

Charge Symmetry Breaking in Spin Dependent Parton Distributions and the Bjorken Sum Rule

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We present the first determination of charge symmetry violation (CSV) in the spin-dependent parton distribution functions of the nucleon. This is done by determining the first two Mellin moments of the spin-dependent parton distribution functions of the octet baryons from $N_f = 2 + 1$ lattice simulations. The results are compared with predictions from quark models of nucleon structure. We discuss the contribution of partonic spin CSV to the Bjorken sum rule, which is important because the CSV contributions represent the only partonic corrections to the Bjorken sum rule.

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I. INTRODUCTION

Charge symmetry refers to the invariance of the strong interaction under a very particular operation in isospin space, namely the interchange of u and d quarks and also protons and neutrons. Technically, the charge symmetry operator P_{CS} corresponds to a rotation of 180° about the 2 axis in isospin space. In nuclear systems, charge symmetry is generally valid to substantially better than 1% [1]. At the partonic level, charge symmetry implies the equality of different parton distribution functions (PDFs), namely

$$u^p(x, Q^2) = d^n(x, Q^2), \quad d^p(x, Q^2) = u^n(x, Q^2), \quad (1)$$

with analogous relations for antiquark PDFs. To date, no experimental violation of charge symmetry has been observed at the partonic level, and current upper limits are consistent with the validity of partonic charge symmetry in the range 5-10% [2, 3].

In this letter we report the first determination of the charge symmetry violation (CSV) in the spin-dependent parton distributions arising from quark mass differences. We begin by extracting the zeroth and first moments of the spin-dependent PDFs of the light baryon octet by varying the light (degenerate u, d) and strange quark masses in a $N_f = 2 + 1$ lattice simulation. We compare these results to quark model predictions for the sign and magnitude of these moments. Finally, we examine

the size of the expected contribution of the spin parton distributions to the Bjorken Sum Rule.

II. CHARGE SYMMETRY VIOLATION

Theoretical models for partonic charge symmetry predict that the spin-independent parton CSV distributions $\delta u^-(x) = u^{p-}(x) - d^{n-}(x)$ and $\delta d^-(x) = d^{p-}(x) - u^{n-}(x)$ should be roughly equal in magnitude and opposite in sign [3], where the minus superscript denotes the valence or C-odd combination of parton distribution functions,

$$q^\pm(x) = q(x) \pm \bar{q}(x). \quad (2)$$

The MRST group included valence CSV in a global analysis of high energy experimental data [4]. The best value obtained in this search was in excellent agreement with quark model calculations of valence parton CSV [5], but with very large errors. A recent lattice calculation was able to probe the magnitude of CSV violation [6]. There, the behaviour of the first moments of the hyperon parton distribution functions were studied as the light and strange quark masses were varied in a $N_f = 2 + 1$ lattice simulation. The first moment of the parton distributions $\delta u^+(x)$ and $\delta d^+(x)$ agreed very well in sign and magnitude with both the quark model results and the best value from the global fit — with the uncertainties on the lattice results substantially smaller than those from

the global fit. Note that the lattice calculation accessed the C-even combinations of partonic CSV distributions, so the lattice results contained some sea quark CSV effects that were not included in the other investigations.

We define the m^{th} moment of the charge symmetry violating spin-dependent quark distributions in the nucleon as

$$\begin{aligned}\delta\Delta u^m &= \int_0^1 dx x^m (\Delta u^p(x) - \Delta d^n(x)), \\ &= \langle x^m \rangle_{\Delta u}^p - \langle x^m \rangle_{\Delta d}^n,\end{aligned}\quad (3)$$

$$\begin{aligned}\delta\Delta d^m &= \int_0^1 dx x^m (\Delta d^p(x) - \Delta u^n(x)), \\ &= \langle x^m \rangle_{\Delta d}^p - \langle x^m \rangle_{\Delta u}^n.\end{aligned}\quad (4)$$

In the limit where the strange and light quarks have nearly equal mass, these CSV spin moments are related to hyperon spin moments by

$$\delta\Delta u^m \sim \langle x^m \rangle_{\Delta u}^{\Sigma} - \langle x^m \rangle_{\Delta s}^{\Xi}, \quad (5)$$

$$\delta\Delta d^m \sim \langle x^m \rangle_{\Delta s}^{\Sigma} - \langle x^m \rangle_{\Delta u}^{\Xi}. \quad (6)$$

III. LATTICE SIMULATION DETAILS

In the numerical calculation of the moments defined in Eqs.(3)-(6), our gauge field configurations have been generated with $N_f = 2 + 1$ flavours of dynamical fermions, using the Symanzik improved gluon action and nonperturbatively $\mathcal{O}(a)$ -improved Wilson fermions [7]. The quark masses are chosen by first finding the $SU(3)_{\text{flavour}}$ -symmetric point where flavour singlet quantities take on their physical values and then varying the individual quark masses while keeping the singlet quark mass $\bar{m}_q = (m_u + m_d + m_s)/3 = (2m_l + m_s)/3$ constant [8, 9]. Simulations are performed on lattice volumes of $24^3 \times 48$ with lattice spacing, $a = 0.083(3)\text{fm}$. A summary of the dynamical configurations used is given in Table I. More details regarding the tuning of the simulation parameters can be found in Refs. [8, 9].

