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Supersymmetric solutions for non-relativistic holography

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Abstract

We construct families of supersymmetric solutions of type IIB and $D = 11$ supergravity that are invariant under the non-relativistic conformal algebra for various values of dynamical exponent $z \geq 4$ and $z \geq 3$, respectively. The solutions are based on five- and seven-dimensional Sasaki-Einstein manifolds and generalise the known solutions with dynamical exponent $z = 4$ for the type IIB case and $z = 3$ for the $D = 11$ case, respectively.

1 Introduction

There has recently been much interest in finding holographic realisations of systems invariant under the non-relativistic conformal algebra starting with the work [1], [2] and discussed further in related work [3]-[32]. Such systems are invariant under Galilean transformations, generated by time and spatial translations, spatial rotations, Galilean boosts and a mass operator, which is a central element of the algebra, combined with scale transformations. If x^+ is the time coordinate, and $\mathbf{x}$ denotes d spatial coordinates, the scaling symmetry acts as

$$\mathbf{x} \to \mu \mathbf{x}, \qquad x^+ \to \mu^z x^+ \ , \tag{1.1}$$

where z is called the dynamical exponent. When $z = 2$ this non-relativistic conformal symmetry can be enlarged to an invariance under the Schrödinger algebra which includes an additional special conformal generator.

The solutions found in [1], [2] with $d = 2$ and $z = 2$ were subsequently embedded into type IIB string theory in [8],[9],[10] and were based on an arbitrary five-dimensional Sasaki-Einstein manifold, SE_5. The work of [9] also constructed type IIB solutions with $d = 2$ and $z = 4$ and again these were constructed using an arbitrary SE_5. It was also shown in [9] that the solutions with $z = 2$ and $z = 4$ can be obtained from a five dimensional theory with a massive vector field after a Kaluza-Klein reduction on the SE_5 space [9]. This procedure was generalised to solutions of $D = 11$ supergravity in [31]: using a similar KK reduction on an arbitrary seven-dimensional Sasaki-Einstein space, SE_7, solutions with non relativistic conformal symmetry with $d = 1$ and $z = 3$ were found.

The type IIB solution of [8],[9],[10] with $z = 2$ do not preserve any supersymmetry [9]. One aim of this note is to show that, by contrast, the type IIB solutions of [9] with $z = 4$ and the $D = 11$ solutions of [31] with $z = 3$ are both supersymmetric and generically preserve two supersymmetries. A second aim is to generalise both of these supersymmetric solutions to different values of z. We will construct new supersymmetric solutions using eigenmodes of the Laplacian acting on one-forms on the SE_5 or SE_7 space. If the eiegenvalue is μ then we obtain type IIB solutions with $z = 1 + \sqrt{1 + \mu}$ and $D = 11$ solutions with $z = 1 + \frac{1}{2}\sqrt{4 + \mu}$. This gives rise to type IIB solutions with $z \geq 4$ and $D = 11$ solutions with $z \geq 3$, respectively. For the case of S^5 we get solutions with $z = 4, 5, \ldots$ while for the case of S^7 we get solutions with $z = 3, 3\frac{1}{2}, 4, \ldots$ and both of these preserve 8 supersymmetries.

Our constructions have some similarities with the construction of type IIB solutions in [24] that were based on eigenmodes of the Laplacian acting on scalar functions on the SE_5 space. Our IIB solutions preserve the same supersymmetry and we show how our solutions can be superposed with those of [24] while maintaining a scaling symmetry. An analogous superposition is possible for the $D = 11$ solutions, which we shall also describe.

2 The type IIB solutions

The ansatz for the type IIB solutions we shall consider is given by

$$\begin{aligned} ds^2 &= \frac{dr^2}{r^2} + r^2\left[2dx^+dx^- + dx_1^2 + dx_2^2\right] + ds^2(SE_5) + 2r^2 C dx^+ \\ F_5 &= 4r^3 dx^+ \wedge dx^- \wedge dr \wedge dx_1 \wedge dx_2 + 4Vol(SE_5) \\ &- dx^+ \wedge \left[*_{CY_3} dC + d(r^4 C) \wedge dx_1 \wedge dx_2\right] \end{aligned} \tag{2.1}$$

