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The Multistatic Sonar Location Problem and Mixed-Integer Programming

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Abstract. A multistatic sonar system consists of one or more sources that are able to emit underwater sound, and receivers that listen to the direct sound as well as its reflected sound waves. From the differences in the arrival times of these sounds, it is possible to determine the location of surrounding objects. The propagation of underwater sound is a complex issue that involves several factors, such as the density and pressure of the water, the salinity and temperature level, as well as the pulse length and volume and the reflection properties of the surface. These effects can be approximated by nonlinear equations. Furthermore, natural obstacles in the water, such as the coastline, need to be taken into consideration. Given a certain area of the ocean that should be endowed with a sonar system for surveillance, we consider the task of determining how many sources and receivers need to be deployed, and where they should be located. We give an integer nonlinear formulation for this problem, and several ways to derive an integer linear formulation from it. These formulations are numerically compared using a test bed from coastlines around the world and a state-of-the-art MIP solver (CPLEX).

Keywords: Integer Nonlinear Programming, Multistatic Sonar, Quadratic Constraints, Linearization, Integer Linear Programming.

1 Introduction to Sonar

Sonar is a technique to detect objects that are under water or at the surface using sound propagation. In active sonar systems, a sound is emitted from a source and its echoes are detected by a receiver, revealing information about nearby objects. Active sonar has been in use for nearly 100 years and has become a key component of undersea detection. The basic operating principle of active sonar is that acoustic energy is emitted from a source and its echoes are detected by a receiver; these echoes reveal information about surrounding objects. In a monostatic system, the source and the receiver are collocated in the same place. Bistatic sonar uses a source and a receiver pair in different locations. Multistatic sonar uses several sources and receivers simultaneously as a network. For the surveillance of a large area of the ocean, a number of both types of devices must be deployed. This leads to an optimization problem to find the least costly multistatic network that is able to cover all of a desired area. No algorithm currently in the literature provides an optimal placement of an arbitrary number of sources and receivers. In a discretized setting, we describe mathematical models designed to determine the minimum-cost sensor layout that will cover a portion of the ocean (a tile) by sonar surveillance, with adequate detection probability throughout the tile. We model the physical properties of sound traveling between sources, target, and receivers, the ocean (temperature, density, salinity) as well as geometrical considerations (obstacles such as islands or coastlines). Details are given in Section 2. We formulate an integer nonlinear program for the multistatic sonar source-receiver location problem and discuss several linearizations in Section 3. We compare these formulations empirically using topological data from coastal areas around the world and a state-of-the-art solver MIP solver and give concluding remarks in Section 4.

2 Input Data

We obtain ocean topography data from [10]. At present, we do not use sea level information and only distinguish in a binary fashion between the ocean (negative elevation value) and the dry land (positive elevation value). A desired part of the ocean and shoreline (a tile) is taken from the database. Since the resolution of the data is too fine to let each data pixel become a possible target/source/receiver location, we aggregate the raw input data into larger rectangular areas (also called grid cells). We then average the elevation data from all pixels within a cell and apply the resulting elevation to the entire cell. Denote the set of rectangles with negative elevation (i.e., those that are underwater) by G (for grid) and the number of elements in G by $n := |G|$.

The sonar signal is characterized by the range of the day ϱ_0, which indicates how quickly the signal diminishes as the target, source, and receiver become farther apart. In a definite range ("cookie-cutter") sensor model, a target in a cell $k \in G$ is detected by a source placed in cell $i \in G$ and a receiver placed in cell $j \in G$ with probability $p_{i,j,k} \in \{0,1\}$. Denote by $d_{i,j}$ the Euclidean distance between (the centers of) cell i and j. Necessary for detection ($p_{i,j,k} = 1$) is that the target k is inside the Cassini oval defined by the equation $d_{i,k} \cdot d_{k,j} \leq \varrho_0^2$, c.f. [7]. If the target is too close to the line from source to receiver, then the original signal and its reflection at the target become indistinguishable at the receiver. This phenomenon is known as the direct blast effect. The pulse length κ_b determines the severity of this effect, since longer pulses are more prone to overlapping with the reflected signal. The direct blast zone is defined by the ellipsoid $d_{i,k} + d_{k,j} \leq d_{i,j} + 2\kappa_b$, c.f. [6]. To account for the direct blast effect, we say that $p_{i,j,k} = 0$, if the target lies within the direct blast zone. Additionally, if an obstacle lies on either straight-line path of source to target, target to receiver, or source to receiver, then $p_{i,j,k} = 0$.

The cost for each source is c_s, and the cost for each receiver is c_r. Typically, $c_s \gg c_r$, i.e., a source is much more costly than a receiver, usually by a factor of 5.

