## Strangeness Contribution to the Proton Spin from Lattice QCD

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We compute the strangeness and light-quark contributions  $\Delta s$ ,  $\Delta u$  and  $\Delta d$  to the proton spin in  $n_f = 2$  lattice QCD at a pion mass of about 285 MeV and at a lattice spacing  $a \approx 0.073$  fm, using the non-perturbatively improved Sheikholeslami-Wohlert Wilson action. We carry out the renormalization of these matrix elements which involves mixing between contributions from different quark flavours. Our main result is the small negative value  $\Delta s^{\overline{\text{MS}}}(\sqrt{7.4} \text{ GeV}) = -0.020(10)(4)$  of the strangeness contribution to the nucleon spin. The second error is an estimate of the uncertainty, due to the missing extrapolation to the physical point.

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Introduction.—The proton spin can be split into a quark spin contribution  $\Delta\Sigma$ , a quark angular momentum contribution  $L_q$  and a gluonic contribution  $\Delta G$  (including spin and angular momentum) [1]:

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta G.$$
(1)

In the naïve non-relativistic SU(6) quark model,  $\Delta \Sigma = 1$ , with vanishing  $L_q$  and  $\Delta G$ . In this case there will be no strangeness contribution  $\Delta s$  to  $\Delta \Sigma = \Delta u + \Delta d + \Delta s + \cdots$ , where, in our notation,  $\Delta q = \Delta \Sigma_q$  contains both, the spin of the quarks q and of the antiquarks  $\bar{q}$ .

Experimentally,  $\Delta s$  is obtained by integrating the strangeness contribution  $\Delta s(x)$  to the spin structure function  $g_1$  over the momentum fraction x. The integral over the range in which data exist agrees with zero; see, e.g., new COMPASS data [2, 3] for  $x \ge 0.004$  or HERMES data [4] for x > 0.02, while global analyses give values [5–7]  $\Delta s \approx -0.12$ , suggesting a large negative  $\Delta s(x)$  at very small x. Pioneering lattice simulations of disconnected matrix elements also indicated values [8, 9]  $\Delta s \approx -0.12$ . However, the errors given in these studies are quite optimistic while the global fits rely on an extrapolation of the integrated experimental  $\Delta\Sigma$  to small x and constrain the axial octet charge  $a_8$  to a value, obtained from hyperon  $\beta$ -decays, assuming SU(3)<sub>F</sub> flavour symmetry. Some time ago, employing heavy baryon chiral perturbation theory, Savage and Walden [10] pointed out that  $SU(3)_F$  symmetry in weak baryonic decays may be violated by as much as 25 % and hence  $\Delta s(x)$  could remain close to zero also for x < 0.001; see also [11].  $SU(3)_F$  symmetry is however supported by lattice simulations of hyperon axial couplings [12–15], albeit within non-negligible errors.

In this Letter, we directly compute the matrix elements that contribute to the  $\Delta q$ , including quark line disconnected diagrams. Preliminary results were presented at conferences [16–18].

Simulation details and methods.—We simulate  $n_{\rm f} = 2$  non-perturbatively improved Sheikholeslami-Wohlert Fermions, using the Wilson gauge action, at  $\beta = 5.29$ and  $\kappa = \kappa_{ud} = 0.13632$ . Setting the scale from the chirally extrapolated nucleon mass [19], we obtain the lattice spacing  $a^{-1} = 2.71(2)(7)$  GeV, where the errors are statistical and from the extrapolation, respectively.

We realize two additional valence  $\kappa$  values,  $\kappa_m = 0.13609$  and  $\kappa_s = 0.13550$ . The corresponding pion masses are  $m_{\text{PS},ud} = 285(3)(7) \text{ MeV}$ ,  $m_{\text{PS},m} = 449(3)(11) \text{ MeV}$  and  $m_{\text{PS},s} = 720(5)(18) \text{ MeV}$ .  $\kappa_s$  was fixed so that the  $m_{\text{PS},s}$  value is close to the mass of a hypothetical strange-antistrange pseudoscalar meson:  $(m_{K^{\pm}}^2 + m_{K^0}^2 - m_{\pi^{\pm}}^2)^{1/2} \approx 686.9 \text{ MeV}$ . We investigate volumes of  $32^364$  and  $40^364$  lattice points, i.e.,  $Lm_{\text{PS},ud} = 3.36$  and 4.20, respectively, where the largest spatial lattice extent is  $L \approx 2.91 \text{ fm}$ .

