# Evading the pulsar constraints on the cosmic string tension in supergravity inflation

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## Abstract

The cosmic string is a useful probe of the early Universe and may give us a clue to physics at high energy scales which particle accelerators cannot reach. Although the most promising tool to observe it is the cosmic microwave background (CMB), the constraint from gravitational waves is becoming so stringent that detecting its signatures in CMB may be impossible. In this paper, we construct a scenario that contains cosmic strings observable in the cosmic microwave background while evading the constraint imposed by the recent pulsar timing data. We argue that cosmic strings with relatively large tension are allowed by diluting loops contributing to the relevant frequency range of the gravitational wave background. We also present a particle physics model to realize such dilution in the context of chaotic inflation in supergravity, where the phase transition occurs during inflation due to the time-dependence of the Hubble induced mass.

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## I.  INTRODUCTION

Cosmic strings [1] are line-like topological defects which may have been generated in a phase transition in the early Universe through the Kibble mechanism [2]*1. Contrary to other topological defects such as monopoles and domain walls, they do not dominate the energy density of the Universe, since they evolve according to a scaling law [4]. At first, they were expected to explain the origin of the primordial density fluctuations that lead to structure formation [5]. Although recent observations of the CMB have ruled out the possibility that cosmic strings alone are responsible for the primordial density fluctuation, they may still contribute to the CMB temperature fluctuation [6–9], which would be a clue to exploring physics at high energy scales.

Thus far, many observations for detecting cosmic strings through the CMB have been done by using the Gott-Kaiser-Stebbins effect [10]. The geodesics of photons passing near a moving string are perturbed, which affects the temperature fluctuations of the CMB. One can thereby constrain the line energy density, or tension, $\mu$. The latest constraint on the string tension is $G\mu \lesssim 10^{-7}$ [6, 7]*2 with $G$ being the gravitational constant. Future observations such as Planck [11, 12] and CMBpol [13] are expected to detect the signatures of strings on small scales or in larger multipoles of the power spectrum with a tension up to $G\mu \sim 10^{-8}$ [8, 9].

There is yet another probe to detect or constrain cosmic strings, the stochastic gravitational wave background (GWB) generated by oscillating string loops [14–19]. When long cosmic strings in a Hubble volume intersect with each other, they reconnect and generate closed string loops. As a consequence, the distribution of infinite strings obeys the scaling rule [20–22]. At the same time, the loops produced in such a way oscillate, emit gravitational waves (GWs), and shrink gradually. Since the frequency of GWs is determined by the size of cosmic string loops that are generated continuously in the cosmic history, the spectrum of GWB, which are the sum of GWs emitted by each loop, are expected to range widely, from $10^{-10}$ Hz to $10^5$ Hz or even higher frequencies [14]. Although their amplitude is typically very small, we may observe their signatures in the GWB through the future gravitational

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*1 See [3], however, for the impossibility of thermal phase transitions to produce strings relevant to structure formation even in the classical big bang cosmology.

*2  To be precise, the constraint on the string tension depends on the model of string evolution.

detectors [19] such as eLISA [23], DECIGO [24], and BBO [25] as well as the ongoing pulsar timing arrays (PTAs) [26, 27]. In PTAs, GWs can be detected through the modulation of arrival of pulses from the millisecond pulsars (MSPs) due to the change of the distance between the Earth and MSPs caused by the GWB.

While the PTAs have not detected the signatures of strings thus far, they have obtained stringent constraints on the cosmic string tension [26, 27]. In particular, the European Pulsar Timing Array (EPTA) has recently reported the most stringent constraint, $G\mu < 10^{-9}$ [27], though it depends on some assumptions regarding string loop formation and GW emission[*3] Therefore, if we believe this constraint, we cannot hope to detect the signature of cosmic strings in the CMB.

In this paper, we reconsider the formation mechanism of cosmic strings and propose a new scenario which does not contradict with the present GW observations while accommodating large enough $G\mu$ to be detectable in the CMB. If they are sufficiently dilute, the loop formation is reduced and hence the amplitude of GWs is suppressed. This can be realized if the phase transition takes place during inflation as proposed in several literatures [3, 30–35]. While previous proposals assumed nontrivial interactions between the string-forming Higgs field and the inflaton field [31–33] or gravity [3, 34], we here point out that the F-term inflation in supergravity can naturally realize a phase transition during inflation thanks to the Hubble induced mass [36], as studied in Refs. [37, 38]. To be concrete, we construct a model that generates cosmic stings during chaotic inflation [39] in supergravity [40]. In this model, the separation of infinite strings is expanded so large that they cannot make loops until long after inflation and hence they enter the scaling regime at a later epoch, for example, around the time of matter-radiation equality. Since sources of GWs do not exist at early times, we can evade the PTA constraint on the GWB.

The rest of this paper is organized as follows. In section II, we briefly review the GWs emitted by cosmic string loops and see the current limit from observation using MSPs. In section III, we propose a new scenario for the generation and evolution of cosmic strings

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[*3] Recently, Ref. [28] claims a mild and conservative constraint, $G\mu < 5 \times 10^{-7}$, for a certain size of the cosmic string loops, which is disfavored by recent numerical simulations [22]. For the size of cosmic string loop favored in Ref. [22], Ref. [28] gives a more stringent constraint $G\mu < 10^{-10}$. Although the size of cosmic string loops is still a matter of debate [29], we here focus on the case where the loop length is large as suggested in Ref. [22] and explore a way to evade the PTA constraints.

which is consist with current observations and discuss the possibility of their detection in the future. In section IV, we construct a model realizing such a scenario, and consider how to determine the parameters of our model. Section V is devoted to discussion.

## II. GRAVITATIONAL WAVES FROM STRING LOOPS AND PTA LIMIT

Here we review the gravitational wave emission from cosmic string loops. In order to estimate the GWB today, first we have to know the size distribution of loops at a given time. Without any intersection and reconnection, the energy density of long string network would fall in proportion to $a^{-2}$. Actually, however, these processes frequently occur on cosmic time scales to transfer a part of the network's energy to loops which decay by radiating gravitational waves if the number density of long strings is large enough. As a result, the system relaxes to a scaling solution and the energy density of long strings decreases in proportion to $d_{\mathrm{H}}^{-2} = H^2$. In terms of the characteristic length of the strings, $\xi = \gamma d_{\mathrm{H}} = \gamma H^{-1}$, where $\gamma$ is a numerical coefficient, we can express the energy density of long strings as

$$\rho = \frac{\xi\mu}{\xi^3} = \frac{\mu}{\xi^2}\,. \tag{1}$$

The coefficient $\gamma$ is obtained by numerical simulation. Recent simulation suggests $\gamma = 0.15$ in the radiation dominant era, and $\gamma = 0.17$ in the matter dominant era [22].

