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# The Multi-Objective Vendor Selection Problem Using Fuzzy Weights

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**Abstract.** Sourcing, which lies at the initial stage of a supply chain, can be made efficient by making an appropriate selection of vendors. The search for the best suited suppliers is among the most important decisions for companies to improve their performance and deliver their own products with the greatest benefits for their customers. The vendor selection in the supply chain is a multi-criteria decision making problem. In our formulation of that problem it includes three primary objectives: Minimizing the ordering cost, minimizing the transportation cost, and minimizing the late deliveries. These objective are optimized subject to some realistic constraints that are necessary in selecting the appropriate vendor. This paper consists of two different types of weighted approach: the relative weights represent the relative importance of the objective functions, whereas the aggregated fuzzy weights represent the relative rating of the fuzzy number to each criterion. Also, the input information of the formulated problem has been considered to be imprecise. Concepts from the fuzzy set theory are used to handle the imprecision by considering all of those parameters as trapezoidal fuzzy numbers and then the $\alpha$-cut approach is used to get the crisp values of the parameters. The multi objective vendor selection problem (MOVSP) is thus formulated in a fuzzy goal environment with two different weighted approaches. They are used to find the allotted quota given to the suppliers. A real life example is presented and solved by the LINGO 13.0 solver.

**Keywords:** Multi-objective Vendor Selection Problem, Weighted Criterion, Fuzzy Sets, Trapezoidal Fuzzy Number, Fuzzy Goal Programming.

## 1 Introduction

A supply chain is a system of organizations, people, technology, activities, information, and resources involved in moving a product or service from between different suppliers finally to the customers. A supply chain is a network of facilities and distribution options that performs the functions of procurement of materials, transformation of these materials into intermediate and finished products, and the distribution of these finished products to customers. Supply chains exist in both service and manufacturing organizations, although the complexity of the chain may vary greatly from industry to industry and firm to firm. In its simplest from and as an example of a very simple supply chain for a single product, consider a raw material that is procured from vendors, transformed into finished goods in a single step and then transported to distribution centers, and ultimately to customers. However, realistic supply chains have multiple end products with shared components, facilities, and capacities (see Figure 1). It was defined by Quinn [1997] that "the supply chain includes all of those activities associated with moving goods from the raw-materials stage through to the end costumer/user". The selection of right supplier/vendor for an organization should not only meet customer/user requirements, but also bring profit to the firm, help in fulfilling various criteria such as cost, delivery, quality objectives and technical specifications. Kumar et al. [2006b] discussed that the nature of the supplier/vendor selection in supply chain is a multi-criterion decision making problem.

Individual supplier may perform differently on different criteria. According to them, supply chain decision faces many constraints, some of these are related to supplier's internal policy and externally imposed system requirements.

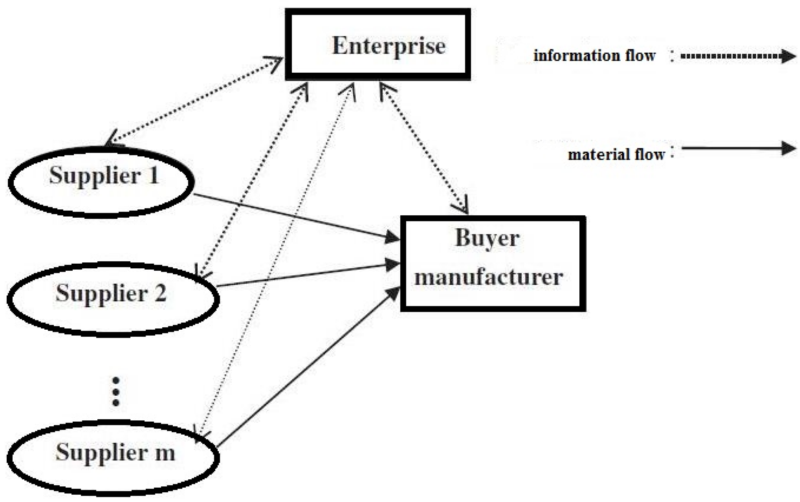


**Fig. 1.** A Supply Chain Network.

Since the 1960s, the criteria for vendor selection and vendor rating were a central main area of research in supply chain management (SCM). Basically there are three quantitative techniques used for supplier selection: (1) multiple attribute decision making, (2) mathematical programming models, and (3) intelligent approaches. Among these quantitative techniques, mathematical programming model were used extensively for the vendor selection problem (VSP). The goal programming developed by Charnes et al. [1968] emerged a powerful technique to solve such multi-criteria decision making problems. Since the commencement of the goal programming technique, it has been enriched by many researchers such as Lee [1972], Ignizio [1976] and many more. Undoubtedly, goal programming was one of the major breakthroughs in dealing with multi-criteria decision making problems. On the other hand, Zadeh [1965] suggested the concept of fuzzy sets as one possible way of improving the modeling of vague parameters. Zimmermann [1978] developed a fuzzy programming approach to solve multi-criteria decision making problems. However, one of the major problems which decision makers face is the modeling of ill conditioned optimization problems or the problems where the coefficients are imprecise and vague in nature. Classical methods of mathematical programming failed to optimization such problems. Bellman and Zadeh [1970] gave a concept that the constraints and goals in such situations may be viewed as fuzzy in nature. Weber and Current [1993] introduced the concept of multi-objective programming technique for selecting the vendors with their order quantities by multiple conflicting criteria. Several authors such as Dahel [2003], Xia and Wu [2007], Pokharel [2008], Tsai and Wang [2010], Rezaei and Davoodi [2011] worked on the multi-objective vendor selection problem (MOVSP). Kumar et al. [2004, 2006b] formulated a fuzzy mixed integer goal programming model for a multiple sourcing supplier selection problem (SSP) including three fuzzy goals: cost, quality, and delivery, and subject to buyer's demand, suppliers' capacity, and others. They used the max-min technique from Zimmermann [1978] to solve the multi-objective problem. Lamberson et al. [1976] gave an idea to develop a systematic vendor selection process for identifying and prioritizing relevant criteria and to evaluate the trade-offs between technical, economic and performance criteria. Kumar et al. [2006a] used a lexicographic goal programming approach for solving a piecewise linear VSP of quantity discounts. In real situations, objectives (or criteria) have various weights related to strategies of the purchasing department, to cope with the problem. Amid et al. [2006, 2009] formulated a fuzzy based model for SSP including three fuzzy goals cost, quality and delivery, and subject to capacity restriction and market demand. In order to deal with the objectives' weight, they used the additive model of Tiwari et al. [1987] for solving their multi criterion model model. Lin [2004] subsequently proposed a weighted max-min (WMM) model for solving fuzzy multi criterion model of supplier selection. This approach was later applied by Amid et al. [2011] to a fuzzy multi-objective supplier selection problem (MOSSP) with three fuzzy goals cost, quality and delivery, and subject to capacity and demand requirement constraints. Liao and Kao [2010] combined the Taguchi loss function, analytical hierarchy process, and multi-choice goal programming model for solving the supplier selection problem. Liao and Kao [2011] also gave a two-stage model for selecting suppliers in a company which is engaged in the watch manufacturing sector by using a fuzzy Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) with triangular fuzzy numbers and multi-choice goal programming (MCGP) to optimize the problem. Wu et al. [2010] used a trapezoidal membership function, and solved the probabilistic multi-criteria vendor selection model by using sequential programming, taking risk factors into consideration. Ozkok and Tiryaki [2011] established a compensatory fuzzy approach to solve a multi-objective linear supplier selection problem with multiple items by using cost, service, and quality as objectives. Arikan [2013] formulated a multi-objective VSP based on price, quality, customer service and delivery criteria. Shirkouhi et al. [2013] developed an interactive two-phase fuzzy multi-objective linear programming model for the supplier selection under multi-price level and multiproducts. Kilic [2013] developed an integrated approach including a fuzzy technique for selecting the best supplier in a multi-item/multi supplier environment. Rouyendegh and Saputro [2014] described an optimum decision making method for selecting supplier and allocating order by applying fuzzy TOPSIS and MCGP. Jadidi et al. [2014] developed a normalized goal programming model with predetermined goals and predetermined weights for solving a multi-objective supplier selection problem. Chang et al. [2010] considered multiple aspiration levels and vague goal relations to help the decision makers for choosing the better suppliers by using multi-choice goal programming (MCGP) with fuzzy approach. Karimi and Rezaeinia [2014] adopted a revised multi-segment goal programming model for selecting the suppliers. Sivrikaya et al. [2015] adopted a fuzzy Analytic Hierarchy Process (AHP) goal programming approach with linguistic variables expressed in trapezoidal fuzzy numbers, which is applied to assess weights and ratings of supplier selection criteria.