On the lattice we compute moments of the spin-dependent quark distribution functions, $\Delta q(x)$

$$\langle x^m \rangle_{\Delta q}^B = \int_0^1 dx x^m (\Delta q^B(x) + (-1)^m \Delta \bar{q}^B(x)), \quad (7)$$

where x is the Bjorken scaling variable associated with baryon B . This involves calculating the matrix elements of local twist-2 operators, namely

$$\begin{aligned}\langle B(\vec{p}) | [\mathcal{O}_q^{5\{\mu_0 \dots \mu_m\}} - \text{Tr}] | B(\vec{p}) \rangle \\ = 2 \langle x^m \rangle_{\Delta q}^B [s^{\{\mu_0} p^{\mu_1} \dots p^{\mu_m\}} - \text{Tr}],\end{aligned}\quad (8)$$

where $\mathcal{O}_q^{5\mu_0 \dots \mu_m} = i^m \bar{q} \gamma_5 \gamma^{\mu_0} \overleftrightarrow{D}^{\mu_1} \dots \overleftrightarrow{D}^{\mu_m} q$. We note that in the case of the unpolarised quark distribution functions the lowest moment is protected by a sum rule

#	κ_l	κ_s	m_π [MeV]	m_K [MeV]
1	0.12083	0.12104	460(17)	401(15)
2	0.12090	0.12090	423(15)	423(15)
3	0.12095	0.12080	395(14)	438(16)
4	0.12100	0.12070	360(13)	451(16)
5	0.12104	0.12062	334(12)	463(17)

TABLE I: Pion and kaon masses on $24^3 \times 48$ lattices with lattice spacing, $a = 0.083(3)\text{fm}$ [8], where the error on the lattice spacing has been included in the errors for m_π and m_K . The first column denotes the ensemble number.

(baryon number conservation). As a result, we only considered the first non-trivial moment, $\langle x \rangle_q$, in our previous calculation of the spin-independent CSV [6]. The lowest moment of the spin-dependent quark distribution functions, however, is not protected by such a sum rule. Hence, in this work we consider the first two ($m = 0, 1$) moments, which, according to Eq. (7), contain one C-even and one C-odd moment. This allows us to better assess the impact of the sea distribution in our results.

In this paper we only consider the quark-line connected contributions to the first two moments, $\langle 1 \rangle_{\Delta q}$, $\langle x \rangle_{\Delta q}$, which means that we only include the part of \bar{q}^B coming from quark-line connected backward moving quarks, the so-called Z-graphs. While the contributions from disconnected insertions are expected to be small [10, 11], in the following analysis we will focus on differences of baryons and so these contributions will cancel in the $SU(3)_{\text{flavour}}$ limit and should be negligible for small expansions around this limit, as considered here.

We use a nucleon polarised in the $+z$ -direction with the standard local operators

$$\mathcal{O}_{\Delta q}^{\langle 1 \rangle} = \mathcal{O}_{\Delta q}^{53} \quad \text{and} \quad \mathcal{O}_{\Delta q}^{\langle x \rangle} = \mathcal{O}_{\Delta q}^{5\{43\}}. \quad (9)$$

The matrix elements in Eq. (8) are obtained on the lattice by considering the ratios:

$$\begin{aligned}R(t, \tau, \vec{p}, \mathcal{O}_{\Delta q}^{\langle 1 \rangle}) &= \frac{C_{3\text{pt}}(t, \tau, \vec{p}, \mathcal{O}_{\Delta q}^{\langle 1 \rangle})}{C_{2\text{pt}}(t, \vec{p})} = i \langle 1 \rangle_{\Delta q}, \\ R(t, \tau, \vec{p}, \mathcal{O}_{\Delta q}^{\langle x \rangle}) &= \frac{C_{3\text{pt}}(t, \tau, \vec{p}, \mathcal{O}_{\Delta q}^{\langle x \rangle})}{C_{2\text{pt}}(t, \vec{p})} = -i \frac{m_B}{2} \langle x \rangle_{\Delta q},\end{aligned}\quad (10)$$

where $C_{2\text{pt}}$ and $C_{3\text{pt}}$ are lattice two and three-point functions, respectively, with total momentum, \vec{p} , (in our simulation we consider only $\vec{p} = 0$). The operators $\mathcal{O}_{\Delta q}^{\langle 1 \rangle}$ and $\mathcal{O}_{\Delta q}^{\langle x \rangle}$ are inserted into the three-point function, $C_{3\text{pt}}(t, \tau, \vec{p}, \mathcal{O})$ at time, τ , between the baryon source located at time, $t = 0$, and sink at time, t .