Now let us determine the loop distribution function $n(t, \ell)$, which represents the number density of loops with size $\ell$ at time $t$. At the moment of loop formation ($t = t_g$), its size is proportional to the horizon size. We can take the initial length as $\alpha t_g$ with $\alpha$ a constant. The length of a loop becomes smaller with time due to the loss of energy by gravitational wave emission. The energy transferred from a loop into gravitational waves per unit time is expressed as $\Gamma G\mu^2$, where $\Gamma$ is a constant around 50 [21]. Thus, the length of a loop at time $t$ generated at $t_g$ can be written as

$$\ell = \alpha t_g - \Gamma G\mu(t - t_g)\,. \tag{2}$$

In other words, loops with size $\ell$ at time $t$ were formed at

$$t_g = \frac{\ell + \Gamma G\mu t}{\alpha + \Gamma G\mu}\,, \tag{3}$$

while those with size $\ell + d\ell$ at time $t$ were formed at

$$t_g + dt_g = \frac{\ell + d\ell + \Gamma G\mu t}{\alpha + \Gamma G\mu}. \tag{4}$$

In the flat FRW universe dominated by a perfect fluid with its equation of state given by $P = w\rho$, the scale factor evolves as $a \propto t^{\frac{2}{3(1+w)}}$, and the Hubble parameter is given by $H = \frac{2}{3(1+w)t}$. Taking these facts into account, we obtain the following functional form.

$$
\begin{aligned}
n(t,\ell) &= \frac{8(1+3w)}{27(1+w)^3\gamma^2 t_g^3} \times \frac{1}{\alpha(\ell + \Gamma G\mu t)} \times \left(\frac{a(t_g)}{a(t)}\right)^3 \\
&\approx \frac{10(\alpha + \Gamma G\mu)^3}{\alpha(\ell + \Gamma G\mu t)^4} \times \left(\frac{a(t_g)}{a(t)}\right)^3.
\end{aligned}
\tag{5}
$$

The last approximation is valid for loops which were generated in both the radiation- ($w = 1/3$) and matter-dominated ($w = 0$) regimes.

Now we briefly summarize the formulation to estimate the energy density of GWB. This was originally done by Damour and Vilenkin [15], and some improvements were proposed in Ref. [18], whose notation we follow. Taking the effect of cusps into account, the Fourier mode of amplitude, $h$, of gravitational radiation emitted by a loop with length $\ell$ at redshift $z$ is given by

$$h(f, z, \ell) = \frac{G\mu H_0 \ell^{\frac{2}{3}}}{(1+z)^{\frac{1}{3}}\varphi_r(z)} |f|^{-\frac{4}{3}}\Theta[1 - \theta_m(f, z, \ell)]. \tag{6}$$

Here $\varphi_r(z)$ represents the comoving distance to the loop defined as

$$r = \frac{1}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_\Lambda + \Omega_{m,0}(1+z)^3 + \Omega_{r,0}(1+z)^4}} \equiv \frac{1}{H_0}\varphi_r(z), \tag{7}$$

and $\theta_m$ is defined by $\theta_m \equiv \left(\frac{(1+z)f\ell}{2}\right)^{-\frac{1}{3}}$. The argument of the Heaviside function $\Theta$ corresponds to the lowest frequency of loop oscillation. The energy density of GWB is obtained by integrating the contribution from various loops,

$$
\begin{aligned}
\Omega_{\mathrm{GW}}(f) &= \frac{4\pi^2}{3H_0^2} f^3 \int dz \int dl\, h^2(f, z, \ell) \frac{d^2 R(z, \ell)}{dz d\ell} \\
&= \frac{4\pi^2}{3H_0^2} f^3 \int dz \int dh\, h^2 \frac{d^2 R(z, h)}{dz dh}.
\end{aligned}
\tag{8}
$$

Here $\frac{d^2 R(z, \ell)}{dz d\ell}$ is the observable burst rate per length per redshift

$$\frac{d^2 R(z, \ell)}{dz d\ell} = \frac{1}{H_0^3}\varphi_V(z)\frac{1}{1+z}\frac{2n\left(t(z), \ell\right)}{\ell}\frac{\theta_m^2(f, z, \ell)}{4}\Theta[1 - \theta_m(f, z, \ell)], \tag{9}$$

where $H_0^{-3}\varphi_V(z)dz$ represents the spatial volume corresponding to $z \sim z + dz$. Its detailed derivation is found in Ref. [18].

Next, let us consider the observational constraint on the GWB. Currently the strongest limit comes from the Pulsar Timing Array (PTA) experiment, which aims at the detection of the GWB using MSPs. (Detailed explanation and current status is found in Ref. [27].) This experiment has sensitivity at $f = 10^{-9} \sim 10^{-7}$Hz, which is determined by the duration of the observation. At this very low frequency, considering the effect of cusps, one can conclude that the GWB spectrum of the energy density, $\Omega_{\mathrm{GW}} = \dfrac{1}{\rho_{cr}}\dfrac{d\rho_{\mathrm{GW}}}{d\log f}$, is proportional to $f^{-\frac{1}{3}}$, coming from loops which are generated during the matter-dominated era.

According to Ref. [27] the latest limit is

$$\Omega_{\mathrm{GW}}(f = 1\mathrm{yr}^{-1})h^2 \leq \begin{cases} 10^{-8.4}(2\sigma), \\ 10^{-9.0}(1\sigma). \end{cases} \tag{10}$$

This has been translated to the upper limit on the line density of cosmic string as [27],

$$G\mu \leq 4.0 \times 10^{-9} . \tag{11}$$

Note that this constraint has been obtained using the method of Damour and Vikenkin [15] taking $\alpha > \Gamma G\mu$ and incorporating all the relevant modes.[*4] On the other hand, analysis based on the EPTA data has been done in Ref. [28] incorporating possible variations of cosmic string properties. They reported the most conservative constraint on the cosmic string tension as

$$G\mu < 5.3 \times 10^{-7} \tag{12}$$

which is obtained in the case of $\alpha$ taking a small value, $\alpha \simeq 10^{-5}$, and assuming the effect of cusps is neglegible. However, recent studies [15–18] emphasize that the effect of cusps, which emit higher mode waves, is unignorable. Moreover, recent simulations [22] indicate that the typical value of $\alpha$ is much larger than $10^{-5}$. If we take the effect of cusps into account and assume $\alpha \simeq 0.1$ as in Ref. [28] the constraint of the cosmic string tension turns out to be much more severe, $G\mu < 10^{-10}$. Here we propose a method by which the EPTA constraint can be evaded even in the case where $\alpha$ is not small and when the effect of cusps are taken into account.