Motivated from the above literature, we extend a non-fuzzy MOVSP model from Kumar et al. [2006b] to imprecise parameters which are considered as fuzzy numbers. The so extended model is then transformed with the $\alpha$-cut approach into a series of classical (non-fuzzy) linear programs, which are solved by standard Linear Programming techniques and numerical solvers. We demonstrate this approach using a numerical example.

The remainder of this paper is organized as follows. Section 2 provides an introduction to the preliminaries of the subject to make the study self-contained. Section 3 deals with the $\alpha$-cut presentation for the multi-objective goal programming model. In Section 4, the description of multi objective vendor selection problem is discussed. Section 5 provides the goal programming formulation with $\alpha$-cuts for solving MOVSP. In Section 6, the computational procedure for solving the above model is presented. In Section 7, conclusions are drawn regarding the effectiveness of the developed solution procedure.

## 2 Preliminaries

Before formulating the problem of interest, we introduce the basic definitions of fuzzy sets and fuzzy numbers, which are reproduced here from Khan et al. [2016] and Abbasbandy and Asady [2004].

### 2.1 Fuzzy Sets

A fuzzy set $\widetilde{A}$ in a universe of discourse $X$ is defined as a set of pairs $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)) : x \in X\}$, where $\mu_{\widetilde{A}} : X \to [0,1]$ is a mapping called the membership function of the fuzzy set $\widetilde{A}$. $\mu_{\widetilde{A}}(x)$ is called the membership value or degree of membership of $x \in X$ in the fuzzy set $\widetilde{A}$. The larger the value of $\mu_{\widetilde{A}}$, the stronger (or higher) the grade of membership in $\widetilde{A}$.

### 2.2 Fuzzy Numbers

A fuzzy set $\widetilde{A}$ in $\mathbb{R}$ is called a fuzzy number if it satisfies the following conditions:

i. $\widetilde{A}$ is convex and normal, that is, $\mu_{\widetilde{A}}(x) = 1$ for some $x \in \mathbb{R}$.
ii. The $\alpha$-cut $\widetilde{A}_\alpha := \{x \mid \mu_{\widetilde{A}}(x) \geq \alpha, x \in X\}$ is a closed interval for every $\alpha \in (0,1]$.
iii. The support of $\widetilde{A}$ is bounded, where $\mathrm{supp}(\widetilde{A}) := \{x \in \mathbb{R} : \mu_{\widetilde{A}}(x) > 0\}$.

### 2.3 Triangular Fuzzy Numbers

A fuzzy number $\widetilde{A}$ is a triangular fuzzy number denoted by $(\delta, m, \beta)$, where $\delta, m$ and $\beta$ are real numbers with $\delta < m < \beta$ and its membership function $\mu_{\widetilde{A}}(x)$ is given as

$$\mu_{\widetilde{A}}(x) := \begin{cases} 0, & x \leq \delta, \\ \frac{x-\delta}{m-\delta}, & \delta \leq x \leq m, \\ 1, & x = m, \\ \frac{\beta-x}{\beta-m}, & m \leq x \leq \beta, \\ 0, & x \geq \beta. \end{cases} \tag{1}$$

$\mu_{\widetilde{A}}(x)$ satisfies the following conditions (c.f. Figure 2):

i. $\mu_{\widetilde{A}}(x)$ is a continuous mapping from $\mathbb{R}$ to closed interval $[0,1]$,
ii. $\mu_{\widetilde{A}}(x) = 0$ for every $x \in (-\infty, \delta]$,
iii. $\mu_{\widetilde{A}}(x)$ is strictly increasing and continuous on $[\delta, m]$,
iv. $\mu_{\widetilde{A}}(x) = 1$ for every $x = m$,
v. $\mu_{\widetilde{A}}(x)$ is strictly decreasing and continuous on $[\beta, m]$,
vi. $\mu_{\widetilde{A}}(x) = 0$ for every $x \in [\beta, \infty)$.

### 2.4 Trapezoidal Fuzzy Numbers

A fuzzy number $\widetilde{A}$ is a trapezoidal fuzzy number denoted by $(\delta, m, n, \beta)$ where $\delta, m, n$ and $\beta$ are real numbers with $\delta < m < n < \beta$ and its membership function

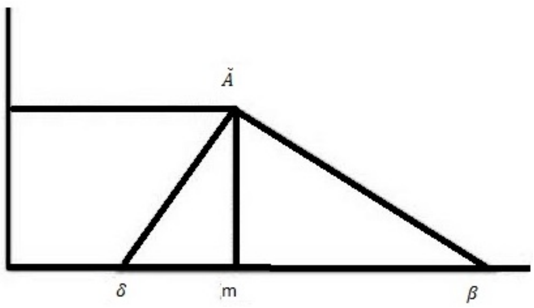


**Fig. 2.** Triangular Fuzzy number.

$\mu_{\widetilde{A}}(x)$ is given as:

$$\mu_{\widetilde{A}}(x) := \begin{cases} 0, & x \leq \delta, \\ \frac{x-\delta}{m-\delta}, & \delta \leq x \leq m, \\ 1, & m \leq x \leq n, \\ \frac{n-x}{\beta-n}, & n \leq x \leq \beta, \\ 0, & x \geq \beta. \end{cases}$$

$\mu_{\widetilde{A}}(x)$ satisfies the following conditions (c.f. Figure 3):

i. $\mu_{\widetilde{A}}(x)$ is a continuous mapping from $\mathbb{R}$ to closed interval $[0,1]$,
ii. $\mu_{\widetilde{A}}(x) = 0$ for every $x \in (-\infty, \delta]$,
iii. $\mu_{\widetilde{A}}(x)$ is strictly increasing and continuous on $[\delta, m]$,
iv. $\mu_{\widetilde{A}}(x) = 1$ for every $x \in [m, n]$,
v. $\mu_{\widetilde{A}}(x)$ is strictly decreasing and continuous on $[n, \beta]$,
vi. $\mu_{\widetilde{A}}(x) = 0$ for every $x \in [\beta, \infty)$.