where SE_5 is an arbitrary five-dimensional Sasaki-Einstein space and the metric $ds^2(SE_5)$ is normalised so that the Ricci tensor is equal to four times the metric (i.e. the same normalisation as that of a unit radius five-sphere). Recall that the metric cone over the SE_5,

$$ds^2(CY_3) = dr^2 + r^2 ds^2(SE_5)\ , \tag{2.2}$$

is Calabi-Yau. The Kähler form on the CY_3 is denoted ω_{ij} and the complex structure is defined[1] by $J_i{}^j = \omega_{ik} g^{kj}$, where g_{ij} is the Calabi-Yau cone metric. We will define the one-form η, which is dual to the Reeb vector on SE_5 by

$$\eta_i = -J_i{}^j\,(d\log r)_j\ . \tag{2.3}$$

The one-form C is a one-form on the CY_3 cone. When $C = 0$ we have the standard $AdS_5 \times SE_5$ solution of type IIB which, in general, preserves eight supersymmetries (four Poincaré and four superconformal), corresponding to an $N = 1$ SCFT in $d = 4$. More generally, we can deform this solution by choosing $C \neq 0$ provided that dC is co-closed on CY_3:

$$d *_{CY} dC = 0\ . \tag{2.4}$$

With this condition, F_5 is closed and in fact it is also sufficient for the type IIB Einstein equations to be satisfied. As we will show these solutions preserve one

[1] While this is standard in the physics literature, often in the maths literature $J_i{}^j = -\omega_{ik} g^{kj}$.

half of the Poincaré supersymmetries. Note that the solution is invariant under the transformation

$$x^- \to x^- - \Lambda, \qquad C \to C + d\Lambda \tag{2.5}$$

for some function Λ on the CY cone. Thus, if $dC = 0$, we can remove C, at least locally, by such a transformation.

We will look for solutions where the one-form C has weight λ under the action of $r\partial_r$. Then it is straightforward to check, following [1] and [2] that our solution is invariant under non-relativistic conformal transformations with two spatial dimensions x^1, x^2 and dynamical exponent $z = 2 + \lambda$. For example the scaling symmetry is acting as in (1.1) combined with $r \to \mu^{-1}r$, $x^- \to \mu^{2-z}x^-$. Following the analysis of closed and co-closed two forms on cones (such as dC) in appendix A of [33] we consider solutions constructed from a co-closed one-form β on the SE_5 space that is an eigenmode of the Laplacian $\Delta_{SE} = (d^\dagger d + dd^\dagger)_{SE}$:

$$C = r^\lambda \beta, \qquad \Delta_{SE}\beta = \mu\beta, \qquad d^\dagger\beta = 0 \ . \tag{2.6}$$

It is straightforward to check that dC is co-closed providing that $\mu = \lambda(\lambda + 2)$. For our applications we choose the branch $\lambda = -1 + \sqrt{1 + \mu}$ leading to solutions with

$$z = 1 + \sqrt{1 + \mu} \ . \tag{2.7}$$

A general result valid for any five-dimensional Einstein space, normalised as we have, is that for co-closed 1-forms $\mu \geq 8$ and $\mu = 8$ holds iff the 1-form is dual to a Killing vector (see section 4.3 of [34]). Thus in general our construction leads to solutions with

$$z \geq 4 \ . \tag{2.8}$$

Since all SE_5 manifolds have at least the Reeb Killing vector, dual to the one-form η, this bound is always saturated. Indeed the solution of [9] with $z = 4$ is in our class. Specifically it can be obtained by setting $C = \sigma r^2 \eta$ (and redefining $x^- \to -x^-/2$): one can explicitly check that η is co-closed on SE_5 and is an eigenmode of Δ_{SE} with eigenvalue $\mu = 8$. Note that for this solution the two-form dC is proportional to the Kähler-form of the Calabi-Yau cone: $dC = 2\sigma\omega$.

On S^5 the spectrum of Δ_{S^5} acting on one-forms is well known and we have $\mu = (s+1)(s+3)$ for $s = 1, 2, 3\ldots$ (see for example [35] eq (2.20)) leading to $\lambda = s+1$ and hence new classes of solutions with $z = 4, 5, 6\ldots$. Note that these solutions come in families, transforming in the $SO(6)$ irreps **15**, **64**, **175**, $\ldots$. To obtain similar results for $T^{1,1}$ one can consult [36].

