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WISPy Cold Dark Matter

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Abstract. Very weakly interacting slim particles (WISPs), such as axion-like particles (ALPs) or hidden photons (HPs), may be non-thermally produced via the misalignment mechanism in the early universe and survive as a cold dark matter population until today. We find that, both for ALPs and HPs whose dominant interactions with the standard model arise from couplings to photons, a huge region in the parameter spaces spanned by photon coupling and ALP or HP mass can give rise to the observed cold dark matter. Remarkably, a large region of this parameter space coincides with that predicted in well motivated models of fundamental physics. A wide range of experimental searches – exploiting haloscopes (direct dark matter searches exploiting microwave cavities), helioscopes (searches for solar ALPs or HPs), or light-shining-through-a-wall techniques – can probe large parts of this parameter space in the foreseeable future.

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1 Introduction

The two most relevant features of dark matter (DM) particles are their feeble interactions with standard model particles and their cosmological stability. In addition DM is required to be sufficiently cold in order to allow for efficient structure formation.

A realization of all these features are weakly interacting massive particles (WIMPs). Thermally produced in the early Universe their large, of order TeV scale mass ensures that by now they are non-relativistic and therefore sufficiently cold. Their interactions are small due to the large mass of the mediator particles (such as W or Z bosons) which makes the interaction very short ranged. Despite this, their large mass and the correspondingly large phase space is at odds with the required stability on cosmological time-scales. In order to ensure this stability one is therefore forced to introduce symmetries that conserve the number of these DM particles. However, motivating these symmetries on theoretical grounds is non-trivial. Global symmetries may be broken in quantum gravity whereas local symmetries lead to additional interactions which may cause conflicts with the required weakness of the DM interactions. Nevertheless good candidates exist. Two of the most famous examples are: the lightest supersymmetric particle in supersymmetric models with R-parity (see, e.g. [1] for a review), or the lightest Kaluza-Klein modes in models with conserved parity in extra dimensions (see, e.g. [2] for a review), both models very appealing because of their connection with more fundamental theories of space and time, and their discovery potential at the LHC.

Although it is way too early to make a final judgement it is nevertheless noteworthy that the initial searches at LHC have not given any indication of the existence of WIMPs. Because of this and the mentioned theoretical issues it is interesting and timely to consider alternative ways to realize the essential features of DM.

Sufficient stability of the DM particles can also be achieved by combining the weakness of their interactions with a sufficiently small mass. The latter drastically reduces the phase space (and the number/type of possible decay products) thereby increasing the lifetime. This is the road we want to pursue in this paper: we will concentrate on very weakly interacting slim particles (WISPs) as DM candidates.

Yet, a thermally produced light DM candidate can run foul of the required coldness of DM and can prevent structure formation. More precisely, the free-streaming length (the distance a DM particle can travel before getting trapped in a potential well) would increase with decreasing mass, and therefore at some point these DM particles would be inconsistent with the existence of dwarf galaxies, galaxies, clusters, superclusters and so on. This argument can be used for instance to rule out standard neutrinos as DM. This reasoning is extremely powerful in light of the increasingly precise cosmological data at our disposal and even subdominant components of thermally produced light DM can be ruled out, the case of eV mass axions being a prime example (see, e.g. [3]).

However, there are non-thermal means for producing sufficiently cold dark matter made of light particles. One of the most interesting is the misalignment mechanism, discussed mostly in the case of the QCD axion [4–6] or (recently) string axions [7–10] and the central topic of this paper. Very recently, this mechanism has also been proposed to produce cold dark matter (CDM) out of hidden photons (HPs) [11]. This mechanism is extremely general and generates CDM out of essentially any field, if it satisfies some general conditions.

In this paper, we shall revisit the misalignment mechanism for both cases: we treat first axion-like particles (ALPs - which may arise as pseudo-Nambu-Goldstone bosons in field theory and appear generically in all string compactifications) and then hidden photons. Our conclusions turn out to be extremely motivating. Once produced, a population of very light cold dark matter particles is extremely difficult to reabsorb by the primordial plasma. Therefore, we find that in both cases, ALPs and HPs, a huge region in parameter space spanned by their masses and their couplings to standard model particles can give rise to the observed dark matter. The novelty in this work is that we shall provide new constraints and expose interesting regions of parameter space relevant for direct and indirect searches.

