

# The threshold region for Higgs production in gluon fusion

Marco Bonvini

*Deutsches Elektronen-Synchrotron, DESY, Notkestraße 85, D-22603 Hamburg, Germany*

Stefano Forte

*Dipartimento di Fisica, Università di Milano and INFN,  
 Sezione di Milano, Via Celoria 16, I-20133 Milano, Italy*

Giovanni Ridolfi

*Dipartimento di Fisica, Università di Genova and INFN,  
 Sezione di Genova, Via Dodecaneso 33, I-16146 Genova, Italy*

We provide a quantitative determination of the effective partonic kinematics for Higgs production in gluon fusion in terms of the collider energy at the LHC. We use the result to assess, as a function of the Higgs mass, whether the large  $m_t$  approximation is adequate and whether Sudakov resummation is advantageous. We argue that our results hold to all perturbative orders. Based on it, we conclude that the full inclusion of finite top mass corrections is likely to be important for accurate phenomenology for a light Higgs with  $m_H \sim 125$  GeV at the LHC with  $\sqrt{s} = 14$  TeV.

An accurate computation of the Higgs boson production cross-section [1] is essential in the search for this particle, which might be on the verge of being discovered at the LHC [2]. If the Higgs is light, and in particular in the region  $m_H \sim 125$  GeV, perhaps favored by LHC data, the dominant Higgs production mechanism is gluon fusion, which starts at leading order  $\mathcal{O}(\alpha_s^2)$  through a (predominantly top) quark loop. Higher order corrections, which turn out to be quite large, are accordingly difficult to compute, and the full next-to-next-to-leading order (NNLO) result is known either in the large  $m_t$  limit, at the fully differential level [3], or as an expansion in inverse powers of  $m_t$  for the fully inclusive cross-section [4]. As  $m_t \rightarrow \infty$  the quark loop shrinks to a point (pointlike approximation) and the LO process becomes a tree-level process of an effective theory.

At NLO, where the exact result is known, the large  $m_t$  approximation turns out to work surprisingly well, even up to values of the Higgs mass at and above the top pair production threshold. This result can at least in part be understood based on the observation that the partonic cross-section is dominated by logarithmically enhanced contributions related to soft gluon radiation which are independent of  $m_t$ , so the pointlike approximation becomes exact, up to an overall factor which starts at NNLO [5]. This soft dominance should take place when  $m_H$  is raised at fixed  $s$  so the energy  $\hat{s}$  of the partonic production subprocess approaches threshold  $\hat{s} \sim m_H^2$ .

Close enough to threshold it is advantageous to resum these logarithmically enhanced terms (threshold resummation), and it has indeed been observed that this resummation significantly corrects and stabilizes the perturbative result in regions in which the pointlike approximation holds to satisfactory accuracy [6]. It is important to understand that this may happen even if the expansion in powers of the strong coupling  $\alpha_s$  behaves in a perturbative way, i.e. if the size of higher-order logarithmically

enhanced contributions decreases with the perturbative order, so an actual all-order resummation is not really necessary. For this, it is sufficient that these enhanced contributions approximate the missing higher orders well enough that their inclusion actually improves the accuracy of the computation, and the desirability of resummation should thus be judged by the accuracy of the logarithmic approximation.

Even so, this can only be possible if the center of mass energy of the partonic collision  $\hat{s}$  is significantly lower than the hadronic one  $s$ , which at the LHC is very far from threshold. Because the gluon distribution is strongly peaked at small values of the momentum fraction of each hadron carried by the gluon itself, this is especially likely in a gluon fusion channel. A quantitative assessment of this effect is thus important in order to determine the accuracy of the pointlike limit, and also whether threshold resummation is advantageous.