IV. LATTICE RESULTS

The operators used for determining the moments of the spin-dependent PDFs need to be renormalised, preferably using a nonperturbative method such as RI'-MOM

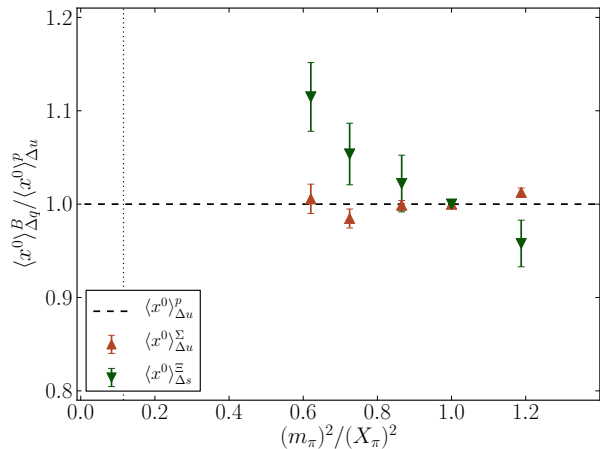


FIG. 1: Ratio of doubly-represented quark contributions to a baryon's spin, $\langle 1 \rangle_{\Delta u}^\Sigma / \langle 1 \rangle_{\Delta u}^p$ and $\langle 1 \rangle_{\Delta u}^\Xi / \langle 1 \rangle_{\Delta u}^p$ as a function of m_π^2 / X_π^2 , where the pion masses are normalised to the lattice determination of X_π . The vertical dotted line indicates the physical pion mass.

#	$\langle 1 \rangle_{\Delta u}^\Sigma / \langle 1 \rangle_{\Delta u}^p$	$\langle 1 \rangle_{\Delta s}^\Sigma / \langle 1 \rangle_{\Delta d}^p$	$\langle 1 \rangle_{\Delta s}^\Xi / \langle 1 \rangle_{\Delta u}^p$	$\langle 1 \rangle_{\Delta u}^\Xi / \langle 1 \rangle_{\Delta d}^p$
1	1.013(5)	0.964(22)	0.958(25)	1.028(42)
2	1.0	1.0	1.0	1.0
3	0.999(5)	1.036(15)	1.022(30)	1.009(51)
4	0.985(10)	1.034(26)	1.054(33)	0.972(52)
5	1.006(16)	1.064(43)	1.115(37)	0.949(66)

TABLE II: Ratios of the zeroth moment of the C-even, spin-dependent hyperon PDFs.

[12–14]. Here, however, we will only present results for ratios of the first two moments so that the renormalisation constants cancel. In Fig. 1 we present results for the ratio of the $u(s)$ -quark contribution to the spin of the $\Sigma(\Xi)$ baryon to the contribution of the u in the proton, as a function of m_π^2 , normalised with the centre-of-mass of the pseudoscalar meson octet, $X_\pi = \sqrt{(2m_K^2 + m_\pi^2)/3} = 411$ MeV. They are also given in Table II for each ensemble. We see that the ratio of the contribution from the u -quark to the spin of the Σ and the proton is roughly constant as the quark mass is decreased. We also observe that the contribution from the strange quark to the spin of the Ξ -baryon is greater than that of the u -quark in the proton and increases as the mass of the light (strange) quark is decreased (increased).

Unlike the unpolarised case in [6], there is no sum rule to preserve the total spin-dependent quark contributions. This implies that the strange quark contribution to the spin of the Σ doesn't necessarily have to be the same as the d -quark to the proton, and in fact we see in Fig. 2 that $\langle 1 \rangle_{\Delta s}^\Sigma > \langle 1 \rangle_{\Delta d}^p$. Conversely, we note in Fig. 2 that the u -quark in the Ξ baryon feels the effect of the two heavier strange quarks and $\langle 1 \rangle_{\Delta u}^\Xi < \langle 1 \rangle_{\Delta d}^p$ decreases as we approach the physical point.

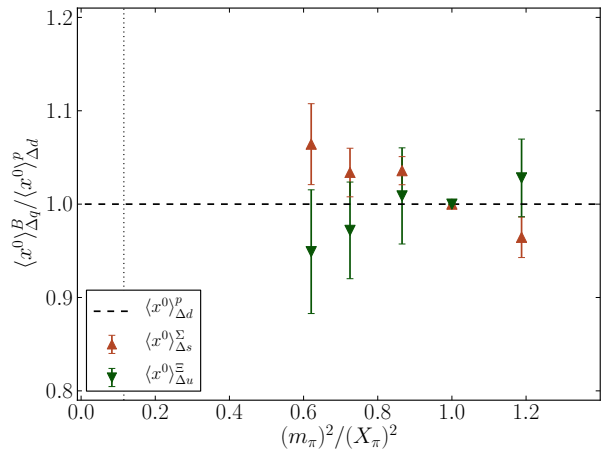


FIG. 2: Ratio of singly-represented quark contributions to a baryon's spin, $\langle 1 \rangle_{\Delta s}^\Sigma / \langle 1 \rangle_{\Delta d}^p$ and $\langle 1 \rangle_{\Delta u}^\Xi / \langle 1 \rangle_{\Delta d}^p$ as a function of m_π^2 / X_π^2 , where the pion masses are normalised to the lattice determination of X_π . The vertical dotted line indicates the physical pion mass.