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[*4] The formalism developed in Ref. [18] would yield a slightly more stringent constraint on the line density.

This turns to be possible by considering a different mechanism of string formation. Since the dominant contribution to $\Omega_{\mathrm{GW}}(f = 1\mathrm{yr}^{-1})$ in the standard scenario comes from the loops which were generated during late radiation and early matter-dominated eras, we can evade the PTA constraint if the number density of such loops can be reduced. In this case, we can explain the PTA constraint without setting $G\mu$ to be small.

## III.   A NEW SCENARIO OF STRING FORMATION

In the standard cosmic string scenario, where strings are formed by spontaneous symmetry breaking after reheating or at the end of hybrid inflation [41, 42], the string network soon relaxes to the scaling solution and it is impossible to reduce the number of small loops relevant to the PTA constraint. However, the number of loops can be reduced in a scenario where cosmic strings are formed not after inflation but *during* inflation. This type of scenario was also studied by one of us in a different context [3].

Let us see the detail of this scenario. Since the mean separation of strings becomes larger and larger as inflation proceeds, they have to wait for a long time to cross. Strings can neither intersect with each other nor form loops until their mean separation falls well below the horizon distance. In this scenario, we can remove the sources of the GWB (cosmic string loops) at early times, so we can satisfy the observational constraints imposed by GWB with a relatively big value of $G\mu$.

We calculate $\Omega_{\mathrm{GW}}$ in such a scenario where loops start to form only after the time $t_{\mathrm{sc}}$, when strings begin to satisfy the scaling law. Although some loops may have been formed before $t_{\mathrm{sc}}$, such loops would be rare and hence we here evaluate the GWB spectrum assuming that they are negligible and the GW emission starts at $t_{sc}$ suddenly.

The calculation can be done as follows. We perform $z$ integral in Eq. (8) to $z_{\mathrm{sc}}$, which corresponds to the redshift of $t_{\mathrm{sc}}$. The relation between cosmic time $t$ and $z$ is given by

$$t = \frac{1}{H_0} \int_z^\infty \frac{dz'}{(1 + z')\sqrt{\Omega_{\Lambda 0} + (1 + z')^3 \Omega_{\mathrm{m}0} + (1 + z')^4 \Omega_{\mathrm{r}0}}} \equiv \frac{1}{H_0} \varphi_t(z), \qquad (13)$$

$$z = \varphi_z(H_0 t) \quad \text{(the inverse function of } \varphi_t(z)\text{)}. \qquad (14)$$

Some modifications are needed in the $h$ integral in Eq. (8) . Shortly after $t_{\mathrm{sc}}$, at time $t_1$, the minimum loop size is $\ell_1 \equiv \alpha t_{\mathrm{sc}} - \Gamma G\mu(t_1 - t_{\mathrm{sc}})$ since very small loops are absent. Besides, the $\Theta$-function means that only the strings with $\ell > \frac{2}{f(1+z)} \equiv \ell_2$ contribute to the GWB.

Therefore the $h$ integral runs not from 0 but form $\max(h_1, h_2)$, where $h_1$ and $h_2$ correspond to the amplitude of GWs emitted from the strings with the length $\ell_1$ and $\ell_2$, respectively. The upper limit of $h$ corresponds to the maximum loop size. Although the authors of Ref. [17] insist that one can remove the contribution from rare bursts by introducing the cutoff in the $h$ integral, the value of the cutoff is ambiguous. Therefore we do not introduce it and include "all bursts".

Figure 1 shows the density parameter of GWBs from cosmic strings at the frequency $f = 1$ year$^{-1}$ as a function of $z_{\rm sc}$ for various values of $\alpha$. We can read off the condition $z_{\rm sc} \simeq 1 - 10^3$ should satisfy for each value of $\alpha$ and $G\mu$ to be consistent with the PTA observation. Figure 2 shows the maximum $z_{\rm sc}$ (or the earliest time for strings to start forming loops) in order not to contradict with the PTA data for $\alpha = 10^{-1}$. We find an approximate formula for $z_{\rm sc}$, which evades the PTA limit as

$$z_{\rm sc} \lesssim 3 \times 10^3 \left( \frac{G\mu}{10^{-8}} \right)^{-0.94} . \tag{15}$$

Therefore, we still have the opportunity to detect features of cosmic string using the future CMB observations with small enough $z_{\rm sc}$ since strings with $G\mu \lesssim 10^{-7}$ are allowed if loop formation is delayed.


## IV. COSMIC STRINGS AND INFLATION IN SUPERGRAVITY

We present a new mechanism to realize the above scenario in supergravity. While the previous models introduced a nonminimal coupling of the scalar curvature to the string-forming scalar field [3, 34] or direct coupling with the inflaton [31–33], our model does not require any additional interactions, since it is automatically provided by gravitationally suppressed interaction arising from supergravity.

The F-term potential in supergravity is written as

$$V = e^{\frac{K}{M_{\rm G}^2}} \left[ (D_i W) K^{ij\,-1} (D_j W)^* - \frac{3}{M_G^2} |W|^2 \right] , \tag{16}$$

where $K$ is the Kähler potential and $W$ is the superpotential. Here we have defined