The necessary formalism was presented in Ref. [7], and applied to Drell-Yan production. The cross-section for Higgs production is a function of the scale  $m_H^2$  and a scaling variable  $\tau = m_H^2/s$ , given by the convolution

$$\frac{\sigma(\tau, m_H^2)}{\tau} = \int_{\tau}^1 \frac{dz}{z} \mathcal{L}\left(\frac{\tau}{z}, m_H^2\right) \frac{\hat{\sigma}(z, \alpha_s(m_H^2))}{z} \quad (1)$$

of a partonic cross-section  $\hat{\sigma}\left(\frac{m_H^2}{\hat{s}}, \alpha_s(m_H^2)\right)$  and a parton luminosity  $\mathcal{L}\left(\frac{\hat{s}}{s}, m_H^2\right)$ ; a sum over relevant partonic subprocesses is understood. In our case at LO the only subprocess is  $gg \rightarrow H$ , so

$$\mathcal{L}(z, \mu^2) = \int_z^1 \frac{dy}{y} g(y, \mu^2) g\left(\frac{z}{y}, \mu^2\right) \quad (2)$$

where  $g(z, \mu^2)$  is the gluon parton distribution (PDF) in the proton. It is convenient to write the partonic cross-section in terms of a dimensionless coefficient function

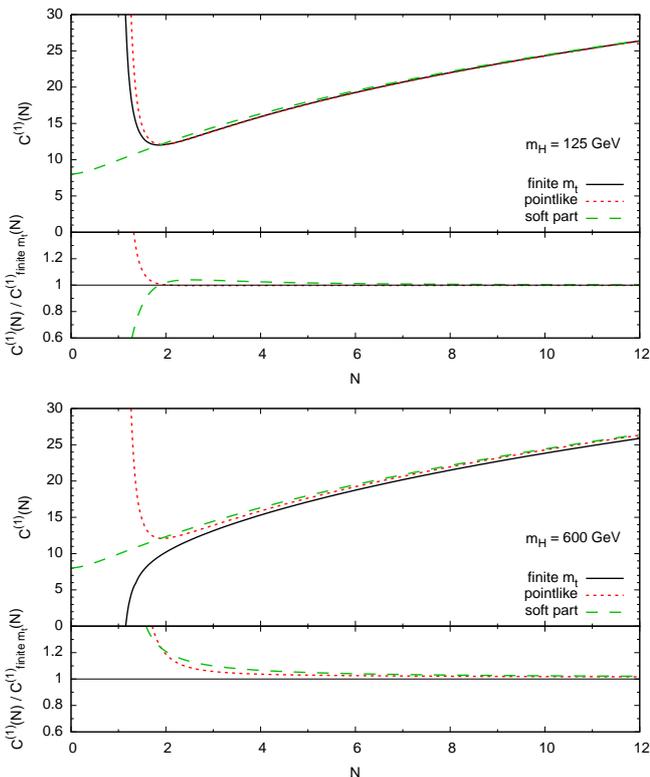


FIG. 1: The  $\mathcal{O}(\alpha_s)$  contribution to the coefficient function Eq. (4) as a function of  $N$ , for  $m_H = 125$  GeV (upper plot) and  $m_H = 600$  GeV (lower plot). In each case we show the exact result and the pointlike and logarithmic approximations, as well as the ratio of the latter two to the exact result.

$$C(z, \alpha_s)$$

$$\hat{\sigma}(z, \alpha_s) = \sigma_0 z C(z, \alpha_s), \quad (3)$$

where  $\sigma_0$  is the LO partonic cross-section, so the  $gg$  coefficient function has an expansion in powers of  $\alpha_s$

$$C(z, \alpha_s) = \delta(1-z) + \alpha_s C^{(1)}(z) + \alpha_s^2 C^{(2)}(z) + \mathcal{O}(\alpha_s^3). \quad (4)$$

Upon Mellin transformation

$$\sigma(N, m_H^2) = \int_0^1 d\tau \tau^{N-1} \frac{\sigma(\tau, m_H^2)}{\tau}, \quad (5)$$

$$C(N, \alpha_s) = \int_0^1 dz z^{N-1} C(z, \alpha_s), \quad (6)$$

$$\mathcal{L}(N, \mu^2) = \int_0^1 dz z^{N-1} \mathcal{L}(z, \mu^2) \quad (7)$$

the convolution in Eq. (1) turns into an ordinary product:

$$\sigma(N, m_H^2) = \sigma_0 \mathcal{L}(N, m_H^2) C(N, \alpha_s). \quad (8)$$

The Mellin transformation maps the large  $\tau \rightarrow 1$  region into the large  $N \rightarrow \infty$  region, and the small  $\tau \rightarrow 0$  region

