0/\sqrt{N_s}$ as $N_s > 1$. Therefore, if compared with its behavior at $N_s = 1$, we check, as expected from the model [3], that r becomes closer to its asymptotic DLA limit $r_0 = N_c/C_F = 9/4$, as depicted in Fig. 2. Setting $N_s = 1$ in (9), one recovers the appropriate limits in the vacuum [4, 9, 10]. Finally, the gluon jets are still more

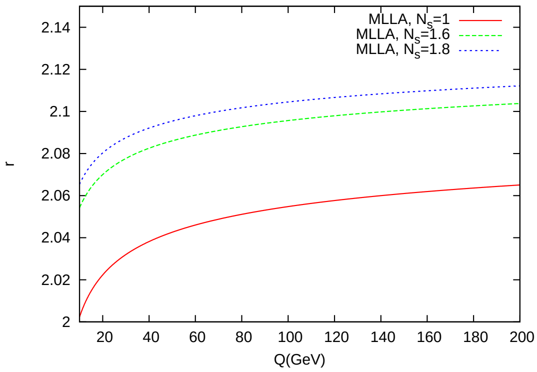

Figure 2: MLLA ratio r (9) as a function of $Q = E\Theta$ in the vacuum ($N_s = 1$) and in the medium ($N_s = 1.6$ and $N_s = 1.8$) for $n_f = 3$.

active than the quark jets in producing secondary particles and the shape of the curves are roughly the same.

The normalized second multiplicity correlator $A_2 = \langle N_A(N_A - 1)\rangle / N_A^2$ defines the width of the multiplicity distribution and is related to its dispersion by the formula $D_A^2 = (A_2 - 1)N_A^2 + N_A$ [9]. These moments, which are less inclusive than the average multiplicity, prove to be $\mathcal{K}$-independent and therefore provide a pure test of multiparticle production. The medium-modified system of two-coupled evolution equations for this observable follows from the MLLA master equation for the azimuthally averaged generating functional [4] and can be written in the convenient form

$$\begin{aligned}\frac{d}{dY}(N_g^{(2)} - N_g^2) = \int_0^1 dx\gamma_0^2\Phi_g^g\Big[N_g^{(2)}(Y+\ln x) + \Big(N_g^{(2)}(Y+\ln(1-x)) - N_g^{(2)}(Y)\Big)\\ +\Big(N_g(Y+\ln x) - N_g(Y)\Big)\Big(N_g(Y+\ln(1-x)) - N_g(Y)\Big)\Big]\\ +n_f\int_0^1 dx\gamma_0^2\Phi_g^q\Big[2\Big(N_q^{(2)}(Y+\ln x) - N_q^2(Y+\ln x)\Big) - \Big(N_g^{(2)}(Y) - N_g^2(Y)\Big)\\ +\Big(2N_q(Y+\ln x) - N_g(Y)\Big)\Big(2N_q(Y+\ln(1-x)) - N_g(Y)\Big)\Big],\end{aligned} \tag{10}$$

$$\begin{aligned}\frac{d}{dY}(N_q^{(2)} - N_q^2) = \int_0^1 dx\gamma_0^2\Phi_q^g\Big[N_g^{(2)}(Y+\ln x) + \Big(N_q^{(2)}(Y+\ln(1-x)) - N_q^{(2)}(Y)\Big)\\ +2\Big(N_g(Y+\ln x) - N_q(Y)\Big)\Big(N_q(Y+\ln(1-x)) - N_q(Y)\Big)\Big],\end{aligned} \tag{11}$$

which proves to be more suitable for obtaining analytical solutions in the following. We use a new method to compute solutions at MLLA by replacing $N_A^{(2)} = A_2 N_A^2$ on both sides of the expanded equations at $x \sim 1 - x \sim 1$. The notations in (10,11) follow the same logic as those in (5,6). Applying the analysis that led to the system (5,6), we obtain from (10,11)

$$\frac{d^2}{dY^2}\left(N_g^{(2)} - N_g^2\right) = \gamma_0^2\left(N_s - a_1\frac{d}{dY}\right)N_g^{(2)} + (a_1 - b_1)\gamma_0^2\frac{d}{dY}N_g^2, \tag{12}$$

$$\frac{d^2}{dY^2}\left(N_q^{(2)} - N_q^2\right) = \frac{C_F}{N_c}\gamma_0^2\left(N_s - \tilde{a}_1\frac{d}{dY}\right)N_g^{(2)}, \tag{13}$$

where

$$b_1 = \frac{1}{4N_c}\left[\frac{11}{3}N_c - 4\frac{T_R}{N_c}\left(1 - 2\frac{C_F}{N_c}\right)^2\right].$$

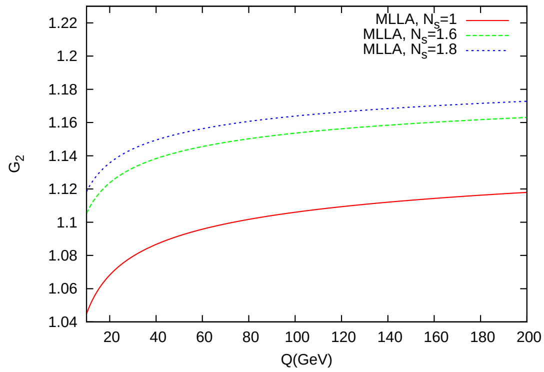

Figure 3: MLLA second multiplicity correlator inside a gluon jet (15) as a function of $Q = E\Theta$ in the vacuum ($N_s = 1$) and in the medium ($N_s = 1.6$ and $N_s = 1.8$ for $n_f = 3$.