We now discuss a construction that can be used when the spectrum of the Laplacian acting on functions is known, but not acting on one-forms. For example, the scalar Laplacian was studied in [40] for the $Y^{p,q}$ metrics [41], but as far as we know it has not been discussed acting on one-forms. Specifically we construct $(1,1)$ forms dC on the CY cone using scalar functions Φ on the cone as follows. We write

$$C_i = J_i^{\ j} \partial_j \Phi \tag{2.9}$$

for some function Φ on CY_3. A short calculation shows that if

$$\nabla^2_{CY} \Phi = \alpha \tag{2.10}$$

for some constant α then dC is co-closed. The two-form dC is a $(1,1)$ form on CY_3 and it is primitive, $J^{ij} dC_{ij} = 0$, if and only if $\alpha = 0$. Observe that the solution of [9] with $z = 4$ fits into this class by taking $\Phi = -\sigma r^2/2$ and $\alpha = -6\sigma$, leading to $C = \sigma r^2 \eta$.

We now consider solutions with $\alpha = 0$, corresponding to harmonic functions[2] on the CY cone with dC $(1,1)$ and primitive. We next write

$$\Phi = r^\lambda f \tag{2.11}$$

where f is a function on the SE_5 space satisfying

$$-\nabla^2_{SE_5} f = k f \tag{2.12}$$

with $k = \lambda(\lambda + 4)$ (see e.g. [37]). For the solutions of interest we choose the branch $\lambda = -2 + \sqrt{4+k}$ leading to $z = \sqrt{4+k}$. For the special case of the five-sphere we can check with the results that we obtained above. The eigenfunctions f on the five-sphere are given by spherical harmonics with $k = l(l+4)$, $l = 1, 2, \ldots$ and hence $z = l + 2$. The $l = 1$ harmonic appears to violate the bound (2.8). However, it is straightforward to see that the construction for $l = 1$ leads to $dC = 0$ for which C can be removed by a transformation of the form (2.5). Thus for S^5 we should consider $l \geq 2$ leading to solutions with $z = 4, 5, \ldots$, as above. It is worth pointing out that for higher values of l some of the eigenfunctions will also lead to closed C: if we consider the harmonic function on $\mathbb{R}^6$ given by $x^{i_1} \ldots x^{i_l} c_{i_1 \ldots i_l}$ where c is symmetric and traceless then, with $J = dx^1 \wedge dx^2 + dx^3 \wedge dx^4 + dx^5 \wedge dx^6$ we see that $dC = 0$ if $J_{[i}^{\ j} c_{k]j i_3 \ldots i_l} = 0$.

[2]Note that in general the one-form C defined in (2.9) has a component in the dr direction, unlike in (2.6). However, locally we can remove it by a transformation of the form (2.5). Also, one can directly show that the resulting one-form β is co-closed on the SE_5 space.

2.1 Supersymmetry

We introduce the frame

$$\begin{aligned} e^+ &= r dx^+ \\ e^- &= r(dx^- + C) \\ e^2 &= r dx_1 \\ e^3 &= r dx_2 \\ e^4 &= \frac{dr}{r} \\ e^m &= e^m_{SE}, \qquad m = 5, \ldots, 9 \end{aligned} \tag{2.13}$$

where e^m_{SE} is an orthonormal frame for the SE_5 space. We can write

$$F_5 = B_5 + *_{10} B_5 \tag{2.14}$$

$$B_5 = 4e^+ \wedge e^- \wedge e^2 \wedge e^3 \wedge e^4 - r e^+ \wedge dC \wedge e^2 \wedge e^3 \tag{2.15}$$

where we have chosen $\epsilon_{+-23456789} = +1$. The Killing spinor equation can be written

$$D_M \epsilon + \frac{i}{16} \not{F} \Gamma_M \epsilon = D_M \epsilon + \frac{i}{2} \not{B} \Gamma_M \epsilon = 0 \ . \tag{2.16}$$

We are using the conventions for type IIB supergravity [42][43] as in [44] and in particular, $\Gamma_{11} = \Gamma_{+-23456789}$ with the chiral IIB spinors satisfying $\Gamma_{11}\epsilon = -\epsilon$.