The quark polarizations are extracted from the largetime behaviour of ratios of three-point over two-point functions. We create a polarized proton at a time  $t_0 = 0$ , probe it with an axial current at a time t and destroy the zero momentum proton at  $t_f > t > 0$ . Quark line connected and disconnected terms contribute:

$$R^{\rm con}(t_{\rm f},t) = \frac{\langle \Gamma^{\alpha\beta}_{\rm pol} C^{\beta\alpha}_{3pt}(t_{\rm f},t) \rangle}{\langle \Gamma^{\alpha\beta}_{\rm unpol} C^{\beta\alpha}_{2pt}(t_{\rm f}) \rangle},\tag{2}$$

$$R^{\rm dis}(t_{\rm f},t) = -\frac{\langle \Gamma^{\alpha\beta}_{\rm pol} C^{\beta\alpha}_{2pt}(t_{\rm f}) \sum_{\mathbf{x}} {\rm Tr}[\gamma_j \gamma_5 M^{-1}(\mathbf{x},t;\mathbf{x},t)] \rangle}{\langle \Gamma^{\alpha\beta}_{\rm unpol} C^{\beta\alpha}_{2pt}(t_{\rm f}) \rangle}$$

Here M is the lattice Dirac operator,  $\Gamma_{\text{unpol}} = \frac{1}{2}(\mathbb{1} + \gamma_4)$ is a parity projector and  $\Gamma_{\text{pol}} = i\gamma_j\gamma_5\Gamma_{\text{unpol}}$  projects out the difference between the two polarizations (in direction  $\hat{j}$ ). We average over j = 1, 2, 3 to increase statistics. For the up and down quark matrix elements we compute the sum of connected and disconnected terms while only  $R^{\text{dis}}$ contributes to  $\Delta s$ .

For disconnected contributions we fix the time distance between the source and the current insertion  $t = 4a \approx$ 0.29 fm and vary  $t_{\rm f}$ . Both t and the distance between current and sink  $t_{\rm f} - t$  should be taken large, to suppress excited state contributions. Using the sink and source smearing described in [20], we find the asymptotic limit to be effectively reached for  $t_{\rm f} \simeq 6a-7a$ ; see Fig. 1 for an example. The saturation into a plateau at  $t_{\rm f} \leq 2t$  and the convergence of the point sink data towards the same value demonstrate that t = 4a was reasonably chosen. To be on the safe side, we only fit the  $t_{\rm f} \geq 8a \approx 0.58$  fm smearedsmeared ratios. Building upon previous experience [21], the connected part, for which the statistical accuracy is less of an issue, is obtained at the larger, fixed value  $t_{\rm f} = 15a$ , varying t.

The disconnected contribution is computed with the stochastic estimator methods described in [17, 22], employing time partitioning, a second order hopping parameter expansion and the truncated solver method. We compute the Green functions for four equidistant source times on each gauge configuration. We also construct backwardly propagating nucleons, replacing the positive parity projector  $\frac{1}{2}(\mathbb{1} + \gamma_4)$  by  $\frac{1}{2}(\mathbb{1} - \gamma_4)$ , seeding the noise vectors on eight (four times two) timeslices. In addition to the 48 (four times spin times colour) solves for smeared conventional sources, that are necessary to construct the two-point functions, we run the Conjugate Gradient (CG) algorithm on  $N_1 = 730$  complex  $\mathbb{Z}_2$  noise sources for  $n_{\rm t} = 40$  iterations. The bias from this truncation is corrected for [22] by  $N_2 = 50$  BiCGstab solves that are run to convergence. We analyse a total of 2024 thermalized trajectories on each of the two volumes where we bin the data to eliminate autocorrelations.

Renormalization.—Non-singlet axial currents renormalize with a renormalization factor  $Z_A^{ns}(a)$  that only depends on the lattice spacing. This was determined non-perturbatively for the action and lattice spacing in use [23]:  $Z_A^{ns} = 0.76485(64)(73)$ .