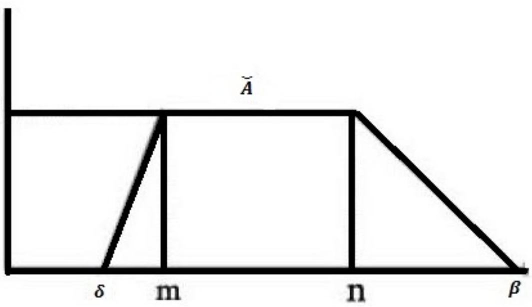


**Fig. 3.** Trapezoidal Fuzzy number.

### 2.5 Trapezoidal LR-Fuzzy Numbers

A trapezoidal LR-fuzzy number $\widetilde{A}$, denoted by $\widetilde{A} = (a, b, \beta, \gamma)$ with defuzzifier $a, b$, left fuzziness $\beta > 0$ and right fuzziness $\gamma > 0$ is defined by its membership function

$$\mu_{\widetilde{A}}(x) := \begin{cases} 0, & x \leq a - \beta, \\ \frac{x-a+\beta}{\beta}, & a - \beta \leq x \leq a, \\ 1, & a \leq x \leq b, \\ \frac{b-x+\gamma}{\gamma}, & b \leq x \leq b + \gamma, \\ 0, & x \geq b + \gamma. \end{cases}$$

A plot of $mu_{\widetilde{A}}$ is shown in Figure 4.

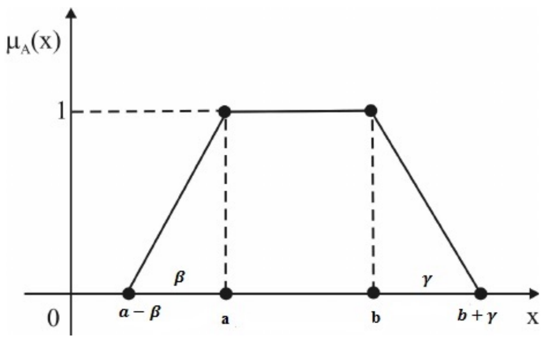


**Fig. 4.** Trapezoidal LR-Fuzzy number.

### 2.6 $\alpha$-Cut of a Trapezoidal Fuzzy Number

If the fuzzy set is a trapezoidal fuzzy number, $\widetilde{A} = (\delta, m, n, \beta)$, then its $\alpha$-cut is defined as

$$(\widetilde{A})_\alpha = \left[(\widetilde{A})^L_\alpha, (\widetilde{A})^U_\alpha\right] = [\delta + (m - \delta)\alpha, -(\beta - n)\alpha + n].$$

If the fuzzy set is a trapezoidal LR-fuzzy number, $\widetilde{A} = (a, b, \beta, \gamma)$, then its $\alpha$-cut is defined as

$$(\widetilde{A})_\alpha = \left[(\widetilde{A})^L_\alpha, (\widetilde{A})^U_\alpha\right] = [(\alpha - \beta) + \beta\alpha, (b + \gamma) - \gamma\alpha].$$

where $(\widetilde{A})^L_\alpha$ and $(\widetilde{A})^U_\alpha$ are the lower bound and upper bound of the $\alpha$-cut of $\widetilde{A}$, respectively, such that

$$(\widetilde{A})^L_\alpha \leq (\widetilde{A})_\alpha \leq (\widetilde{A})^U_\alpha.$$

The cut or $\alpha$-level set of a fuzzy set is called a crisp set.

## 3 Multi-Objective Fuzzy Goal-Programming Formulation with $\alpha$-Cut

Goal programming (GP) is an important technique for decision-makers (DM) to consider simultaneously the conflicting objectives when searching for a set of acceptable solutions. GP is among the most widely used technique for solving multi-criteria and multi-objective decision making problems. Nishad and Singh [2015] declared that there might be a situation where the DMs would like to make a decision on the problem, with the goal that can be achieved from some specific aspiration levels.

Let us consider a fuzzy multi-objective optimization problem with $n$ decision variable, $I = \{1, \ldots, m\}$ constraints (where in $I_1 \subseteq I$ we subsume the less-than constraints, in $I_2 \subseteq I$ the greater-than constraints, and in $I_3 \subseteq I$ the equality constraints), and $K$ objective functions, i.e.,

$$\begin{aligned}
&\min \ Z(X) = \{\widetilde{C}_1 x, \widetilde{C}_2 x, \widetilde{C}_3 x, \ldots, \widetilde{C}_K x\}, \\
&\text{subject to} \\
&\qquad \widetilde{A}_i x \geq \widetilde{B}_i, \quad i \in I_1, \\
&\qquad \widetilde{A}_i x \leq \widetilde{B}_i, \quad i \in I_2, \\
&\qquad \widetilde{A}_i x = \widetilde{B}_i, \quad i \in I_3, \\
&\qquad x_j \geq 0, \quad j = 1, 2, 3, \ldots, n.
\end{aligned} \tag{2}$$

Since problem (2) has fuzzy coefficients which have a probability distribution in an uncertain interval, it can be approximated in terms of its $\alpha$-cut interval. The $\alpha$-cut interval for $\widetilde{C}_k$ is defined as

$$(\widetilde{C}_k)_\alpha = [(\widetilde{C}_k)^L_\alpha, (\widetilde{C}_k)^U_\alpha].$$

Then the objective function can be defined as

$$[(Z_k(x))_\alpha]^L = \sum_{j=1}^{n} (\widetilde{C}_k)^L_\alpha x_j,$$

$$[(Z_k(x))_\alpha]^U = \sum_{j=1}^{n} (\widetilde{C}_k)^U_\alpha x_j.$$

In a similar manner the inequality constraints can also be approximated as

$$\sum_{j=1}^{n} (\widetilde{A}_{i,j}) x_j \geq \widetilde{B}_i, \quad i \in I_1,$$

$$\sum_{j=1}^{n} (\widetilde{A}_{i,j}) x_j \leq \widetilde{B}_i, \quad i \in I_2.$$

They can be expressed in terms of their $\alpha$-cut interval as

$$\sum_{j=1}^{n}(\widetilde{A}_{ij})_{\alpha}^{U}x_j \geq (\widetilde{B}_i)_{\alpha}^{L}, \quad i \in I_1,$$

$$\sum_{j=1}^{n}(\widetilde{A}_{ij})_{\alpha}^{L}x_j \leq (\widetilde{B}_i)_{\alpha}^{U}, \quad i \in I_2.$$

The fuzzy equality constraint

$$\sum_{j=1}^{n}(\widetilde{A}_{ij})x_j = \widetilde{B}_i, \quad i \in I_3$$

can be transformed into two inequalities as

$$\sum_{j=1}^{n}(\widetilde{A}_{ij})_{\alpha}^{L}x_j \leq (\widetilde{B}_i)_{\alpha}^{U}, \quad i \in I_3,$$

$$\sum_{j=1}^{n}(\widetilde{A}_{ij})_{\alpha}^{U}x_j \geq (\widetilde{B}_i)_{\alpha}^{L}, \quad i \in I_3.$$