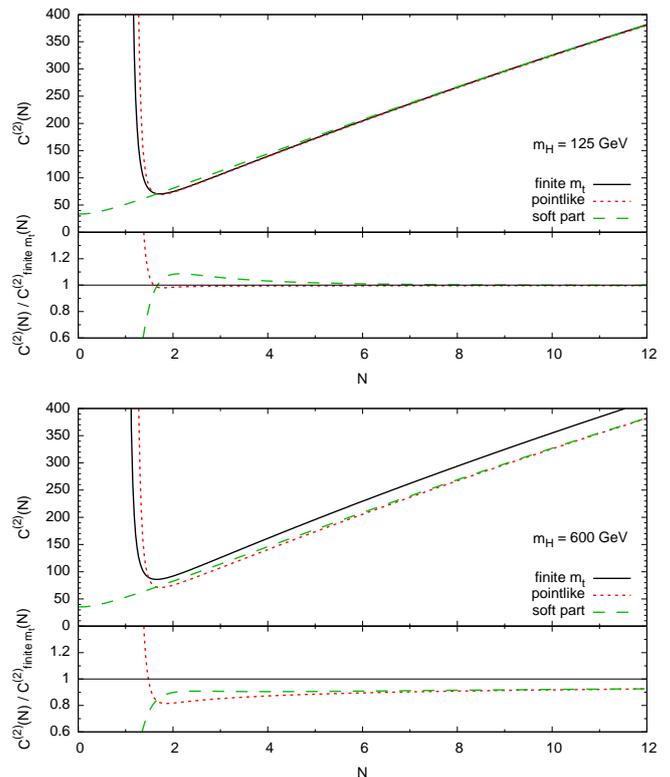


FIG. 2: Same as Fig. 1, but to  $\mathcal{O}(\alpha_s^2)$ .

into the small  $N \rightarrow N_s$  region, with  $N_s$  the rightmost singularity of  $\sigma(N, m_H^2)$  (i.e. the convergence abscissa of the Mellin transform). For gluon-initiated processes,  $N_s = 1$  to all perturbative orders.

The dominant partonic kinematic region can then be determined through a saddle point argument, by computing the value of  $N$  which provides the dominant contribution to the Mellin inversion integral:

$$\frac{\sigma(\tau, m_H^2)}{\tau} = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dN \tau^{-N} \sigma(N, m_H^2) \quad (c > N_s). \quad (9)$$

Namely, we define

$$E(N, \tau, m_H^2) \equiv N \ln \frac{1}{\tau} + \ln \sigma(N, m_H^2), \quad (10)$$

so that

$$\frac{\sigma(\tau, m_H^2)}{\tau} = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dN e^{E(N, \tau, m_H^2)}. \quad (11)$$

In the saddle point approximation, the integral Eq. (11) is dominated by the value of the exponent  $E(N, \tau, m_H^2)$  at its stationary point  $N_0$

$$\left. \frac{\partial E(N, \tau, m_H^2)}{\partial N} \right|_{N=N_0} = 0, \quad (12)$$

and by the behaviour in its vicinity. The position of the saddle point is a function  $N_0 = N_0(\tau, m_H^2)$ , solution of Eq. (12). Hence, for any value of the physical kinematics, the questions whether the partonic cross-section is well approximated by its pointlike limit or resummation is advantageous are answered by verifying whether this is the case for  $C(N_0, \alpha_s)$ .

A unique real saddle point is present due to the drop of the cross-section  $\sigma(N, m_H^2)$  as  $N$  grows. This drop is driven by the parton luminosity  $\mathcal{L}(N, \mu^2)$ , which thus controls the position of the saddle  $N_0$ . The drop of the luminosity at large  $N$  (and its growth at small  $N$ ) reflects in turn the drop of the PDFs and luminosity as  $z \rightarrow 1$  (and their growth as  $z \rightarrow 0$ ). However, for large  $N$ ,  $C(N, \alpha_s)$  actually grows with  $N$ . This growth, which is due to the logarithmically enhanced contributions, is only possible because the partonic cross-section is a distribution, rather than an ordinary function: the Mellin transform of an ordinary positive function is easily proven to be a decreasing function of  $N$ . Nevertheless, the parton luminosity always offsets this increase, because the physical cross-section  $\sigma(\tau, m_H^2)$  is an ordinary positive function and thus must decrease with  $N$ . The faster the growth of the cross-section, the better the saddle point approximation, which is thus especially good for gluon-dominated processes, as the gluon is more peaked at small  $z$  than the quark, and thus the gluon-gluon luminosity drops faster than quark luminosity.