However, due to the axial anomaly, the renormalization constant of singlet currents,  $Z_A^{\rm s}(\mu, a)$ , acquires an anomalous dimension. To first non-trivial order this



FIG. 1. The disconnected ratio  $R^{\text{dis}}$  versus  $t_{\text{f}}$  on the 40<sup>3</sup>64 volume at  $\kappa_{\text{val}} = \kappa_{\text{cur}} = \kappa_s$  for smeared-smeared (SS) and smeared-point (SP) source-sink combinations.

reads [24, 25]  $\gamma_A^s(\alpha_s) = -6C_{\rm F}n_{\rm f}[\alpha_s/(4\pi)]^2$ .  $Z_A^s$  deviates from  $Z_A^{\rm ns}$  starting at  $\mathcal{O}(\alpha_s^2)$  in perturbation theory. Both factors have been calculated to this order, with the result for the conversion into the  $\overline{\rm MS}$  scheme at a scale  $\mu$  [26]

$$z(\mu, a) = Z_A^{\rm s}(\mu, a) - Z_A^{\rm ns}(a)$$
  
=  $C_{\rm F} n_{\rm f} \left[ 15.8380(8) - 6\ln(a^2\mu^2) \right] \left(\frac{\alpha_s}{4\pi}\right)^2$ , (3)

where we have set the Sheikholeslami-Wohlert parameter  $c_{\rm SW} = 1$  to be consistent to this order in perturbation theory. To this first non-trivial order no scale enters the coupling parameter  $\alpha_s$ . Since perturbation theory in terms of the bare lattice parameter  $\alpha_0 = 6/(4\pi\beta)$ is known to converge poorly, we substitute  $\alpha_s$  by a coupling defined from the measured average plaquette  $\alpha_s = -3 \ln \langle U_{\Box} \rangle / (4\pi) = 0.14278(5)$ , where we have used the chirally extrapolated value [27]  $\langle U_{\Box} \rangle = 0.54988(11)$ .

No dimension-four operator can be constructed that mixes with the relevant forward matrix element of  $\bar{q}\gamma_{\mu}\gamma_{5}q$ and that cannot be removed, using the equations of motion [28]. This also holds for the singlet case [29], such that we only need to replace

$$Z_A^{\rm ns} \mapsto Z_A^{\rm ns}(1+b_A am), \quad Z_A^{\rm s} \mapsto Z_A^{\rm s}(1+b_A^{\rm s} am), \quad (4)$$

to achieve full  $\mathcal{O}(a)$  improvement. The factor  $b_A$  is known to  $\mathcal{O}(\alpha_s)$  [28]:  $b_A = b_A^s + \mathcal{O}(\alpha_s^2) \approx 1 + 18.02539 C_{\mathrm{F}} \frac{\alpha_s}{4\pi}$ . We obtain the values

$$1 + b_A am = \begin{cases} 1.0324(3)(47) & (m_s, \kappa = 0.13550) \\ 1.0041(3)(5) & (m_{ud}, \kappa = 0.13632) \end{cases},$$
(5)

where the first error is due to the uncertainty in the quark mass and the second error corresponds to 50% of the one-loop correction. Considering the small size of this correction it is unlikely that the (two-loop) difference between singlet and non-singlet  $b_A$ -factors will result in any noticeable effect, and in particular not at the light-quark mass  $m_{ud}$ , where it will be needed [see Eq. (11) below].



FIG. 2. Volume and valence quark mass dependence of the unrenormalized  $\Delta s^{\text{lat}}$ .

For  $n_{\rm f} = 2$  we get

$$z(\sqrt{7.4}\,\text{GeV}) = 0.0055(1)(27)\,,$$
 (6)

at the renormalization scale  $\mu^2 = 7.4 \,\text{GeV}^2 = 1.01(5) \,a^{-2}$ . We again include a 50 % systematic er-

In the  $n_{\rm f} = 1 + 1 + 1$  theory the matrix elements renormalize as follows:

$$g_A = \Delta T_3 = (\Delta u - \Delta d)^{\text{MS}}$$
$$= Z_A^{\text{ns}}(a)(\Delta u - \Delta d)^{\text{lat}}(a), \qquad (7)$$

$$a_8 = \Delta T_8 = (\Delta u + \Delta d - 2\Delta s)^{\overline{\text{MS}}}$$
$$= Z_A^{\text{ns}}(a)(\Delta u + \Delta d - 2\Delta s)^{\text{lat}}(a), \quad (8)$$

$$a_0 = \Delta \Sigma^{\overline{\mathrm{MS}}}(\mu) = (\Delta u + \Delta d + \Delta s)^{\overline{\mathrm{MS}}}(\mu)$$
$$= Z_A^{\mathrm{s}}(\mu, a)(\Delta u + \Delta d + \Delta s)^{\mathrm{lat}}(a). \quad (9)$$

We remark that for non-equal quark masses the nonsinglet combinations Eqs. (7) and (8) also receive contributions from disconnected quark line diagrams.