Thus the minimization problem (2) can be expressed as

$$\min\ [(Z_k(x))_{\alpha}]^{L} = \sum_{j=1}^{n}(\widetilde{C}_k)_{\alpha}^{L}x_j, \quad k = 1, 2, 3, \ldots, K,$$

subject to

$$\begin{aligned}
&\sum_{j=1}^{n}(\widetilde{A}_{ij})_{\alpha}^{U}x_j \geq (\widetilde{B}_i)_{\alpha}^{L}, && i \in I_1,\\
&\sum_{j=1}^{n}(\widetilde{A}_{ij})_{\alpha}^{L}x_j \leq (\widetilde{B}_i)_{\alpha}^{U}, && i \in I_2,\\
&\sum_{j=1}^{n}(\widetilde{A}_{ij})_{\alpha}^{L}x_j \leq (\widetilde{B}_i)_{\alpha}^{U}, && i \in I_3,\\
&\sum_{j=1}^{n}(\widetilde{A}_{ij})_{\alpha}^{U}x_j \geq (\widetilde{B}_i)_{\alpha}^{L}, && i \in I_3,\\
&x_j \geq 0, && j = 1, 2, 3, \ldots, n.
\end{aligned} \tag{3}$$

Next, we consider the transformation of the objectives to fuzzy goals by assigning an aspiration level to each of them. Problem (3) can be transformed into a fuzzy goal programm by taking certain aspiration levels and introducing variables $d_k^- \geq 0$ that measure the lower deviation for each of them. Then the objective function is transformed into the new constraint

$$\frac{u_k - \sum_{j=1}^{n} (\widetilde{C}_k)_\alpha^L}{u_k - g_k} + d_k^- \geq 1. \tag{4}$$

Here $g_k$ are the aspiration levels for the $k$-th goal (also the lowest acceptable level for the $k$-th goal or the ideal solution). The upper acceptable level $u_k$ is an anti-ideal (worst-case) solution. For a given and fixed value of $\alpha \in [0, 1]$ they are computed as follows:

$$g_k = \min \sum_{j=1}^{n} (\widetilde{C}_k)_\alpha^L x_j, \quad k = 1, 2, \ldots, K,$$
$$u_k = \max \sum_{j=1}^{n} (\widetilde{C}_k)_\alpha^U x_j, \quad k = 1, 2, \ldots, K.$$

When we define $w_k$ for $k = 1, 2, \ldots, K$ as weight coefficients for the objective functions, then the multi-objective optimization problem is turned into the following single-objective optimization problem with $n$ decision variable and $m$ constraints:

$$\min \ Z = \sum_{k=1}^{K} w_k d_k^-$$

subject to

$$\begin{aligned}
&\frac{u_k - \sum_{j=1}^{n} (\widetilde{C}_k)_\alpha^L}{u_k - g_k} + d_k^- \geq 1 && \\
&\sum_{j=1}^{n} (\widetilde{A}_{ij})_\alpha^U x_j \geq (\widetilde{B}_i)_\alpha^L, && i \in I_1, \\
&\sum_{j=1}^{n} (\widetilde{A}_{ij})_\alpha^L x_j \leq (\widetilde{B}_i)_\alpha^U, && i \in I_2, \\
&\sum_{j=1}^{n} (\widetilde{A}_{ij})_\alpha^L x_j \leq (\widetilde{B}_i)_\alpha^U, && i \in I_3, \\
&\sum_{j=1}^{n} (\widetilde{A}_{ij})_\alpha^U x_j \geq (\widetilde{B}_i)_\alpha^L, && i \in I_3, \\
&x_j \geq 0, && j = 1, 2, 3, \ldots, n, \\
&d_k^- \geq 0, && k = 1, \ldots, K.
\end{aligned}$$

## 4 Multi-Objective Vendor Selection Problem (MOVSP)

This model is an extension of an model given by Kumar et al. [2006b], where a hypothetical system of $n$ vendors with deterministic parameter is considered (for details we refer to Kumar et al. [2006b]). In Table 1 we summarize the symbols used in the model formulation.

**Table 1.** List of symbols used in the model formulation.

| | |
|---|---|
| $i$ | index for vendor, for all $i = 1, 2, ...., n$ |
| $k$ | index for objectives, for all $k = 1, 2, 3, ...., K$ |
| $x_i$ | order quantity given to vendor $i$ |
| $D$ | aggregate demand of the item over a fixed planning period |
| $n$ | number of vendors competing for selection |
| $p_i$ | price per unit of item for ordered quantity $x_i$ to the vendor $i$ |
| $t_i$ | Transportation cost of a unit item of the ordered from the vendor $i$ |
| $l_i$ | percentage of the late delivered units by the vendor $i$ |
| $U_i$ | upper limit of the quantity available for vendor $i$ |
| $B_i$ | budget constraint allocated to each vendor. |
| $q_i$ | percentage of the rejected units delivered by the vendor $i$ |
| $r_i$ | vendor rating value for vendor $i$ |
| $P$ | least total purchasing value that a vendor can have |
| $f_i$ | vendor quota flexibility for vendor $i$ |
| $F$ | least value of flexibility in supply quota that a vendor should have |
| $Q$ | maximum rejection that supplier can afford |

In this model we consider the following three objectives: The first objective is to minimize the net cost for ordering the aggregate demand, i.e.,

$$\min Z_1 = \sum_{i=1}^{n} p_i x_i. \tag{5}$$

The second objective is to minimize the net transportation cost for all the items, i.e.,

$$\min Z_2 = \sum_{i=1}^{n} t_i x_i.$$

The third objective is to minimize the late delivered items of the vendors, i.e.,

$$\min Z_3 = \sum_{i=1}^{n} l_i x_i.$$

The set of feasible solutions to the problem is described by the following constraints. There is a restriction due to the aggregate demand of the items, i.e.,

$$\sum_{i=1}^{n} x_i = D.$$

There are restrictions due to the maximum capacity of the vendors, i.e.,

$$x_i \leq U_i \quad \text{for all} \quad i = 1, 2, \ldots, n.$$

There are restriction on the budget allocated to the vendors for supplying the items, i.e.,

$$p_i(x_i) \leq B_i \quad \text{for all} \quad i = 1, 2, \ldots, n.$$

The total item purchasing value is bounded, i.e.,

$$\sum_{i=1}^{n} r_i(x_i) \geq P.$$

Some flexibility is needed with the vendors quota, i.e.,

$$\sum_{i=1}^{n} f_i(x_i) \leq F.$$

There is a restriction on number of rejected items from the supplier, i.e.,

$$\sum_{i=1}^{n} q_i(x_i) \leq Q.$$

Finally, there are non-negativity restriction on the vendor, i.e.,

$$x_i \geq 0 \quad \text{for all} \quad i = 1, 2, \ldots, n. \tag{6}$$

The assumptions made by Kumar et al. [2006b] in this formulation are as follows:

i. Only one item is purchased from one vendor.
ii. Quantity discounts are not taken into consideration.
iii. No shortage of the item is allowed for any of the vendors.
iv. Lead-time and demand of the item are constant and known with certainty.