The outline of this paper goes as follows: in Sect. 2 we review the essentials of the misalignment mechanism. In Sects. 3 and 4 we elaborate on two particular cases, axion-like particles and hidden photons, respectively. We discuss the cosmological constraints, also noting some misconceptions in the results of [11]. In Sect. 5 we discuss the direct detection of ALP and HP CDM in microwave cavity experiments. We conclude in Sect. 6.

2 Essentials of the misalignment mechanism

The misalignment mechanism relies on assuming that fields in the early universe have a random initial state (such as as one would expect, for example, to arise from quantum fluctuations during inflation) which is fixed by the expansion of the universe; fields with mass m evolve on timescales $t \sim m^{-1}$. After such a timescale, the fields respond by attempting to minimize their potential, and consequently oscillate around the minimum. If there is no significant damping via decays, these oscillations can behave as a cold dark matter fluid since their energy density is diluted by the expansion of the universe as $\rho \propto a^{-3}$, where a is the universe scale factor.

In order to be more quantitative, let us revisit this mechanism for the simple case of a real scalar field with Lagrangian,

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m_\phi^2 \phi^2 + \mathcal{L}_I, \tag{2.1}$$

where $\mathcal{L}_I$ encodes interactions of the scalar field with itself and the rest of particles in the primordial bath. We assume that the universe underwent a period of inflation at a value of the Hubble expansion parameter $H = d \log a/dt$ larger than the scalar mass. After inflation the field shall be approximately spatially uniform and the initial state is characterized by a single initial value, ϕ_i. After inflation a period of reheating occurs, followed by a period of radiation dominated expansion. The equation of motion for ϕ in the expanding Universe is

$$\ddot{\phi} + 3H\dot{\phi} + m_\phi^2 \phi = 0. \tag{2.2}$$

In general, the mass receives thermal corrections from $\mathcal{L}_I$ which might be crucial, thus $m_\phi = m_\phi(t)$ should be understood.

The solution of this equation can be separated into two regimes. In a first epoch, $3H \gg m_\phi$, so ϕ is an overdamped oscillator and gets frozen, $\dot{\phi} = 0$. At a later time, t_1, characterized by $3H(t_1) = m_\phi(t_1) \equiv m_1$, the damping becomes undercritical and the field can roll down the potential and start to oscillate. During this epoch, the mass term is the leading scale in the equation and the solution can be found in the WKB approximation,

$$\phi \simeq \phi_1 \left(\frac{m_1 a_1^3}{m_\phi a^3} \right)^{1/2} \cos \left(\int_{t_1}^{t} m_\phi \, dt \right), \tag{2.3}$$

where $\phi_1 \sim \phi_i$ since up to t_1 the evolution is frozen. Note that in obtaining this solution we have not assumed a particular form for H but just its definition, and so it is valid for the radiation, matter, and vacuum energy dominated phases of the universe and their transitions.

The solution corresponds to fast oscillations with a slow amplitude decay. Let us call this amplitude $\mathcal{A}(t) = \phi_1 (m_1 a_1^3/m_\phi a^3)^{1/2}$ and the phase $\alpha(t) = \int^t m_\phi(t) dt$. The energy density of the scalar field is

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2} m_\phi^2 \phi^2 = \frac{1}{2} m_\phi^2 \mathcal{A}^2 + \dots\, , \tag{2.4}$$

where the dots stand for terms involving derivatives of $\mathcal{A}$, which by assumption are much smaller than m_ϕ ($m_\phi \gg H$ in this regime). Let us also consider the pressure,

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2} m_\phi^2 \phi^2 = -\frac{1}{2} m_\phi^2 \mathcal{A}^2 \cos(2\alpha) - \mathcal{A}\dot{\mathcal{A}} m_\phi \sin(2\alpha) + \dot{\mathcal{A}}^2 \cos^2(\alpha)\, . \tag{2.5}$$

At times $t \gg t_1$[1] the oscillations in the pressure occur at time scales $1/m_\phi$ much much faster than the cosmological evolution. We can therefore average over these oscillations, giving

$$\langle p_\phi \rangle = \langle \dot{\mathcal{A}}^2 \cos^2(\alpha) \rangle = \frac{1}{2}\dot{\mathcal{A}}^2. \tag{2.6}$$

[1] When the field just starts to oscillate the averaging employed in the following is not a good approximation and the equation of state is a non-trivial and strongly time dependent function. Depending on when the transition occurs this may have interesting cosmological effects on, e.g., structure formation.