The constant N_s only affects the leading double logarithmic term of the equations. The terms proportional to a_1, $(a_1 - b_1)$ and $\tilde{a}_1$ are hard vacuum corrections, which *partially* account for energy conservation, indeed $\gamma_0^2 \frac{dN}{dY} \approx \sqrt{N_s}\gamma_0^3$ and the relative correction to DLA is $\mathcal{O}(\sqrt{\alpha_s/N_s})$.
Setting $N_g^{(2)} = G_2 N_g^2$ in (12) and making use of (8), the system can be solved iteratively by taking terms up to $\mathcal{O}(\alpha_s)$ into consideration. The analytical solution reads,

$$G_2 - 1 = \frac{1 - \left(\frac{2}{3}a_1 + 2b_1\right)\frac{\gamma_0}{\sqrt{N_s}} + \mathcal{O}\left(\frac{\gamma_0^2}{N_s}\right)}{3 - (4a_1 - \beta_0)\frac{\gamma_0}{\sqrt{N_s}} + \mathcal{O}\left(\frac{\gamma_0^2}{N_s}\right)}, \tag{14}$$

while its expansion in the form $1 + \gamma_0/\sqrt{N_s}$ leads to

$$G_2 - 1 \approx \frac{1}{3} - c_1\frac{\gamma_0}{\sqrt{N_s}} + \mathcal{O}\left(\frac{\gamma_0^2}{N_s}\right), \tag{15}$$

where the linear combination of color factors can be written in the form

$$c_1 = \frac{1}{4N_c}\left(\frac{55}{9} - 4\frac{T_R}{N_c} + \frac{112}{9}\frac{T_R}{N_c}\frac{C_F}{N_c} - \frac{32}{3}\frac{T_R}{N_c}\frac{C_F^2}{N_c^2}\right). \tag{16}$$

We use (15) and (9) and substitute $N_q^{(2)} = Q_2 N_q^2$ into (13) such that the solution reads

$$Q_2 - 1 \approx \frac{N_c}{C_F}\left(\frac{1}{3} - \tilde{c}_1\frac{\gamma_0}{\sqrt{N_s}}\right) + \mathcal{O}\left(\frac{\gamma_0^2}{N_s}\right), \tag{17}$$

where we obtain the combination of color factors

$$\tilde{c}_1 = \frac{1}{4N_c}\left(\frac{55}{9} + \frac{4}{9}\frac{T_R}{N_c}\frac{C_F}{N_c} - \frac{8}{3}\frac{T_R}{N_c}\frac{C_F^2}{N_c^2}\right). \tag{18}$$

Setting $N_s = 1$ in (15) and (17) we get a perfect agreement with the vacuum results [9]. In Fig. 3 and Fig. 4, we compare our results for the medium-modified second multiplicity correlators (15) and (17) with predictions in the vacuum ($N_s = 1$) [9] in the limiting spectrum approximation inside the typical range $10 \leq Q(\text{GeV}) \leq 200$ for RHIC and LHC phenomenology. Similarly to the MLLA ratio $r(N_s)$, Eq. (9), the hard corrections $\mathcal{O}(\gamma_0)$ are suppressed by a factor $1/\sqrt{N_s}$. As expected from the model,

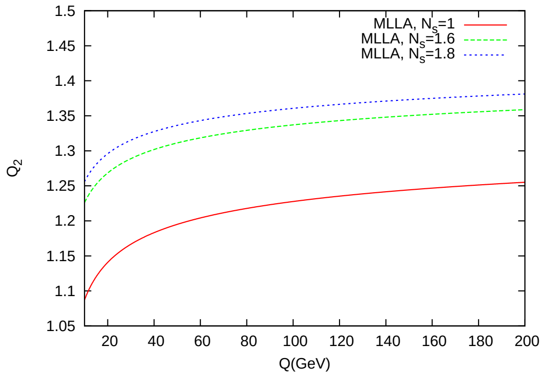

Figure 4: MLLA second multiplicity correlator inside a quark jet (17).

we check that these results approach their DLA limits when N_s increases; moreover, the multiplicity fluctuations of individual events must be larger for quark jets as compared to gluon jets just like in the vacuum [9]. Another interesting feature of these observables concerns the shape of the curves. They are roughly identical and prove not to depend on the medium parameter N_s. Moreover, there exists evidence for a flattening of the slopes as the hardness of the jet $Q = E\Theta$ increases for $N_s \geq 1$ (vacuum and medium). This kind of scaling behavior is known as the Koba-Nielsen-Olsen (KNO) scaling [11]: it was discovered by Polyakov in quantum field theory [12] and experimentally confirmed by e^+e^- measurements [13] for the second and higher order multiplicity correlators.