If ϵ are the Killing spinors for the $AdS_5 \times SE_5$ solution, then we find that we must also impose that

$$\begin{aligned} \Gamma^{+-23}\epsilon &= i\epsilon \\ \Gamma^+ \epsilon &= 0 \ . \end{aligned} \tag{2.17}$$

The first condition maintains the Poincaré supersymmetries but breaks all of the superconformal supersymmetries (this can be explicitly checked using, for example, the results of [45]). The second condition breaks a further half of these[3]. Thus when $dC \neq 0$, we preserve two Poincaré supersymmetries for a generic SE_5 and this is increased to eight Poincaré supersymmetries for S^5.

[3] That we preserve the Poincaré supersymmetries suggests that we can extend our solutions away from the near horizon limit of the D3-branes. This is indeed the case but we won't expand upon that here.

3 The $D = 11$ solutions

The construction of the $D = 11$ solutions is very similar. We consider the ansatz for D=11 supergravity solutions:

$$\begin{aligned} ds^2 &= \frac{d\rho^2}{4\rho^2} + \rho^2\left[2dx^+dx^- + dx^2\right] + ds^2(SE_7) + 2\rho^2 C dx^+ \\ G &= -3\rho^2 dx^+ \wedge dx^- \wedge d\rho \wedge dx + dx^+ \wedge dx \wedge d(\rho^3 C) \end{aligned} \tag{3.1}$$

where SE_7 is a seven-dimensional Sasaki-Einstein space and $ds^2(SE_7)$ is normalised so that the Ricci tensor is equal to six times the metric (this is the normalisation of a unit radius seven-sphere). It is convenient to change coordinates via $\rho = r^2$ to bring the solution to the form

$$\begin{aligned} ds^2 &= \frac{dr^2}{r^2} + r^4\left[2dx^+dx^- + dx^2\right] + ds^2(SE_7) + 2r^4 C dx^+ \\ G &= -6r^5 dx^+ \wedge dx^- \wedge dr \wedge dx + dx^+ \wedge dx \wedge d(r^6 C)\ . \end{aligned} \tag{3.2}$$

In these coordinates the cone metric

$$ds^2_{CY} = dr^2 + r^2 ds^2(SE_7) \tag{3.3}$$

is a metric on Calabi-Yau four-fold. We will use the same notation for the CY space as in the previous section.

When the one-form C is zero we have the standard $AdS_4 \times SE_7$ solution of $D = 11$ supergravity that, in general, preserves eight supersymmetries. We again find that all the equations of motion are solved if C is a one-form on CY_4 and the two-form dC is co-closed

$$d *_{CY} dC = 0\ . \tag{3.4}$$

The solutions are again invariant under the transformation (2.5). We will consider solutions where the one-form C has weight λ under the action of $r\partial_r$, corresponding to dynamical exponent $z = 2 + \lambda/2$. As before, using the results in appendix A of [33], we consider solutions constructed from a co-closed one-form β on the SE_7 space that is an eigenmode of the Laplacian Δ_{SE}:

$$C = r^\lambda \beta, \qquad \Delta_{SE}\beta = \mu\beta, \qquad d^\dagger\beta = 0\ . \tag{3.5}$$

One can check that dC is co-closed providing that $\mu = \lambda(\lambda+4)$. For our applications we choose the branch $\lambda = -2 + \sqrt{4+\mu}$ leading to solutions with

$$z = 1 + \tfrac{1}{2}\sqrt{4+\mu}\ . \tag{3.6}$$

A general result valid for any seven-dimensional Einstein space, normalised as we have, is that for co-closed 1-forms $\mu \geq 12$ and $\mu = 12$ holds iff the 1-form is dual to a Killing vector (see section 4.3 of [34]). Thus in general our construction leads to solutions with

$$z \geq 3 \tag{3.7}$$

and the bound is again saturated for all SE_7 spaces. Observe that the solutions of [31] with $z = 3$ fit into this class. Specifically they are obtained by setting $C = \sigma r^2 \eta$ (after redefining $x \to x/2$ and $x^- \to -x^-/8$). On S^7 the spectrum of Δ_{S^7} is well known and we have $\mu = s(s+6)+5$ for $s = 1, 2, 3\ldots$ (see for example [34] eq (7.2.5)) leading to $\lambda = 1 + s$ and hence new classes of solutions with $z = 3, 3\frac{1}{2}, 4, \ldots$. These solutions come in families transforming in the $SO8)$ irreps $\mathbf{28}$, $\mathbf{160_v}$, $\mathbf{567_v}$, ... Results on the spectrum of the Laplacian on some homogeneous SE_7 spaces can be found in [46],[47],[48].