As already mentioned $\dot{\mathcal{A}} \ll m_\phi \mathcal{A}$. Thus, at leading order in $\dot{\mathcal{A}}/(\mathcal{A}m)$, the equation of state is just

$$w = \langle p \rangle / \langle \rho \rangle \simeq 0, \tag{2.7}$$

which is exactly that of non-relativistic matter.

It follows from (2.3) that the energy density in a comoving volume, ρa^3, is not conserved if the scalar mass changes in time. The quantity

$$N = \rho a^3 / m_\phi = \frac{1}{2} m_1 a_1^3 \phi_1^2, \tag{2.8}$$

is however constant even in this case, and can be interpreted as a comoving number of non-relativistic quanta of mass m_ϕ. Here, we only need the conservation of N to compute the energy density today as

$$\rho_\phi(t_0) = m_0 \frac{N}{a_0^3} \simeq \frac{1}{2} m_0 m_1 \phi_1^2 \left(\frac{a_1}{a_0}\right)^3, \tag{2.9}$$

where quantities with a 0-subscript are evaluated at present time.

More physics insight is gained using temperatures instead of times and scale factors. First, we use the conservation of comoving entropy $S = sa^3 = 2\pi g_{*S}(T) T^3 a^3/45$ to write $(a_1/a)^3 = g_{*S}(T)T^3/g_{*S}(T_1)T_1^3$. Then we use the expression for the Hubble constant in the radiation dominated era $H = 1.66\sqrt{g_*(T)}T^2/m_{\rm Pl}$ and the definition of T_1, $3H(T_1) = m_1$ to express T_1 in terms of m_1 and the Planck mass $m_{\rm Pl} = 1.22 \times 10^{19}$ GeV. The functions g_* and g_{*S} are the effective numbers of energy and entropy degrees of freedom defined in [12]. The dark matter density today, (2.9), can then be expressed as

$$\rho_{\phi,0} \simeq 0.17 \frac{\rm keV}{\rm cm^3} \times \sqrt{\frac{m_0}{\rm eV}} \sqrt{\frac{m_0}{m_1}} \left(\frac{\phi_1}{10^{11}\,{\rm GeV}}\right)^2 \mathcal{F}(T_1), \tag{2.10}$$

where $\mathcal{F}(T_1) \equiv (g_*(T_1)/3.36)^{\frac{3}{4}} (g_{*S}(T_1)/3.91)^{-1}$ is a smooth function ranging from 1 to ~ 0.3 in the interval $T_1 \in (T_0, 200{\rm GeV})$. The abundance is most sensitive to the initial amplitude of the oscillations, $\propto \phi_1^2$, and to a lesser degree to today's mass m_0. The factor $\propto 1/\sqrt{m_1}$ reflects the damping of the oscillations in the expanding universe: the later the oscillations start, i.e. the smaller T_1 and therefore H_1 and m_1, the less damped they are for a given m_0.

If we compare the above estimate with the DM density measured by WMAP and other large scale structure probes [13],

$$\rho_{\rm CDM} = 0.76(3) \frac{\rm keV}{\rm cm^3}, \tag{2.11}$$

it is clear that we need very large values of ϕ_1 to account for all the dark matter. However, a relatively small ϕ_1 could be compensated by a small $m_1 \ll m_0$.

If we want the condensate to mimic the behaviour of standard cold dark matter we should ensure that, at latest at matter-radiation equality, at a temperature $T_{\rm eq} \sim 1.3$ eV, the mass attains its current value m_0 and therefore the DM density starts to scale truly as $1/a^3$. In particular, at this point the field should already have started to oscillate. This corresponds to a lower limit on m_1, $m_1 > 3H(T_{\rm eq}) = 1.8 \times 10^{-27}$ eV, which implies an upper bound on $\rho_{\phi,0}$,

$$\rho_{\phi,0} < 0.76 \frac{\rm keV}{\rm cm^3} \times \frac{m_0}{\rm eV} \left(\frac{\phi_1}{43\,{\rm TeV}}\right)^2. \tag{2.12}$$

In other words if we want these particles to be the DM, we need that $(m_0/\text{eV})(\phi_1/43\,\text{TeV})^2 > 1$, giving us a constraint on the required initial field value as a function of the mass today.