$$K^{ij} = \frac{\partial^2 K}{\partial \phi_i \partial \phi_j^*} , \tag{17}$$

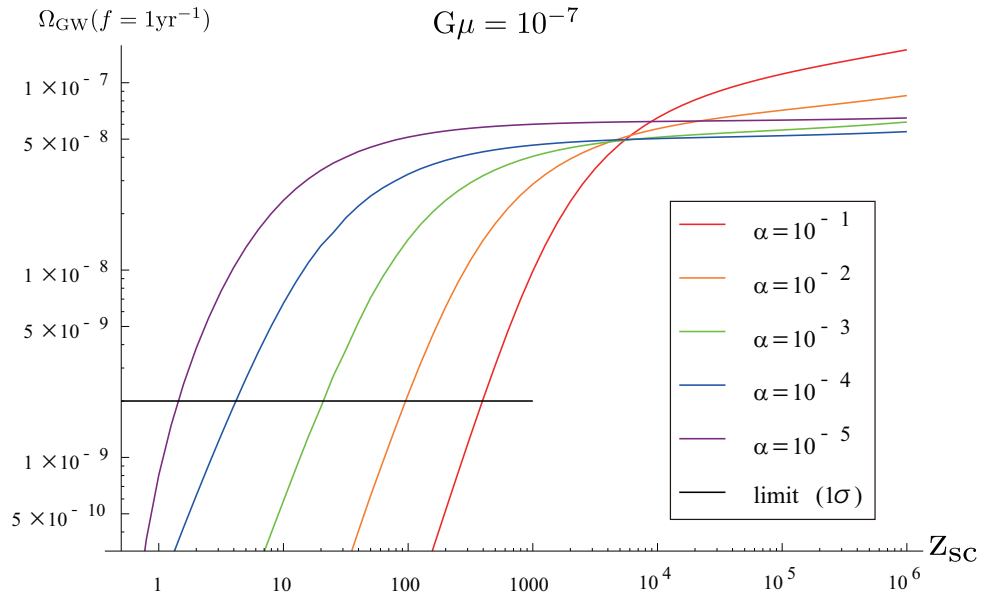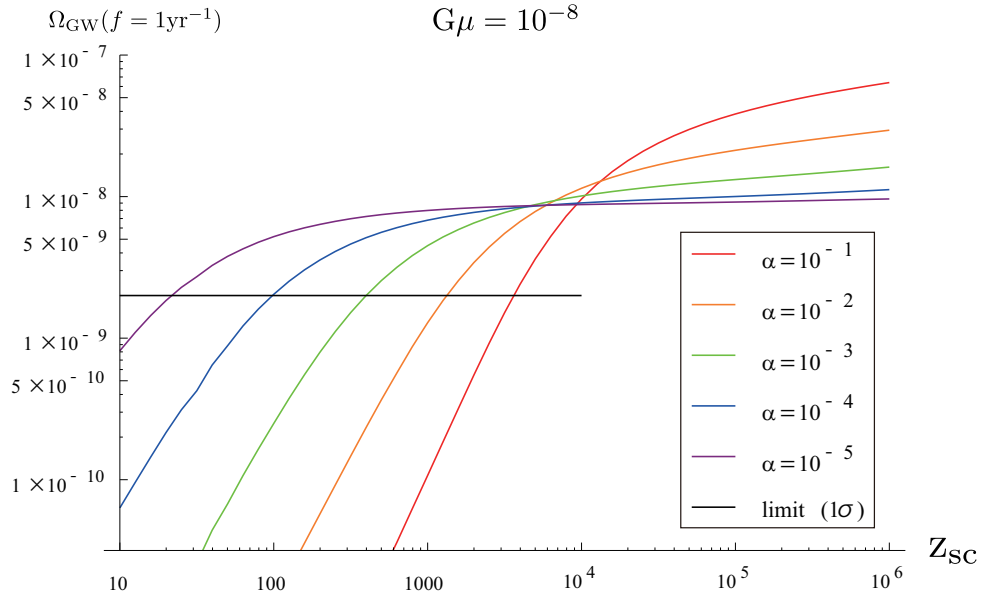$$D_i W = \frac{\partial W}{\partial \phi_i} + \frac{1}{M_G^2} \frac{\partial K}{\partial \phi_i} W , \tag{18}$$

8

$\Omega_{\mathrm{GW}}(f = 1\mathrm{yr}^{-1})$     $\mathrm{G}\mu = 10^{-8}$

$1 \times 10^{-7}$
$5 \times 10^{-8}$

$1 \times 10^{-8}$
$5 \times 10^{-9}$

$1 \times 10^{-9}$
$5 \times 10^{-10}$

$1 \times 10^{-10}$

| | |
|---|---|
| —— | $\alpha = 10^{-1}$ |
| —— | $\alpha = 10^{-2}$ |
| —— | $\alpha = 10^{-3}$ |
| —— | $\alpha = 10^{-4}$ |
| —— | $\alpha = 10^{-5}$ |
| —— | limit $(1\sigma)$ |

$10$     $100$     $1000$     $10^4$     $10^5$     $10^6$     $z_{\mathrm{sc}}$

$\Omega_{\mathrm{GW}}(f = 1\mathrm{yr}^{-1})$     $\mathrm{G}\mu = 10^{-7}$

$1 \times 10^{-7}$
$5 \times 10^{-8}$

$1 \times 10^{-8}$
$5 \times 10^{-9}$

$1 \times 10^{-9}$
$5 \times 10^{-10}$

| | |
|---|---|
| —— | $\alpha = 10^{-1}$ |
| —— | $=\alpha = 10^{-2}$ |
| —— | $\alpha = 10^{-3}$ |
| —— | $\alpha = 10^{-4}$ |
| —— | $\alpha = 10^{-5}$ |
| —— | limit $(1\sigma)$ |

$1$     $10$     $100$     $1000$     $10^4$     $10^5$     $10^6$     $z_{\mathrm{sc}}$

FIG. 1: Energy density of GWB at $f = 1\mathrm{year}^{-1}$, $\Omega_{\mathrm{GW}}$, as a function of $z_{\mathrm{sc}}$, the redshift at the onset of the scaling solution for various values of $\alpha$. The horizontal line shows the upper bound imposed by the PTA observation.
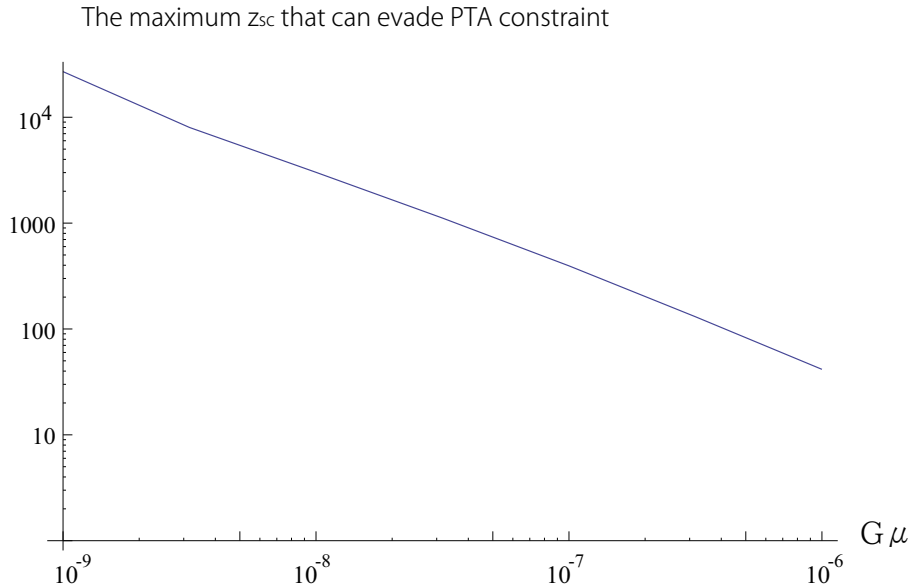
The maximum $z_{\text{sc}}$ that can evade PTA constraint