In this paper we have dealt with the medium-modified average multiplicity and the medium-modified second multiplicity correlator in quark and gluon jets at RHIC and LHC energy scales. The starting point of our calculations is based on the Borghini-Wiedemann work [3], which models parton energy loss in a nuclear medium. The average multiplicity is found to be enhanced by the factor $\sqrt{N_s}$ acting on the exponential leading contribution (8); this leads in particular to the rescaling of the anomalous dimension γ_{med} ($\gamma \to \gamma_{\text{med}} \approx \sqrt{N_s}\gamma_0$) or equivalently, to the enhancement of the in medium coupling constant. Since hard corrections are suppressed by the extra factor $1/\sqrt{N_s}$, it is straightforward to check that r, G_2 and Q_2 approach the asymptotic DLA limits $r_0 = N_c/C_F$, $G_2 = 4/3$ and $Q_2 = 1 + N_c/3C_F$ [4] when N_s increases. The previously mentioned KNO-scaling experienced by G_2 and Q_2 proves no special sensibility to the model and should normally hold like in the vacuum.

Finally, since these results are model-dependent, they may still be improved in the future, specially after the N_s-dependence of the non-singular parts of the splitting functions (4) has been exactly computed.

Perspective: Many experimental characterizations of the medium-modified intrajet structure in heavy-ion collisions at RHIC and at the LHC require a soft momentum cut-off p_T^{cut}, with $Q > p_T^{\text{cut}}$ to remove the effects of the high multiplicity background. In [3], the soft background was subtracted by integrating the single inclusive differential distribution $\frac{dN}{d\ln p_T}$ ("hump-backed plateau") over the range $Q \geq p_T \geq p_T^{\text{cut}}$, with $p_T^{\text{cut}} > \Lambda_{QCD}$. Accordingly, the equivalent computation should be performed for the second multiplicity correlator by integrating the double differential inclusive distribution (two-particle correlation) $\frac{d^2N}{d\ln p_{1,T}\, d\ln p_{2,T}}$ over $p_{i,T}$, with the lower bounds of integration $p_{i,T}^{\text{cut}} > \Lambda_{QCD}$ $(i = 1, 2)$. Imposing such a cut-off in our calculations will affect the normalization rather than the behavior and the shape of these observables as a function of N_s and the jet energy scale of the process Q [14].

References

[1] K. Adcox et al. (PHENIX Collab.), Phys. Rev. Lett. **88** (2002) 022301;
S.S. Adler et al. (PHENIX Collab.), Phys. Rev. Lett. **91** (2003) 072301;
C. Adler et al. (STAR Collab.), Phys. Rev. Lett **89** (2002) 202301.

[2] F. Arleo, hep-ph/0810.1193 and references therein;
S. Peigné & A.V. Smilga, hep-ph/0810.5702.

[3] N. Borghini & U.A. Wiedemann, hep-ph/0506218.

[4] Yu.L. Dokshitzer, V.A. Khoze, A.H. Mueller & S.I. Troyan, Basics of Perturbative QCD, Editions Frontières, Paris (1991).

[5] S.Sapeta & U.A Wiedemann, Eur. Phys. J. **C55** (2008) 293.

[6] Ya.I. Azimov, Yu.L. Dokshitzer, V.A. Khoze & S.I. Troian, Z. Phys. **C** 27 (1985) 65; Yu.L. Dokshitzer, V.A. Khoze & S.I. Troian, J. Phys. **G** 17 (1991) 1585.

[7] I.M. Dremin, V.A. Nechitailo, Mod. Phys. Lett. A **9** (1994) 1471; JETP Lett. **58** (1993) 945.

[8] M. Heinz for the STAR Collaboration, nucl-exp/0809.3769.

[9] E.D. Malaza & B.R. Webber, Phys. Lett. **B** 149 (1984) 501; E.D. Malaza & B.R. Webber, Nucl. Phys. **B** 267 (1986) 702.

[10] V.A. Khoze & W. Ochs, Int. J. Mod. Phys. A **12** (1997) 2949.

[11] Yu.L. Dokshitzer, Phys. Lett. B **305** (1993) 295.

[12] A.M. Polyakov, Sov. Phys. JETP **32** (1971) 296.

[13] HRS Coll., Phys. Rev. D **34** (1986) 3304; AMY Coll., Phys. Rev. D **42** (1990) 737; DELPHI Coll., Z. Phys. C-Particles and Fields **50** (1991) 185.

[14] N. Borghini & R. Perez-Ramos, in preparation.