#	$\langle x \rangle_{\Delta u}^\Sigma / \langle x \rangle_{\Delta u}^p$	$\langle x \rangle_{\Delta s}^\Sigma / \langle x \rangle_{\Delta d}^p$	$\langle x \rangle_{\Delta s}^\Xi / \langle x \rangle_{\Delta u}^p$	$\langle x \rangle_{\Delta u}^\Xi / \langle x \rangle_{\Delta d}^p$
1	1.036(7)	0.907(29)	0.989(28)	1.025(69)
2	1.0	1.0	1.0	1.0
3	0.992(7)	1.064(24)	1.035(40)	1.029(79)
4	0.956(11)	1.095(51)	1.043(34)	1.000(104)
5	0.960(16)	1.257(93)	1.067(43)	0.995(117)

TABLE III: Ratios of the first moment of the C-odd spin-dependent hyperon PDFs.

Similar effects are seen in the $m = 1$ (or x -) moments given in Table III and shown in Fig. 3 and 4, with the exception that in this case, $\langle x \rangle_{\Delta u}^\Sigma < \langle x \rangle_{\Delta u}^p$, i.e. the u -quark in the Σ feels the effect of the heavier strange quark being present in the baryon.

To infer the level of CSV relevant to the nucleon, we only need to consider a small expansion about the $SU(3)_{\text{flavour}}$ symmetric point, for which linear flavour expansions prove to work extremely well [8]. For instance, we can write

$$\begin{aligned} \delta\Delta u^0 &= m_\delta \left(-\frac{\partial \langle 1 \rangle_{\Delta u}^p}{\partial m_u} + \frac{\partial \langle 1 \rangle_{\Delta u}^p}{\partial m_d} \right) + \mathcal{O}(m_\delta^2), \\ \delta\Delta u^1 &= m_\delta \left(-\frac{\partial \langle x \rangle_{\Delta u}^p}{\partial m_u} + \frac{\partial \langle x \rangle_{\Delta u}^p}{\partial m_d} \right) + \mathcal{O}(m_\delta^2), \end{aligned} \quad (11)$$

where $m_\delta \equiv (m_d - m_u)$ and we have already made use of charge symmetry by equating $\partial \langle x^m \rangle_{\Delta d}^n / \partial m_d = \partial \langle x^m \rangle_{\Delta u}^p / \partial m_u$ and $\partial \langle x^m \rangle_{\Delta d}^n / \partial m_u = \partial \langle x^m \rangle_{\Delta u}^p / \partial m_d$. Similar expressions hold for $\delta\Delta d^0$ and $\delta\Delta d^1$.

Near the $SU(3)_{\text{flavour}}$ symmetric point, we note that the up quark in the proton is equivalent to an up quark in a Σ^+ or a strange quark in a Ξ^0 , which we describe collectively as the “doubly-represented” quark [15].

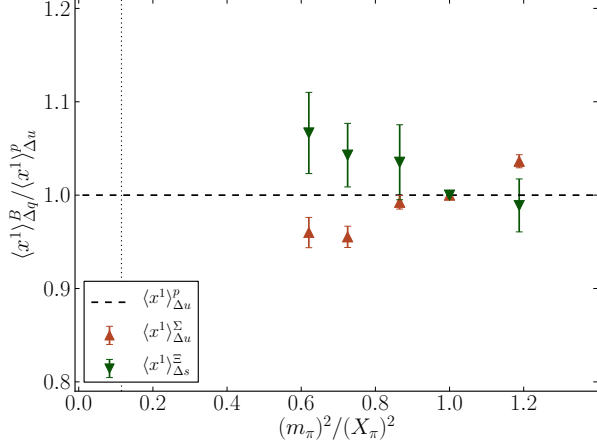


FIG. 3: Ratio of doubly represented spin-dependent quark momentum fractions, $\langle x \rangle_{\Delta u}^{\Sigma} / \langle x \rangle_{\Delta u}^P$ and $\langle x \rangle_{\Delta s}^{\Xi} / \langle x \rangle_{\Delta u}^P$ as a function of m_π^2 / X_π^2 , where the pion masses are normalised to the lattice determination of X_π . The vertical dotted line indicates the physical pion mass.