The stationary point is determined by the interplay in Eq. (10) of the rise of the first term and the drop of the hadronic cross-section  $\sigma(N, m_H^2)$ , which in turn is the product of the coefficient function and the luminosity. The shape of the NLO and NNLO contributions to the coefficient function is shown in Figs. 1 and 2, for two values of the Higgs mass which are allowed by present data [2]. Clearly, when the partonic cross-section rises monotonically with  $N$ , it is the drop of the parton luminosity which determines the drop of the hadronic cross-section and thus the position of the saddle, and even when it drops, the decrease of its hadronic counterpart is much stronger in the presence of a luminosity, so the location of the saddle is substantially larger.

We have determined the position of the saddle at NLO and NNLO, using NNPDF2.1 parton distributions [8] at the corresponding order. Results are shown in Fig. 3, which is the main result of this paper. The position of the saddle  $N_0$  depends on two independent variables, which can be chosen out of the three variables  $m_H$ ,  $s$ , and  $\tau = m_H^2/s$ . If results for  $N_0$  are shown as a function of  $\tau$ , the dependence on the other variable (be it  $m_H$  or  $s$ ) becomes very slight, because it enters only through the scale dependence of  $\alpha_s$  and the parton distributions: this is explicitly seen in the top plot of Fig. 3, where results are shown for two different values of  $m_H$ ; the dependence would be similar if  $\sqrt{s}$  were varied instead by a comparable factor. The dependence of the position of the saddle

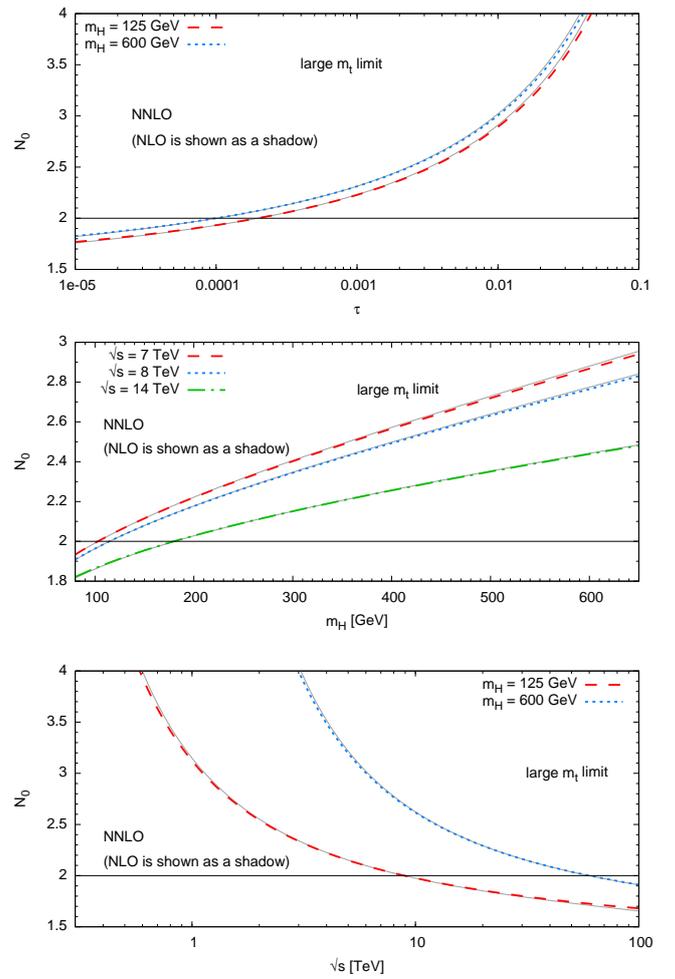


FIG. 3: The position of the saddle point  $N_0$  for the Mellin inversion integral Eq. (9) as a function of:  $\tau = m_H^2/s$  (top);  $m_H$  for three different values of the collider energy  $s$  (middle);  $s$  for two different values of  $m_H$  (bottom). The curves in the top plot depend very weakly on either  $m_H$  (shown) or  $s$ . Both the NLO and NNLO are shown, computed using the appropriate NNPDF2.1 PDF set [8] in each case.