As before we can construct $(1,1)$ co-closed two-forms dC using scalar functions Φ on CY_4 We write

$$C_i = J_i{}^j \partial_j \Phi, \qquad \nabla^2_{CY} \Phi = \alpha \quad . \tag{3.8}$$

and dC is again primitive if and only if $\alpha = 0$. The solutions of [31] with $z = 3$ arise by taking $\Phi = \sigma r^2$ and $\alpha = -8\sigma$ leading to $C = \sigma r^2 \eta$. We now focus on solutions with $\alpha = 0$, corresponding to harmonic functions on the CY cone. We take

$$\Phi = r^\lambda f \tag{3.9}$$

where f is a function on the SE_7 space satisfying

$$-\nabla^2_{SE_7} f = kf \tag{3.10}$$

with $k = \lambda(\lambda + 6)$. For our applications we choose the branch $\lambda = -3 + \sqrt{9+k}$ leading to solutions with $z = \frac{1}{2} + \frac{1}{2}\sqrt{9+k}$. For example, on the seven-sphere the eigenfunctions f are given by spherical harmonics with $k = l(l+6)$ with $l = 1, 2, \ldots$ and hence $z = 2 + l/2$. Excluding the $l = 1$ harmonic, as it can be removed by a transformation of the form (2.5), for S^7 we are left with solutions with $z = 3, 7/2, 4, \ldots$, as above.

3.1 Supersymmetry

We introduce a frame

$$\begin{aligned} e^+ &= r^2 dx^+ \\ e^- &= r^2(dx^- + C) \\ e^2 &= r^2 dx \\ e^3 &= \frac{dr}{r} \\ e^m &= e^m_{SE}, \qquad m = 4, \ldots, 10 \ . \end{aligned} \tag{3.11}$$

We thus have

$$\begin{aligned} G &= 6e^+ \wedge e^- \wedge e^2 \wedge e^3 + r^2 e^+ \wedge e^2 \wedge dC \\ *_{11} G &= -6Vol(SE_7) + dx^+ *_{CY} dC \end{aligned} \tag{3.12}$$

where we have chosen the orientation $\epsilon_{+-23....10} = +1$.

The Killing spinor equation can be written as

$$\nabla_M \epsilon + \frac{1}{288}[\Gamma_M{}^{N_1N_2N_3N_4} - 8\delta^{N_1}_M \Gamma^{N_2N_3N_4}] G_{N_1N_2N_3N_4}\epsilon = 0 \ . \tag{3.13}$$

We are using the conventions for $D = 11$ supergravity [49] as in [50] and in particular $\Gamma_{+-2345678910} = +1$.

If ϵ are the Killing spinors arising for the $AdS_4 \times SE_7$ solution, then we find that we must also impose that

$$\begin{aligned} \Gamma^{+-2}\epsilon &= -\epsilon \\ \Gamma^+\epsilon &= 0 \ . \end{aligned} \tag{3.14}$$

The first condition maintains the Poincaré supersymmetries but breaks all of the superconformal supersymmetries. The second condition breaks a further half of these. Thus when $dC \neq 0$, we preserve two Poincaré supersymmetries for a generic SE_7 and this is increased to eight Poincaré supersymmetries for S^7.

3.2 Skew-Whiffed Solutions

If $AdS_4 \times SE_7$ is a supersymmetric solution of $D = 11$ supergravity, then if we "skew-whiff" by reversing the sign of the flux (or equivalently changing the orientation of SE_7) then apart from the special case when the SE_7 space is the round S^7, all supersymmetry is broken [51]. Despite the lack of supersymmetry, such solutions are known to be perturbatively stable [51]. Similarly, if we reverse the sign of the flux in our new solutions (3.2), we will obtain solutions of $D = 11$ supergravity that will generically not preserve any supersymmetry.