The local derivatives required for $\delta\Delta u^m$ can be obtained by varying the masses of the up and down quarks independently. Within the present calculation, we note that the difference $\langle x^m \rangle_{\Delta s}^{\Xi} - \langle x^m \rangle_{\Delta u}^P$ measures precisely the variation of the doubly-represented quark matrix element with respect to the doubly-represented quark mass (while holding the singly-represented quark mass fixed). Similar variations allow us to evaluate the other required derivatives, where we write

$$\begin{aligned} \frac{\partial \langle x^m \rangle_{\Delta u}^P}{\partial m_u} &\simeq \frac{\langle x^m \rangle_{\Delta s}^{\Xi^0} - \langle x^m \rangle_{\Delta u}^P}{m_s - m_l}, \\ \frac{\partial \langle x^m \rangle_{\Delta u}^P}{\partial m_d} &\simeq \frac{\langle x^m \rangle_{\Delta u}^{\Sigma^+} - \langle x^m \rangle_{\Delta u}^P}{m_s - m_l}, \\ \frac{\partial \langle x^m \rangle_{\Delta d}^P}{\partial m_u} &\simeq \frac{\langle x^m \rangle_{\Delta u}^{\Xi^0} - \langle x^m \rangle_{\Delta d}^P}{m_s - m_l}, \\ \frac{\partial \langle x^m \rangle_{\Delta d}^P}{\partial m_d} &\simeq \frac{\langle x^m \rangle_{\Delta s}^{\Sigma^+} - \langle x^m \rangle_{\Delta d}^P}{m_s - m_l}. \end{aligned} \quad (12)$$

With these expressions and Eq. (11), we obtain the relevant combinations for our determination of CSV in the nucleon

$$\begin{aligned} \delta\Delta u^m &= m_\delta \frac{\langle x^m \rangle_{\Delta u}^{\Sigma^+} - \langle x^m \rangle_{\Delta s}^{\Xi^0}}{m_s - m_l}, \\ \delta\Delta d^m &= m_\delta \frac{\langle x^m \rangle_{\Delta s}^{\Sigma^+} - \langle x^m \rangle_{\Delta u}^{\Xi^0}}{m_s - m_l}. \end{aligned} \quad (13)$$

By invoking the Gell-Mann–Oakes–Renner relation and normalising to the total nucleon isovector $m = 0, 1$

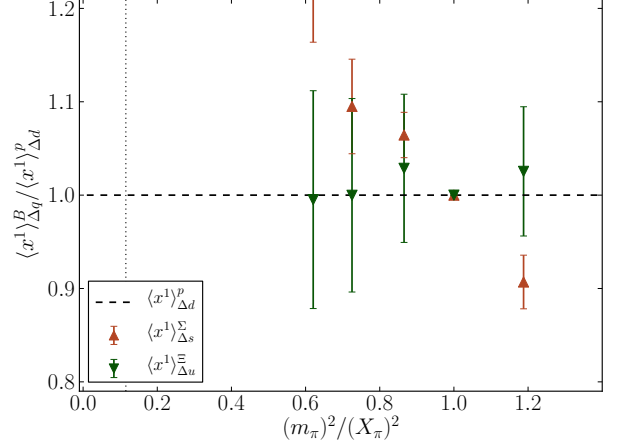


FIG. 4: Ratio of singly represented spin-dependent quark momentum fractions, $\langle x \rangle_{\Delta s}^{\Sigma} / \langle x \rangle_{\Delta d}^P$ and $\langle x \rangle_{\Delta u}^{\Xi} / \langle x \rangle_{\Delta d}^P$ as a function of m_π^2 / X_π^2 , where the pion masses are normalised to the lattice determination of X_π . The vertical dotted line indicates the physical pion mass.

spin-dependent moments, we write

$$\frac{\delta\Delta u^m}{\langle x^m \rangle_{\Delta u-\Delta d}^P} = \frac{m_\delta}{\bar{m}_q} \frac{(\langle x^m \rangle_{\Delta u}^{\Sigma^+} - \langle x^m \rangle_{\Delta s}^{\Xi^0}) / \langle x^m \rangle_{\Delta u-\Delta d}^P}{(m_K^2 - m_\pi^2) / X_\pi^2}, \quad (14)$$

$$\frac{\delta\Delta d^m}{\langle x^m \rangle_{\Delta u-\Delta d}^P} = \frac{m_\delta}{\bar{m}_q} \frac{(\langle x^m \rangle_{\Delta s}^{\Sigma^+} - \langle x^m \rangle_{\Delta u}^{\Xi^0}) / \langle x^m \rangle_{\Delta u-\Delta d}^P}{(m_K^2 - m_\pi^2) / X_\pi^2}. \quad (15)$$

Written in this way, the fractional spin-dependent CSV terms are just the slopes of the curves shown in Figs. 5 and 6 (evaluated at the symmetry point) multiplied by the ratio m_δ / \bar{m}_q . By fitting the slopes, we obtain

$$\frac{\delta\Delta u^{0+}}{\langle 1 \rangle_{\Delta u+\Delta d}^P} = \frac{m_\delta}{\bar{m}_q} (-0.137 \pm 0.028), \quad (16)$$

$$\frac{\delta\Delta d^{0+}}{\langle 1 \rangle_{\Delta u+\Delta d}^P} = \frac{m_\delta}{\bar{m}_q} (-0.0433 \pm 0.013), \quad (17)$$

$$\frac{\delta\Delta u^{1-}}{\langle x \rangle_{\Delta u-\Delta d}^P} = \frac{m_\delta}{\bar{m}_q} (-0.161 \pm 0.035), \quad (18)$$

$$\frac{\delta\Delta d^{1-}}{\langle x \rangle_{\Delta u-\Delta d}^P} = \frac{m_\delta}{\bar{m}_q} (-0.068 \pm 0.016). \quad (19)$$