In the above discussed MOVSP, the parameters are assumed to take deterministic values but in most of the practical situation these may take imprecise value for some possible reasons as listed below:

i. The price of the item might depend upon the interest of the decision maker. Sometimes he/she might decide to spend more or less for the quantity ordered.
ii. Transportation cost may vary in the lot.
iii. Late delivery of items may vary in the lot.
iv. Vendor rating and vendor quota flexibility may vary depending upon the above listed reason.

## 5 Fuzzy Goal Programming with $\alpha$-Cut Approach for the MOVSP

In view of the above equations from (5) to (6), the mathematical formulation of the MOVSP with $n$ decision variables and $m$ constraints is of the following form:

$$\min\left(Z_1=\sum_{i=1}^{n}\widetilde{p}_i x_i, Z_2=\sum_{i=1}^{n}\widetilde{t}_i x_i, Z_3=\sum_{i=1}^{n}\widetilde{l}_i x_i\right),$$

subject to

$$\begin{aligned}
&\sum_{i=1}^{n} x_i = D,\\
&x_i \le \widetilde{U}_i, \quad \forall i=1,2,\ldots,n,\\
&\widetilde{p}_i(x_i) \le \widetilde{B}_i, \quad \forall i=1,2,\ldots,n,\\
&\sum_{i=1}^{n}\widetilde{q}_i(x_i) \le Q,\\
&\sum_{i=1}^{n}\widetilde{r}_i(x_i) \ge \widetilde{P},\\
&\sum_{i=1}^{n}\widetilde{f}_i(x_i) \le \widetilde{F},\\
&x_i \ge 0, \quad \forall i=1,2,\ldots,n.
\end{aligned} \tag{7}$$

For simplicity all the fuzzy parameters in problem (7) are considered as trapezoidal LR-fuzzy number of the $(a,b,\beta,\gamma)$-type. The problem has fuzzy coefficients which have a probability distribution in uncertain intervals, hence the problem can be written in terms of $\alpha$-cut intervals. The lower and upper bound for the respective $\alpha$-cut intervals of the objective function are defined as

$$\left\{[(Z_1(x))_\alpha]^L=\sum_{i=1}^{n}(\widetilde{p}_i)_\alpha^L x_i,\ [(Z_1(x))_\alpha]^U=\sum_{i=1}^{n}(\widetilde{p}_i)_\alpha^U x_i\right\},$$

$$\left\{[(Z_2(x))_\alpha]^L=\sum_{i=1}^{n}(\widetilde{t}_i)_\alpha^L x_i,\ [(Z_2(x))_\alpha]^U=\sum_{i=1}^{n}(\widetilde{t}_i)_\alpha^U x_i\right\},$$

$$\left\{[(Z_3(x))_\alpha]^L=\sum_{i=1}^{n}(\widetilde{l}_i)_\alpha^L x_i,\ [(Z_3(x))_\alpha]^U=\sum_{i=1}^{n}(\widetilde{l}_i)_\alpha^U x_i\right\}.$$

In the next step, we construct a membership function for the minimization type objective function, and then replaced it by the lower bound of its $\alpha$-cut interval.

For first minimize objective $\left\{[(Z_1(x))_\alpha]^L=\sum_{i=1}^{n}(\widetilde{p}_i)_\alpha^L x_i\right\}$, the membership function is given as:

$$\mu_1(x)=\begin{cases}1, & Z_1(x)\le g_1,\\ \frac{u_1-[(Z_1(x))_\alpha]^L}{u_1-l_1}, & g_1\le Z_1(x)\le u_1,\\ 0, & Z_1(x)\ge u_1.\end{cases}$$

For second minimize objective $\left\{[(Z_2(x))_\alpha]^L = \sum_{i=1}^n (\widetilde{t}_i)_\alpha^L x_i\right\}$, the membership function is given as:

$$\mu_2(x) = \begin{cases} 1, & Z_2(x) \le g_2, \\ \frac{u_2 - [(Z_2(x))_\alpha]^L}{u_2 - l_2}, & g_2 \le Z_2(x) \le u_2, \\ 0, & Z_2(x) \ge u_2. \end{cases}$$

For third minimize objective $\left\{[(Z_3(x))_\alpha]^L = \sum_{i=1}^n (\widetilde{l}_i)_\alpha^L x_i\right\}$, the membership function is given as:

$$\mu_3(x) = \begin{cases} 1, & Z_3(x) \le g_3, \\ \frac{u_3 - [(Z_3(x))_\alpha]^L}{u_3 - l_3}, & g_3 \le Z_3(x) \le u_3, \\ 0, & Z_3(x) \ge u_3. \end{cases}$$

The constraints are transformed in terms of $\alpha$-cut values as

$$\begin{aligned} x_i &\le (\widetilde{U}_i)_\alpha^U, \quad i = 1, 2, \ldots, n, \\ (\widetilde{p}_i)_\alpha^L (x_i) &\le (\widetilde{B}_i)_\alpha^U, \quad i = 1, 2, \ldots, n, \\ \sum_{i=1}^n (\widetilde{q}_i)_\alpha^L (x_i) &\le Q, \\ \sum_{i=1}^n (\widetilde{r}_i)_\alpha^U (x_i) &\ge (\widetilde{P})_\alpha^L, \\ \sum_{i=1}^n (\widetilde{f}_i)_\alpha^L (x_i) &\le (\widetilde{F})_\alpha^U. \end{aligned}$$

Summing it up, the MOVSP (7) is transformed into the following form:

$$\min[(Z_1(x))_\alpha]^L = \sum_{i=1}^n (\widetilde{p}_i)_\alpha^L x_i,$$

$$\min[(Z_2(x))_\alpha]^L = \sum_{i=1}^n (\widetilde{t}_i)_\alpha^L x_i,$$

$$\min[(Z_3(x))_\alpha]^L = \sum_{i=1}^n (\widetilde{l}_i)_\alpha^L x_i,$$

subject to

$$\sum_{i=1}^n x_i = D,$$

$$x_i \le (\widetilde{U}_i)_\alpha^U, \quad \forall i = 1, 2, \ldots, n, \tag{8}$$

$$(\widetilde{p}_i)^L_\alpha(x_i) \le (\widetilde{B}_i)^U_\alpha, \quad \forall i = 1, 2, \ldots, n,$$

$$\sum_{i=1}^{n} (\widetilde{q}_i)^L_\alpha(x_i) \le Q,$$

$$\sum_{i=1}^{n} (\widetilde{r}_i)^U_\alpha(x_i) \ge (\widetilde{P})^L_\alpha,$$

$$\sum_{i=1}^{n} (\widetilde{f}_i)^L_\alpha(x_i) \le (\widetilde{F})^U_\alpha,$$

$$x_i \ge 0, \quad \forall i = 1, 2, \ldots, n.$$

As we considered trapezoidal LR-fuzzy numbers of the $(a, b, \beta, \gamma)$-type, problem (8) can also be represented as