FIG. 2: The maximum $z_{\text{sc}}$ that can evade the PTA constraint as a function of $G\mu$ for $\alpha = 10^{-1}$.

and $M_G = 1/\sqrt{8\pi G}$ is the reduced Planck mass. During the standard slow-roll inflation, the energy density $\rho$ is dominated by the potential energy, so the Friedmann equation reads $3H^2 M_G^2 = \rho \cong V$. For $K \ll M_G^2$ we find

$$3M_G^2 H^2 \cong V \cong (D_i W) K^{ij\,-1} (D_j W)^* - \frac{3}{M_G^2}|W|^2 \tag{19}$$

in F-term inflation. Since the kinetic terms of the scalar fields are given by $-K^{ij}\partial_\mu \phi_i \partial^\mu \phi_j^*$, the minimal model assumes $K = |\phi_i|^2$ which yields canonical kinetic terms. Here we can expand the exponential factor after canonical normalization and diagonalization,

$$e^{\frac{K}{M_G^2}} = 1 + \frac{\phi_i \phi^{i*}}{M_G^2} + \cdots . \tag{20}$$

Therefore, the scalar potential $V$ contains the term $3H^2 \phi_i \phi^{i*}$. This means that the field $\phi_i$ receives an additional Hubble-scale mass, which is called the Hubble-induced mass[*5]. If the Hubble parameter is large enough at the beginning of inflation, these fields fall down to the origin quickly and the symmetry is restored during inflation. Moreover, in inflation models in

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[*5] The inflaton should not acquire the Hubble induced mass; otherwise the slow-roll condition cannot be satisfied. In order to realize inflation, one must assume shift symmetry [40] or adopt contrived models [43].

|        | $\phi$ | $X$ | $s$ | $\psi$ | $\bar{\psi}$ |
|--------|--------|-----|-----|--------|--------------|
| $U(1)_S$ | 0    | 0   | 0   | +1     | −1           |
| $R$      | 0    | +2  | +2  | 0      | 0            |
| $Z_2$    | −1   | −1  | 0   | 0      | 0            |

TABLE I: Charge assignments on superfields in the model under the $U(1)_S$ symmetry, the $R$-symmetry, and $Z_2$-symmetry.

which the Hubble parameter changes gradually during inflation, such as the chaotic inflation model [39], it is possible for the phase transition to take place during inflation. Here we adopt the following superpotential,

$$W = W_I + W_S, \tag{21}$$

$$W_I = M\phi X, \tag{22}$$

$$W_S = m^2 s - \lambda s \psi \bar{\psi}, \tag{23}$$

and the Kähler potential

$$K = \frac{1}{2}(\phi + \phi^*)^2 + |s|^2 + |X|^2 + |\psi|^2 + |\bar{\psi}|^2 \,, \tag{24}$$

to realize chaotic inflation, where $W_I$ and $W_S$ are superpotentials of the inflation sector and the string sector, respectively. Here $\phi$ is the inflaton, $X$ is an additional singlet, $\psi$ and $\bar{\psi}$ are the symmetry-breaking Higgs fields that break $U(1)_S$ symmetry, $s$ is a singlet which destabilizes the Higgs field at the origin, $M$ is the inflaton mass, $m$ is the symmetry breaking scale, and $\lambda$ is a coupling constant. Note that we impose $R$-symmetry and discrete $Z_2$ symmetry in order to suppress all other unwanted interactions such as $X\phi^2, s\phi$, and so on[*6]. Charge assignments on the fields are shown in Table I.

This model can realize chaotic inflation naturally thanks to the shift symmetry, $\phi \to \phi + c$ with $c$ being a real parameter, along the imaginary part of $\phi$ which plays the role of the

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[*6] Note that the term $\epsilon s \phi^2$ in the superpotential cannot be forbidden by these symmetries but it breaks the shift symmetry of $\phi$. Since we can expect that the inflaton mass $M$ arises as an order parameter of the shift symmetry breaking, the coupling constant $\epsilon$ should be suppressed enough, say $\epsilon \sim M^2 \sim 10^{-10}$ in Planck units. Thus, the following discussion does not change.

inflaton [40]. Setting $\phi = \frac{1}{\sqrt{2}}(\chi + i\varphi)$, the potential for these fields is

$$
\begin{aligned}
V &= e^{\frac{K}{M_G^2}} \left[ (D_i W) K^{ij\,-1} (D_j W)^* - \frac{3}{M_G^2} |W|^2 \right] \\
&= \exp\left[ \frac{1}{M_G^2} \left( \chi^2 + |X|^2 + |s|^2 + |\psi|^2 + |\bar{\psi}|^2 \right) \right] \\
&\quad \times \left[ \frac{1}{2} M^2 (\chi^2 + \varphi^2) + M^2 |X|^2 + m^4 - \lambda m^2 (\psi\bar{\psi} + \psi^*\bar{\psi}^*) \right. \\
&\quad\quad + \lambda^2 |\psi|^2 |\bar{\psi}|^2 + \lambda^2 |s|^2 |\bar{\psi}|^2 + \lambda^2 |s|^2 |\psi|^2 \\
&\quad\quad \left. + \frac{1}{M_G^2} \left\{ \left( \frac{3}{2} M^2 \chi^2 - \frac{1}{2} M^2 \varphi^2 \right) |X|^2 + \cdots \right\} \right] .
\end{aligned}
\tag{25}
$$

Note that we do not have to consider all of these terms because the fields $\chi$, $s$, $X$, $\psi$, and $\bar{\psi}$ have very steep potential due to the factor $e^{\frac{K}{M_G^2}}$ at the field values larger than $M_G$. This means they cannot have field values much bigger than $M_G$. Since $\chi$ and $s$ have larger masses than the Hubble parameter, they are practically fixed at the origin during inflation. Although $X$ has a mass of the order of the inflaton mass, its contribution to the total potential energy (or the dynamics of inflation) and the density fluctuation are negligible due to its much smaller field value than the inflaton. Since only the inflaton $\varphi$ can have a value much larger than $M_G$, this model reduces to a simple chaotic inflation model, $V = M^2\varphi^2/2$.