Chiral perturbation theory yields the quark mass ratio $m_\delta / \bar{m}_q = 0.066(7)$ [16] while the experimentally determined moments are $\langle 1 \rangle_{\Delta u+\Delta d}^P = g_A = 1.2695(29)$ [17] and $\langle x \rangle_{\Delta u-\Delta d}^P = 0.190(8)$ [18] in the $\overline{\text{MS}}$ scheme at 4 GeV^2 . We note that in principle the zeroth moments in Eqs. (16) and (17) will receive their scale dependence from an additional term

$$\frac{z(\mu, a)}{3} \frac{(\delta\Delta u^0 + \delta\Delta d^0)}{\langle 1 \rangle_{\Delta u-\Delta d}^P}, \quad (20)$$

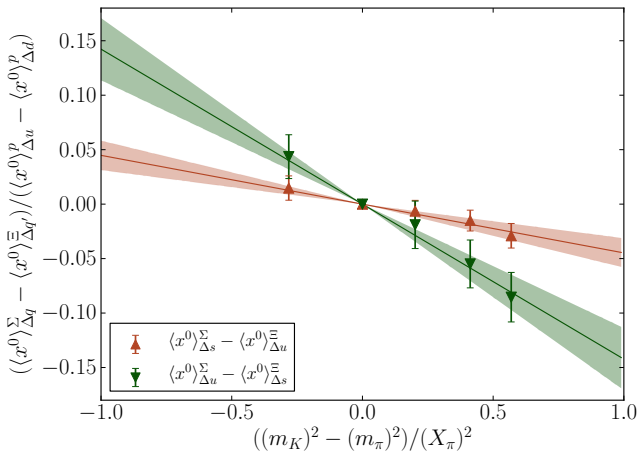


FIG. 5: The zeroth moment of the spin difference between doubly and singly represented quarks in the Σ and Ξ as a function of the strange/light quark mass difference. We deduce $\delta\Delta u^0$ and $\delta\Delta d^0$, respectively, from the slopes of these curves (c.f. Eqs. (16) - (19)).

where $z(\mu, a)$ is the difference between the (scale-dependent) singlet and (scale-independent) nonsinglet axial-vector current renormalisation constants. At order $\mathcal{O}(\alpha_s^2)$ in perturbation theory this results in a correction of $< 1\%$ at $\mu^2 = a^{-2} = 6 \text{ GeV}^2$ [11, 19] and, due to its small anomalous dimension, also at other scales, e.g. $\mu^2 = 4 \text{ GeV}^2$.

Substituting these values into Eqs. (16)-(19) yields the first lattice QCD estimates of the spin CSV moments

$$\begin{aligned} \delta\Delta u^{0+} &= -0.0116(27), & \delta\Delta d^{0+} &= -0.0036(11), \\ \delta\Delta u^{1-} &= -0.0020(5), & \delta\Delta d^{1-} &= -0.0009(2). \end{aligned} \quad (21)$$

We can make several observations regarding these spin CSV moments. First, the fractional spin CSV for both moments and both flavours are similar in magnitude and all have the same (negative) sign. Second, we can compare the first moments of the spin CSV distributions with the corresponding first moments of the spin-independent CSV distributions that were reported in Ref. [6], namely

$$\delta u^+ = -0.0023(6), \quad \delta d^+ = +0.0020(3). \quad (22)$$

The first moments of the spin-independent CSV results have roughly equal magnitudes but opposite signs, with δu being negative and δd positive, in both qualitative and quantitative agreement with quark model predictions [5, 20] and with the best-fit values from a global fit that included valence CSV [4].

Next we note that the zeroth moments of the spin-dependent CSV distributions are larger than the first moments. Lastly, we have estimated the CSV associated only with the $u-d$ mass difference. It is important to also find a method to investigate the CSV induced by electromagnetic effects which, at least in the unpolarised case, is expected to be of a similar size [21, 22].

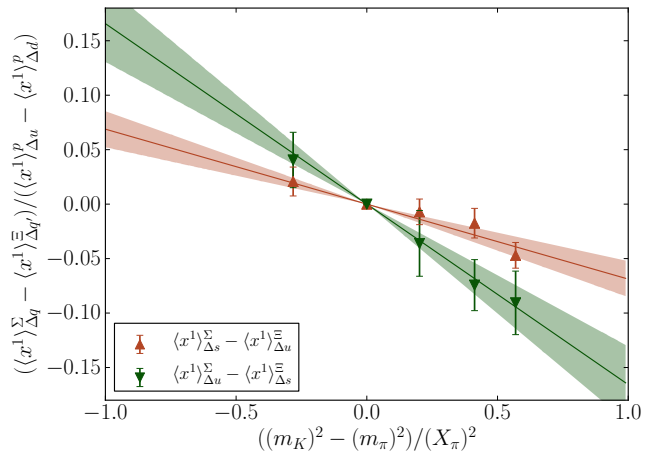


FIG. 6: The first moment of the spin difference in the Σ and Ξ vs. the strange/light quark mass difference, from which we deduce $\delta\Delta u^1$ and $\delta\Delta d^1$.