We employ  $n_{\rm f} = 2$  sea quarks so that our singlet current is  $\Delta u + \Delta d$  rather than the  $\Delta \Sigma$  of Eq. (9). This modifies the renormalization pattern:

$$\begin{split} & \Delta u(\mu) \\ \Delta d(\mu) \\ \Delta s(\mu) \end{split} \overset{\overline{\mathrm{MS}}}{=} \begin{pmatrix} Z_A^{\mathrm{ns}}(a) + \frac{z(\mu,a)}{2} & \frac{z(\mu,a)}{2} & 0 \\ \frac{z(\mu,a)}{2} & Z_A^{\mathrm{ns}}(a) + \frac{z(\mu,a)}{2} & 0 \\ \frac{z(\mu,a)}{2} & \frac{z(\mu,a)}{2} & Z_A^{\mathrm{ns}}(a) \end{pmatrix} \begin{pmatrix} \Delta u(a) \\ \Delta d(a) \\ \Delta s(a) \end{pmatrix}^{\mathrm{lat}}. \end{split}$$
(10)

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 $\Delta s^{\overline{\text{MS}}}$  receives light-quark contributions but the  $\Delta u^{\overline{\text{MS}}}$ and  $\Delta d^{\overline{\text{MS}}}$  remain unaffected by the (quenched) strange quark. Obviously, unitarity is violated, due to this quenching. The combination  $\Delta T_8$  still transforms with  $Z_A^{ns}$  [Eq. (8)] while Eq. (9) is violated, as it should be; instead, the  $n_{\rm f} = 2$  singlet operator  $\Delta u + \Delta d$  renormalizes with  $Z_A^s$ . We remark that the above renormalization pattern is similar to that of the scalar matrix element in the  $n_{\rm f} = 2$  theory [20, 30, 31]. Note that in spite of the quenched strange quark the mismatch between directly converting the result into the  $\overline{\text{MS}}$  scheme at a scale  $\mu$ , using  $z(n_{\rm f} = 2)/2$ , and first converting into the  $\overline{\text{MS}}$  scheme at another scale  $\mu'$  and subsequently running within the  $\overline{\text{MS}}$  scheme with  $\ln(\mu/\mu')\gamma_A^s(n_{\rm f} = 3)/3$  to the scale  $\mu$  is tiny.

Results and systematics.—In Fig. 2 we display the volume and (light) valence quark mass dependence of our unrenormalized  $\Delta s^{\text{lat}}$ . There are no statistically significant finite size or mass effects.

Using Eqs. (10) and (4) we can renormalize

$$\Delta q^{\overline{\mathrm{MS}}}(\mu) = Z_A^{\mathrm{ns}}(1 + b_A a m_q) \Delta q^{\mathrm{lat}} + \frac{z(\mu)}{2} (\Delta u + \Delta d)^{\mathrm{lat}}$$
(11)

for  $q \in \{u, d, s\}$ . As discussed above, we omit the  $\mathcal{O}(a)$  improvement factor  $(b_A^s Z_A^s - b_A Z_A^{ns}) a m_{ud}$  of the  $(\Delta u + \Delta d)^{\text{lat}}$  term. This is of  $\mathcal{O}(\alpha_s^2 a m_{ud})$  and numerically negligible. We display the bare lattice numbers for the connected and disconnected contributions to the proton spin and the renormalized  $\mathcal{O}(a)$  improved values in Table I, for the two volumes. The  $\Delta u^{\overline{\text{MS}}}$  and  $\Delta d^{\overline{\text{MS}}}$  values are reduced by about 0.035, due to the sea quark contributions while  $\Delta s^{\overline{\text{MS}}}$  increases by 0.002 (< 10 %), due to the mixing with light-quark flavours.

The uncertainties associated to the renormalization are much smaller than the statistical errors. Below we will only quote large volume results, with statistical and renormalization errors added in quadrature. Error sources that have so far not been accounted for are the missing continuum limit extrapolation, the quenching of the strange quark and simulating at a light sea quark mass value that is four times bigger than the physical one. There are no indications of radical quark mass effects: the flavour mixing effects within the renormalization are small in spite of the comparatively large  $\Delta u$  and  $\Delta d$  values. The dependence on the valence quark mass is small too; see Fig. 2.