3 Model Formulations

All model formulations below have in common the binary decision variables $s_i, r_i \in \{0,1\}$ for each $i \in G$, which model the decision whether to place a source ($s_i = 1$) or a receiver ($r_i = 1$) in cell i. The objective (in all formulations) is to minimize the total deployment cost, which we calculate as follows:

$$c_s \sum_{i \in G} s_i + c_r \sum_{j \in G} r_j. \tag{1}$$

An Integer Nonlinear Model. In the first nonlinear formulation the binary variables s_i and r_j are multiplied in order to represent the joint decision of placing a source at i **and** a receiver in j:

$$\sum_{i \in G} \sum_{j \in G} p_{i,j,k} s_i r_j \geq 1, \quad \forall k \in G. \tag{2}$$

Each constraint of (2) is a quadratic knapsack constraint. In general, for any given $k \in G$ the non-negative matrix $(p_{i,j,k})_{i,j}$ is indefinite. The solver CPLEX is able to process constraints of this form since version 12.6 [2]. Thus, the base run for comparison with the other reformulation approaches is to solve the model:

$$\min\{(1)|(2); s, r \in \{0,1\}^G\}. \tag{3}$$

The Oldest Linearization Technique. The first documented linearization of a product of binaries $s_i r_j$ by [3, 1] (and independently by others later on) introduces a new binary variable $h_{i,j} \in \{0,1\}$ with $h_{i,j} = 1$ if and only if $s_i = 1$ and $r_j = 1$. In this method the constraints $2h_{i,j} \leq s_i + r_j$ and $s_i + r_j \leq 1 + h_{i,j}$ (for all $i, j \in G$) are a linear description of this relationship. In our case, because of the non-negativity of all $p_{i,j,k}$, only the first constraint is necessary. Thus the first linear version of (3) is

$$\min\ (1),\ \text{s.t.} \sum_{i \in G} \sum_{j \in G} p_{i,j,k} h_{i,j} \geq 1, \quad \forall k \in G, \tag{4a}$$

$$2h_{i,j} \leq s_i + r_j, \quad \forall i, j \in G, \tag{4b}$$

$$s \in \{0,1\}^G, r \in \{0,1\}^G, h \in \{0,1\}^{G \times G}. \tag{4c}$$

Compared to the nonlinear integer formulation (3), this binary linear model has an additional n^2 binary variables and n^2 constraints.

Standard Linearization of the Model. A linearization for $s_i r_j$ similar to the previous one from [5] introduces continuous auxiliary variables $h_{i,j} \in [0,1]$ together with the constraints $h_{i,j} \leq s_i, h_{i,j} \leq r_j$ and $s_i + r_j \leq 1 + h_{i,j}$. This is perhaps the first and most natural formulation to come to mind (and for good reason: Padberg [9] showed that the constraints are facet defining), and is hence called "standard linearization." As before, the third constraint is not required in our case. Then, the second linear version of (3) is

$$\min\ (1),\ \text{s.t.}\ \sum_{i\in G}\sum_{j\in G} p_{i,j,k} h_{i,j} \geq 1, \quad \forall k \in G, \tag{5a}$$

$$\left.\begin{array}{l} h_{i,j} \leq s_i \\ h_{i,j} \leq r_j \end{array}\right\}, \quad \forall i,j \in G, \tag{5b}$$

$$s \in \{0,1\}^G, r \in \{0,1\}^G, h \in [0,1]^{G\times G}. \tag{5c}$$

Compared to the nonlinear binary formulation (3), this mixed-integer linear model has an additional n^2 continuous variables and $2n^2$ constraints.

Glover's Linearization. To adapt a linearization technique from Glover [4], we set $L_{j,k} := \sum_{i\in G} p_{i,j,k}$ for all $j,k \in G$, and the model reads:

$$\min\ (1),\ \text{s.t.}\ \sum_{j\in G} z_{j,k} \geq 1, \quad \forall k \in G, \tag{6a}$$

$$\left.\begin{array}{r} \sum_{i\in G} p_{i,j,k} s_i \geq z_{j,k} \\ L_{j,k} r_j \geq z_{j,k}, \end{array}\right\}, \ \forall j,k \in G, \tag{6b}$$

$$s \in \{0,1\}^G, r \in \{0,1\}^G, z \in \mathbb{R}_+^{G\times G}. \tag{6c}$$

This model introduces n^2 additional continuous variables and $2n^2$ additional constraints (compared to (3)).

Oral-Kettani's Linearization. Oral and Kettani [8] proposed two formulations that come with n^2 additional continuous variables, but fewer constraints compared to Glover's formulation; namely, only n^2 (not counting the trivial bound on $z_{j,k}$ as constraint). The first of the two formulations is:

$$\min\ (1),\ \text{s.t.}\ \sum_{j\in G} (L_{j,k} r_j - z_{j,k}) \geq 1, \quad \forall k \in G, \tag{7a}$$

$$\left.\begin{array}{l} z_{j,k} \geq L_{j,k} r_j - \sum_{i\in G} p_{i,j,k} s_i \\ L_{j,k} \geq z_{j,k}, \end{array}\right\}, \quad \forall j,k \in G, \tag{7b}$$

$$s \in \{0,1\}^G, r \in \{0,1\}^G, z \in \mathbb{R}_+^{G\times G}. \tag{7c}$$

The second Oral-Kettani linearization is:

$$\min\ (1),\ \text{s.t.}\ \sum_{j\in G} \left(\sum_{i\in G} p_{i,j,k} s_i - z_{j,k} \right) \geq 1, \quad \forall k \in G, \tag{8a}$$