Now we investigate the phase transition during inflation. Diagonizing the mass matrix of $\psi$ and $\bar{\psi}$, we obtain the following canonically normalized fields:

$$
\begin{cases}
\psi_1 = \dfrac{1}{\sqrt{2}} (\psi + \overline{\psi}^*) , \\
\psi_2 = \dfrac{1}{\sqrt{2}} (\psi - \overline{\psi}^*) .
\end{cases}
\tag{26}
$$

The relevant terms that control the dynamics of the phase transition during inflation are

$$
\begin{aligned}
V &= \frac{1}{2} M^2 \varphi^2 + \left[ \frac{1}{M_G^2} \left( \frac{1}{2} M^2 \varphi^2 + m^4 \right) - \lambda m^2 \right] |\psi_1|^2 + \frac{\lambda^2}{4} |\psi_1|^4 \\
&\approx \frac{1}{2} M^2 \varphi^2 + \left( 3H^2 - \lambda m^2 \right) |\psi_1|^2 + \frac{\lambda^2}{4} |\psi_1|^4 .
\end{aligned}
\tag{27}
$$

The first term is the potential for the inflaton. The mass term of $\psi_1$ changes its sign from positive to negative during inflation at $H^2 = \lambda m^2/3$. As a result, $\psi_1$ may be destabilized and the $U(1)_S$ symmetry is spontaneously broken during inflation, while $\psi_2$ has always positive mass squared.

Our model contains the parameters $\lambda$, $m$, and $M$. $M$ is related to the primordial power

spectrum of curvature fluctuations via

$$\Delta_{\mathcal{R}}^2(k) = \frac{1}{24\pi^2} \frac{V}{M_G^4} \frac{1}{\epsilon}\Big|_{k=aH} \approx \frac{1}{6\pi^2} \frac{M^2}{M_G^2} \mathcal{N}^2, \tag{28}$$

with $\mathcal{N} \simeq 55$ being the number of $e$-folds when observable scales exit the horizon. The observed value $\Delta_{\mathcal{R}}^2(k) = 2.4 \times 10^{-9}$[44] determines the inflaton mass to be $M \cong 10^{13}$GeV. We assume $\lambda$ is of order of unity and $m$ is much smaller than $M_G$.

The line energy density of cosmic strings is related to the vacuum expectation value of the string-generating field as [*7]

$$\mu \cong 2\pi \langle \psi_1^2 \rangle = 4\pi \frac{m^2}{\lambda} . \tag{29}$$

Therefore one can determine the value of $m^2/\lambda$ by fixing $G\mu$.

The crucial issue is whether strings with $G\mu \geq 10^{-8}$, whose tension is large enough to be detectable by CMB observations [9], can be produced with appropriate density so that they can evade the PTA constraint but are observable using CMB. It is therefore important to study when the phase transition takes place to clarify the mean separation and the correlation length of the string network.

In our scenario, the phase transition is triggered by quantum fluctuation during inflation rather than thermal fluctuations and we can make use of the results of [3, 32, 35, 38] to analyze its properties. Calculations based on the stochastic inlfation method [45, 46] show that the phase of the string-forming field $\psi_1$ is fixed when the classical potential force surpasses the effect of stochastic quantum fluctuations [35]. This occurs $\Delta\mathcal{N} = \sqrt{c/2}$ e-folds after $\psi_1 = 0$ has become classically unstable with $3H^2 = \lambda m^2$ where $c \equiv \lambda m^2/M^2$ [35, 38]. The number of e-folds from this epoch to the end of inflation is given by

$$\mathcal{N}_D = \frac{1}{2}(c - \sqrt{2c} - 1), \tag{30}$$

which gives the extra dilution which is absent in the case where phase transitions occur after or without inflation. The Hubble parameter at this epoch, which we denote with a suffix $f$, is given by

$$H_f = \left(\frac{c}{3} - \frac{\sqrt{2c}}{3}\right)^{1/2} M. \tag{31}$$

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[*7] To be precise, one should treat the coefficient as a function depending on $\lambda$, rather than as a constant, $2\pi$. However within the range of interesting values of $\lambda$ in our model, we can approximate this coefficient as $2\pi$[42].

The typical separation between strings at this epoch can be estimated as in [3, 32]. First, choose an arbitrary point $\boldsymbol{x}_0$ with an arbitrary field value of $\psi_1(\boldsymbol{x}_0)$. One can always perform a gauge transformation to make this real: $\psi_1(\boldsymbol{x}_0) \equiv \psi_{1R}(\boldsymbol{x}_0) + i\psi_{1I}(\boldsymbol{x}_0) = \psi_{1R}(\boldsymbol{x}_0)$. Then, since a string is a locus of $\psi_{1R}(\boldsymbol{x}) = \psi_{1I}(\boldsymbol{x}) = 0$, the nearby string segment can be found by searching for a point (in fact, a line) with $\psi_{1R}(\boldsymbol{x}) = 0$ on the surface of $\psi_{1I}(\boldsymbol{x}) = 0$. To the lowest-order Taylor expansion, the condition for a string to exist within a distance $r$ from the point $\boldsymbol{x}_0$ is given by

$$r|\boldsymbol{e}_I \times \nabla\psi_{1R}(\boldsymbol{x}_0)| > |\psi_{1R}(\boldsymbol{x}_0)|, \tag{32}$$

where $\boldsymbol{e}_I \equiv \nabla\psi_{1I}(\boldsymbol{x}_0)/|\nabla\psi_{1I}(\boldsymbol{x}_0)|$ is the unit normal vector to the surface $\psi_{1I} = 0$. Since $\psi_{1R}(\boldsymbol{x}_0)$ and $\nabla\psi_{1R}(\boldsymbol{x}_0)$ are statistically independent, almost Gaussian variables, one can express the probability for the inequality to be satisfied at a distance $r$, $P(r)$, in terms of the variances $\langle\psi_{1R}^2\rangle \equiv \sigma^2$ and $\langle(\nabla\psi_{1I})^2\rangle/3 = \langle(\partial_x\psi_{1I})^2\rangle \equiv \sigma_{g'}^2$, which will be evaluated later. Without loss of generality one can take $z$-axis along $\boldsymbol{e}_I$. Then writing $\nabla\psi_{1R} \equiv (u_x, u_y, u_z)$, $P(r)$ is given by