To conclude this section let us note that dark matter generated by the misalignment mechanism may have interesting properties beyond those of cold dark matter. At the time of their production, particles from the misalignment mechanism are semi-relativistic. Their momenta are of the order of the Hubble constant $p \sim H_1 \ll T_1$; accordingly we have (outside of gravitational wells) a velocity distribution with a very narrow width of roughly,

$$\delta v(t) \sim \frac{H_1}{m_1}\left(\frac{a_1}{a_0}\right) \ll 1. \tag{2.13}$$

Combined with the high number density of particles, $n_{\phi,0} = N/a_0^3 = \rho_{\text{CDM}}/m_0$, this narrow distribution typically leads to very high occupation numbers for each quantum state,

$$\mathcal{N}_{\text{occupation}} \sim \frac{(2\pi)^3}{4\pi/3}\frac{n_{\phi,0}}{m_0^3\delta v^3} \sim 10^{42}\left(\frac{m_1}{m_0}\right)^{3/2}\left(\frac{\text{eV}}{m_0}\right)^{5/2}, \tag{2.14}$$

where we have used $a_0/a_1 \sim T_1/T_0 \sim \sqrt{m_1 m_{\text{Pl}}/T_0}$. If the interactions are strong enough to achieve thermalisation, as argued in Refs. [14, 15] for the case of axions, this high occupation number can lead to the formation of a Bose-Einstein condensate. This could lead to interesting properties which may also lead to interesting signatures in cosmological observations [14–18]. Although we will not investigate this intriguing possibility here, we note that these features could also be realized for the more general light DM particles discussed in this paper.

In the following we discuss two particularly interesting possibilities for DM from the misalignment mechanism, axion-like particles and hidden photons.

3 Axion-like particles

In this section we will focus on a specific type of WISP, namely axion-like particles (ALPs). By this we shall mean particles with only derivative couplings to matter, and in particular an interaction with photons given by

$$\mathcal{L} \supset -\frac{1}{4}g\phi F_{\mu\nu}\tilde{F}^{\mu\nu}, \tag{3.1}$$

where ϕ is the ALP and g is a dimensionful coupling. The chief examples of ALPs are pseudo Nambu-Goldstone bosons (pNGBs) and string "axions" which can be treated together. For concreteness we will focus in this paper on particles of these types.

The cosmology of the ALP condensate depends on the type of interaction generating its mass and in particular how this mass changes through the evolution of the universe. In the following we will go through a variety of possibilities for the DM formation as well as its cosmological viability for different scenarios. The regions which allow for viable ALP DM are then assembled in Fig. 1 in the $m_\phi - g$ plane.

3.1 Axion-like particles from pNGBs and string theory

When a continuous global symmetry is spontaneous broken, massless particles appear in the low energy theory: Nambu-Goldstone bosons (NGBs). They appear in the Lagrangian as phases of the high energy degrees of freedom. Since phases are dimensionless the canonically normalised theory at low energies always involves the combination ϕ/f_ϕ, where ϕ is the NGB field and f_ϕ is a scale close to the spontaneous symmetry breaking (SSB) scale. The range for ϕ/f_ϕ is $(-\pi, \pi)$ and therefore the natural values for ϕ_1 are $\sim f_\phi$. String axions on the other hand appear in all compactifications. They share these properties (having a shift symmetry and being periodic) but with the natural size of f_ϕ being the string scale (in type II compactifications this can be somewhat modified by a factor of the typical length scale of the compactification).

Indeed, all of the global symmetries in the standard model are broken[2]. Furthermore the black hole no hair theorem and what we know about quantum gravity tell us that this should ultimately also occur to any additional global symmetries. Hence we should have pseudo Nambu-Goldstone Bosons (pNGBs) instead of NGBs. They then have a mass, and can be a dark matter candidate.

There are many possibilities for breaking the shift symmetry, explicitly or spontaneously, perturbatively or non-perturbatively; for stringy axions, the shift symmetry is exact to all orders in perturbation theory and is only broken non-perturbatively, for instance from a non-abelian anomaly, gaugino condensation or stringy instantons. The ALP potential can typically be parametrized as

$$V(\phi) = m_\phi^2 f_\phi^2 \left(1 - \cos\frac{\phi}{f_\phi}\right). \tag{3.2}$$

The mass of the ALP is in general unrelated to the QCD axion mass and in particular will be independent of the temperature unless generated by a sector that is thermalised. The ALP will satisfy the equation of motion (2.2) as long as ϕ/f_ϕ is small, i.e. few oscillations after t_1. The inaccuracy of the quadratic approximation can be cured by an additional correction factor to (2.10). This is normally an $\mathcal{O}(1)$ factor except if we fine tune the initial condition to $\phi = \pi f_\phi$.