$$\min[(Z_1(x))_\alpha]^L = \sum_{i=1}^{n} ((b_p + \gamma_p) - \gamma_p \alpha)\, x_i,$$
$$\min[(Z_2(x))_\alpha]^L = \sum_{i=1}^{n} ((b_t + \gamma_t) - \gamma_t \alpha)\, x_i,$$
$$\min[(Z_3(x))_\alpha]^L = \sum_{i=1}^{n} ((b_l + \gamma_l) - \gamma_l \alpha)\, x_i,$$

subject to

$$\begin{aligned}
&\sum_{i=1}^{n} x_i = D, \\
&x_i \le ((a_U - \beta_U) + \beta_U \alpha), \quad \forall i = 1, 2, \ldots, n, \\
&((b_p + \gamma_p) - \gamma_p \alpha)\, x_i \le ((a_B - \beta_B) + \beta_B \alpha), \quad \forall i = 1, 2, \ldots, n, \\
&\sum_{i=1}^{n} ((b_q + \gamma_q) - \gamma_q \alpha)\, x_i \le Q, \\
&\sum_{i=1}^{n} ((a_r - \beta_r) + \beta_r \alpha)\, x_i \ge ((b_P + \gamma_P) - \gamma_p \alpha), \\
&\sum_{i=1}^{n} ((b_f + \gamma_f) - \gamma_f \alpha)\, x_i \le ((a_F - \beta_F) + \beta_F \alpha), \\
&x_i \ge 0 \quad \forall i = 1, 2, \ldots, n,
\end{aligned} \tag{9}$$

where $\alpha \in [0, 1]$ is an arbitrary chosen fixed number.

Note that the above given formulation with trapezoidal LR-fuzzy numbers can easily be transformed into non-LR trapezoidal fuzzy numbers by using $(a - \beta, a, b, b + \gamma)$, which is equivalent to $(\delta, m, n, \beta)$.

Next, we consider the conversion of the objective functions to fuzzy goals by means of assigning an aspiration level to them. Thus, problem (8) is transformed into a fuzzy goal program by taking certain aspiration levels and introducing variables for the deviation from below to the objective function also. The minimization type objective function in (9) is transformed into

$$\frac{u_k - \sum_{j=1}^{n}(Z_k(x)_\alpha)^L}{u_k - g_k} + d_k^- \geq 1, \quad k = 1, 2, \ldots, K.$$

where

$$g_k = \min[(Z_1(x))_\alpha]^L, \quad u_k = \max[(Z_1(x))_\alpha]^U, \quad \forall k = 1, 2, \ldots, K.$$

Now using the above described goal programming method, the model is converted into single objective problem as follows:

$$\begin{aligned}
&\min Z = \sum_{k=1}^{K} w_k d_k^-,\\
&\text{subject to}\\
&\qquad \frac{u_k - \sum_{j=1}^{n}(Z_k(x)_\alpha)^L}{u_k - g_k} + d_k^- \geq 1,\\
&\qquad d_k^- \geq 0, \quad k = 1, 2, 3, \ldots, K,\\
&\qquad \text{and the set of constraints of problem (8).}
\end{aligned}$$

Here, $Z$ represents the achievement function. Weights $w_k$ are the relative weights attached to the respective objective functions. For calculating these weights we introduce two methods.

## 5.1 Weights measuring the target deviation

Weights attached to each of the objectives to measure the relative importance of a deviation from their respective target:

$$w_k = \begin{cases} \dfrac{1}{g_k - l_k} & \text{for the maximizing case,} \\ \dfrac{1}{u_k - g_k} & \text{for the minimizing case,} \end{cases}$$

for all $k = 1, 2, 3, \ldots, K$.

## 5.2 Aggregated weights attached to the fuzzy numbers itself

Awasthi et al. [2014] developed the aggregated fuzzy weights for triangular fuzzy number. Motivated by his study we developed the aggregated fuzzy weights for trapezoidal fuzzy number. The aggregated fuzzy weights $w_k$ of each objective function $k$ are calculated as follows. We first compute aggregated values as

$$\begin{aligned}
\widetilde{w}_{k,1} &= \min\{\widetilde{A}_{k,1} : k = 1, 2, 3\}, \\
\widetilde{w}_{k,2} &= \frac{1}{4} \sum_{k=1}^{3} \widetilde{A}_{k,2}, \\
\widetilde{w}_{k,3} &= \frac{1}{4} \sum_{k=1}^{3} \widetilde{w}_{k,3}, \\
\widetilde{w}_{k,4} &= \max\{\widetilde{w}_{k,j,4} : k = 1, 2, 3\}.
\end{aligned} \tag{10}$$

Then we defuzzify the elements for the criteria weights and the alternatives into crisp values by employing the following equation (see Yong [2006]):

$$w_k = \frac{\widetilde{w}_{k,1} + 2\widetilde{w}_{k,2} + 2\widetilde{w}_{k,3} + \widetilde{w}_{k,4}}{6} \tag{11}$$

## 6 Numerical Illustration

In order to illustrate the developed method, we consider an example of the Vendor Selection Problem from Kumar et al. [2006b] with some extensions. Some additional data is also assumed by us to illustrate the situation of uncertainty in the parameters of the MOVSP. The instance consists of four vendors $s \in \{1, 2, 3, 4\}$. Their respective profiles with fuzzy parameters are shown in Table 2. If the purchasers are following a 95% ($2\sigma$ limits) of the accepted policy, then the maximum limit of rejections should not exceed 5% of the demand. Hence, the maximum rejection that a purchaser can afford is $25000 \cdot 0.05 = 1250$. The least value of flexibility in vendors' quota and the least total purchase value of supplied items are policy decisions and are depending on the demand. The least value of flexibility in the suppliers' quota is given as $F = \widetilde{f} \cdot D$, and the least total purchase value of supplied items is given as $P = \widetilde{r} \cdot D$. The overall fuzzy flexibility $\widetilde{f}$ for the vendor is $(0.03, 0.04, 0.005, 0.006)$ on a scale from 0 to 1, and the overall fuzzy vendor rating $\widetilde{r}$ for the vendor is $(0.93, 0.95, 0.04, 0.06)$ on the scale of 0 to 1. The aggregated demand $D$ is 25000 units.

**Table 2.** Instance data for the Vendor Selection Problem

| Vendor $s$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $\widetilde{p}_i$ | (110, 130, 10, 15) | (305, 325, 15, 20) | (250, 265, 13, 18) | (355, 370, 12, 20) |
| $\widetilde{t}_i$ | (15, 17, 2, 3) | (11, 12, 1, 2) | (5, 7, 0.9, 1.8) | (21, 24, 4, 6) |
| $\widetilde{l}_i$ | (2, 2.5, 0.3, 0.7) | (3, 4.5, 0.6, 0.9) | (9, 11, 0.8, 1.1) | (4, 6, 0.5, 0.7) |
| $\widetilde{U}_i$ | (5600, 5800, 200, 400) | (16500, 16900, 600, 750) | (7000, 7900, 250, 450) | (5500, 5800, 310, 420) |
| $\widetilde{B}_i$ | (1250000, 1300000, 50000, 60000) | (5000000, 5500000, 65000, 70000) | (1750000, 1800000, 40000, 45000) | (300000, 325000, 10000, 15000) |
| $\widetilde{q}_i$ | (4, 6, 0.8, 1.2) | (4, 5.5, 0.6, 0.8) | (1, 2, 0.1, 0.2) | (7.5, 8.5, 1.5, 1.8) |
| $\widetilde{f}_i$ | (0.05, 0.06, 0.001, 0.002) | (0.02, 0.03, 0.002, 0.003) | (0.07, 0.09, 0.001, 0.003) | (0.03, 0.04, 0.001, 0.002) |
| $\widetilde{r}_i$ | (0.87, 0.88, 0.01, 0.02) | (0.90, 0.94, 0.04, 0.06) | (0.91, 0.93, 0.03, 0.05) | (0.89, 0.90, 0.08, 0.09) |