$$\left.\begin{array}{l} z_{j,k} \geq \sum_{i\in G} p_{i,j,k} s_i - L_{j,k} r_j \\ L_{j,k} \geq z_{j,k}, \end{array}\right\}, \quad \forall j,k \in G, \tag{8b}$$

$$s \in \{0,1\}^G, r \in \{0,1\}^G, z \in \mathbb{R}_+^{G\times G}. \tag{8c}$$

4 Computational Results and Conclusions

We compare the above six formulations on a test set of 22 instances. The ocean topography data from various regions all over the world were extracted from a global map, collected by Ryan et al. [10]. The computations were carried out on a 2014 MacBookPro with 16 GB RAM and a 2.8 GHz Intel Core i7 processor. We set a time limit of 1,000 seconds and default settings of the solver IBM ILOG CPLEX 12.7.1 otherwise. The results can be found in Table 1, with the second Oral-Kettani formulation slightly ahead that of Glover, and CPLEX failing to solve most instances within the time limit. An example result appears in Figure 1.

Instance	n	(3)	(4)	(5)	(6)	(7)	(8)
BabAlMandabStrait	29	1000.01	2.03	1.02	0.85	1.76	0.53
ChoctawhatcheeBay	31	1000.01	2.5	0.8	0.53	1.9	0.49
Dardanelles	19	3.04	0.24	0.07	0.1	0.07	0.04
EnglishChannel	48	1000.35	11.32	9.44	4.1	13.24	7.5
Falklandsund	57	1005.49	66.23	296.46	65.5	36.6	59.9
GulfOfAkaba	22	39.74	0.15	0.1	0.09	0.16	0.07
GulfOfFinland	37	1000.09	582.37	4.65	1.91	12.58	2.29
GulfOfSirte	45	1002.06	295.68	70.69	16.04	48.42	9.71
KarkinytskaGulf	34	1000.01	3.5	0.33	0.69	1.95	0.38
KerchStrait	36	1000.01	1.05	0.25	0.33	0.55	0.29
LagoDeMaracaibo	48	1000.02	4.45	1.46	1.44	2.35	1.6
Lesbos	30	1000.02	1.88	0.43	0.4	1.09	0.69
MontereyPeninsular	45	1000.26	12.19	5.06	4.29	12.74	2.66
NewYork	38	1000.52	6.59	1.25	1.57	8.27	2.83
OpenSea-Biscaya	54	1002.77	22.52	151.63	24.59	22.34	30.56
Oresund	71	1000.84	33.69	20.45	40.03	37.98	16.08
Ruegen	37	1000.02	34	7.3	2.94	45.32	1.19
Smalandsfarvandet	58	1000.73	229.88	31.74	26.09	32.02	7.53
Storebaelt	40	1000.25	57.58	10.63	2.26	12.66	2.57
StraitOfGibraltar	52	1000.49	28.66	43.62	7.72	70.45	15.63
StraitOfHormuz	41	1000.02	0.97	0.58	0.99	2.64	0.5
TaedongGang	39	1000.02	6.76	2.8	1.77	4.88	3.13
SUM		20056.77	1404.24	660.76	204.23	369.97	166.17
RANK		6	5	4	2	3	1

Table 1. Computational Results.

When facing a bilinear constraint of the type $x^T Ay \leq b$ with binary variable vectors x, y and an indefinite matrix A, several techniques for their linearization were developed by researchers over the last five decades. Today, classical MILP solvers (such as CPLEX) offer features to automatically deal with such nonlinear constraints, lifting the burden of going to the library from the user. As our results demonstrate, it is still worthwhile to consider the knowledge of the past, and not to blindly rely on the solver. Since it is unclear to determine a priori which of

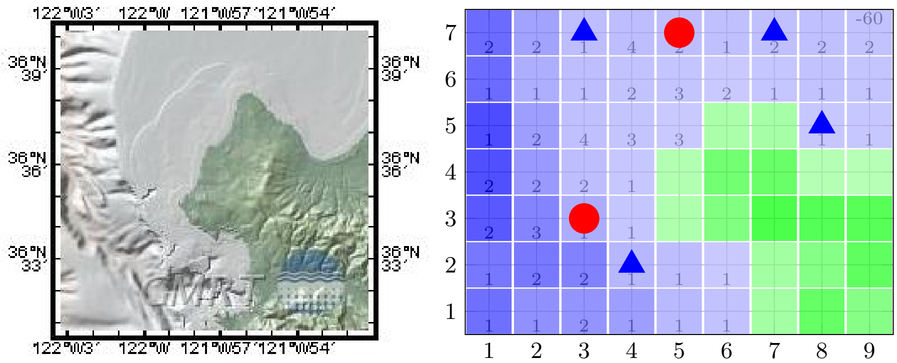

Fig. 1. Left: The Monterey Peninsula area tile [10] as raw input data (365 cols, 285 rows). Right: Optimal placement of 2 sources (red circles) and 4 receivers (blue triangles) on a 9x7 grid. Numbers ≥ 1 at each coordinate show multiplicity of coverage.

the method outperforms the others, it is necessary to implement and test all of them.

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