V. QUARK MODEL COMPUTATION

We can compare the lattice results with estimates of valence quark spin-dependent PDFs obtained from quark model calculations. Schreiber, Signal and Thomas [23] calculated parton spin distributions from bag models. Sather [24] derived an analytic approximation giving valence parton CSV distributions in terms of derivatives of phenomenological PDFs. Sather's equations are valid for parton distributions at a low Q^2 scale appropriate for quark models, and should also be valid for CSV spin distributions. In this approximation, the valence parton CSV spin distributions are

$$\begin{aligned} \delta\Delta d^-(x) &= -\frac{\delta M}{M} \frac{d}{dx} [x\Delta d^-(x)] - \frac{\delta m}{M} \frac{d}{dx} \Delta d^-(x), \\ \delta\Delta u^-(x) &= \frac{\delta M}{M} \left(-\Delta u^-(x) + (1-x) \frac{d}{dx} \Delta u^-(x) \right), \end{aligned} \quad (23)$$

where δM is the $n-p$ mass difference and δm is the diquark mass difference $m_{dd} - m_{uu}$ which is determined rather accurately to be 4 MeV [25]. The zeroth moment of the spin dependent CSV distributions is overly sensitive to the small- x behaviour of these PDFs, a region where the quark model results are less reliable. Therefore we compare only with the first moments of the CSV spin-dependent distributions. Using the model of Schreiber, Signal and Thomas we find

$$\delta\Delta u^{1-} = -0.0008, \quad \delta\Delta d^{1-} = -0.0011. \quad (24)$$

Alternatively, if we use the spin-dependent PDFs from a Nambu-Jona-Lasinio model calculation [26, 27] together with Eqs. (23), we find

$$\delta\Delta u^{1-} = -0.0003, \quad \delta\Delta d^{1-} = -0.0007. \quad (25)$$

These phenomenological model predictions agree with the lattice results insofar as the first spin moments are all the same (negative) sign, and have similar magnitudes. As for the spin independent case, the result for the down distribution, which is dominated by the diquark mass shift, is in better agreement than that for the up quark where there are a number of small corrections, not all included in the Sather approximation.

VI. BJORKEN SUM RULE

Finally, the spin-dependent CSV distributions contribute to the Bjorken sum rule [28], which has the form

$$\int_0^1 dx [g_1^p(x) - g_1^n(x)] = \frac{G_A}{6G_V} \left[1 - \frac{\alpha_S(Q^2)}{\pi} \right] = \left\langle \frac{\Delta u^+(x) - \Delta d^+(x)}{6} + \frac{4\delta\Delta d^+(x) + \delta\Delta u^+(x)}{18} \right\rangle. \quad (26)$$

In the first line of Eq. (26) we write the Bjorken sum rule in terms of the difference of the spin-dependent structure functions g_1 for the proton and neutron, integrated over all x . In the second line of Eq. (26) we write the sum rule in terms of the first moment of spin-dependent parton distributions. This quantity is correct up to terms of order $\mathcal{O}(\alpha_S)$ (there are also higher-twist terms of order $\mathcal{O}(1/Q^2)$). We see that, except for the CSV corrections, this ratio is given by the zeroth moment of the difference of the C-even spin distributions Δu^+ and Δd^+ integrated over all x .

We have included the contribution from partonic spin CSV in Eq. (26) which is noteworthy for several reasons. First, with the exception of corrections arising from partonic spin CSV terms, there are essentially no other partonic corrections to the Bjorken sum rule at leading twist (this is one reason why it is so important to obtain precise values for this sum rule). Second, the correction involves the zeroth moments of $\delta\Delta u^+$ and $\delta\Delta d^+$. At present the Bjorken sum rule is best determined from a recent COMPASS experiment at $Q^2 = 3 \text{ GeV}^2$ to a precision of about 8% [29]. Using the zeroth moment obtained from our lattice calculations (see Eq. (21)) we estimate that the spin CSV terms contribute approximately 1% to the Bjorken sum rule. At the present measured precision it is not

possible to observe such a small contribution. However, the Bjorken sum rule could in principle be measured at a future electron collider, where one could imagine aiming for 1% precision [30]. With such precision it is possible that the spin CSV contributions calculated here would be sufficiently large to make a measurable difference in the sum rule.

VII. CONCLUSION

In summary, we have performed the first lattice determinations of the polarised quark moments of the hyperons, Σ and Ξ in $N_f = 2 + 1$ lattice QCD. By examining the $SU(3)_{\text{flavour}}$ -breaking effects in these momentum fractions, we are able to extract the first QCD determination of the size and sign of charge-symmetry violation in the spin-dependent parton distribution functions in the nucleon, $\delta\Delta u$ and $\delta\Delta d$. We compare our results with estimates of the first moment of the parton spin CSV from a quark model calculation, obtaining qualitative agreement with the quark model results. Finally, we estimate the contribution of partonic spin CSV to the Bjorken sum rule, and show that spin CSV effects should change the Bjorken sum rule by approximately 1%.