We solve this instance of the MOVSP problem by a fuzzy goal programming with the $\alpha$-cut approach as described in Section 5. To this end, we first replace the fuzzy numbers by their $\alpha$-cut and thus the MOVSP is transformed into the following form

$$\begin{aligned}
\min Z_1 &= (100+10\alpha)x_1 + (290+15\alpha)x_2 \\
&\quad + (237+13\alpha)x_3 + (343+12\alpha)x_4, \\
\min Z_2 &= (13+2\alpha)x_1 + (10+\alpha)x_2 + \\
&\quad (4.1+0.9\alpha)x_3 + (17+4\alpha)x_4, \\
\min Z_3 &= (0.017+0.003\alpha)x_1 + (0.024+0.006\alpha)x_2 \\
&\quad + (0.082+0.008\alpha)x_3 + (0.035+0.005\alpha)x_4,
\end{aligned}$$

subject to

$$\begin{aligned}
& x_1 + x_2 + x_3 + x_4 = 25000, \\
& x_1 \leq (6200 + 400\alpha), \\
& x_2 \leq (17650 + 750\alpha), \\
& x_3 \leq (8350 + 450\alpha), \\
& x_4 \leq (6220 + 420\alpha), \qquad (12) \\
& (100+10\alpha)x_1 \leq (1360000 - 60000\alpha), \\
& (290+15\alpha)x_2 \leq (5570000 - 70000\alpha), \\
& (237+13\alpha)x_3 \leq (1845000 - 45000\alpha), \\
& (343+12\alpha)x_4 \leq (340000 - 15000\alpha),
\end{aligned}$$

$$
\begin{aligned}
&(0.032 + 0.008\alpha)x_1 + (0.034 + 0.006\alpha)x_2 \\
&\quad + (0.009 + 0.001\alpha)x_3 + (0.06 + 0.015\alpha)x_4 \leq 1250, \\
&(0.062 - 0.002\alpha)x_1 + (0.0033 - 0.003\alpha)x_2 + (0.093 - 0.003\alpha)x_3 \\
&\quad + (0.042 - 0.002\alpha)x_4 \geq (0.025 + 0.005\alpha) \cdot 25000, \\
&(0.86 + 0.01\alpha)x_1 + (0.86 + 0.04\alpha)x_2 + (0.88 + 0.03\alpha)x_3 \\
&\quad + (0.81 + 0.008\alpha)x_4 \leq (1.01 - 0.06\alpha) \cdot 25000, \\
&x_1, x_2, x_3, x_4 \geq 0.
\end{aligned}
$$

The aspiration levels of goals $Z_1, Z_2$, and $Z_3$ are calculated and summarized in Table 3.

**Table 3.** Aspiration level for the goals.

| $\alpha$ | $g_1$ | $g_2$ | $g_3$ |
|---|---|---|---|
| 0 | 5659405 | 204069.6 | 576.71 |
| 0.1 | 5703132 | 206854.4 | 601.5638 |
| 0.2 | 5746888 | 209636.4 | 626.4187 |
| 0.3 | 5790673 | 212415.8 | 651.2767 |
| 0.4 | 5834488 | 215555.1 | 676.1383 |
| 0.5 | 5878332 | 218808 | 701.0042 |
| 0.6 | 5922205 | 222082.9 | 725.8752 |
| 0.7 | 5966109 | 225379.7 | 750.7519 |
| 0.8 | 6010042 | 228698.2 | 775.6349 |
| 0.9 | 6054006 | 232038.3 | 800.5249 |
| 1.0 | 6098000 | 235400 | 825.4225 |

**Method 1.** After establishing the target levels which represent optimistic aspiration levels for each objective function, we calculate the fractional weight attached to the lower-deviation variables $d_k^-$ by using the method described in Paragraph 5.1. For some discrete choices of $\alpha \in [0, 1]$ this yields the numerical values shown in Table 4.

**Table 4.** Weights determination.

| $\alpha$ | $w_1$ | $w_2$ | $w_3$ |
|---|---|---|---|
| 0 | 0.000000766 | 0.000014178 | 0.002058829 |
| 0.1 | 0.000000773 | 0.000014234 | 0.002091673 |
| 0.2 | 0.000000781 | 0.000014292 | 0.002125826 |
| 0.3 | 0.000000789 | 0.000014353 | 0.002161371 |
| 0.4 | 0.000000797 | 0.000014493 | 0.002198403 |
| 0.5 | 0.000000805 | 0.000014664 | 0.002237002 |
| 0.6 | 0.000000813 | 0.000014846 | 0.002277273 |
| 0.7 | 0.000000822 | 0.000015041 | 0.002319329 |
| 0.8 | 0.000000831 | 0.000015250 | 0.002363283 |
| 0.9 | 0.000000840 | 0.000015473 | 0.002409272 |
| 1.0 | 0.000000849 | 0.000015711 | 0.002457428 |

The compromise solution values for $Z_1, Z_2$ and $Z_3$ and an optimum allocation of order quantities for the vendors for the same discrete values for $\alpha \in [0, 1]$ as above are summarized in Table 5.

**Table 5.** Compromise objective values corresponding to different weights.

| $\alpha$ | objective values $(Z_1, Z_2, Z_3)$ | vendors' allocations $(x_1, x_2, x_3, x_4)$ |
|---|---|---|
| 0 | (6116096, 274598.9, 577) | (6200, 17650, 159, 991) |
| 0.1 | (6150822, 277105.3, 602) | (6160, 17575, 282, 983) |
| 0.2 | (6185543, 279596.4, 627) | (6120, 17500, 405, 975) |
| 0.3 | (6220260, 282072.3, 651) | (6080, 17425, 528, 967) |
| 0.4 | (6255078, 284547, 676) | (6040, 17350, 650, 960) |
| 0.5 | (6289786, 286992.7, 701) | (6000, 17275, 773, 952) |
| 0.6 | (6324595, 289437.8, 726) | (5960, 17200, 895, 945) |
| 0.7 | (6359294, 291853.2, 751) | (5920, 17125, 1018, 937) |
| 0.8 | (6394094, 294268.8, 776) | (5880, 17050, 1140, 930) |
| 0.9 | (6428784, 296654, 800) | (5840, 16975, 1263, 922) |
| 1.0 | (6463575, 299040, 825) | (5800, 16900, 1385, 915) |

**Method 2.** We calculate the aggregated fuzzy weights $\widetilde{w}_k$ attached to the fuzzy criterion by using equations (10) and (11). The value of $\widetilde{p}_i, \widetilde{t}_i$ and $\widetilde{l}_i$ of Table 2 are revised by using non-LR fuzzy numbers