TABLE I. The connected and disconnected contributions to  $\Delta q^{\rm lat}$  as well as the renormalized spin content at a scale  $\mu = \sqrt{7.4}$  GeV. (The  $\Delta T_i$  are scale-independent.) The first error is statistical, the second is from the renormalization. In addition an overall 20 % systematic error needs to be added.

q	V, L	$\Delta q_{ m con}^{ m lat}$	$\Delta q_{ m dis}^{ m lat}$	$\Delta q^{\overline{\mathrm{MS}}}(\mu)$
u		1.065(22)	-0.034(16)	0.794(21)(2)
d		-0.344(14)	-0.034(16)	-0.289(16)(1)
s	$V = 32^{3}64$	0	-0.031(12)	-0.023(10)(1)
$T_3$	$L\approx 2.33{\rm fm}$	1.409(24)	0	1.082(18)(2)
$T_8$		0.721(26)	-0.006(18)	0.550(24)(1)
$\Sigma$		0.721(26)	-0.098(42)	0.482(38)(2)
u		1.071(15)	-0.049(17)	0.787(18)(2)
d		-0.369(9)	-0.049(17)	-0.319(15)(1)
s	$V = 40^{3}64$	0	-0.027(12)	-0.020(10)(1)
$T_3$	$L\approx 2.91{\rm fm}$	1.439(17)	0	1.105(13)(2)
$T_8$		0.702(18)	-0.044(19)	0.507(20)(1)
Σ		0.702(18)	-0.124(44)	0.448(37)(2)

Nevertheless, having simulated only at one lattice spacing and sea quark mass, we cannot extrapolate our results to the physical point. Consequently, we underestimate the value [32]  $g_A = 1.2670(35)$  from neutron  $\beta$ -decays by 13 % and find  $\Delta T_3 = 1.105(13)$  instead. Our prediction  $\Delta T_8 = 0.507(20)$  differs by the same 13 % from the phenomenological estimate [32]  $a_8 = 0.585(25)$ . We take this as an indication of the size of the remaining systematics and add an additional 20 % error to all our results.

Conclusions.—We determined the first moments of proton flavour singlet and non-singlet polarized parton distributions from  $n_{\rm f} = 2$  lattice QCD, at a pion mass of 285 MeV, at a single lattice spacing  $a \approx 0.073$  fm. We found  $\Delta \Sigma = \Delta u + \Delta d + \Delta s = 0.45(4)(9)$  and a small negative  $\Delta s = -0.020(10)(4)$ , in the  $\overline{\rm MS}$  scheme, at a scale  $\mu = \sqrt{7.4}$  GeV. We underestimated both  $g_A$  and  $a_8$  by similar factors  $\approx 0.87$  and this may suggest that some of the systematics cancel when considering ratios of matrix elements. Nevertheless, we emphasize that there is a considerable uncertainty in the  $a_8$  value [10] and our  $\Delta \Sigma$  is already relatively large, due to the small difference  $\Delta T_8 - \Delta \Sigma = -3\Delta s = 0.059(29)(12)$ .

Interestingly, our results are in remarkable agreement with the cloudy bag model prediction of [11]. The small (unrenormalized)  $\Delta s^{\text{lat}}$  value obtained recently in [31] is also consistent with our study. Our  $\Delta \Sigma$  value is larger than previously expected, however, it is compatible with the latest COMPASS number [2]  $a_0(\sqrt{3} \text{ GeV}) =$ 0.35(3)(5). The experimental number may increase further once smaller *x*-values become accessible. We suggest relaxing the weak hyperon decay SU(3)<sub>F</sub> constraint on  $a_8$  in determinations of polarized parton distribution functions [5–7], and including our  $\Delta s$  prediction instead. Acknowledgments.— This work is supported by the EU (Grant No. 238353, ITN STRONGnet) and by the DFG Grant No. SFB/TR 55. S.C. acknowledges support from the Claussen-Simon-Foundation (Stifterverband für die Deutsche Wissenschaft) and J.Z. from the Australian Research Council (Grant No. FT100100005). Computations were performed on the SFB/TR55 QPACE supercomputers, the BlueGene/P (JuGene) and the Nehalem Cluster (JuRoPA) of the Jülich Supercomputer Center, the IBM BlueGene/L at the EPCC (Edinburgh), the SGI Altix ICE machines at HLRN (Berlin/Hannover) and Regensburg's Athene HPC cluster. The CHROMA software suite [33] was used and gauge configurations were generated with the BQCD code [34].

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