The dimensionful coupling parameter g in (3.1) can be parametrised as

$$g \equiv \frac{\alpha}{2\pi}\frac{1}{f_\phi}\mathcal{N}. \tag{3.3}$$

In the simplest case $\mathcal{N}$ is an integer, but this is not true in general when the ALP mixes, either kinetically or via symmetry-breaking effects with other ALPs or with pseudoscalar mesons.

We can then represent the allowed regions of ALP dark matter in the m_ϕ–g plane by using

$$\phi_1 = \theta_1 \frac{\alpha\mathcal{N}}{2\pi g} \tag{3.4}$$

with $\theta_1 = |\phi_1|/f_\phi$, the initial misalignment angle whose range is restricted to values between $-\pi$ and π. The model dependent factor $\mathcal{N}$ will from now on be taken to be unity, but the reader should keep in mind that in principle it can be very different in particular constructions.

3.2 ALP dark matter from the misalignment mechanism

The value of ϕ_1 that determines the DM abundance depends on the behaviour of the ALP field during inflation. For a pseudo-Nambu-Goldstone boson, the spontaneous symmetry

[2] Assuming that neutrinos are Majorana fermions, otherwise $B - L$ is an exception.

breaking (SSB) could take place before or after inflation: the pNGB effectively exists only after SSB and it is during the associated phase transition that its initial values are set. For a string axion, provided the inflationary scale is below the string scale (or, equivalently, the decay constant) we should have control over the field theory, and so it will behave like a pNGB with symmetry broken before inflation. Assuming its mass to be much smaller than the Hubble scale at the time of SSB, $H_{\rm SSB}$, the ALP field will take random values in different causally disconnected regions of the universe. The initial size of these domains cannot be larger than

$$L_{i,\rm dom} \sim \frac{1}{H_{\rm SSB}} \sim \frac{m_{\rm Pl}}{f_\phi^2 \sqrt{g_*(f_\phi)}}. \tag{3.5}$$

For a string axion, or for a pNGB whose associated SSB happens before inflation, the initial domains are stretched over a size larger than the current size of the universe. Consequently the initial field value is the same for every point within our current horizon. The current DM density then depends on this initial field value, leaving an additional parameter to tune the DM density.

On the other hand, if the SSB happens after inflation, the DM density has inhomogeneities of order $\mathcal{O}(1)$ at length scales $\lesssim L_{i,\rm dom}$. Non-linear effects, due to the attractive self-interaction caused by higher order terms in the expansion of the potential (3.2), drive the overabundances to form peculiar DM clumps that are called miniclusters [19–22]. These miniclusters act like seeds enhancing the successive gravitational clumping that leads to structure formation. The minicluster mass is set by the dark matter mass inside the Hubble horizon $d_H = H^{-1}$ when the self-interaction freezes-out, i.e. $M_{\rm mc} \sim \rho_\phi(T_\lambda) d_H(T_\lambda)^3$ for the freeze-out temperature T_λ. Long-range interactions will be exponentially suppressed at distances longer than $1/m_\phi$ so we can expect T_λ to be of the order T_1, with at most a logarithmic dependence on other parameters. This is indeed the case for QCD axions, for which the miniclustering is quenched soon after the QCD phase transition that turns on the potential (3.2) [14] giving $M_{\rm mc} \sim 10^{-12} M_\odot$, where $M_\odot = 1.116 \times 10^{57}$ GeV is the solar mass, and a radius $R_{\rm mc} \sim 10^{11}$ cm [23]. In the case of ALPs, $M_{\rm mc}$ can be larger if the mass is lighter. The authors of [22] pointed out that the present data on the CDM power spectrum constrain $M_{\rm mc} \lesssim 4 \times 10^3 M_\odot$ which translates into a lower bound in temperature $T_\lambda > 2 \times 10^{-5}$ GeV and in the ALP mass $m_\phi > H(T = 2 \times 10^{-5}\ {\rm GeV}) \sim 10^{-20}$ eV.

If some of these miniclusters survive the tidal disruption during structure formation they should be observable in forthcoming lensing experiments [22, 23]. In any case, at larger scales, the DM density averages to a constant value corresponding to $\langle \phi_1^2 \rangle \sim \pi^2 f_\phi^2/3$ bearing the mentioned $\mathcal{O}(1)$ correction due to the non-harmonic behaviour of large initial phases.