**Table 6.** Data for the Vendor Selection Problem.

| Vendors | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $\widetilde{p}_i$ | (100, 110, 130, 145) | (290, 305, 325, 345) | (237, 250, 265, 283) | (343, 355, 370, 390) |
| $\widetilde{t}_i$ | (13, 15, 17, 20) | (10, 11, 12, 14) | (4.1, 5,7, 8.8) | (17, 21, 24, 30) |
| $\widetilde{l}_i$ | (1.7, 2, 2.5, 3.2) | (2.4, 3, 4.5, 5.4) | (8.1, 9, 11, 12.1) | (3.5, 4, 6, 6.7) |

For criterion $Z_1$, the aggregated fuzzy weights are given as

$$\begin{aligned}
\widetilde{w}_{1,1} &= \min(100, 110, 130, 145) = 100,\\
\widetilde{w}_{1,2} &= \frac{1}{4}(290 + 305 + 325 + 345) = 316,\\
\widetilde{w}_{1,3} &= \frac{1}{4}(237 + 250 + 265 + 283) = 258.75,\\
\widetilde{w}_{1,4} &= \max(343, 355, 370, 390) = 390.
\end{aligned}$$

Using equation (8) to transform the aggregated fuzzy weights into crisp number $w_1$, we have that

$$w_1 = \frac{100 + 2 \cdot 316 + 2 \cdot 258.75 + 390}{6} = 273.25.$$

Similarly, the aggregate weights for the two other criteria are $w_2 = 13.1583333$ and $w_3 = 0.06025$. Using the above calculated weights, the solutions of model (12) for some discrete choices for $\alpha \in [0, 1]$ are given in Table 7.

**Table 7.** Compromise objective values corresponding to fixed weights.

| $\alpha$ | Objective values $(Z_1, Z_2, Z_3)$ | Vendor's Allocation $(x_1, x_2, x_3, x_4)$ |
|---|---|---|
| 0 | (5659448, 222674.4, 1008) | (6200, 11016, 7784, 0) |
| 0.1 | (5688620, 225697.5, 1014) | (6160, 11117, 7723, 0) |
| 0.2 | (5746929, 229225, 1031) | (6120, 11218, 7662, 0) |
| 0.3 | (5790713, 232484.1, 1042) | (6080, 11318, 7602, 0) |
| 0.4 | (5834507, 235730.6, 1054) | (6040, 11417, 7484, 0) |
| 0.5 | (5878364, 238970.2, 1066) | (6000, 11516, 7484, 0) |
| 0.6 | (5922231, 242197, 1077) | (5960, 11614, 7426, 0) |
| 0.7 | (5966161, 245417, 1089) | (5920, 11712, 7368, 0) |
| 0.8 | (6010045, 248618.2, 1101) | (5880, 11808, 7312, 0) |
| 0.9 | (6054046, 251818.5, 1112) | (5840, 11905, 7255, 0) |
| 1.0 | (6098000, 255000, 1124) | (5800, 12000, 7200, 0) |

## 7 Discussion and Result Analysis

The solution of the problem instance given in Section 6 was solved by LINGO 13.0. The following results were generated which indicate that most of the goals are attainable with some minor and major improvement in the set of targets. The result analysis of vendor selection for $\alpha \in [0, 1]$ is given in Figure 5.

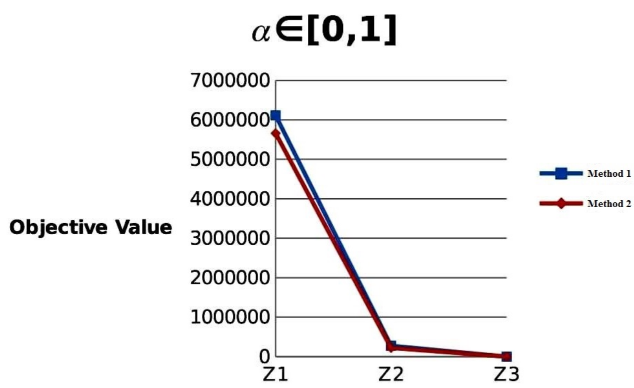


**Fig. 5.** Graphical presentation of the results.

From Table 5 and Table 7 at $\alpha = 0$, we can see that the results obtained from method 2 are very close to the aspiration goal set by the decision maker. Method 2 provides a better objective value (in comparison with Method 1) of the solution for an aggregate demand of 25000 units that yields a minimum net cost as 5659448, a minimum transportation cost as 222674.4 and a minimum late delivered units as 1008. Thus, the DM of the manufacturer can decide to purchase 11016 units form vendor 2, 7784 units form vendor 3, 6200 units from vendor 3 and does not purchase any item from vendor 4 due to their most inferior performance on the criteria set (viz. highest percentage rejections, high percentage late deliveries, less vendor rating value, less quota flexibility value, etc.).

From Table 5 and Table 7 at $\alpha = 1.0$, we can see that the result obtained from method 2 are very close to the aspiration goal set by the decision maker. Method 2 provides a better objective value (in comparison with Method 1) of the solution for an aggregate demand of 25000 units that yields the minimum net cost 6098000, minimum transportation cost 255000 and minimum late delivered units 1124. Thus, the DM at the manufacturer can decide to purchase 12000 units form vendor 2, 7200 units form vendor 3, 25800 units from vendor 3 and does not purchase any item from vendor 4 due to the most inferior performance on the criteria set (viz. highest percentage rejections, high percentage late deliveries, less vendor rating value, less quota flexibility value, etc.).

A similar conclusion can given for other values of $\alpha$. Notable, the objective values and order quantities associated for different $\alpha$ values does not need to be the same.

## 8 Conclusion

In multi-criteria decision-making problem, there may be situations where a decision maker has to be satisfied with a solution of a fuzzy goal programming problem where some of the fuzzy goals are achieved and some are not because these fuzzy goals are subject to the function of realistic constraints. In this paper, we have considered a vagueness in the parameters. One more constraint of quantity rejection is added to the problem that was originally formulated by Kumar et al. [2006b]. The fuzziness of the data is handled by an $\alpha$-cut approaches. The resulting crisp form of the MOVSP is solved by a fuzzy goal programming technique with two different weighted criteria approaches in order to get the optimal result of an order allocation given to the suppliers.

In multi-objective optimization problems, it is a difficult task to set priority weights for various goals. The situation becomes even more tedious, when the goals are naturally conflicting each other. To this end, we considered two different types of weighted approaches. Firstly, the relative weights represent the relative importance of the objective functions, and secondly, the aggregated fuzzy weights represent the relative rating of the fuzzy number to each criterion. The result obtained by using the aggregated fuzzy weights approach satisfies the fuzzy goals better (compared to the weighted approach) in the sense that the solutions are very close to the aspiration level for different levels of $\alpha \in [0, 1]$. We found that at $\alpha = 0$ we get an optimal solution of the formulated MOVSP. As the value of $\alpha$ changes, there was a slightly increase in the value of the objective functions.

In our future work we plan to apply the proposed method also to reformulate and solve other real-world multi-objective fuzzy optimization problems, in particular, those with discrete decision variables.

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