on the perturbative order is completely negligible (as also shown), and so is the dependence on whether the pointlike approximation is used or not. We have also checked that varying the renormalization and factorization scales by a factor two changes the value of  $N_0$  by 1% or less, even though more dramatic scheme changes which affect the infrared behaviour of the PDF such as suggested in Ref. [9] might have somewhat larger effects. The impact of changing the PDF set or the value of  $\alpha_s$  in a reasonable range is rather less than that.

The size of the region around the value of  $N_0$  which dominates the integral can be estimated by computing the second derivative of  $\ln \sigma(N, m_H^2)$ , which gives the width of the gaussian integral which approximates the Mellin inversion in the complex  $N$  plane. We find that

a one-sigma region corresponds to a variation of  $N_0$  by about 25% about its central value. So, within the accuracy of our saddle point approximation, the distinction between different curves at fixed  $\tau$  is of little import. For clarity and completeness, in Fig. 3 we show the position of the saddle as a function of the Higgs mass for the three values of  $s$  relevant for the LHC, and as a function of the center of mass energy for two values of the Higgs mass.

We can now assess both the adequacy of the pointlike approximation, and the desirability of resummation for given values of  $s$  and  $m_H$ : first, using Fig. 3 the given hadronic kinematics can be translated into a value of  $N_0$ . Then, for this value of  $N_0$  we can check whether the pointlike approximation is accurate, and threshold resummation is advisable. To this purpose, in Figs. 1–2 we compare the exact NLO and NNLO contributions to the coefficient function to its various approximations. At NLO, we use the implementation of Ref. [10] of the original exact result of Refs. [11, 12]; at NNLO a full exact computation is not available, so for light Higgs we use the expansion of Ref. [4] matched to the exact small  $z$  limit of Ref. [13], while for heavy Higgs, where the expansion of Ref. [4] is unstable, the “finite  $m_t$ ” curve of Fig. 2 is merely given by correcting the pointlike result through the inclusion of the first order correction in  $\frac{m_t}{m_H}$  from Ref. [4] (and it thus only indicates the location of the region where the finite  $m_t$  corrections are likely relevant).

The pointlike results at NLO [14, 15] and NNLO [16] have been long known. As a soft part, we show all contributions which survive the  $N \rightarrow \infty$  limit, defined as the exact Mellin transform of all contributions of the form  $z \left( \frac{\ln^k \left( \frac{1-z}{\sqrt{z}} \right)}{1-z} \right)_+$  and all contributions proportional to  $\delta(1-z)$  to the NLO and NNLO coefficient functions. While the coefficients of these terms are fixed by soft resummation, there is a certain latitude in defining which subleading terms to include. Our choice reproduces the exact soft kinematics [7, 17] thereby optimizing the agreement with the exact result to all orders. The region in which the soft limit defined in this way is close to the full result is the region in which one expects soft resummation to improve the accuracy of the computation, even when all-order resummation is not mandatory.

Inspection of Figs. 1–2 shows that the region in which the pointlike approximation is adequate is very close to the region in which the soft approximation is good, so indeed the success of the former might be explained by the accuracy of the latter. Both at NLO and NNLO the relevant region is roughly  $N \gtrsim 2$ . This is the region where the partonic cross-section starts to rise with  $N$ , driven by logarithmically enhanced contributions. The apparent failure of both approximations for heavy Higgs at NNLO is likely to be due to the fact that in this case the “finite  $m_t$ ” computation is in fact only approximate.

The fact that the same behaviour is observed at NLO and NNLO is not accidental. On the one hand, the po-

sition of the saddle is largely determined by the parton luminosity, which is perturbatively very stable at NLO and beyond [8]. On the other hand, the shape of the coefficient functions and the dominance of soft terms are mostly controlled by the location of the leading small- and large- $N$  singularities, which can be checked to be stable even upon all-order resummation [7]. This supports the expectation that the desirability of resummation and the all-order reliability of the pointlike approximation can be assessed on the basis of the known low orders.