$$\begin{aligned} P(r) &= \int_{r^2(u_x^2+u_y^2)>\psi_{1R}^2} P(\psi_{1R}, u_x, u_y, u_z) d\psi_{1R} d^3u \\ &= \frac{1}{(2\pi)^2 \sigma\sigma_{g'}^3} \int \exp\left(-\frac{\psi_{1R}^2}{2\sigma^2} - \frac{u_x^2 + u_y^2 + u_z^2}{2\sigma_{g'}^2}\right) d\psi_{1R} d^3u \\ &= \frac{\sigma_{g'} r}{(\sigma^2 + \sigma_{g'}^2 r^2)^{1/2}}, \end{aligned} \tag{33}$$

where $P(\psi_{1R}, u_x, u_y, u_z)$ is the one-point probability distribution function of $\psi_{1R}$ and $\nabla\psi_{1R}$. Thus the typical distance to reach a string, which also gives an estimate of the mean separation of strings, is given by $r \simeq \sigma/\sigma_{g'}$.

Next we calculate the variances at $t = t_f$, which is obtained from the power spectrum of $\psi_1$ [38]

$$\mathcal{P}_{\psi_1} = |\psi_{1k}(t_f)|^2 \simeq \frac{H^2(t_k)}{2k^3} \left(\frac{S(t_k)}{S(t_f)}\right) \exp\left[-\frac{9}{2M^2}\left(H(t_k) - \frac{2M}{3}\sqrt{c - \frac{1}{2}}\right)^2 + \frac{(2 - \sqrt{3})^2 c}{4}\right], \tag{34}$$

where

$$S(t) \equiv \left(\lambda m^2 - \frac{M^2}{2} - \frac{3H^2(t)}{4}\right)^{1/2}, \tag{35}$$

and $t_k$ is the epoch when the comoving mode $k$ exits the horizon. We can evaluate the ratio

of the variances as

$$\frac{\sigma^2}{\sigma_{g'}^2} = \frac{\int |\psi_{1k}(t_f)|^2 \frac{d^3 k}{(2\pi)^3}}{\int \frac{k^2}{3} |\psi_{1k}(t_f)|^2 \frac{d^3 k}{(2\pi)^3}} \equiv \left(\frac{\tilde{r}(c)}{H_f}\right)^2$$

$$\simeq 15\sqrt{2\pi}(x_f - x_d) \exp\left[\frac{15}{2}(x_f - x_d)^2 + \frac{c}{5} - \sqrt{2c} + \frac{2}{5}\right] \frac{1}{H_f^2} \qquad (36)$$

where

$$x_d \equiv \frac{2}{5}\left(c - \frac{1}{2}\right)^{1/2}, \quad x_f \equiv \left(\frac{c}{3} - \frac{\sqrt{2c}}{3}\right)^{1/2}. \qquad (37)$$

Now we get the typical separation of strings. In addition to the expression (36), we also numerically evaluate $\tilde{r}(c)$ as a function of $c$ and get $\tilde{r}(c) \simeq 400$ at $c = 100$.

After the initial configuration of the string network is determined at $t_f$ during inflation, the typical separation simply scales as the scale factor until it falls below the Hubble radius. At redshift $z$, it reads

$$d(z) = r e^{\mathcal{N}_D} \frac{a_{\rm R}}{a_{\rm end}} \frac{a_{\rm eq}}{a_{\rm R}} \frac{a(z)}{a_{\rm eq}}$$

$$= 3 \times 10^{18} \frac{\tilde{r}(c)}{x_f(c)} \exp\left[\frac{1}{2}(c - \sqrt{2c} - 1)\right] \frac{1}{z + 1} \left(\frac{T_{\rm R}}{10^6 \, {\rm GeV}}\right)^{-\frac{1}{3}} {\rm GeV}^{-1}, \qquad (38)$$

where the suffixes end, R, and eq represent the epochs of the end of inflation, reheating and radiation-matter equality, respectively. If we assume the network enters the scaling regime immediately after the string separation becomes smaller than the characteristic length of a scaling network $d(z) \lesssim \xi(z)$, where,

$$\xi(z) = \gamma d_{\rm H} = \gamma H^{-1} = \frac{\gamma \times 7 \times 10^{41}}{\sqrt{\Omega_{\Lambda 0} + \Omega_{\rm m0}(1 + z)^3 + \Omega_{\rm r0}(1 + z)^4}} {\rm GeV}^{-1}, \qquad (39)$$

then we can get a relation between the model parameres and $z_{\rm sc}$ by equating (38) and (39) to determine the region of the parameter space where traces of cosmic strings are detectable by future CMB observations without conflicting with the PTA constraints. Figure 3 shows allowed region of the model parameters where the redshift at the onset of the scaling regime has been obtained from an equality

$$d(z_{\rm sc})/\xi(z_{\rm sc}) = 1. \qquad (40)$$

We must admit, however, that this estimate has some uncertainty. Unlike in the standard scenario where the string network approaches the scaling solution as the string density decreases, in the present case it is reached as the number of long strings per the Hubble volume

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increases to the scaling value. To our knowledge, such a situation has not been numerically simulated, and it is an open issue when the system relaxes to the scaling solution. Fortunately, however, there exists a finite section of parameter space where all the requirements are met, even if we change our criterion of the epoch when the scaling law is achieved.

Indeed, if we assume the scaling is achieved when an equality $d(z_{\rm sc})/\xi(z_{\rm sc}) = K$ is satisfied, we find that the two vertical lines and the oblique line representing the gravitational-wave constraint in Figure 3 shift to the right by

$$\frac{2}{100} \left( \frac{\sqrt{2c}}{\sqrt{2c}-1} \frac{c-\sqrt{2c}}{c-\sqrt{2c}-1} \right) \log K \qquad (41)$$

$$\simeq 2.2 \times 10^{-2} \log K \quad (\text{at } c = 100)$$

so that the shape and the size of the allowed region remain intact. Note that the parameter $K$ can also accommodate possible variation of the initial string separation, too. Therefore, we conclude that there are certainly interestingly large parameter space where signatures of the cosmic strings are detectable by the future CMB observation evading the PTA constraints.