During the spontaneous symmetry breaking of a global symmetry topological defects such as cosmic strings [24] and domain walls can be formed. Strings have a thickness $\delta \sim 1/f_\phi$ and typical sizes of the order of the horizon, $L \sim t$. As strings enter into the horizon they can rapidly reconnect, form loops and decay into pNGBs. If the SSB happens after inflation, we have to consider also their contribution to the energy budget of the universe. Axions resulting from string decay are known to contribute significantly to their cold DM density, but the exact amount is subject to a long-standing controversy [12]. The debate is focused around the axion emission spectrum. Some authors argue that the decay proceeds very fast, with an emission spectrum $1/k$ with high and low energy cutoff of order respectively $1/\delta$ and $1/L$. In this case the contribution to n_ϕ is similar to that from the misalignment mechanism [25, 26]. Others put forward that the string decays happen after many oscillations, with a radiation spectrum peaked around $2\pi/L$, which enhances the contribution to cold

DM by a multiplicative factor of $\log(L/\delta) \sim \log(f_a/m_a) \sim \mathcal{O}(60)$ [27–33]. Once the axion potential builds up at the QCD phase transition, domain walls build up. If the axion potential has only one minimum these domain walls can still efficiently decay into axions, leading to a third axion population which is thought however not to be significant. If different exactly degenerate vacua exist the domain walls are persistent and can very easily run in conflict with observations. Therefore one assumes a small explicit breaking of the Peccei-Quinn symmetry, which breaks the degeneracy and allows domain walls to decay. It is possible although somehow fine-tuned to do so without compromising the solution to the strong-CP problem. For a recent review on axion cosmology see e.g. [34].

We expect the same type of behavior for ALPs with similar characteristics than axions, i.e. ALPs whose mass is generated at a late phase transition due to a hidden sector which becomes strongly interacting. In this case we should keep in mind the controversy of the string decay spectrum and assume an uncertainty of order $\log(f_\phi/m_\phi)$ in the DM abundance. The domain wall problem can in principle be solved by strong enough explicit breaking, and their contribution to the DM appears to be subdominant as well. In models where the cosmological evolution of the ALP mass is different, the situation can differ from the above. These models have to be studied case by case, which is beyond the scope of this paper.

3.3 Sufficient production of dark matter

As we have seen in Sect. 2 a general constraint arises from the fact that we get a sufficient amount of DM but at the same time the mass at matter radiation equality has to be greater than the Hubble constant. This is the bound Eq. (2.12). For pNGB ALPs we however also have that the field value itself cannot be larger than πf_ϕ which itself is connected to the coupling to photons. Combining these two restrictions gives us a way to constrain the viable regions. The light red region in Fig. 1 labelled "$m_1 > 3H(T_{\rm eq})$" corresponds to this general bound with $\phi_1 \leq f_\phi$ and $\mathcal{N}$=1.

Any ALP model should satisfy this bound for its zero mode to behave as DM before matter-radiation equality. Realistic models attempting to saturate this bound will have problems either fitting the cosmic microwave background (CMB) data or with the WKB approximation we have used. In this sense this bound is very conservative. Importantly, for $\mathcal{N} \sim 1$, it seems to exclude the possibility of providing DM from the type of ultralight ALP field that has been invoked to explain the puzzlingly small opacity of the universe for $\sim$ TeV gamma rays (see Ref. [36] and references therein) in terms of photon $\leftrightarrow$ ALP conversions in astrophysical magnetic fields, requiring[3] $g \sim 10^{-11}\text{GeV}^{-1}$ and $m_\phi \lesssim 1$ neV, see [37–40]. To allow an ALP to explain these observations and simultaneously to be dark matter requires $\mathcal{N} \gtrsim 10$ which is still conceivable.

Let us now turn to the stronger constraints that can be obtained for specific time/temperature dependencies of the ALP mass. The simplest realization of an ALP model has m_ϕ constant throughout the universe expansion. In this case, we can infer the DM yield from Eq. (2.10) using $m_1 = m_0$. The region of ALP DM in this case is depicted in pink in Fig. 1 and labelled "Standard ALP CDM ($m_1 = m_0$)". We have assumed $\mathcal{N} = 1$ and used the a-priori unknown value of θ_1 to tune the right DM abundance. The upper bound on g reflects the fact that θ_1 cannot be larger than π, and thus assumes $\theta_1 \sim \pi$. Moving to lower values of g requires inflation happening after SSB in order to have a homogeneous small value of θ_1 which is increasingly fine-tuned to zero. In this sense the values closest to

[3]The required coupling is determined by the extragalactic background light and is therefore plagued by sizeable uncertainties.