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- [1] G. A. Miller, A. K. Opper and E. J. Stephenson, *Ann. Rev. Nucl. Part. Sci.* **56** (2006) 253 [nucl-ex/0602021].
 - [2] J. T. Londergan and A. W. Thomas, *Prog. Part. Nucl. Phys.* **41** (1998) 49 [hep-ph/9806510].
 - [3] J. T. Londergan, J. C. Peng and A. W. Thomas, *Rev. Mod. Phys.* **82** (2010) 2009 [arXiv:0907.2352 [hep-ph]].
 - [4] A. D. Martin, R. G. Roberts, W. J. Stirling and R. S. Thorne, *Eur. Phys. J. C* **35** (2004) 325 [hep-ph/0308087].
 - [5] E. N. Rodionov, A. W. Thomas and J. T. Londergan, *Mod. Phys. Lett. A* **9** (1994) 1799.
 - [6] R. Horsley *et al.* (CSSM and QCDSF/UKQCD Collaborations), *Phys. Rev. D* **83** (2011) 051501 [arXiv:1012.0215 [hep-lat]].
 - [7] N. Cundy *et al.*, *Phys. Rev. D* **79** (2009) 094507 [arXiv:0901.3302 [hep-lat]].
 - [8] W. Bietenholz *et al.*, *Phys. Lett. B* **690** (2010) 436 [arXiv:1003.1114 [hep-lat]].

- [9] W. Bietenholz *et al.*, Phys. Rev. D **84** (2011) 054509 [arXiv:1102.5300 [hep-lat]].
- [10] M. Deka *et al.*, Phys. Rev. D **79** (2009) 094502 [arXiv:0811.1779 [hep-ph]].
- [11] G. S. Bali *et al.*, arXiv:1112.3354 [hep-lat].
- [12] G. Martinelli *et al.*, Nucl. Phys. B **445** (1995) 81 [hep-lat/9411010].
- [13] M. Göckeler *et al.*, Nucl. Phys. B **544** (1999) 699 [hep-lat/9807044].
- [14] M. Göckeler *et al.*, Phys. Rev. D **82** (2010) 114511 [arXiv:1003.5756 [hep-lat]].
- [15] D. B. Leinweber, Phys. Rev. D **53** (1996) 5115 [hep-ph/9512319].
- [16] H. Leutwyler, Phys. Lett. B **378** (1996) 313 [hep-ph/9602366].
- [17] K. Nakamura *et al.* [Particle Data Group Collaboration], J. Phys. G **37** (2010) 075021.
- [18] J. Blümlein and H. Böttcher, Nucl. Phys. B **841** (2010) 205 [arXiv:1005.3113 [hep-ph]].
- [19] A. Skouroupathis and H. Panagopoulos, Phys. Rev. D **79** (2009) 094508 [arXiv:0811.4264 [hep-lat]].
- [20] J. T. Londergan and A. W. Thomas, Phys. Lett. B **558** (2003) 132 [hep-ph/0301147].
- [21] A. D. Martin, R. G. Roberts, W. J. Stirling and R. S. Thorne, Eur. Phys. J. C **39** (2005) 155 [hep-ph/0411040].
- [22] M. Glück, P. Jimenez-Delgado and E. Reya, Phys. Rev. Lett. **95** (2005) 022002 [hep-ph/0503103].
- [23] A. W. Schreiber, A. I. Signal and A. W. Thomas, Phys. Rev. D **44** (1991) 2653.
- [24] E. Sather, Phys. Lett. B **274** (1992) 433.
- [25] R. P. Bickerstaff and A. W. Thomas, J. Phys. G **15** (1989) 1523.
- [26] I. C. Cloët, W. Bentz and A. W. Thomas, Phys. Lett. B **621**, 246 (2005) [hep-ph/0504229].
- [27] I. C. Cloët, W. Bentz and A. W. Thomas, Phys. Rev. Lett. **95**, 052302 (2005) [nucl-th/0504019].
- [28] J. D. Bjorken, Phys. Rev. **148** (1966) 1467; Phys. Rev. D **1** (1970) 1376.
- [29] M. G. Alekseev *et al.* [COMPASS Collaboration], Phys. Lett. B **690** (2010) 466 [arXiv:1001.4654 [hep-ex]].
- [30] E. Sichtermann, 6th Circum Pan-Pacific Symposium on High Energy Spin Physics, Vancouver 2007, unpublished.
- [31] R. G. Edwards and B. Joó (SciDAC), Nucl. Phys. Proc. Suppl. **140** (2005) 832 [hep-lat/0409003].
- [32] P. A. Boyle, Comp. Phys. Comm. **180**, (2009) 2739.