4 Further Generalisation

We now discuss a further generalisation of the solutions that we have considered so far, preserving the same amount of supersymmetry, which incorporate the construction of [24]. For type IIB the metric is now given by

$$ds^2 = \frac{dr^2}{r^2} + r^2 \left[2dx^+dx^- + dx_1^2 + dx_2^2\right] + ds^2(SE_5) + r^2 \left[2Cdx^+ + h(dx^+)^2\right] \tag{4.1}$$

with the five-form unchanged from (2.1). The conditions on the one-form C are as before and we demand that h is a harmonic function on the CY_3 cone:

$$\nabla^2_{CY} h = 0 \ . \tag{4.2}$$

Choosing h to have weight λ' under $r\partial_r$ we take

$$h = r^{\lambda'} f' \ , \tag{4.3}$$

where f' is an eigenfunction of the Laplacian on SE_5 with eigenvalue k'

$$-\nabla^2_{SE_5} f' = k' f' \tag{4.4}$$

with $k' = \lambda'(\lambda' + 4)$. If we set $C = 0$ and choose the branch $\lambda' = -2 + \sqrt{4 + k'}$ then these are the solutions constructed in section 5 of [24] and have dynamical exponent $z = \frac{1}{2}\sqrt{4 + k'}$. As noted in [24] an application of Lichnerowicz's theorem [52],[53] implies that these solutions have $z \geq 3/2$ with $z = 3/2$ only possible for S^5. Now if there is a scalar eigenfunction with eigenvalue k' and a one-form eigenmode of the Laplacian on SE_5 with eigenvalue μ that satisfy $z = \frac{1}{2}\sqrt{4 + k'} = 1 + \sqrt{1 + \mu}$ then we can superpose the solution with h as in (4.3) and the one-form C as in (2.6) and have a solution with scaling symmetry with this value of z. For example on S^5, using the notation as before, we have $k' = l'(l' + 4)$, $l' = 1, 2, \ldots$ and $\mu = (s + 1)(s + 3)$, $s = 1, 2, \ldots$ and hence we must demand that $l' = 2(s + 2)$, $s = 1, 2, \ldots$, giving solutions with $z = 3 + s$.

The story for $D = 11$ is very similar. The metric is now given by

$$ds^2 = \frac{dr^2}{r^2} + r^4 \left[2dx^+dx^- + dx^2\right] + ds^2(SE_7) + r^4 \left[2Cdx^+ + h(dx^+)^2\right] \tag{4.5}$$

with the four-form unchanged from (3.2). The conditions on the one-form C are as before and we demand that h is a harmonic function on the CY_4 cone:

$$\nabla^2_{CY} h = 0 \ . \tag{4.6}$$

Choosing h to have weight λ' under $r\partial_r$ we take

$$h = r^{\lambda'} f' \ , \tag{4.7}$$

where f' is an eigenfunction of the Laplacian on SE_7 with eigenvalue k'

$$-\nabla^2_{SE_7} f' = k' f' \tag{4.8}$$

with $k' = \lambda'(\lambda' + 6)$. If we set $C = 0$ and chose the branch $\lambda' = -3 + \sqrt{9 + k'}$ then these solutions have dynamical exponent $z = \frac{1}{4}(1 + \sqrt{9 + k'})$. Lichnerowicz's theorem [52],[53] implies that these solutions have $z \geq 5/4$ with $z = 5/4$ only possible for S^7. If there is a scalar eigenfunction with eigenvalue k' and a one-form eignemode of the Laplacian on SE_7 with eigenvalue μ that satisfy $z = \frac{1}{4}(1 + \sqrt{9 + k'}) = 1 + \frac{1}{2}\sqrt{4 + \mu}$ then we can superpose the solution with h as in (4.7) and the one-form C as in (3.5) and have a solution with scaling symmetry with this value of z. For example on S^7, using the notation as before, we have $k' = l'(l' + 6)$, $l' = 1, 2, \ldots$ and $\mu = s(s + 6) + 5$, $s = 1, 2, \ldots$ and hence we must demand that $l' = 2(s + 3)$, $s = 1, 2, \ldots$, giving solutions with $z = \frac{1}{2}(5 + s)$.

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