At the LHC,  $\tau$  is quite small: if  $m_H \sim 125$  GeV,  $\tau \sim 10^{-4}$  and if  $m_H \sim 600$  GeV,  $\tau \sim 10^{-3} - 10^{-2}$ , thereby leading to values of  $N_0$ , close to the transition value  $N \sim 2$ , for which resummation is at best desirable, but certainly not mandatory, as  $\alpha_s \ln^2 N_0 \ll 1$ . Nevertheless, Fig. 3, in which the  $N = 2$  line has been drawn for ease of reference, shows that for a heavy Higgs with  $m_H \sim 600$  GeV the pointlike approximation is adequate, and Sudakov resummation clearly advantageous, for any collider with center of mass energy up to  $\sqrt{s} \lesssim 30$  TeV, and thus certainly at the LHC.

Even for a light Higgs with  $m_H \sim 125$  GeV the pointlike approximation is fine for a collider with energy of 7 TeV, but it may start failing as the energy is raised, and becomes inadequate at the LHC with  $\sqrt{s} = 14$  TeV, where the saddle drops at  $N_0 \approx 1.9$  so the finite-mass corrections to  $C^{(1)}$  Fig. 1 reach the percent level, though care should be taken because, close to the region where the approximation breaks down, the conclusion may depend on various details and approximations. However, because the breakdown of the pointlike approximation is in significant part due to the presence of spurious high-energy double logs [13], it is expected to be more noticeable in less inclusive observables. Correspondingly, with this higher center of mass energy, Sudakov resummation is likely to stop being advantageous for a light Higgs.

In summary, we have provided a way of assessing whether the pointlike approximation is adequate and whether Sudakov resummation is desirable based on the behaviour of the known first few orders of the partonic cross-section. Based on it, we conclude that for a light Higgs with  $m_H \sim 125$  GeV the full inclusion of finite top mass corrections to its cross-section is likely to be important for accurate phenomenology at the LHC with  $\sqrt{s} = 14$  TeV.

We thank F. Tackmann for a critical reading of the manuscript, and R. Harlander and A. Vicini for providing computing code.

- 
- [1] S. Dittmaier *et al.* [LHC Higgs Cross Section Working Group Collaboration], arXiv:1101.0593 [hep-ph].
  - [2] G. Aad *et al.* [ATLAS Collaboration], Phys. Lett. B **710** (2012) 49;

- S. Chatrchyan *et al.* [CMS Collaboration], arXiv:1202.1488 [hep-ex].
- [3] C. Anastasiou, K. Melnikov and F. Petriello, Nucl. Phys. B **724** (2005) 197.
- [4] R. V. Harlander and K. J. Ozeren, JHEP **0911** (2009) 088.
- [5] M. Kramer, E. Laenen and M. Spira, Nucl. Phys. B **511** (1998) 523.
- [6] S. Catani *et al.*, JHEP **0307** (2003) 028.
- [7] M. Bonvini, S. Forte and G. Ridolfi, Nucl. Phys. B **847** (2011) 93.
- [8] R. D. Ball *et al.*, Nucl. Phys. B **849** (2011) 296;  
R. D. Ball *et al.* [The NNPDF Collaboration], Nucl. Phys. B **855** (2012) 153.
- [9] E. G. de Oliveira, A. D. Martin and M. G. Ryskin, arXiv:1206.2223 [hep-ph].
- [10] R. Bonciani, G. Degrossi and A. Vicini, JHEP **0711** (2007) 095.
- [11] D. Graudenz, M. Spira and P. M. Zerwas, Phys. Rev. Lett. **70** (1993) 1372.
- [12] M. Spira *et al.*, Nucl. Phys. B **453** (1995) 17 [hep-ph/9504378].
- [13] S. Marzani *et al.*, Nucl. Phys. B **800** (2008) 127.
- [14] S. Dawson, Nucl. Phys. B **359** (1991) 283.
- [15] A. Djouadi, M. Spira and P. M. Zerwas, Phys. Lett. B **264** (1991) 440.
- [16] C. Anastasiou and K. Melnikov, Nucl. Phys. B **646** (2002) 220.
- [17] S. Forte and G. Ridolfi, Nucl. Phys. B **650** (2003) 229.