We have shown in Figure 3 the parameter corresponding to $z_{\rm sc} = 1100$ and $d(z = 0) = H_0^{-1}$ which means there would be only one string inside our present horizon. We find $m$ should be about $10^{14}$GeV, and so use $m_{14} \equiv m/10^{14}$GeV. Therefore $c$ becomes $100\lambda m_{14}^2$. The horizontal axis is related to the time when strings start to scale and produce loops. According to the results of the previous section, an upper limit is imposed on $z_{\rm sc}$ in order not to contradict the MSP observation. This upper limit on $z_{\rm sc}$ yields a lower limit on $\lambda m_{14}^2$. The value of $G\mu$ is proportional to the value of vertical axis.

## V.  DISCUSSION

In this paper, we have studied a new mechanism to evade the PTA constraints on the cosmic string tension. We have found that the PTA constraints can be evaded if the loop formation starts at a later epoch, say, $z \lesssim 10^4$, depending on the value of $\alpha$ and the tension of the cosmic strings. We have also shown that this scenario can be realized if the phase transition and string formation take place during inflation. As a specific example, we have shown there is a finite region of parameter space where we can expect to detect the signatures of cosmic strings through CMB observations while accommodating the PTA constraints in the F-term chaotic inflation model in supergravity [40].
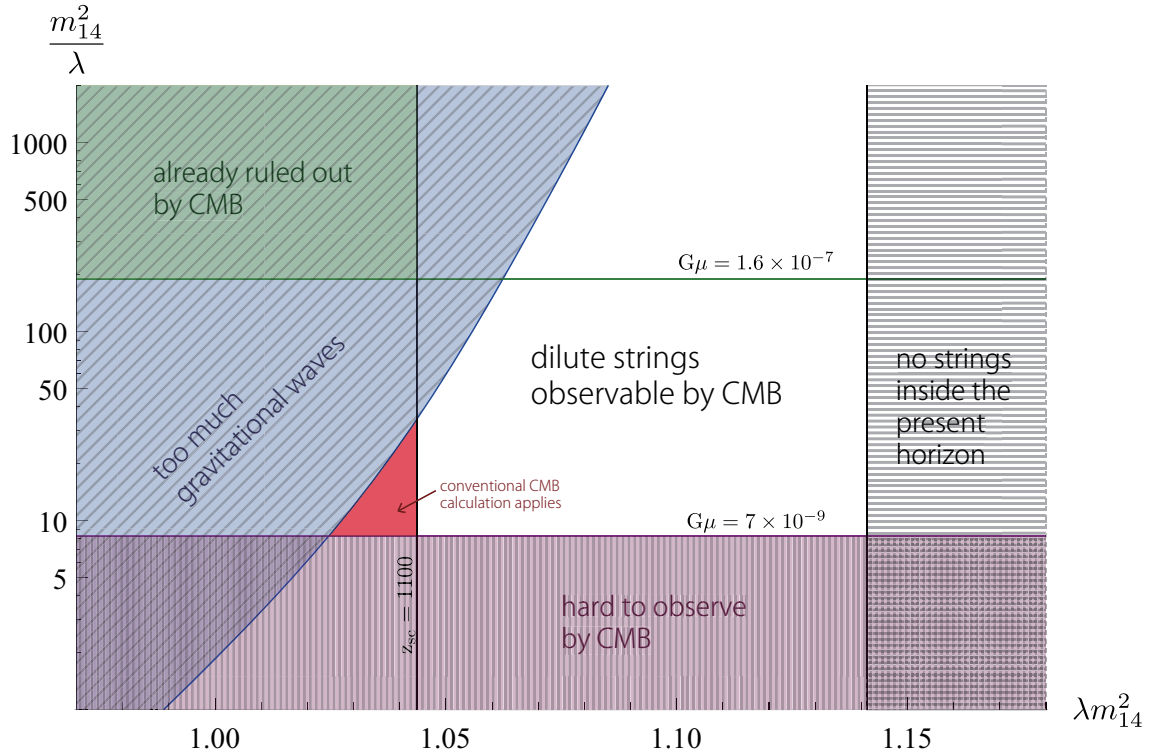
FIG. 3: The allowed parameter region of this model is shown. In the blue-painted region, the network of strings starts to produce loops so early that the gravitational waves emitted by them contradict observations. Though determining this region requires more statistical analysis of bursts and a more realistic treatment of $\alpha$, we emphasize that some region remains allowed. The most interesting region, which represents the case strings in which can not only evade the PTA constraint but also contribute to the anisotropy of the CMB as much as ordinary strings do. The white region corresponds to the case where strings are observable through the CMB but at densities smaller than in the conventional scenario. As a recent constraint on $G\mu$, we used the value obtained in [7]. According to [9], strings with $G\mu < 7 \times 10^{-9}$ are not observable.

As seen in Figure 1, the value of $\alpha$ is very important to determine the constraint on $z_{\rm sc}$. Recent simulation [22] indicates that loops are formed with various sizes ranging from $\alpha \simeq 10^{-1}$ to $\alpha < 10^{-5}$ in the standard scenario. In our scenario, however, the mean separation of cosmic strings is much larger than the horizon initially, so it is expected that larger loops are produced more frequently than in the ordinary scenario. In addition, inflation also erases small-scale fluctuations on long strings. Hence strings do not develop

small-scale structure and the correlation length remains of the order of the Hubble radius for some time, even after $z_{\mathrm{sc}}$. As a result, we expect $\alpha$ to take a relatively large value, $\alpha \sim 10^{-1}$, at the onset of the scaling regime.

Though we assume that the cosmic string network enters the scaling regime when the mean separation of cosmic strings becomes comparable to the characteristic length of the scaling network, the time of the onset of the scaling regime is still ambiguous. However, we can expect that the earliest candidate for the onset of the scaling regime is when the smallest scale of the cosmic strings, $(a(z)/a_f)H_f^{-1}$, becomes comparable to the characteristc length of the scaling network $\xi(z)$. We confirmed that, even in this case, there is a finite but interesting parameter space. Therefore, we conclude that there is certainly a region of parameter space where we can expect to detect the signatures of cosmic strings by future CMB observations.

In our scenario, the effect of the CMB may need to be reconsidered. If inflation did not last so long after strings were formed, stings start to satisfy the scaling rule at an earlier epoch. As a consequence, their separation would already become smaller than the horizon at the recombination. Then there would be as many strings as in the standard scenario near the last scattering surface of the CMB photons. The effect on the CMB would be the same as that has already been analyzed in the literature [47]. On the other hand, if inflation lasted very long after strings had been formed, they could not start to scale until very late time. Therefore, their separation would be larger than the horizon at the recombination. In this case, their effect on the CMB would become smaller. For a quantitative understanding, numerical simulations of the string network with appropriate initial conditions are desired.

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